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Prediction of Fracture Parameters of High Strength and Ultra-high Strength Concrete Beam using Gaussian Process Regression and Least Squares

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Abstract: This paper studies the applicability of Gaussian Process Regression (GPR) and Least Squares Support Vector Machines (LSSVM) to predict fracture parameters and failure load (P_{max}) of high strength and ultra-high strength concrete beams. Fracture characteristics include fracture energy (G_F), critical stress intensity factor (K_{IC}) and critical crack tip opening displacement ($CTOD_C$) Mathematical models have been developed in the form of relation between several input variables such as beam dimensions, water cement ratio, compressive strength, split tensile strength, notch depth, modulus of elasticity and output fracture parameters. Four GPR and four LSSVM models have been developed using MATLAB software for training and prediction of fracture parameters. A total of 87 data sets (input-output pairs) are used, 61 of which are used to train the model and 26 are used to test the models. The data-sets used in this study are derived from experimental results. The developed models have also been compared with the Artificial Neural Networks (ANN), Support Vector Regression (SVR) and Multivariate Adaptive Regression Splines (MARS). From the overall study, it is observed that the concept of GPR and LSSVM can be successfully applied to predict fracture parameters of high strength and ultra high strength concrete.

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1 Introduction

Concrete is of high excellence in terms of strength and long term performance are considered to be ideal requirements for special applications. Concretes of strengths exceeding 80 MPa are now commonly used in the construction of high-rise buildings, long span bridges and offshore structures. The major problems being faced by civil engineers are maintenance, retrofitting and preservation of these structures. Ultra High Strength Concrete (UHSC) is a highly engineered material with several chemical and mineral admixtures materials. It has been successfully applied in the field for the construction of Sherbrook Pedestrian Bridge, Canada, The Glenmore/Legs by Pedestrian, Alberta, Canada and P shaped UHPC beams installed in footbridges in Auckland, New Zealand[Seibert (2008); Rebentrost and Wight (2009)].Since UHSC is a relatively new material, the fracture behavior of this material is not well understood [Richard and Cheyrezy (1994,1995); Mingzhe et al. (2010); Goltermann et al. (1997)].

Concrete being a quasi-brittle materials exhibit a nonlinear region before the peak of the stress–strain relationship and substantial post-peak strain softening. Linear elastic fracture mechanics cannot be applied directly to the quasi-brittle materials[7]. Due to high heterogeneity nature in concrete, cracks follow the weakest matrix links in the material. They lead their way through the weak bonds, voids, mortar and get arrested on encountering a hard aggregate, forming crack face bridges. Micro cracking, crack bridging and aggregate interlocking are a few of many specific mechanisms that absorb energy during fracture process. These mechanisms contribute to the tendency of the main crack to follow a tortuous path [Bazant (2000); Barenblatt (1959); Dugdale (1960)]. This tortuous nature of the crack causes difficulty in computing the fracture energy. Therefore, modeling the exact nature of the fracture surface poses a new challenge to the researchers. In these days, most theoretical works in fracture mechanics are based on the fundamental assumption that cracks have smooth surfaces. This assumption is helpful to use analytical models in the field of fracture mechanics.

Over the past few years, researchers have used different statistical modelling methods such as Artificial Neural Network Support Vector Regression, Multivariate Adaptive Regression Splines and Relevance Vector Machine for prediction of fracture characteristics of concrete. Yuvaraj et al. (2013) used Support vector regression, Artificial Neural Network [Yuvaraj et al. (2012)] and Multivariate Adaptive Regression Splines [Yuvaraj et al. (2013)] to predict the fracture characteristics of concrete beams. Though the performance of ANN is acceptable, its results are hard to interpret. Support vector machines do not directly provide probability estimates and in the case of MARS, parameter confidence intervals and other checks on the model cannot be calculated directly, unlike linear regression models. This study uses Gaussian Process Regression (GPR) and Least Squares Support Vector Machines (LSSVM) for the prediction of fracture characteristics of concrete. GPR (Gaussian Process Regression) gives a non-parametric modelling approach and probabilistic Bayesian framework which can be applied to various engineering problems. The probabilistic GP chooses hyper parameters directly from the training data, gives a probabilistic measure of the uncertainty of the model prediction and obtain a relatively good model when only a small set of training data is available [Azman and Kocijan (2007); Pal and Deswal (2010); Likar and Kocijan (2007); Yuan et al. (2008)]. In comparison to back-propagation neural networks, Gaussian processes are conceptually simpler to understand and implement in practice. The LSSVM is a statistical learning theory which adopts a least squares linear system as a loss functions instead of the quadratic program in original support vector machine (SVM) [Suykans and Vandewalle (1999); Baesens et al. (2000); Espinoza et al. (2003); Lu et al. (2003); Mitra et al. (2007)]. It is closely related to Gaussian processes and regularization networks. It requires solving a set of only linear equations (linear programming), which is much easier and computationally very simple. Both GPR and LSSVM have a strong potential for predicting the facture characteristics with high correlation and precision to the experimental value. They differ from most of the other black-box identification approaches as it does not try to approximate the modeled system by fitting the parameters of the selected basis functions but rather searches for the relationship among measured data.

The aim of this study is to predict the fracture characteristics of high strength concrete beams (HSC) and Ultra high strength concrete beams (UHSC). This paper presents development and validation of models based on concept of Gaussian Process Regression (GPR) and Least Squares Support Vector Machines (LSSVM) to predict fracture parameters and failure load (P_{max}) of high strength and ultra-high strength concrete beams.

2 Gaussian Process Regression

This study incorporates Gaussian Process Regression (GPR) for prediction of Failure load (Pmax), fracture energy (G_F), critical stress intensity factor (K_{IC}) and critical crack tip opening displacement ($CTOD_C$). In GPR, the learning of data is modeled as Bayesian estimation problem. It is assumed that the parameters of GPR are random variables. GPR has been successfully adopted for solving different problems in engineering [Yuan et al. (2008); Pal and Deswal (2010)]

Let us consider the following set of samples

$$L = \{\chi_i, y_i\}_{i=1}^{D} \chi_i \in R^N, \quad y_i \in R \tag{1}$$

Where x is input variable, y is output, R^N is N-dimensional vector space and R is one dimensional vector space. This article uses Beam length (L), Cross-section area (A), Notch depth (a), water-cement ratio (w/c), compressive strength (f_{ck}), split tensile strength (σ_t) and modulus of elasticity (E)as input variables. The output of GPR is failure load (P_{max}), fracture energy (G_F), critical stress intensity factor (K_{IC}) and critical crack tip opening displacement ($CTOD_C$). So, x= [L, A, a, w/c, f_{ck}, σ_t, E] and y= [P_{max}, G_F, K_{IC}, $CTOD_C$].

GPR uses the following expression for prediction of y.

$$y_i = f(\chi_i) + \in_i \tag{2}$$

where f(x_i) is latent function and ε_i is Gaussian noise. GPR treats f(x_i) as random variable.

The joint distribution of y is given by the following equation.

$$P(y) = N(0, K(\chi,\chi) + \sigma^2 I) \tag{3}$$

Where K(x, x) is kernel function and I is identity matrix.

The predictive distribution of y_{D+1} corresponding to a new given input x_{D+1} is given by the following expression.

$$\begin{pmatrix} y \\ y_{D+1} \end{pmatrix} \sim N(0, K_{D+1}) \tag{4}$$

Where K_{D+1} is covariance matrix and its expression is given by

$$K_{D+1} = \begin{bmatrix} K(\chi,\chi) + \sigma^2 I & K(\chi_{D+1}) \\ K(\chi_{D+1})^T & K(\chi_{D+1}) \end{bmatrix} \tag{5}$$

The distribution of y_{D+1} is Gaussian with mean and variance (Williams, 1998):

$$\mu = K(\chi_{D+1})^T [K(\chi,\chi) + \sigma^2 I]^{-1} y \tag{6}$$

$$\sum = K(\chi_{D+1}) - K(\chi_{D+!})[K(\chi,\chi) + \sigma^2 I]^{-1} K(\chi_{D+1}) \tag{7}$$

To develop GPR, a covariance function is required. The details of GPR is given by Williams and Rasmussen (1996). Radial basis function has been used a covariance function.

3 Least Squares Support Vector Machine (LSSVM)

SVM is a novel machine tool and especially useful for the classification and prediction with small sample cases. This novel approach motivated by statistical learning theory led to a class of algorithms characterized by the use of nonlinear kernels, high generalization of abilities and the sparseness of solution. Unlike the classical neural network approach the SVM formulation of the learning problem leads to quadratic programming with linear constraints. However the size of the matrix involved in the QP problem is directly proportional to the number of training points. Hence to reduce the complexity of optimization processes, a modified version called LS-SVM is proposed by taking equality instead of inequality constraints to obtain a linear set of equations instead of a QP problem in dual space. Instead of solving a QP problem by SVM, LS-SVM can obtain the solution of a set of linear equations. The formation of LS-SVM introduced is as follows. The following regression model can be constructed by using non-linear mapping function $\varphi(x)$.

$$y(x) = w^T \varphi(x) + b \tag{8}$$

where w is the weight vector and b is the bias term. By mapping the original input data into a high dimensional space, the nonlinear separable problem becomes linearly separable in space. Then the following cost function is formulated in the framework of empirical risk minimization

$$\min J(w,e) = \frac{1}{2} w^T w + \gamma \frac{1}{2} \sum_{k=1}^{N} e_k^2 \tag{9}$$

subject to equality constraints

$$y_k = w^T \varphi(x_k) + b + e_k \quad (k = 1, 2, 3, \ldots., N) \tag{10}$$

where e_k is the random error and gamma is a regularization parameter in determining the trade-off between minimizing the training error and minimizing the model complexity. To solve this optimization problem Lagrange function is constructed as:

$$L(w,b,e;\alpha) = J(w,e) - \sum_{k=1}^{N} \alpha_k \{w^T \varphi(x_k) + b + e_k - y_k\} \tag{11}$$

where a_k are Lagrange multiplier. The solution of equation (9) can be obtained by partially differentiating it with respect to w, b, e_k and a_k

$$\frac{\partial L}{\partial w} = 0 \rightarrow w = \sum_{K=1}^{N} \alpha_k \varphi(x_k) \tag{12}$$

$$\frac{\partial L}{\partial b} = 0 \rightarrow \sum_{k=1}^{N} \alpha_k = 0 \tag{13}$$

$$\frac{\partial L}{\partial e_k} = 0 \rightarrow \alpha_k = \gamma e_k \quad k = 1, ..., N \tag{14}$$

$$\frac{\partial L}{\partial \alpha_k} = 0 \rightarrow w^T \varphi(x_k) + b + e_k - y_k = 0 \quad k = 1, ..., N \tag{15}$$

The equations (10)-(13) can be rewritten as:

$$\begin{bmatrix} 0 & \vec{1} \\ \vec{1} & \Omega + y^{-1} I \end{bmatrix} \tag{16}$$

where,

$$y = [y_1, ...y_n]$$

$$\vec{1} = [1, ..., 1]$$

$$\alpha = [\alpha_1, ..., \alpha_n]$$

$$\Omega_{kl} = \varphi(x_k)^T \varphi(x_1) ....k, l = 1, ..., N$$

Finally b and a_k can be obtained by the solution to the linear equation:

$$\vec{b} = \frac{\vec{1}^T \left(\Omega + \gamma^{-1} I_n\right)^{-1} y}{\vec{1}^T \left(\Omega + \gamma^{-1} I_n\right)^{-1} \vec{1}} \tag{17}$$

According to mercer's theorem the LS-SVM model can be expressed as:

$$y(x) = \sum_{k=1}^{N} \alpha_k K(x, x_k) + b \tag{18}$$

where K(x, x_k) is the nonlinear kernel function. In comparison with some other feasible kernel functions, the RBF function a more compact supported kernel and is able to reduce a more computational complexity of the training process and improve generalization performance of LS-SVM. As a result RBF kernel was selected as kernel function as:

$$K(X, X_k) = \exp\left(\|X - X_k\|^2 2\sigma^{-2}\right) \tag{19}$$

where σ is the scale factor for tuning.

4 Development of GPR and LSSVM Models

Out of the 87 data sets which are available, 61 datasets are used to train the models and 26 datasets are used to test the accuracy of the models [Yuvaraj et al. (2013)]. Tables 1 and 2 show the training and testing data-sets respectively. The data was normalized between 0 and 1 before being used in the model as following:

$$D_{norm} = \frac{D - D_{\min}}{D_{\max} - D_{\min}} \tag{20}$$

The assessment of the model is done on the basis of coefficient of regression value R which is calculated using the formula:

$$R = \frac{\sum_{i=1}^{n}\left(E_{ai} - \overline{E}_{a}\right)\left(E_{pi} - E_{p}\right)}{\sqrt{\sum_{i=1}^{n}\left(E_{ai} - \overline{E_{a}}\right)}\sqrt{\sum_{i=1}^{n}\left(E_{pi} - \overline{E_{p}}\right)}} \tag{21}$$

where E_{ai} and E_{pi} are the actual and predicted values, respectively, $\bar{E}_a$ and $\bar{E}_p$ are mean of actual and predicted E values. For an effective and good model the R value should be close to one. Also while comparing the models the values of R is compared and the model with R value closer to one and higher than the other is considered better and used.

Note:

L- length, A- c/s area, a_0- Notch depth, w/c-Water- cementations material ratio, f_{ck}-compressive strength,σ_t-Split tensile strength, E- modulus of elasticity, P_{max}-Ultimate load, G_F- Fracture energy, K_{IC}- critical stress intensity factor, $CTOD_C$-Critical crack tip opening displacement.

The success of GPR depends on the value of ε and s. The design values of ε and s have been determined by trial and error approach. The best values of ε and s for each of the GPR models has been given in Table 3.

To achieve a high level performance with LS-SVM models, some parameters have to be tuned including regularization parameters γ and the kernel parameter corresponding to the kernel type, i.e. σ. These parameters have been determined using trial and error approach. The best values of γ and σ for each of the LSSVM models has been given in Table 4.

5 Results and Discussions

The GPR and LSSVM models have been developed using the MATLAB software for training and prediction of the fracture characteristics of high strength and ultra-high strength beams. The models have been trained using 61 data sets and 26 data

Table 1: Training data-sets

S. No	L (mm)	A (cm^2)	a_0 (mm)	w/c	f_{ck} (MPa)	σ_t (MPa)	E (GPa)	P_{max} (KN)	G_F (N/m)	K_{IC} (Mpa $\sqrt{m}$)	$CTOD_C$ (mm)
1	250	25	5	0.45	57.14	3.96	35.78	2.71	115.84	1.126	0.031
2	250	25	4	0.45	57.14	3.96	35.78	2.62	123.31	1.129	0.03
3	250	25	10	0.45	57.14	3.96	35.78	1.98	91.12	1.092	0.0183
4	250	25	9	0.45	57.14	3.96	35.78	1.98	86.65	1.08	0.0186
5	250	25	10	0.45	57.14	3.96	35.78	1.84	74.32	1.083	0.0178
6	250	25	16	0.45	57.14	3.96	35.78	1.14	55.18	0.0916	0.0081
7	250	25	15	0.45	57.14	3.96	35.78	1.42	68.61	0.00902	0.008
8	500	50	9	0.45	57.14	3.96	35.78	4.53	144.02	1.348	0.049
9	500	50	10	0.45	57.14	3.96	35.78	4.10	130.26	1.349	0.0485
10	500	50	18	0.45	57.14	3.96	35.78	3.79	92.72	1.174	0.035
11	500	50	19	0.45	57.14	3.96	35.78	3.63	115.42	1.173	0.0345
12	500	50	28	0.45	57.14	3.96	35.78	2.58	89.12	0.984	0.0149
13	1000	100	19	0.45	57.14	3.96	35.78	7.27	165.25	1.467	0.1026
14	1000	100	19	0.45	57.14	3.96	35.78	7.32	146.28	1.461	0.1
15	1000	100	19	0.45	57.14	3.96	35.78	6.99	148.25	1.456	0.098
16	1000	100	39	0.45	57.14	3.96	35.78	6.01	135.85	1.224	0.0601
17	1000	100	39	0.45	57.14	3.96	35.78	6.32	140.56	1.201	0.06
18	1000	100	58	0.45	57.14	3.96	35.78	4.54	115.12	1.012	0.0281
19	1000	100	60	0.45	57.14	3.96	35.78	4.70	104.22	0.998	0.026
20	250	25	5	0.33	87.71	15.38	37.89	4.20	4157.28	7.984	0.3434
21	250	25	5	0.33	87.71	15.38	37.89	4.15	4102.2	7.941	0.321
22	250	25	10	0.33	87.71	15.38	37.89	3.37	3464.6	7.398	0.2213
23	250	25	10	0.33	87.71	15.38	37.89	3.26	3880.1	7.362	0.218
24	250	25	15	0.33	87.71	15.38	37.89	2.79	3301.2	6.961	0.098
25	250	25	15	0.33	87.71	15.38	37.89	2.88	3410	6.981	0.1
26	250	25	20	0.33	87.71	15.38	37.89	1.98	2892.06	6.118	0.053

27	250	25	20	0.33	87.71	15.38	37.89	2.05	2988.52	6.3	0.051
28	500	50	10	0.33	87.71	15.38	37.89	8.35	4811	8.479	0.456
29	500	50	10	0.33	87.71	15.38	37.89	8.20	4200.1	8.453	0.45
30	500	50	20	0.33	87.71	15.38	37.89	5.10	4516.1	7.401	0.3268
31	500	50	20	0.33	87.71	15.38	37.89	4.99	4266.5	7.386	0.32
32	500	50	20	0.33	87.71	15.38	37.89	5.07	3828.57	7.365	0.318
33	500	50	30	0.33	87.71	15.38	37.89	3.80	3579.89	6.682	0.203
34	500	50	30	0.33	87.71	15.38	37.89	3.79	3865.2	6.701	0.206
35	500	50	40	0.33	87.71	15.38	37.89	2.99	3970.95	6.201	0.093
36	500	50	40	0.33	87.71	15.38	37.89	3.08	3406.67	6.196	0.09
37	250	25	4	0.23	122.52	20.65	42.987	9.99	10349.24	12.601	0.433
38	250	25	5	0.23	122.52	20.65	42.987	10.01	10376.22	12.652	0.44
39	250	25	10	0.23	122.52	20.65	42.987	7.81	8308.49	11.762	0.281
40	250	25	9	0.23	122.52	20.65	42.987	7.43	7900	11.801	0.279
41	250	25	15	0.23	122.52	20.65	42.987	6.20	6925.54	11.092	0.141
42	250	25	15	0.23	122.52	20.65	42.987	5.99	6694.51	11	0.142
43	250	25	20	0.23	122.52	20.65	42.987	4.07	4386.6	7.581	0.0875
44	250	25	19	0.23	122.52	20.65	42.987	3.99	4306.29	7.412	0.0861
45	250	25	20	0.23	122.52	20.65	42.987	4.18	4511.36	7.51	0.085
46	400	40	9	0.23	122.52	20.65	42.987	14.23	11557.07	13.541	0.483
47	400	40	8	0.23	122.52	20.65	42.987	13.98	11354.02	13.582	0.49
48	400	40	16	0.23	122.52	20.65	42.987	10.85	8888.75	11.949	0.3898
49	400	40	15	0.23	122.52	20.65	42.987	10.62	8700.84	11.892	0.383
50	400	40	25	0.23	122.52	20.65	42.987	7.58	7145.19	11.201	0.2515
51	400	40	24	0.23	122.52	20.65	42.987	7.61	7171.63	11.221	0.249
52	400	40	32	0.23	122.52	20.65	42.987	5.56	5021.25	8.471	0.1216
53	400	40	31	0.23	122.52	20.65	42.987	5.60	5058.14	8.45	0.12
54	650	65	13	0.23	122.52	20.65	42.987	19.49	12052.38	13.984	0.581
55	650	65	12	0.23	122.52	20.65	42.987	19.31	11944.13	13.801	0.563
56	650	65	25	0.23	122.52	20.65	42.987	13.37	8076	12.013	0.3069
57	650	65	25	0.23	122.52	20.65	42.987	13.51	8892.69	12	0.301
58	650	65	39	0.23	122.52	20.65	42.987	10.12	6965.9	11.321	0.181
59	650	65	39	0.23	122.52	20.65	42.987	10.30	7085.13	11.103	0.172
60	650	65	52	0.23	122.52	20.65	42.987	7.46	5919.23	9.691	0.094
61	650	65	52	0.23	122.52	20.65	42.987	7.69	6109.05	9.598	0.093

sets are used to validate the model. The performance of GPR models for P_{max}, G_F, K_{IC} and $CTOD_C$ has been respectively shown in Figures 1, 2, 3 and 4, and the same for LSSVM models has been shown in Figures 5, 6, 7 and 8. The values of R for training and testing data-sets of each models, and the corresponding Root mean squared error (RMSE) values has been shown in Table 5.

The developed LSSVM models give the following equations for the Prediction of

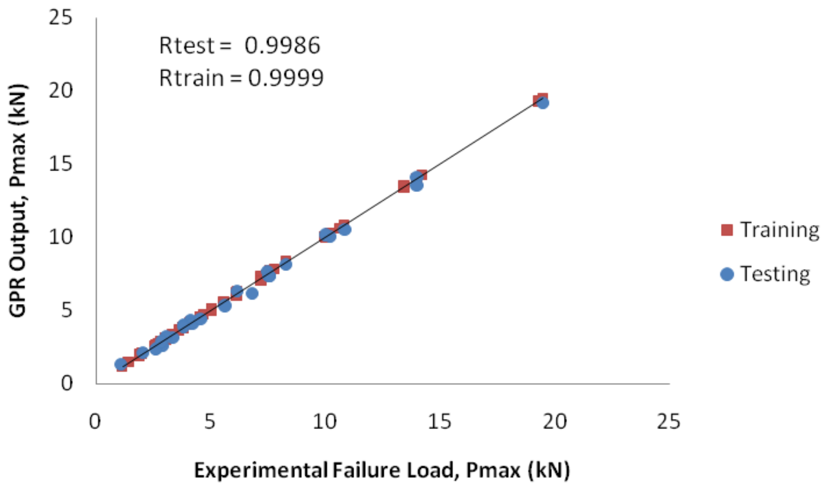

Figure 1: Performance of GPR (P_{max} Model).

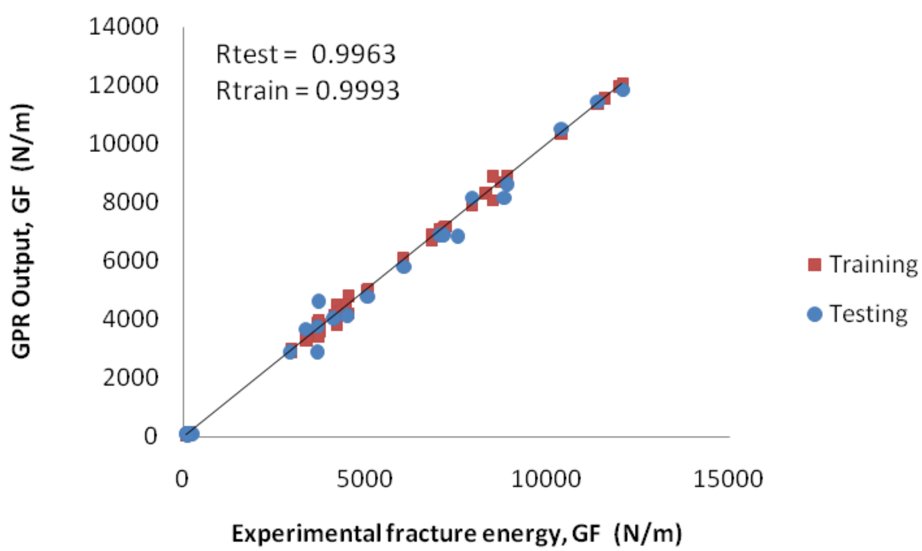

Figure 2: Performance of GPR (G_F Model).

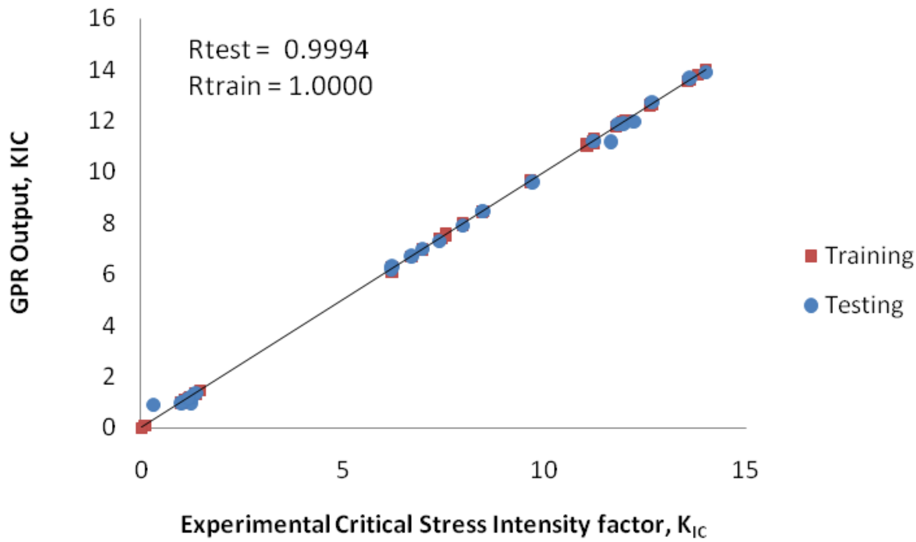

Figure 3: Performance of GPR (K_{IC} Model).

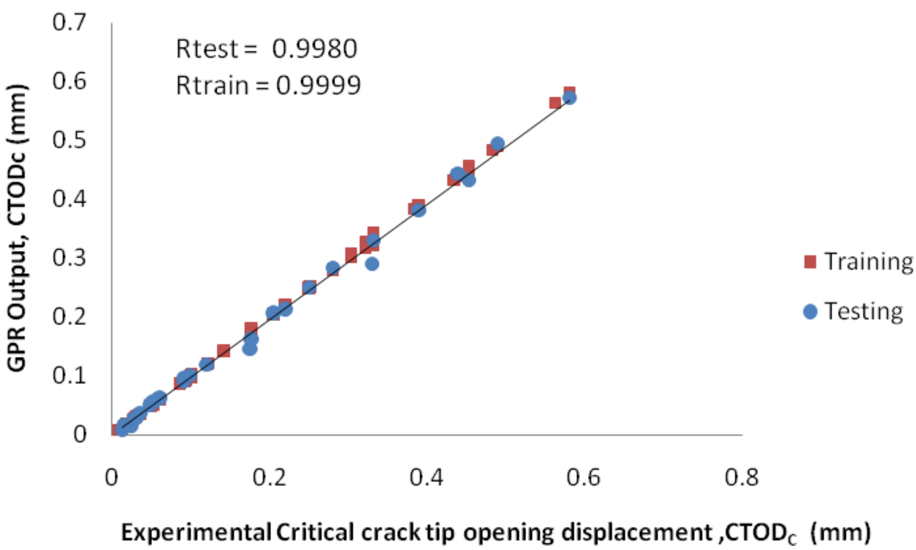

Figure 4: Performance of GPR ($CTOD_C$ Model).

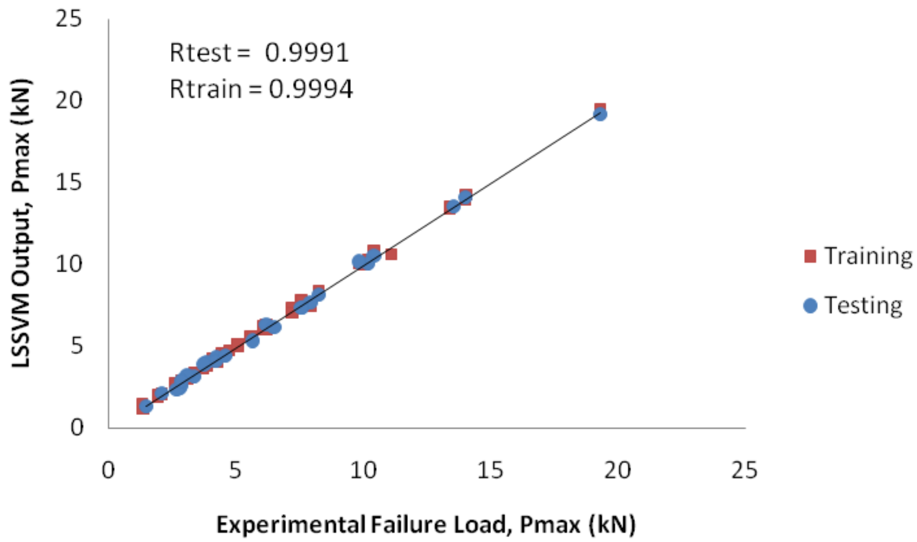

Figure 5: Performance of LSSVM (P_{max} Model).

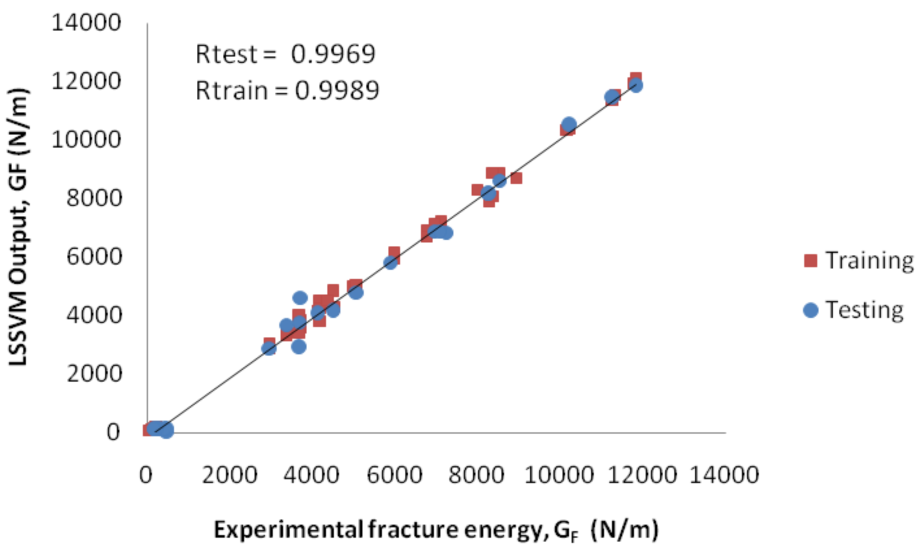

Figure 6: Performance of LSSVM (G_F Model).

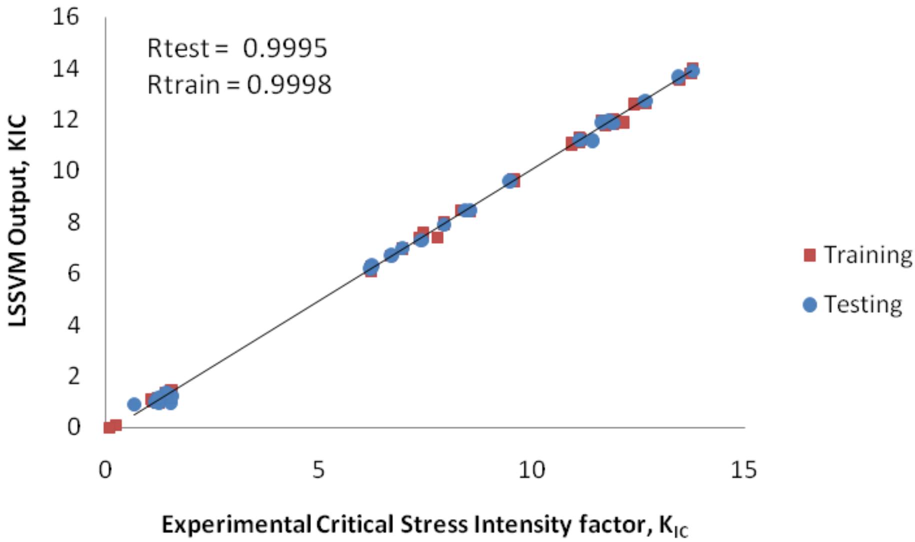

Figure 7: Performance of LSSVM (K_{IC} Model).

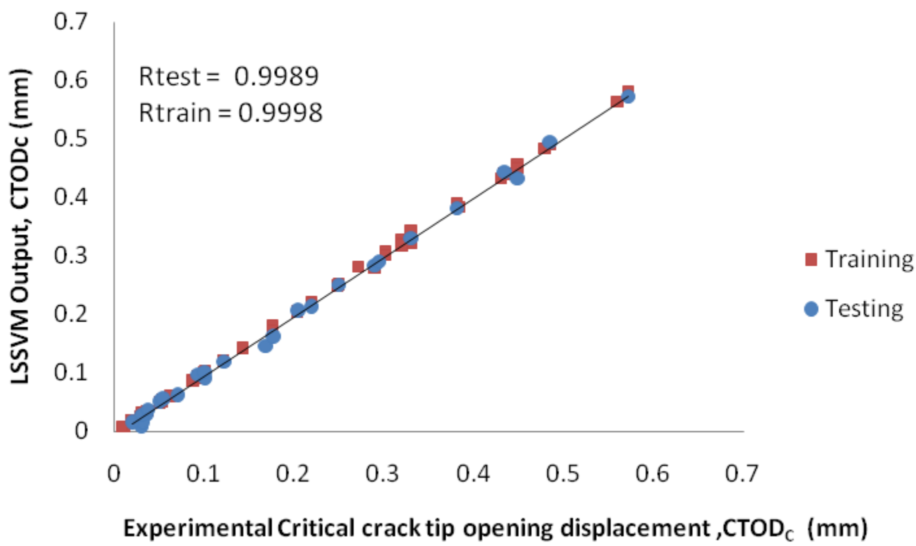

Figure 8: Performance of LSSVM ($CTOD_C$ Model).

Table 2: Testing data-sets.

L (mm)	A (cm^2)	a_0 (mm)	w/c	f_{ck} (MPa)	σ_t (MPa)	E (GPa)	P_{max} (kN)	G_F (N/m)	K_{IC} (Mpa $\sqrt{m}$)	$CTOD_C$ (mm)
250	25	4	0.45	57.14	3.96	35.78	2.412	114.9	1.121	0.029
250	25	17	0.45	57.14	3.96	35.78	1.321	47.4	0.923	0.008
500	50	29	0.45	57.14	3.96	35.78	2.575	96.2	0.998	0.015
500	50	28	0.45	57.14	3.96	35.78	2.321	100.3	0.979	0.015
500	50	10	0.33	87.71	15.38	37.89	8.102	4142.2	8.462	0.432
1000	100	40	0.45	57.14	3.96	35.78	6.278	110.2	1.234	0.062
500	50	10	0.45	57.14	3.96	35.78	4.312	137.0	1.356	0.051
250	25	10	0.33	87.71	15.38	37.89	3.121	3763.1	7.312	0.213
650	65	51	0.23	122.52	20.65	42.987	7.312	5806.5	9.601	0.091
500	50	30	0.33	87.71	15.38	37.89	3.991	4623.5	6.721	0.206
250	25	9	0.23	122.52	20.65	42.987	7.667	8155.0	11.857	0.283
250	25	14	0.23	122.52	20.65	42.987	6.128	6844.0	11.183	0.145
400	40	8	0.23	122.52	20.65	42.987	14.08	11435.2	13.655	0.494
400	40	16	0.23	122.52	20.65	42.987	10.514	8613.2	11.901	0.38
650	65	24	0.23	122.52	20.65	42.987	13.498	8155.1	11.98	0.289
650	65	13	0.23	122.52	20.65	42.987	19.126	11829.1	13.882	0.571
650	65	39	0.23	122.52	20.65	42.987	10.013	6889.1	11.201	0.162
250	25	5	0.23	122.52	20.65	42.987	10.136	10504.7	12.716	0.443
250	25	20	0.33	87.71	15.38	37.89	2.102	2894.0	6.317	0.055
400	40	31	0.23	122.52	20.65	42.987	5.312	4797.2	8.463	0.119
250	25	5	0.33	87.71	15.38	37.89	4.101	4056.4	7.912	0.33
500	50	40	0.33	87.71	15.38	37.89	3.194	2897.9	6.214	0.096
1000	100	58	0.45	57.14	3.96	35.78	4.412	111.9	1	0.027
400	40	25	0.23	122.52	20.65	42.987	7.31	6887.1	11.198	0.25
500	50	18	0.45	57.14	3.96	35.78	3.87	105.3	1.176	0.036
250	25	15	0.33	87.71	15.38	37.89	2.841	3685.1	6.993	0.101

Table 3: Values of ε and s for the GPR models.

MODEL	ε	S
Failure load (P_{max})	0.01	0.9
Fracture energy (G_F)	0.01	0.9
Critical stress intensity factor (K_{IC})	0.01	0.9
Critical crack tip opening displacement ($CTOD_C$)	0.01	0.9

P_{max}, G_F, K_{IC} and $CTOD_C$ respectively:

$$P_{\max} = \sum_{k=1}^{61} \alpha_k e^{\frac{-(x_k-x)(x_k-x)^T}{0.020808}} - 0.0069 \quad (22)$$

Table 4: Values of γ and σ for the LSSVM models.

MODEL	γ	σ
Failure load (P_{max})	45	0.102
Fracture energy (G_F)	20	0.059
Critical stress intensity factor (K_{IC})	20	0.073
Critical crack tip opening displacement ($CTOD_C$)	30	0.039

Table 5: Values of R and RMSE.

	GPR				LSSVM			
	P_{max}	G_F	K_{IC}	$CTOD_C$	P_{max}	G_F	K_{IC}	$CTOD_C$
Rtrain	0.9999	0.9993	1.0000	0.9999	0.9994	0.9989	0.9998	0.9998
Rtest	0.9986	0.9963	0.9994	0.9980	0.9991	0.9969	0.9995	0.9989
RMSE	0.257201	339.848	0.169122	0.011913	0.206242	316.9103	0.182939	0.009509

$$G_F = \sum_{k=1}^{61} \alpha_k e^{\frac{-(x_k-x)(x_k-x)^T}{0.006962}} - 0.0175 \tag{23}$$

$$K_{IC} = \sum_{k=1}^{61} \alpha_k e^{\frac{-(x_k-x)(x_k-x)^T}{0.010658}} - 0.013 \tag{24}$$

$$CTOD_C = \sum_{k=1}^{61} \alpha_k e^{\frac{-(x_k-x)(x_k-x)^T}{0.003042}} - 0.0138 \tag{25}$$

The corresponding α_k values for each of the LSSVM models are shown in Figures 9, 10, 11 and 12. The comparison between Artificial Neural Network (ANN) Support Vector Regression (SVR), Multivariate Adaptive Regression Splines(MARS), GPR and LSSVM models for prediction of fracture parameters in terms of Correlation Coefficient (R) is shown in Figure 13.

These results prove that the GPR and LSSVM models are more accurate and reliable for the prediction of the fracture parameters of high strength and Ultra-high strength concretes.

6 Summary and Conclusions

This study shows the efficient and feasible use of GPR and LSSVM based approach for the prediction of fracture parameters of high strength and Ultra-high strength concrete mixes. Brief description has been outlines for GPR and LSSVM. Experimental data of high strength concrete and ultra high strength concrete has been

Figure 9: Values of α_k for LSSVM (P_{max} model).

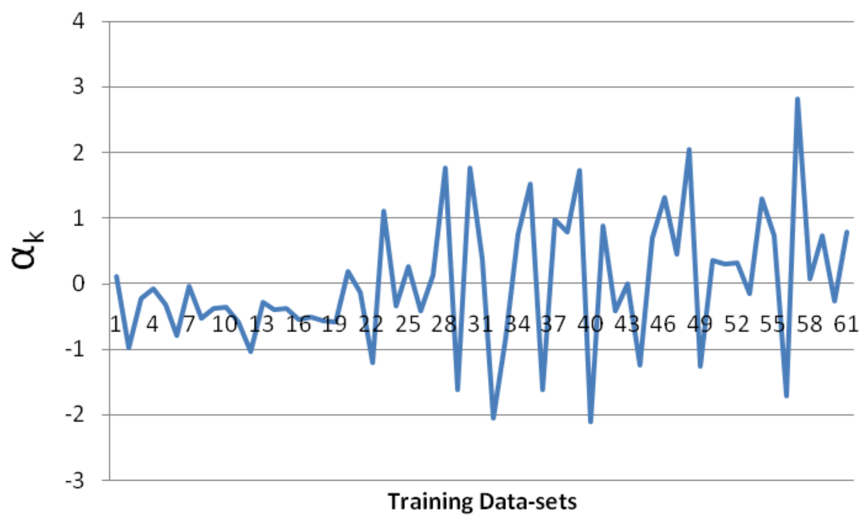

Figure 10: Values of α_k for LSSVM (G_F model).

Figure 11: Values of α_k for LSSVM (K_{IC} model).

Figure 12: Values of α_k for LSSVM ($CTOD_C$ model).

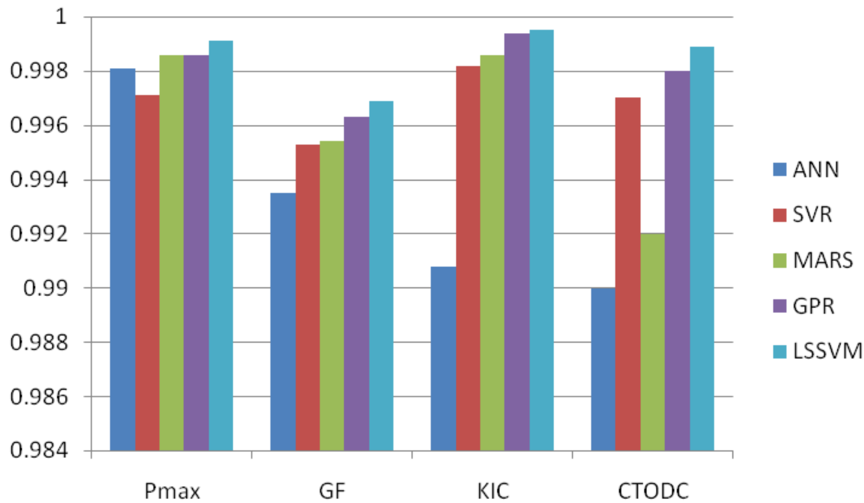

Figure 13: Comparison of ANN, SVR, MARS, GPR and LSSVM in terms of R.

used to develop and validate the GPR and LSSVM based models. Performance of the models has been verified with other popular models such as ANN, SVR and MARS models and it is found that LSSVM is one of the efficient models due to its better coefficient of correlation (R). The developed equations by LSSVM can be used by the users for determination of fracture parameters of high strength and ultra high strength concrete mixes, which in turn will be useful for remaining life prediction and residual strength of concrete structural components.

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