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An Approach with Haar Wavelet Collocation Method for Numerical Simulations of Modified KdV and Modified Burgers Equations

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Abstract: In this paper, an efficient numerical schemes based on the Haar wavelet method are applied for finding numerical solution of nonlinear third-order modified Korteweg-de Vries (mKdV) equation as well as modified Burgers' equations. The numerical results are then compared with the exact solutions. The accuracy of the obtained solutions is quite high even if the number of calculation points is small.

Keywords: Haar wavelets, modified Burgers' equation, modified KdV equation.

1 Introduction

Generalized modified KdV equation [Wazwaz (2009)] is a nonlinear partial differential equation of the form

$$u_t + qu^2 u_x + ru_{xxx} = 0, \quad 0 \le x \le 1, t \ge 0 \tag{1}$$

where q and r are parameters.

Generalized modified Burgers' equation [Irk (2009)] is a nonlinear partial differential equation of the form

$$u_t + u^p u_x - \nu u_{xx} = 0, \quad 0 \le x \le 1 \tag{2}$$

where p and ν are parameters.

The modified Korteweg-de Vries (mKdV) equations are most popular soliton equations and have been extensively investigated. The modified KdV equation is of important significance in many branches of nonlinear science field. The mKdV equa-

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tion appears in many fields such as Alfvén waves in a collisionless plasma, acoustic waves in certain anharmonic lattices, models of traffic congestion, transmission lines in Schottky barrier, ion acoustic soliton, elastic media etc. [Yan (2008)]

Similarly, the modified Burgers' equation [Bratsos (2011)] has the strong nonlinear aspects of the governing equation in many practical transport problems such as nonlinear waves in medium with low frequency absorption, wave processes in thermoelastic medium, turbulence transport, ion reflection at quasi perpendicular shocks, transport and dispersion of pollutants in rivers and sediment transport etc.

Various mathematical methods such as Petrov-Galerkin method [Roshan and Bhamra (2011)], Quintic spline method [Ramadan and El-Danaf (2005)], Sextic B-spline collocation method [Irk (2009)], local discontinuous Galerkin method [Zhang, Yu and Zhao (2013)], and Lattice Boltzmann model [Duan, Liu and Jiang (2008)] have been used in attempting to solve modified Burgers' equations. Dehghan et al. have applied mixed finite difference and Galerkin methods for solving the Burgers [Dehghan, Saray and Lakestani (2014)] and Burgers-Huxley [Dehghan, Saray and Lakestani (2012)] equations. Generalized Benjamin–Bona–Mahony–Burgers equation [Dehghan, Abbaszadeh and Mohebbi (2014)] and KdV equations [Dehghan and Shokri (2007)] have also been investigated by Dehghan et al. via the method of radial basis functions. In 2008, Alipanah and Dehghan have solved the population balance equations by applying rationalized Haar functions.

Zhi-Zhong et al. (2008) improve a numerical method based on two types of wavelets viz. the Haar wavelet and biorthogonal wavelet to compute the band structures of 2D phononic crystals consisting of general anisotropic materials. In 2011, Zhou et al. proposed an efficient wavelet-based algorithm for solving a class of fractional vibration, diffusion and wave equations with strong nonlinearities. Yi and Chen (2012) and Wang et al. (2013) applied Haar wavelet operational matrix method to solve a class of fractional partial differential equations. Using the Haar wavelet operational matrix of fractional order differentiation, the fractional partial differential equations have been reduced to Sylvester equation. Wei et al. (2012) present a computational method for solving space-time fractional convection-diffusion equations with variable coefficients which is based on the Haar wavelets operational matrix of fractional order differentiation. They also exhibit error analysis in order to show the efficiency of the method. Saha Ray and Gupta (2013) proposed Haar wavelet collocation method for solving generalized Burger-Huxley and Huxley equations.

The Haar wavelet method consists of reducing the problem to a set of algebraic equation by expanding the term, which has maximum derivative. Our aim in the present work is to implement the Haar wavelet method to stress its power in handling nonlinear equations, so that one can execute it to various types of strong nonlinear equations.

This paper is systematized as follows: in Section 1, introduction to modified KdV and modified Burgers' equation is discussed. In Section 2, the mathematical preliminaries of Haar wavelet are presented. Sections 3 and 5 define the mathematical models of modified KdV and modified Burgers' equation respectively. The Haar wavelet method has been applied to solve modified KdV and modified Burgers' equation in Sections 4 and 6 respectively. The convergence of Haar wavelet method is discussed in Section 7. The numerical results and discussions are discussed in Section 8 and Section 9 concludes the paper.

2 Haar wavelets and the operational matrices

The Haar wavelet family for $x \in [0,\ 1)$ is defined as follows [Debnath (2002); Lepik (2007); Saha Ray (2012)]

$$h_i(x) = \begin{cases} 1 & x \in [\xi_1, \xi_2) \\ -1 & x \in [\xi_2, \xi_3) \\ 0 & \text{elsewhere} \end{cases} \tag{3}$$

where

$$\xi_1 = \frac{k}{m}, \quad \xi_2 = \frac{k+0.5}{m}, \quad \xi_3 = \frac{k+1}{m}.$$

In these formulae integer $m = 2^j$, $j = 0,\ 1,\ 2, ..., J$ indicates the level of the wavelet; $k = 0,\ 1,\ 2, ..., m-1$ is the translation parameter. Maximum level of resolution is J. The index i is calculated from the formula $i = m+k+1$; in the case of minimal values $m = 1, k = 0$ we have $i = 2$. The maximum possible value of $i = 2M = 2^{J+1}$. It is assumed that the value $i = 1$ corresponds to the scaling function for which

$$h_i(x) = \begin{cases} 1 & \text{for } x \in [0,\ 1) \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

In the following analysis, integrals of the wavelets are defined as

$$p_i(x) = \int_0^x h_i(x)\,dx, \quad q_i(x) = \int_0^x p_i(x)\,dx, \quad r_i(x) = \int_0^x q_i(x)\,dx$$

This can be done with the aid of (3)

$$p_i(x) = \begin{cases} x-\xi_1 & \text{for } x \in [\xi_1, \xi_2) \\ \xi_3 - x & \text{for } x \in [\xi_2, \xi_3) \\ 0 & \text{elsewhere} \end{cases} \tag{5}$$

$$q_i(x) = \begin{cases} 0 & \text{for } x \in [0, \xi_1) \\ \frac{1}{2}(x-\xi_1)^2 & \text{for } x \in [\xi_1, \xi_2) \\ \frac{1}{4m^2} - \frac{1}{2}(\xi_3 - x)^2 & \text{for } x \in [\xi_2, \xi_3) \\ \frac{1}{4m^2} & \text{for } x \in [\xi_3, 1) \end{cases} \tag{6}$$

$$r_i(x) = \begin{cases} \frac{1}{6}(x-\xi_1)^3 & \text{for } x \in [\xi_1,\ \xi_2) \\ \frac{1}{4m^2}(x-\xi_2) + \frac{1}{6}(\xi_3 - x)^3 & \text{for } x \in [\xi_2, \xi_3) \\ \frac{1}{4m^2}(x-\xi_2) & \text{for } x \in [\xi_3, 1) \\ 0 & \text{elsewhere} \end{cases} \tag{7}$$

The collocation points are defined as

$$x_l = \frac{l-0.5}{2M}, \quad l = 1,\ 2,\ \dots,\ 2M$$

It is expedient to introduce the $2M \times 2M$ matrices H, P, Q and R with the elements $H(i,l) = h_i(x_l)$, $P(i,l) = p_i(x_l)$, $Q(i,l) = q_i(x_l)$ and $R(i,l) = r_i(x_l)$.

3 Generalized modified KdV equation

Consider the generalized modified KdV equation [Kaya (2005); Wazwaz (2004)]

$$u_t + qu^2 u_x + ru_{xxx} = 0, \quad 0 \le x \le 1, t \ge 0 \tag{8}$$

with initial condition

$$u(x,0) = \sqrt{\frac{-6r}{q}} \tanh\ (x), \tag{9}$$

The exact solution of eq. (8) is given by [Bekir (2009)]

$$u(x,t) = \sqrt{\frac{-6r}{q}} \tanh(x+2rt), \tag{10}$$

where q and r are parameters.

This exact solution satisfies the following boundary conditions

$$u(0,t) = \sqrt{\frac{-6r}{q}} \tanh\ (2rt), \quad t \ge 0$$

$$u(1,t) = \sqrt{\frac{-6r}{q}} \tanh\ (1+2rt), \quad t \ge 0 \tag{11}$$

4 Application of Haar wavelet method for solving modified KdV equation

Haar wavelet solution of $u(x,t)$ is sought by assuming that $\dot{u}'''(x,t)$ can be expanded in terms of Haar wavelets as

$$\dot{u}'''(x,t) = \sum_{i=1}^{2M} a_s(i)\, h_i(x) \text{ for } t \in [t_s,\ t_{s+1}] \tag{12}$$

where "." and " $'$" stand for differentiation with respect to t and x respectively.

Integrating eq. (12) with respect to t from t_s to t and thrice with respect to x from 0 to x, the following equations are obtained

$$u'''(x,t) = (t-t_s)\sum_{i=1}^{2M} a_s(i)\, h_i(x) + u'''(x,t_s), \tag{13}$$

$$u''(x,t) = (t-t_s)\sum_{i=1}^{2M} a_s(i)\, p_i(x) + u''(x,t_s) + u''(0,t) - u''(0,t_s), \tag{14}$$

$$u'(x,t) = (t-t_s)\sum_{i=1}^{2M} a_s(i)\, q_i(x) + u'(x,t_s) + x\left[u''(0,t) - u''(0,t_s)\right] + u'(0,t) - u'(0,t_s), \tag{15}$$

$$\begin{aligned} u(x,t) = {} & (t-t_s)\sum_{i=1}^{2M} a_s(i)\, r_i(x) + u(x,t_s) + \frac{x^2}{2}\left[u''(0,t) - u''(0,t_s)\right] \\ & + x\left[u'(0,t) - u'(0,t_s)\right] + u(0,t) - u(0,t_s), \end{aligned} \tag{16}$$

$$\dot{u}(x,t) = \sum_{i=1}^{2M} a_s(i)\, r_i(x) + x\dot{u}'(0,t) + \frac{x^2}{2}\dot{u}''(0,t) + \dot{u}(0,\ t), \tag{17}$$

Using finite difference method

$$\dot{u}(0,\ t) = \frac{u(0,\ t) - u(0,\ t_s)}{(t-t_s)},$$

Equation (17) becomes

$$\begin{aligned} \dot{u}(x,t) = {} & \sum_{i=1}^{2M} a_s(i)\, r_i(x) + \frac{x^2}{2}\left[\frac{u''(0,t) - u''(0,t_s)}{t-t_s}\right] + x\left[\frac{u'(0,t) - u'(0,t_s)}{t-t_s}\right] \\ & + \left[\frac{u(0,\ t) - u(0,\ t_s)}{t-t_s}\right], \end{aligned} \tag{18}$$

By using the boundary condition at $x = 1$, eq. (15) becomes

$$u'(1,t) = (t-t_s)\sum_{i=1}^{2M} a_s(i)\,q_i(1) + u'(1,t_s) - u'(0,t_s) + u'(0,\ t) + \left[u''(0,t) - u''(0,t_s)\right],$$

This implies

$$\begin{aligned} & u''(0,t) - u''(0,t_s) \\ & = -(t-t_s)\sum_{i=1}^{2M} a_s(i)\,q_i(1) + \left[u'(1,t) - u'(1,t_s)\right] - \left[u'(0,\ t) - u'(0,t_s)\right], \end{aligned} \tag{19}$$

Substituting eq. (19) in eqs. (14), (15), (16) and (18), we have

$$\begin{aligned} u''(x,t) &= (t-t_s)\sum_{i=1}^{2M} a_s(i)\,p_i(x) + u''(x,t_s) \\ &\quad + \left[-(t-t_s)\sum_{i=1}^{2M} a_s(i)\,q_i(1) + \left[u'(1,t) - u'(1,t_s)\right] - \left[u'(0,\ t) - u'(0,t_s)\right]\right], \end{aligned}$$

$$\begin{aligned} u'(x,t) &= (t-t_s)\sum_{i=1}^{2M} a_s(i) q_i(x) + u'(x,t_s) - u'(0,t_s) + u'(0,t) \\ &\quad + x\left[-(t-t_s)\sum_{i=1}^{2M} a_s(i)\,q_i(1) + \left[u'(1,t) - u'(1,t_s)\right] - \left[u'(0,t) - u'(0,t_s)\right]\right] \end{aligned} \tag{20}$$

$$\begin{aligned} & u(x,t) = (t-t_s)\sum_{i=1}^{2M} a_s(i)\,r_i(x) \\ & + \frac{x^2}{2}\left[-(t-t_s)\sum_{i=1}^{2M} a_s(i)\,q_i(1) + \left[u'(1,t) - u'(1,t_s)\right] - \left[u'(0,\ t) - u'(0,t_s)\right]\right] \\ & + u(x,t_s) + x\left[u'(0,t) - u'(0,t_s)\right] + u(0,t) - u(0,t_s), \\ & \dot{u}(x,t) = \sum_{i=1}^{2M} a_s(i)\,r_i(x) \\ & + \frac{x^2}{2(t-t_s)}\left[-(t-t_s)\sum_{i=1}^{2M} a_s(i)\,q_i(1) + \left[u'(1,t) - u'(1,t_s)\right] - \left[u'(0,\ t) - u'(0,t_s)\right]\right] \\ & + \frac{x}{t-t_s}\left[u'(0,t) - u'(0,t_s)\right] + \frac{1}{t-t_s}\left[u(0,\ t) - u(0,\ t_s)\right], \end{aligned}$$

It is obtained from eq. (6) that,

$$q_i(1) = \begin{cases} 0.5 & \text{if } i = 1 \\ \frac{1}{4\mathrm{m}^2} & \text{if } i > 1 \end{cases}$$

Discretising the above results by assuming $x \to x_l$, $t \to t_{s+1}$, we obtain

$$u'''(x_l, t_{s+1}) = (t_{s+1} - t_s) \sum_{i=1}^{2M} a_s(i) h_i(x_l) + u'''(x_l, t_s),$$

$$u''(x_l, t_{s+1}) = (t_{s+1} - t_s) \sum_{i=1}^{2M} a_s(i) p_i(x_l) + u''(x_l, t_s)$$
$$+ \left[-(t_{s+1} - t_s) \sum_{i=1}^{2M} a_s(i) q_i(1) + \left[u'(1, t_{s+1}) - u'(1, t_s)\right] - \left[u'(0,\ t_{s+1}) - u'(0, t_s)\right] \right],$$

$$u'(x_l, t_{s+1}) = (t_{s+1} - t_s) \sum_{i=1}^{2M} a_s(i) q_i(x_l) + u'(x_l, t_s) + u'(0, t_{s+1}) - u'(0, t_s)$$
$$+ x_l \left[-(t_{s+1} - t_s) \sum_{i=1}^{2M} a_s(i) q_i(1) + \left[u'(1, t_{s+1}) - u'(1, t_s)\right] - \left[u'(0,\ t_{s+1}) - u'(0, t_s)\right] \right],$$

$$u(x_l, t_{s+1}) = (t_{s+1} - t_s) \sum_{i=1}^{2M} a_s(i) r_i(x_l) + u(x_l, t_s) + u(0, t_{s+1}) - u(0, t_s)$$
$$+ x_l \left[u'(0, t_{s+1}) - u'(0, t_s)\right]$$
$$+ \frac{x_l^2}{2} \left[-(t_{s+1} - t_s) \sum_{i=1}^{2M} a_s(i) q_i(1) + \left[u'(1, t_{s+1}) - u'(1, t_s)\right] - \left[u'(0,\ t_{s+1}) - u'(0, t_s)\right] \right],$$

$$\dot{u}(x_l, t_{s+1}) = \sum_{i=1}^{2M} a_s(i) r_i(x_l) + \frac{1}{t_{s+1} - t_s} \left[u(0,\ t_{s+1}) - u(0,\ t_s)\right]$$
$$+ \frac{x_l}{t_{s+1} - t_s} \left[u'(0, t_{s+1}) - u'(0, t_s)\right]$$
$$+ \frac{x_l^2}{2(t_{s+1} - t_s)} \left[-(t_{s+1} - t_s) \sum_{i=1}^{2M} a_s(i) q_i(1) + \left[u'(1, t_{s+1}) - u'(1, t_s)\right] \right.$$
$$\left. - \left[u'(0, t_{s+1}) - u'(0, t_s)\right] \right],$$

Substituting the above equations in eq. (8), we have

$$\sum_{i=1}^{2M} a_s(i)\, r_i(x_l) + \frac{x_l^2}{2(t_{s+1}-t_s)}\left[-(t_{s+1}-t_s)\sum_{i=1}^{2M} a_s(i)\, q_i(1) + \left[u'(1,t_{s+1}) - u'(1,t_s)\right]\right.$$
$$\left. - \left[u'(0,t_{s+1}) - u'(0,t_s)\right]\right]$$
$$+ \frac{x_l}{t_{s+1}-t_s}\left[u'(0,t_{s+1}) - u'(0,t_s)\right] + \frac{1}{t_{s+1}-t_s}\left[u(0,\ t_{s+1}) - u(0,\ t_s)\right]$$
$$= 0.001u'''(x_l,\ t_s) - 6\left[u(x_l,\ t_s)\right]^2\left[u'(x_l,\ t_s)\right],$$

Therefore,

$$\sum_{i=1}^{2M} a_s(i)\left[r_i(x_l) - \frac{x_l^2}{2} q_i(1)\right] = 0.001u'''(x_l,\ t_s)$$
$$-6\left[u(x_l,\ t_s)\right]^2\left[u'(x_l,\ t_s)\right] - \frac{x_l^2}{2(t_{s+1}-t_s)}\left[u'(1,t_{s+1}) - u'(1,t_s)\right]$$
$$+ \frac{x_l^2}{2(t_{s+1}-t_s)}\left[u'(0,t_{s+1}) - u'(0,t_s)\right] - \frac{x_l}{t_{s+1}-t_s}\left[u'(0,t_{s+1}) - u'(0,t_s)\right]$$
$$- \frac{1}{t_{s+1}-t_s}\left[u(0,\ t_{s+1}) - u(0,\ t_s)\right],$$

From the above equation, the wavelet coefficients $a_s(i)$ can be successively calculated. This process starts with

$$u(x_l,t_0) = \sqrt{\frac{-6r}{q}}\ \tanh(x_l),$$

$$u'(x_l,t_0) = \sqrt{\frac{-6r}{q}}\ \sec h^2(x_l)$$

5 Modified Burgers' equation

Consider the generalized modified Burgers' equation [Roshan and Bhamra (2011)]

$$u_t + u^p u_x - \nu u_{xx} = 0, \quad 0 \le x \le 1 \tag{21}$$

where p is a positive constant and $\nu\ (>0)$ can be interpreted as viscosity.

To show the effectiveness and accuracy of proposed scheme, we consider two test examples taking $p = 2$. The numerical solutions thus obtained are compared with the analytical solutions as well as available numerical results.

The initial condition associated with eq. (21) will be

$u(x,t_0) = f(x), \quad 0 \le x \le 1$

with boundary conditions $u(0,t) = u(1,t) = 0$, $t \ge t_0$

6 Haar wavelet based scheme for modified Burgers' equation

It is assumed that $\dot{u}''(x,t)$ can be expanded in terms of Haar wavelets as

$$\dot{u}''(x,t) = \sum_{i=1}^{2M} a_s(i)\, h_i(x) \quad \text{for} \quad t \in [t_s, t_{s+1}] \tag{22}$$

where "." and " $'$ " stand for differentiation with respect to t and x respectively.

Now, integrating eq. (22) with respect to t from t_s to t and twice with respect to x from 0 to x the following equations are obtained

$$u''(x,t) = (t - t_s) \sum_{i=1}^{2M} a_s(i)\, h_i(x) + u''(x,t_s), \tag{23}$$

$$u'(x,t) = (t - t_s) \sum_{i=1}^{2M} a_s(i)\, p_i(x) + u'(x,t_s) + u'(0,t) - u'(0,t_s), \tag{24}$$

$$u(x,t) = (t - t_s) \sum_{i=1}^{2M} a_s(i)\, q_i(x) + u(x,t_s) + x\left[u'(0,t) - u'(0,t_s)\right] + u(0,t) - u(0,t_s), \tag{25}$$

$$\dot{u}(x,t) = \sum_{i=1}^{2M} a_s(i)\, q_i(x) + x\dot{u}'(0,t) + \dot{u}(0,t), \tag{26}$$

By using the boundary condition at $x = 1$, from eq. (26) we have

$$\dot{u}'(0,t) = -\sum_{i=1}^{2M} a_s(i)\, q_i(1), \tag{27}$$

and from eq. (25), we obtain

$$u'(0,t) - u'(0,t_s) = -(t - t_s) \sum_{i=1}^{2M} a_s(i)\, q_i(1), \tag{28}$$

Substituting equation (27) and (28) in eqs. (24), (25) and (26), we have

$$u'(x,t) = (t-t_s)\sum_{i=1}^{2M} a_s(i)\left[p_i(x) - q_i(1)\right] + u'(x,t_s), \tag{29}$$

$$u(x,t) = (t-t_s)\sum_{i=1}^{2M} a_s(i)\left[q_i(x) - xq_i(1)\right] + u(x,t_s),$$

$$\dot{u}(x,t) = \sum_{i=1}^{2M} a_s(i)\left[q_i(x) - xq_i(1)\right],$$

From Eq. (6), it is obtained that

$$q_i(1) = \begin{cases} 0.5 & \text{if } i=1 \\ \frac{1}{4\mathrm{m}^2} & \text{if } i>1 \end{cases} \tag{30}$$

Discretising the above results by assuming $x \to x_l$, $t \to t_{s+1}$, we obtain

$$u''(x_l,t_{s+1}) = (t_{s+1}-t_s)\sum_{i=1}^{2M} a_s(i)\, h_i(x_l) + u''(x_l,t_s), \tag{31}$$

$$u'(x_l,t_{s+1}) = (t_{s+1}-t_s)\sum_{i=1}^{2M} a_s(i)\left[p_i(x_l) - q_i(1)\right] + u'(x_l,t_s), \tag{32}$$

$$u(x_l,t_{s+1}) = (t_{s+1}-t_s)\sum_{i=1}^{2M} a_s(i)\left[q_i(x_l) - x_l\, q_i(1)\right] + u(x_l,t_s), \tag{33}$$

$$\dot{u}(x_l,t_{s+1}) = \sum_{i=1}^{2M} a_s(i)\left[q_i(x_l) - x_l\, q_i(1)\right], \tag{34}$$

Substituting equations (31), (32), (33) and (34) in eq. (21), we have

$$\begin{aligned} \sum_{i=1}^{2M} a_s(i)\left[q_i(x_l) - x_l q_i(1)\right] &= \nu \left[(t_{s+1}-t_s)\sum_{i=1}^{2M} a_s(i)\, h_i(x_l) + u''(x_l,t_s)\right] \\ &- \left[(t_{s+1}-t_s)\sum_{i=1}^{2M} a_s(i)\left[q_i(x_l) - x_l\, q_i(1)\right] + u(x_l,t_s)\right]^2 \\ &\left[(t_{s+1}-t_s)\sum_{i=1}^{2M} a_s(i)\left[p_i(x_l) - q_i(1)\right] + u'(x_l,t_s)\right] \end{aligned} \tag{35}$$

From eq. (35), the wavelet coefficients $a_s(i)$ can be successively calculated. This process starts with

$$u(x_l, t_0) = f(x_l)$$

$$u'(x_l, t_0) = f'(x_l)$$

$$u''(x_l, t_0) = f''(x_l)$$

Example 1. Consider modified Burgers' equation with the following initial and boundary conditions [Roshan and Bhamra (2011); Ramadan and El-Danaf (2005)]

$$u(x,1) = \frac{x}{1 + \frac{1}{c_0} e^{\frac{x^2}{4v}}},$$

$$u(0,t) = u(1,t) = 0, \quad t \geq 1$$

where $c_0 = e^{\frac{1}{8v}}$.

The exact solution of eq. (21) is given by [Roshan and Bhamra (2011); Ramadan and El-Danaf (2005)]

$$u(x,t) = \frac{\frac{x}{t}}{1 + \frac{\sqrt{t}}{c_0} e^{\frac{x^2}{4vt}}}, \quad t \geq 1$$

Example 2. In this example, we consider modified Burgers' equation with initial and boundary conditions in the following form

$$u(x,0) = \sin(\pi x) \quad 0 \leq x \leq 1,$$

$$u(0,t) = u(1,t) = 0, \quad t > 0$$

In case of example 1, the Haar wavelet numerical solutions have been compared with the results obtained by Ramadan et al. (2005), using the collocation method with quintic splines and in case of example 2, the solutions have been compared with the results obtained by Duan et al. (2008), using 2-bit lattice Boltzmann method (LBM). Tables 1 and 2 cite the comparison of Haar wavelet solution with LBM and quintic splines numerical solutions at $t = 0.4$ and $t = 2$, and the numerical solutions at different time stages are exhibited in Fig. 6.

Table 1: Comparison of Haar wavelet solutions with the LBM solutions and 5-Splines solution of modified Burgers' equation (example 2) at $t = 0.4$ and $\nu = 0.01$.

x	Approximate solution using Haar wavelet method (u_{approx})	Approximate solution using lattice Boltzmann method [Duan, Liu, Jiang (2008)]	Approximate solution using Quintic spline method [Ramadan, El-Danaf (2005)]
0.10	0.221423	0.22177116	0.22033034
0.20	0.396841	0.39414890	0.39460783
0.30	0.531256	0.53134565	0.53244922
0.40	0.648350	0.64627793	0.64763455
0.50	0.744936	0.74511632	0.74643231
0.60	0.831235	0.83048713	0.83133318
0.70	0.902641	0.90235089	0.90195203
0.80	0.95132	0.95495434	0.95119837
0.90	0.825329	0.83737688	0.82794559
0.99	0.0623064	0.06214261	0.04674614

Table 2: Comparison of Haar wavelet solutions with the LBM solutions and 5-Splines solution of modified Burgers' equation (example 2) at $t = 2.0$ and $\nu = 0.01$.

x	Approximate solution using Haar wavelet method (u_{approx})	Approximate solution using lattice Boltzmann method [Duan, Liu, Jiang (2008)]	Approximate solution using Quintic spline method [Ramadan, El-Danaf (2005)]
0.10	0.111789	0.11194772	0.11013979
0.20	0.208539	0.20710153	0.20614825
0.30	0.284853	0.28512152	0.28477813
0.40	0.351297	0.35038171	0.35045112
0.50	0.406404	0.40665374	0.40700602
0.60	0.457189	0.45649486	0.45704614
0.70	0.501339	0.50155303	0.50224419
0.80	0.542602	0.54199420	0.54265295
0.90	0.536499	0.53547356	0.53225529
0.99	0.0790367	0.08046491	0.05693884

7 Error of collocation method

From eq. (3), the Haar wavelet family for $x \in [0,\ 1)$ is defined as follows

$$h_i(x) = \begin{cases} 1 & x \in [\xi_1, \xi_2) \\ -1 & x \in [\xi_2, \xi_3) \\ 0 & \text{elsewhere} \end{cases}$$

where

$$\xi_1 = \frac{k}{m}, \quad \xi_2 = \frac{k+0.5}{m}, \quad \xi_3 = \frac{k+1}{m}.$$

Consider

$$u(x,\tilde{t}) = (\tilde{t} - t_s) \sum_{i=1}^{2M} u(i)\ Q_i(x) + \psi(x,\tilde{t}), \quad \tilde{t} \in [t_s,\ t_{s+1}] \tag{36}$$

Define a projection map

$P_m : L^2(\Omega) \to V_J$

by the rule

$$P_m u(x, t_{s+1}) = u_m(x, t_{s+1}) = h \sum_{i=1}^{2M} u(i)\ Q_i(x) \tag{37}$$

where $\Omega = [0,\ 1)$
V_J is a subspace of $L^2(\Omega)$
Now we have to estimate $\|u - P_m u\|$ for arbitrary $u \in L^2(\Omega)$.
Lemma 7.1 Let $u(x,t)$ be defined on $L^2(\Omega)$ and P_m be the projection map defined as above then

$$\|u - P_m u\| \leq \frac{\max |u|}{4M^2}$$

Proof: The integral $\int_0^1 u_m(x,t)\,dx$ is a ramp $\frac{u_i}{4M^2}\left[\frac{1}{2M} + (x - \xi_3)\right]$ on the interval $[0,1)$ with average value $\frac{u_i}{8M^2}\left[\frac{1}{2M} + (1 - \xi_3)\right]$.
The error in approximating the ramp by this constant value over the interval $[0,1)$ is

$$r(x) = \frac{u_i}{8M^2}\left[\frac{1}{2M} + (1 - \xi_3)\right] - \frac{u_i}{4M^2}\left[\frac{1}{2M} + (x - \xi_3)\right]$$

Hence, using $E(x)$ as least square of the error on Ω, we have

$$E^2(x) = \int_0^1 [r(x)]^2 dx$$

$$= \int_0^1 \left(\frac{u_i}{8M^2}\left[\frac{1}{2M} + (1-\xi_3)\right] - \frac{u_i}{4M^2}\left[\frac{1}{2M} + (x-\xi_3)\right]\right)^2 dx$$

$$= \int_0^1 \left(\frac{-u_i}{16M^3} + \frac{u_i}{8M^2} - \frac{u_i\xi_3}{8M^2} - \frac{u_i x}{4M^2} + \frac{u_i\xi_3}{4M^2}\right)^2 dx$$

$$= \int_0^1 \left(\frac{u_i\xi_3}{8M^2} + \frac{u_i}{8M^2} - \frac{u_i}{16M^3} - \frac{u_i x}{4M^2}\right)^2 dx$$

$$= \left(\frac{u_i}{4M^2}\right)^2 \int_0^1 \left(\frac{\xi_3}{2} + \frac{1}{2} - \frac{1}{4M} - x\right)^2 dx$$

$$\leq \left(\frac{u_i}{4M^2}\right)^2$$

$$\Rightarrow |E(x)| \leq \frac{|u_i|}{4M^2} \tag{38}$$

On the interval Ω we have

$$\|u - P_m u\| = \max_{x\in\Omega} E(x) \leq \frac{\max\ |u|}{4M^2} \tag{39}$$

Error Analysis

Let $P_m : L^2(\Omega) \to V_J$ be a projection map and is defined by

$$P_m u(x, t_{s+1}) = u_m(x, t_{s+1}) = h\sum_{i=1}^{2M} u(i)\ Q_i(x) \tag{40}$$

Let us consider the generalized modified Burger's equation

$$\frac{\partial u}{\partial t} + u^2\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2} \tag{41}$$

Suppose that $u_m = P_m u$ be the approximate solution of eq. (41) obtained by wavelet collocation method

$$\frac{\partial u_m}{\partial t} + u_m^2\frac{\partial u_m}{\partial x} = \nu\frac{\partial^2 u_m}{\partial x^2} + e \tag{42}$$

then $\|e\| \leq \frac{A}{4M^2}$

where $A = \max\left|\frac{\partial u}{\partial t}\right| + \nu \max\left|\frac{\partial^2 u}{\partial x^2}\right| + \max\left|u^2 \frac{\partial u}{\partial x}\right| + P_m u^2 \max\left|\frac{\partial u}{\partial x}\right|$

Proof: Subtracting eq. (41) from eq. (42), we have

$$e = \frac{\partial u_m}{\partial t} + u_m^2 \frac{\partial u_m}{\partial x} - \nu \frac{\partial^2 u_m}{\partial x^2} - \frac{\partial u}{\partial t} - u^2 \frac{\partial u}{\partial x} + \nu \frac{\partial^2 u}{\partial x^2}$$

$$= \frac{\partial u_m}{\partial t} - \frac{\partial u}{\partial t} - \nu \left(\frac{\partial^2 u_m}{\partial x^2} - \frac{\partial^2 u}{\partial x^2}\right) + u_m^2 \frac{\partial u_m}{\partial x} - u^2 \frac{\partial u}{\partial x}$$

$$= \frac{\partial (u_m - u)}{\partial t} - \nu \frac{\partial^2 (u_m - u)}{\partial x^2} + (u_m)^2 \frac{\partial (u_m - u)}{\partial x} - \left(u^2 - u_m^2\right) \frac{\partial u}{\partial x}$$

$$= \frac{\partial (P_m - I) u}{\partial t} - \nu \frac{\partial^2 (P_m - I) u}{\partial x^2} + (P_m u)^2 \frac{\partial (P_m - I) u}{\partial x} - (I - P_m) u^2 \frac{\partial u}{\partial x}$$

$$\Rightarrow \|e\| \leq \|P_m - I\| \max\left|\frac{\partial u}{\partial t}\right| + \nu \|P_m - I\| \max\left|\frac{\partial^2 u}{\partial x^2}\right| + \|P_m - I\| (P_m u)^2 \max\left|\frac{\partial u}{\partial x}\right| + \|P_m - I\| \max\left|u^2 \frac{\partial u}{\partial x}\right|$$

$$\leq \|P_m - I\| \left[\max\left|\frac{\partial u}{\partial t}\right| + \nu \max\left|\frac{\partial^2 u}{\partial x^2}\right| + (P_m u)^2 \max\left|\frac{\partial u}{\partial x}\right| + \max\left|u^2 \frac{\partial u}{\partial x}\right|\right]$$

$$\leq \frac{A}{4M^2}$$

where $A = \max\left|\frac{\partial u}{\partial t}\right| + \nu \max\left|\frac{\partial^2 u}{\partial x^2}\right| + (P_m u)^2 \max\left|\frac{\partial u}{\partial x}\right| + \max\left|u^2 \frac{\partial u}{\partial x}\right|$

8 Numerical Results and discussions

The error function is given by

$$Error\ function = \|u_{approx}(x_l, t) - u_{exact}(x_l, t)\| = \sqrt{\sum_{l=1}^{2M} \left(u_{approx}(x_l, t) - u_{exact}(x_l, t)\right)^2}$$

$$Global\ error\ estimate = R.M.S.\ error = \frac{\|u_{approx}(x_l, t) - u_{exact}(x_l, t)\|}{\sqrt{2M}} = \frac{1}{\sqrt{2M}} \sqrt{\sum_{l=1}^{2M} \left(u_{approx}(x_l, t) - u_{exact}(x_l, t)\right)^2} \tag{43}$$

The errors for modified Burgers' equation are measured using two different norms, namely L_2 and L_∞, defined by

$$L_2 = R.M.S.\ error = \frac{1}{\sqrt{2M}}\sqrt{\sum_{l=1}^{2M}\left(\left|u_{approx}(x_l,t) - u_{exact}(x_l,t)\right|\right)^2} \tag{44}$$

$$L_\infty = \max\ \left|u_{approx}(x_l,t) - u_{exact}(x_l,t)\right| \tag{45}$$

The following table exhibits the L_2 and L_∞ error norm for modified Burgers' equation taking $p = 2$ and $\nu = 0.001$ and different values of t. In tables 3, J is taken as 5 i.e. $M = 32$ and Δt is taken as 0.001.

Table 3: L_2 and L_∞ error norm for modified Burgers' equation (example 1) at different values of t with $\nu = 0.001$ and $\Delta t = t_{s+1} - t_s = 0.001$.

Time (sec)	$L_2 \times 10^{-3}$ (Present method)	$L_\infty \times 10^{-3}$ (Present method)	$L_2 \times 10^{-3}$ (Quintic spline) [Ramadan, El-Danaf (2005)]	$L_\infty \times 10^{-3}$ (Quintic spline) [Ramadan, El-Danaf (2005)]
2	0.0755325	0.289254	0.0670395601	0.2796704002
3	0.0711446	0.256515	0.0689577701	0.2514379353
4	0.065229	0.214577	0.0666974605	0.2185661439
5	0.0604007	0.182222	0.0636023977	0.1923643818
6	0.0565458	0.157428	0.0604622308	0.1717652452
7	0.0534117	0.138249	0.0575085655	0.1553318123
8	0.0508078	0.123651	0.0548010376	0.1418932282

The following tables show the comparisons of the exact solutions with the approximate solutions of modified KdV equation taking $q = 6$, $r = -0.001$ and different values of t. In tables 4-7, J is taken as 3 i.e. $M = 8$ and Δt is taken as 0.0001.

In case of $r = -0.001$, the *R.M.S. error* between the numerical solutions and the exact solutions of modified KdV equations for $t = 0.2,\ 0.5,\ 0.8$ and 1 are 0.000017137, 0.0000433416, 0.0000695581 and 0.0000870423 respectively and for $r = -0.1$ and $t = 0.2,\ 0.5,\ 0.8$ and 1 the *R.M.S. error* is found to be 0.00209359, 0.00624177, 0.011631 and 0.0159099 respectively. In the following tables [8-11] also J has been taken as 3 i.e. $M = 8$ and Δt is taken as 0.0001.

Figures 1-4 represent the comparison graphically between the numerical and exact solutions of modified Burgers' equation for different values of t and $\nu = 0.001$. The behaviour of numerical solutions of modified Burgers' equation is cited in figure 5 and 6. Similarly, in case of modified KdV equation, the Figures 7-11 demonstrate

Table 4: The absolute errors for modified KdV equation at various collocation points of x with $t = 0.2$ and $r = -0.001$.

x	**Approximate solution** (u_{approx})	**Exact solution** (u_{exact})	**Absolute Error**
0.03125	0.000975289	0.000975254	3.45313E-8
0.09375	0.00294375	0.00294344	3.10577E-7
0.15625	0.00488976	0.00488889	8.63705E-7
0.21875	0.00679886	0.00679716	1.69770E-6
0.28125	0.00865771	0.00865489	2.81880E-6
0.34375	0.0104545	0.0104502	4.23521E-6
0.40625	0.0121789	0.0121730	5.95649E-6
0.46875	0.0138229	0.0138149	7.99297E-6
0.53125	0.0153800	0.0153696	1.03551E-5
0.59375	0.0168459	0.0168328	1.30530E-5
0.65625	0.0182180	0.0182019	1.60958E-5
0.71875	0.0194955	0.0194760	1.94918E-5
0.78125	0.0206791	0.0206558	2.32477E-5
0.84375	0.0217706	0.0217432	2.73688E-5
0.90625	0.0227729	0.0227410	3.18592E-5
0.96875	0.0236899	0.0236532	3.67252E-5

Table 5: The absolute errors for modified KdV equation at various collocation points of x with $t = 0.5$ and $r = -0.001$.

x	**Approximate solution** (u_{approx})	**Exact solution** (u_{exact})	**Absolute Error**
0.03125	0.000956384	0.000956298	8.64056E-8
0.09375	0.00292541	0.00292463	7.77845E-7
0.15625	0.00487254	0.00487037	2.16512E-6
0.21875	0.00678332	0.00677906	4.25957E-6
0.28125	0.00864442	0.00863734	7.07870E-6
0.34375	0.010444	0.0104333	1.06449E-5
0.40625	0.0121718	0.0121568	1.49840E-5
0.46875	0.0138197	0.0137995	2.01238E-5
0.53125	0.0153812	0.0153552	2.60922E-5
0.59375	0.0168522	0.0168192	3.29164E-5
0.65625	0.0182299	0.0181892	4.06215E-5
0.71875	0.0195135	0.0194643	4.92298E-5
0.78125	0.0207037	0.020645	5.87601E-5
0.84375	0.0218024	0.0217332	6.92277E-5
0.90625	0.0228125	0.0227319	8.06444E-5
0.96875	0.0237378	0.0236448	9.30207E-5

Table 6: The absolute errors for modified KdV equation at various collocation points of x with $t = 0.8$ and $r = -0.001$.

x	Approximate solution (u_{approx})	Exact solution (u_{exact})	Absolute Error
0.03125	0.00093748	0.000937341	1.38295E-7
0.09375	0.00290707	0.00290582	1.24525E-6
0.15625	0.00485531	0.00485185	3.46691E-6
0.21875	0.00676778	0.00676095	6.82219E-6
0.28125	0.00863112	0.00861978	1.13399E-5
0.34375	0.0104335	0.0104164	1.70566E-5
0.40625	0.0121647	0.0121406	2.40144E-5
0.46875	0.0138164	0.0137842	3.22585E-5
0.53125	0.0153825	0.0153406	4.18346E-5
0.59375	0.0168584	0.0168056	5.27869E-5
0.65625	0.0182417	0.0181765	6.51565E-5
0.71875	0.0195315	0.0194525	7.89797E-5
0.78125	0.0207283	0.0206341	9.42876E-5
0.84375	0.0218343	0.0217232	1.11105E-4
0.90625	0.0228522	0.0227227	1.29453E-4
0.96875	0.0237858	0.0236364	1.49345E-4

Table 7: The absolute errors for modified KdV equation at various collocation points of x with $t = 1$ and $r = -0.001$.

x	Approximate solution (u_{approx})	Exact solution (u_{exact})	Absolute Error
0.03125	0.000924876	0.000924703	1.72895E-7
0.09375	0.00289483	0.00289328	1.55692E-6
0.15625	0.00484383	0.0048395	4.33497E-6
0.21875	0.00675741	0.00674888	8.53102E-6
0.28125	0.00862225	0.00860807	1.41814E-5
0.34375	0.0104265	0.0104051	2.13321E-5
0.40625	0.0121599	0.0121299	3.00363E-5
0.46875	0.0138143	0.0137739	4.03506E-5
0.53125	0.0153833	0.015331	5.23326E-5
0.59375	0.0168626	0.0167965	6.60379E-5
0.65625	0.0182496	0.0181681	8.15183E-5
0.71875	0.0195434	0.0194446	9.88196E-5
0.78125	0.0207448	0.0206268	1.17981E-4
0.84375	0.0218555	0.0217165	1.39034E-4
0.90625	0.0228786	0.0227166	1.62005E-4
0.96875	0.0238178	0.0236309	1.86910E-4

Table 8: The absolute errors for modified KdV equation at various collocation points of x with $t = 0.2$ and $r = -0.1$.

x	**Approximate solution** (u_{approx})	**Exact solution** (u_{exact})	**Absolute Error**
0.03125	-0.00276621	-0.00276692	7.13784E-7
0.09375	0.0169876	0.0169809	6.65888E-6
0.15625	0.0366175	0.0365968	2.07298E-5
0.21875	0.0559793	0.0559313	4.79820E-5
0.28125	0.0749397	0.0748436	9.60655E-5
0.34375	0.093380	0.0932052	1.74756E-4
0.40625	0.111199	0.110903	2.95389E-4
0.46875	0.128314	0.127843	4.70239E-4
0.53125	0.144661	0.143950	7.11916E-5
0.59375	0.160198	0.159166	1.03281E-3
0.65625	0.174899	0.173455	1.44463E-3
0.71875	0.188756	0.186798	1.95802E-3
0.78125	0.201774	0.199192	2.58235E-3
0.84375	0.213974	0.210648	3.32555E-3
0.90625	0.225384	0.221190	4.19409E-3
0.96875	0.236043	0.230850	5.19293E-3

Table 9: The absolute errors for modified KdV equation at various collocation points of x with $t = 0.5$ and $r = -0.1$.

x	**Approximate solution** (u_{approx})	**Exact solution** (u_{exact})	**Absolute Error**
0.03125	-0.0217032	-0.0217065	3.31576E-6
0.09375	-0.00194595	-0.0019764	3.04463E-5
0.15625	0.0178593	0.0177691	9.02230E-5
0.21875	0.0375721	0.0373766	1.95531E-4
0.28125	0.0570630	0.0566968	3.66140E-4
0.34375	0.0762172	0.0755894	6.27708E-4
0.40625	0.0949371	0.0939266	1.01047E-3
0.46875	0.113144	0.111596	1.54778E-3
0.53125	0.130779	0.128504	2.27451E-3
0.59375	0.147801	0.144576	3.22569E-3
0.65625	0.164190	0.159755	4.43515E-3
0.71875	0.179941	0.174007	5.93453E-3
0.78125	0.195064	0.187312	7.75255E-3
0.84375	0.209583	0.199668	9.91447E-3
0.90625	0.223529	0.211087	1.24419E-2
0.96875	0.236946	0.221593	1.53524E-2

Table 10: The absolute errors for modified KdV equation at various collocation points of x with $t = 0.8$ and $r = -0.1$.

x	**Approximate solution** (u_{approx})	**Exact solution** (u_{exact})	**Absolute Error**
0.03125	-0.0404827	-0.0404909	8.13685E-6
0.09375	-0.0208453	-0.0209195	7.42051E-5
0.15625	-0.000970764	-0.00118585	2.15085E-4
0.21875	0.0190084	0.018557	4.51366E-4
0.28125	0.0389709	0.0381558	8.15061E-4
0.34375	0.058810	0.0574616	1.34835E-3
0.40625	0.0784361	0.0763344	2.10172E-3
0.46875	0.0977786	0.0946469	3.13176E-3
0.53125	0.116786	0.112288	4.49874E-3
0.59375	0.135428	0.129163	6.26422E-3
0.65625	0.153689	0.14520	8.48888E-3
0.71875	0.171574	0.160343	1.12307E-2
0.78125	0.189101	0.174557	1.45436E-2
0.84375	0.206301	0.187824	1.84763E-2
0.90625	0.223215	0.200143	2.30719E-2
0.96875	0.239891	0.211525	2.83664E-2

Table 11: The absolute errors for modified KdV equation at various collocation points of x with $t = 1$ and $r = -0.1$.

x	**Approximate solution** (u_{approx})	**Exact solution** (u_{exact})	**Absolute Error**
0.03125	-0.0528499	-0.0528626	1.27575E-5
0.09375	-0.0333573	-0.0334734	1.16031E-4
0.15625	-0.0134928	-0.0138262	3.33334E-4
0.21875	0.00661869	0.00592858	6.90107E-4
0.28125	0.0268634	0.0256371	1.22625E-3
0.34375	0.0471421	0.0451472	1.99484E-3
0.40625	0.0673727	0.0643127	3.06007E-3
0.46875	0.0874922	0.0829976	4.49462E-3
0.53125	0.107457	0.101080	6.37673E-3
0.59375	0.127243	0.118455	8.78725E-3
0.65625	0.146843	0.135037	1.18068E-2
0.71875	0.166270	0.150757	1.55134E-2
0.78125	0.185549	0.165569	1.99805E-2
0.84375	0.204718	0.179443	2.52756E-2
0.90625	0.223827	0.192368	3.14593E-2
0.96875	0.24293	0.204347	3.85831E-2

the comparison graphically between the numerical and exact solutions for different values of t and r.

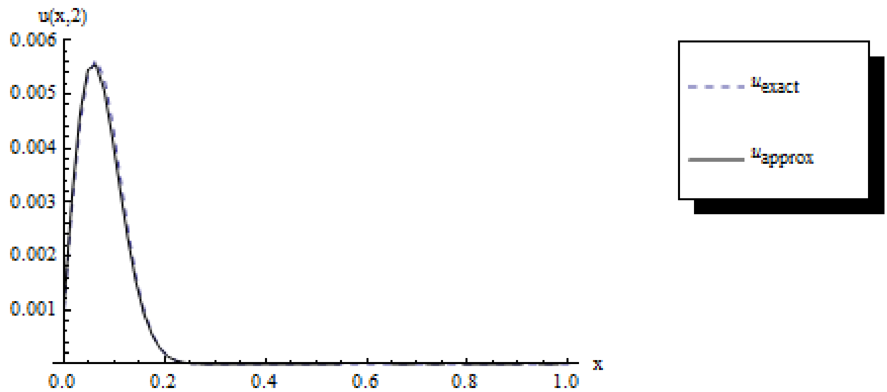

Figure 1: Comparison of Numerical solution and exact solution of modified Burger's equation (example 1) when $t = 2$ and $\nu = 0.001$.

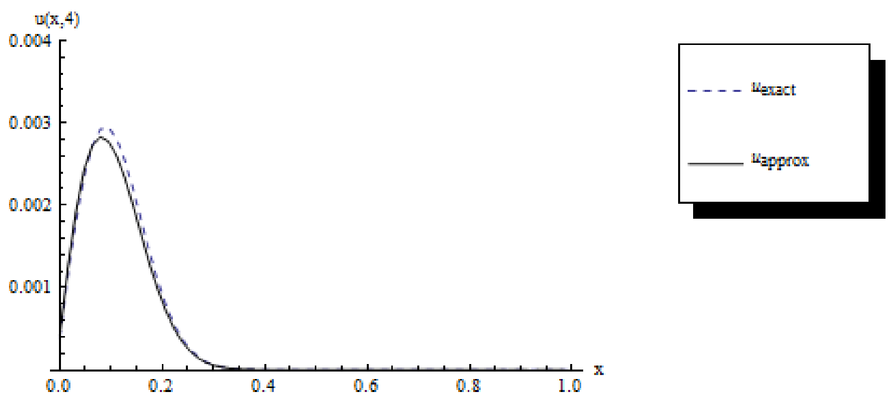

Figure 2: Comparison of Numerical solution and exact solution of modified Burger's equation (example 1) when $t = 4$ and $\nu = 0.001$.

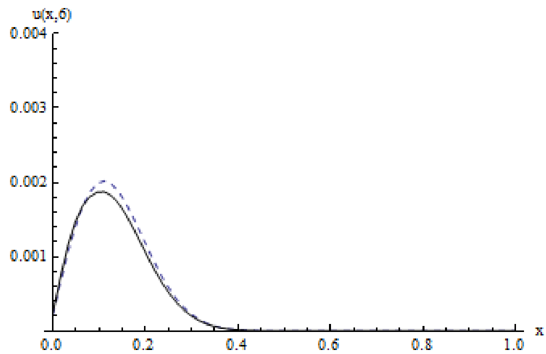

Figure 3: Comparison of Numerical solution and exact solution of modified Burger's equation (example 1) when $t = 6$ and $\nu = 0.001$.

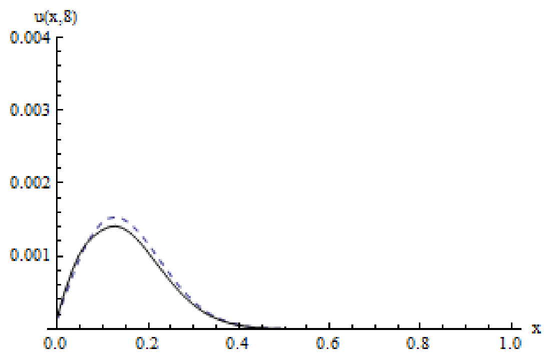

Figure 4: Comparison of Numerical solution and exact solution of modified Burger's equation (example 1) when $t = 8$ and $\nu = 0.001$.

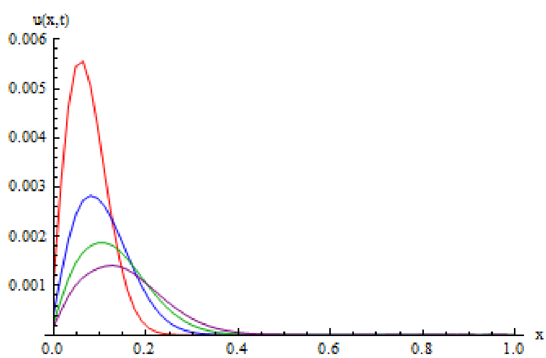

Figure 5: Behaviour of numerical solutions for modified Burgers' equation (example 1) when $\nu = 0.001$ and $\Delta t = 0.001$ at times $t = 2,\ 4,\ 6$ and 8.

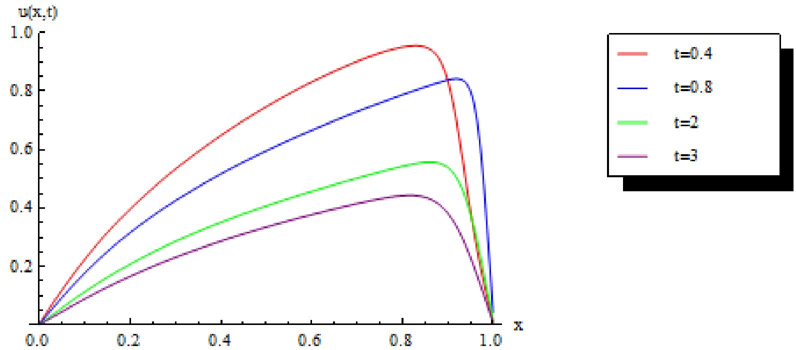

Figure 6: Behaviour of numerical solutions for modified Burgers' equation (example 2) when $\nu = 0.01$ and $\Delta t = 0.001$ at times t=0.4, 0.8, 2 and 3.

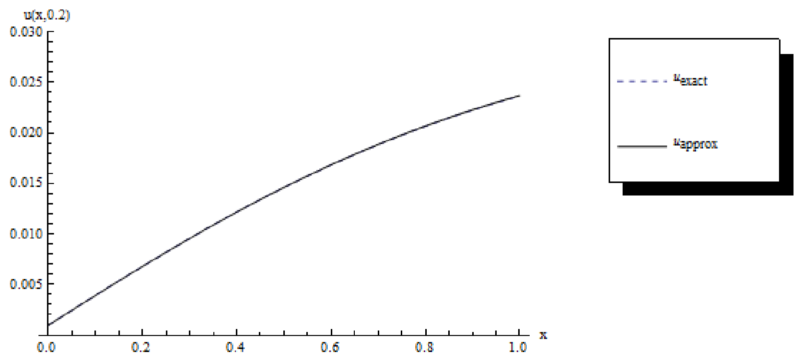

Figure 7: Comparison of Numerical solution and exact solution of modified KdV equation when $t = 0.2$ and $r = -0.001$.

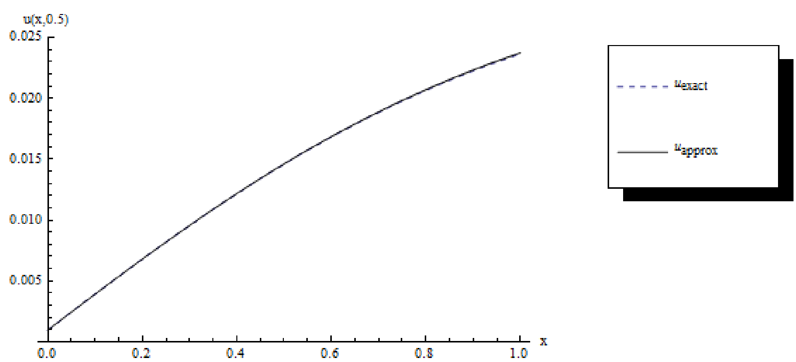

Figure 8: Comparison of Numerical solution and exact solution of modified KdV equation when $t = 0.5$ and $r = -0.001$.

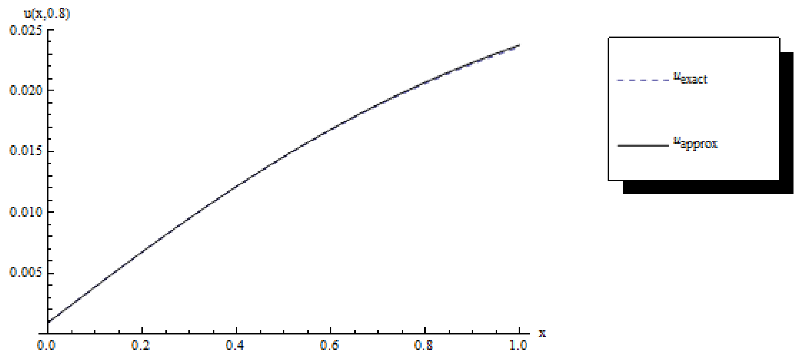

Figure 9: Comparison of Numerical solution and exact solution of modified KdV equation when $t = 0.8$ and $r = -0.001$.

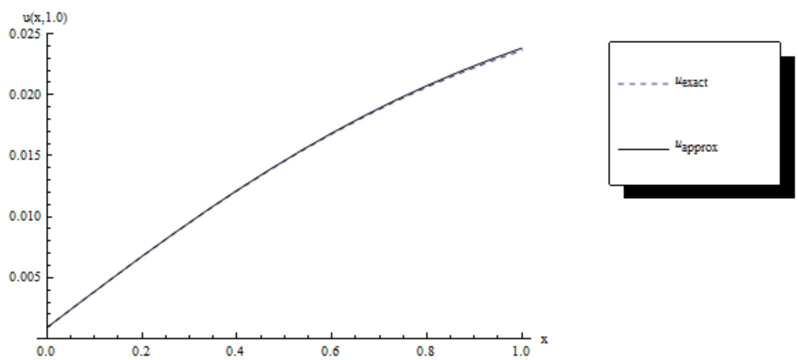

Figure 10: Comparison of Numerical solution and exact solution of modified KdV equation when $t = 1.0$ and $r = -0.001$.

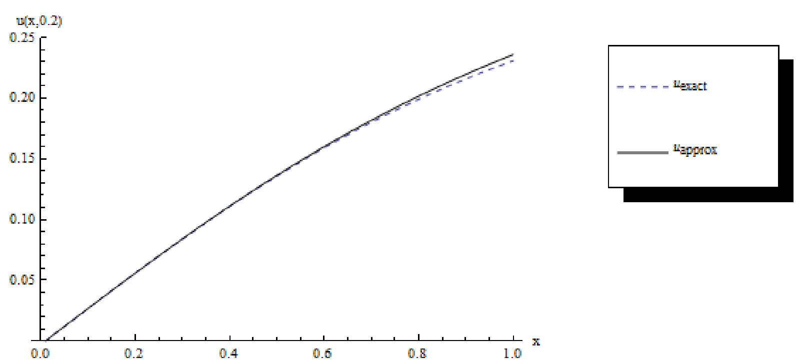

Figure 11: Comparison of Numerical solution and exact solution of modified KdV equation when $t = 0.2$ and $r = -0.1$.

9 Conclusions

In this paper, the modified KdV equation and modified Burgers' equation have been solved by Haar wavelet method. The results thus found are then compared with the exact solutions as well as solutions available in open literature. These have been reported in tables and also have been shown in the graphs. These results demonstrated in Tables justify the accuracy and efficiency of the proposed schemes based on Haar wavelet. The numerical schemes are reliable and convenient for solving modified KdV and modified Burgers' equations. The main advantages of the scheme are its simplicity and applicability. Also it has less computational errors. Moreover, the errors may be reduced significantly if we increase level of resolution which prompts more number of collocation points.

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