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Improved Failure Mode Identification and Reliability Estimates for Electricity Transmission Towers

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Abstract: Studies on the theory of structural system reliability includes identification of main the failure modes and calculation of inclusive failure probabilities for the structural system. The efficient and accurate identification of failure modes in structural systems is difficult and represents a key focus for research in system reliability. The fundamental theory of the branch and bound algorithm for stage critical strength is reviewed in this paper. Some deficiencies in this method are highlighted. Corresponding approaches to overcome these deficiencies are proposed. Calculated system reliability solutions to the classical model, a truss with 10 elements, indicate that the improvement measures proposed in this paper increase the efficiency of recognising the main failure modes of the structural system, and are readily validated. The outcomes of this type of benchmark analysis suggest that the proposed methodology may be capable of representing a suitable basis for the structural system reliability analysis of complex truss-like structures, including transmission towers. Using the proposed approach, the principal failure modes and system reliability of a transmission tower are calculated. Based on practical engineering considerations, effective methods to improve this structural system reliability are proposed.

Keywords: Failure modes; Branch and bound algorithm of stage critical strength; Structural system reliability; Transmission towers.

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1 Introduction

The methodology for the design of transmission towers is based on reliability theory. The adoption of this methodology is driven by the uncertain factors associated with this type of structure [Li, H.; Bai, H. (2007)]. At the same time transmission towers are highly redundant structural systems, meaning that the failure of one element may not necessarily result in the failure of the entire structural system. This may only occur when a series of structural elements become successively fail, resulting in the resistance capacity of whole structure being lost. It is at this point that the structural system may be defined to have failed [Thoft-Christensen; P. Murotsu, Y. (1986)]. It is, therefore, necessary to study the reliability of a structural system when calculating the safety of transmission towers.

Studies on the theory of structural system reliability include two aspects: identification of the principal failure modes and calculation of the global (comprehensive) failure probability. The development of first order and second moment reliability analyses and the Ditlevsen narrow boundary algorithms, form the basis of the practical application of solution methodologies for the assessment of the global failure probability structural systems had its achieved solution in practical application [Rackwitz, R.; Flessler, B. (1978); Ditlevsen O. (1979)]. However, the identification of the failure modes of structural systems efficiently and accurately remains a challenge [Cong D.; Qing-Xiong Y. (1993)]. In the structural system that defines a transmission tower, it is the particular combination of the failed elements within the structural system that leads to of the complete structure collapse, with a corresponding and specific failure mode [Thoft-Christensen; P. Murotsu, Y. (1986)]. It is the failure of different elements and the diversity of the failure sequence creating several different potential failure modes, coupled with the large number of individual structural elements in a transmission tower structure, which results in a large number of alternative failure modes. It is this combination of factors and characteristics that make it difficult to identify all the possible failure modes and to calculate the corresponding failure probability. The system reliability of transmission tower depends on several main failure modes. Clearly, increasing the reliability of the main failure modes can enhance the system reliability of transmission towers. Identification of the main failure modes is, therefore, a key objective in establishing the system reliability of these types of structures. In recent decades, the theory and methods to identify failure modes of structural systems have made great progress with the development of finite element-based methods, and probability network theory. In particular the study of system reliability for mechanical and electronics systems has been adopted in engineering practice [Lu H.; Zhang Y.; Zhao C. (2012); Kuo W.; Prasad V.R.; Tillman F.A. (2011)]. In civil engineering, however, there is often a greater difficulty to efficiently identify the significant structural failure modes due to complexity of environmental loads and structural failure modes.

Methods to identify the failure mode of a structural system under certain performance criteria may be divided into two categories: the ultimate state identification method, and the probability estimate identification method. The first category identifies and selects an element of the main failure mode on the basis of the severity of force effect [Moses F. (1982); Feng Y.S.; Cong D. (1991); Cong D.; Feng Y.S. (1991)], including the maximum criterion of the general ratio between bearing capacity and force, the optimization criterion, the minimum criterion of incremental load approach, and the branch and bound algorithm of critical strength. The second category identifies the main failure mode on the basis of the failure probability of an element [Thoft-Christensen; P. Murotsu, Y. (1986); Murotsu Y.; Okada H.; Taguchi K. (1984); Melchers R. E.; Tang L.K. (1984); Thoft-Christensen P.; Sørensen J.D. (1982)]. Representative methods include the branch and bound algorithm, the truncated enumeration method, β-unzipping method.

Whether an algorithm is successful or not in identifying the main failure mode of structural system, the key lies in building a rational and efficient bound criterion and its associated solution method. In this paper the fundamental principles of the stage critical strength branch and bound algorithm, belonging to the set of ultimate state identification methods, are analysed. Some deficiencies in this method are identified, and corresponding improvement approaches for these deficiencies are proposed. The proposed improved algorithm, which also considers the influence of the variability of the load and the resistance to the bounded parameters, is shown to provide a theoretical basis for establishing reasonable values for the bounded parameters. It can also readily remove redundant and secondary failure modes with larger reliability indices, and improve the efficiency of identifying failure modes without omitting the main failure mode of structural system. A calculation procedure to identify the primary failure modes and the system reliability for complex redundant structures is described, and demonstrated through the analysis of a transmission tower structural system. Calculations of the main failure modes and system reliability of a realistic, practically engineered, transmission tower subjected to wind loading acting at 60° to its primary geometric axes are presented.

The paper is structured to begin with a description of the stage critical strength branch and bound algorithm. A set of improvements to this solution method is proposed and demonstrated using a double layer 10-bar truss as an example structure in Section 2. The developed methodology is subsequently applied to a complex transmission tower and described in Section 3. The outcomes of this analysis and conclusions are presented in Sections 4 & 5, respectively.

2 Fundamental principles of the stage critical strength branch and bound algorithm and proposed improvements

2.1 The fundamental principles of identifying structural failure modes using the stage critical strength branch and bound algorithm

There are many failure modes in structural systems, depending on the configuration of the system shapes and the materials of the member, the loading condition, etc. In order to perform the reliability assessment of the systems, the failure modes and their limit state functions need to be identified. The limit state functions are evaluated at each stage using a deterministic matrix method at the mean values of the basic variables and the uncertainty of the basic variables, to identify the failure paths and hence calculate the system reliability [Thoft-Christensen; P. Murotsu, Y. (1986)].

In the stage critical strength branch and bound algorithm candidate failure elements in every stage are selected, based on the actual stress state of structural member and the criteria for minimizing the critical strength of structural system at the current stage. Because this method considers the failure state and the failure evolutionary process of entire structural system, the branches of failure tree can be effectively controlled in each failure stage and the identification efficiency of the main failure modes can be improved fundamentally [Cong. D. (2001)].

For example, consider a structural system consisting of n elements. In the first stage of identifying the failure mode, the stress σ_j of any element j $(j=1,2,\cdots,n)$ under the external load is:

$$\sigma_j = a_j^{(1)}\Delta F_j^{(1)} \tag{1}$$

where, the coefficient $a_j^{(1)}$ is the stress in element j of the structural system subjected to a external load case; $\Delta F_j^{(1)}$ is the load increment factor for element j in the first stage. When element j reaches a critical state of failure, the stress of this element is equivalent to its strength, i.e.

$$\sigma_j^{\mathrm{cr}} = a_j^{(1)}\Delta F_j^{(1)} = R_j \tag{2}$$

where, R_j is the strength of element j, which may be different for tension and compression members. The critical load increment factor of element j in the first stage is obtained as:

$$\Delta F_j^{(1)} = R_j/a_j^{(1)} \tag{3}$$

The stage critical strength corresponding to the failure of element j is $R_{s(j)}^{(1)} = \Delta F_j^{(1)}$, and the minimum critical strength of all elements in the first stage is then $R_{s(\min)}^{(1)} =$

$\min[R_{s(j)}^{(1)}]$. When the bounding parameter c_k $(1 \le c_k < \infty)$ is defined, the elements that satisfy,

$$R_{s(j)}^{(1)} \le c_k R_{s(\min)}^{(1)} \tag{4}$$

are transferred into the set of failed elements in this stage.

When a total of $k-1$ elements $r_1, r_2, \cdots, r_{k-1}$ have failed one after another, the corresponding load increment factors are respectively recorded as $\Delta F_{r_1}^{(1)}, \Delta F_{r_2}^{(2)}, \cdots,$ $\Delta F_{r_{k-1}}^{(k-1)}$. In stage k of the failure progress, the residual resistance of the failed element i is applied to the nodes of corresponding element in accordance with the stress condition of the failed element. The structural stress state is then reanalysed to take into account this redistribution. The stress σ_{r_k} of non-failed elements $r_k\,[r_k \in (1,2,\cdots,n), r_k \notin (r_1, r_1, \cdots, r_{k-1})]$ can then be obtained as:

$$\sigma_{r_k} = a_{r_k}^{(k)} \Delta F_{r_k}^{(k)} + \sum_{i=1}^{k-1} a_{r_i}^{(i)} \Delta F_{r_i}^{(i)} m_{r_i} \tag{5}$$

where, the coefficient $a_{r_i}^{(i)}$ is the stress of element r_i of structural system that is composed of $n+1-i$ non-failed elements given the element generalized load corresponding to the external load acting in stage i. m_{r_i} is the element material parameter. It reflects the reduction level in the bearing capacity after the element has failed. $F_{r_i}^{(i)}$ is load increment factor of element r_i in stage i. When element r_k reaches its critical state of failure, the stress of this element is equivalent to its R_{r_k}. Meanwhile, the sign function $I_{r_k} = \text{sign}[a_{r_k}^{(k)}]$ is introduced to enable the nature of internal force (tension or compression) in structure to be changed during the load process (arising from a change in the structural system as elements fail), such that,

$$\Delta F_{r_k}^{(k)} = \frac{R_{r_k} - I_{r_k} \times \sum_{i=1}^{k-1} a_{r_i}^{(i)} \Delta F_{r_i}^{(i)} m_{r_i}}{a_{r_k}^{(k)}} \tag{6}$$

In stage k of the failure process, when element r_k is failure along the failure path $r_1 \to r_2 \to \cdots \to r_k$, the stage critical strength $R_{s(r_k)}^{(k)}$ of the structural system is:

$$R_{s(r_k)}^{(k)} = \Delta F_{r_k}^{(k)} + \sum_{i=1}^{k-1} \Delta F_{r_i}^{(i)} m_{r_i} \tag{7}$$

In this stage the minimum critical stage of structural system is $R_{s(\min)}^{(k)} = \min[R_{s(r_k)}^{(k)}]$. Elements that satisfy,

$$R_{s(r_k)}^{(k)} \le c_k R_{s(\min)}^{(k)} \tag{8}$$

are moved into the set of failure elements in stage k of failure process.

Careful selection of bounding parameter c_k not only ensures that the main failure mode of structural system cannot be omitted, but also that the computational effort is reduced. A reasonable range for bounding parameter c_k based its physical significance is suggested to be [1, 2] [Cong. D. (2001)]. From the perspective of engineering application, the meaning of the bounding parameter c_k is somewhat similar to the safety index or partial factor if the correlation between different failure modes is ignored. In general, the obvious correlation between the different failure modes exists. Numerical studies show that the failure modes in which critical strength of system is more than 1.2 times $R^{(k)}_{s(\min)}$, contribute little to failure probability of system [Cong. D. (2001)]. As a result, the bounding parameter c_k is generally adopted to be around 1.2 for practical engineering cases.

2.2 Existing problems of and proposed improvements to the stage critical strength branch and bound algorithm

The following three principal deficiencies exist in the stage critical strength branch and bound algorithm when applied to fundamental engineering problems:

(1) The value of the bounding parameter is normally selected based on practical experience, and it is unable to reflect the influence of the statistical distributions of load and resistance and of the coefficients of variation on the structural reliability.

(2) The redundant failure modes are not removed sufficiently quickly within the analysis.

(3) The minimum critical strength $R^{(k)}_{s(\min)}$ of system in one stage is determined on the basis of the critical strength $R^{(k)}_{s(r_k)}$ in the stage in which the element r_k has failed. The information in the minimum and integrated failure path, which has been formed by the preceding failure elements, cannot be utilized because of a missing relevant feedback mechanism.

2.2.1 Improvement 1: Selection of the bounding parameter

The bounding parameter in the stage critical strength branch and bound algorithm significantly influences the efficiency of identifying the main failure mode. If the value is too small, the main failure modes will be missed. Conversely, the more secondary ones will be eliminated from the analysis [Guoming J.; Bifeng S. (2002)]. At the same time, because the identification of each failure element requires a finite element analysis for the whole structure, the computational efficiency is reduced by overlarge bounding parameters. The relationship between the bounding parameters c_k and the bound of reliability index β is found in this paper by utilizing the central safety factor K. It provides a theoretical foundation for the rational selection of the bounding parameter.

The failure of an element is identified according to its load bearing capacity in relation to the applied load. The bounding parameter is a quantitative index of an element's load bearing capacity, as shown in,

$$c_k = \frac{\mu_R}{\mu_{\min}} \tag{9}$$

The mean value of loads acting on a structure is defined for a given condition, meaning that (9) can be equivalently converted to,

$$c_k = \frac{\mu_R}{\mu_{\min}} = \frac{\mu_R}{\mu_S}\frac{\mu_S}{\mu_{\min}} = \frac{K}{K_{\min}} \tag{10}$$

in which the transformational relation between the bounded parameters c_k and the central safety factor K is also specified. μ_R is the limited value of critical strength satisfied with the bounded norm in structural system. $\mu_{\min}$ is the mean value of minimum critical strength in structural system. μ_S is the mean value of load effect, and μ_S is the fixed value when the load case is determined. K is the central safety factor for every failure path satisfied with the bounded norm in stage k, and $K_{\min}$ is the minimum central safety factor for all failure paths in stage k.

Suppose that the structural performance function is $Z = R - S$. When the probability functions and the coefficients of variation of resistance and loads are known, there is a one-to-one correspondence between the central safety factor K and the reliability index β [Yanghai L.; Weigang B.; Xiuwu G. (1997)]. If the resistance R and load effect S are normally distributed, then the following relationship holds;

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{11}$$

Introducing the central safety factor K, equation (11) can be written as

$$\beta = \frac{K - 1}{\sqrt{K^2\delta_R^2 + \delta_S^2}} \tag{12}$$

or,

$$K = \frac{1 + \beta\sqrt{\delta_R^2 + \delta_S^2 - \beta^2\delta_R^2\delta_S^2}}{1 - \beta^2\delta_R^2} \tag{13}$$

where, δ_R, δ_S are the coefficients of variation of resistance R and load effect S, respectively. Other probability functions can be transformed to equivalent normal distribution at the point of interest using the Rackwitz and Fiessler transformation. The value of the reliability index is limited to the range $[\beta_{\min}, 1.29 + 0.86\beta_{\min}]$ on the basis of the globe β unzipping method [Cong. D. (2001)]. $\beta_{\min}$ is the reliability index corresponding to the failure path in which the mean value of the critical strength is the smallest. The bounded parameter c_k can be obtained by substituting (13) into (10). As a result of considering the probability distribution and the coefficients of variation for the resistance and load terms when determining the bounded parameters c_k, the value of c_k is more reasonable, being linked directly to the variability of the input parameters and the structure safety index.

2.2.2 Improvement 2: Removing redundant failure modes

Many failing elements are included in a failure mode. If some failed elements are removed and the remaining failing elements continue to constitute a failure mode, then these removed failing elements are termed "redundant failing elements" and the failure mode containing the remaining failed elements is referred to as the "redundant failure mode". The contribution of the redundant failure mode to the structural failure probability may be ignored [Cong. D. (2001); Andrzej S.; Nowak, T.C. (2007)] and the redundant failure mode should be removed. Ji Guoming *et al.* selected the main failure mode using the minimal cut set method and enhanced the identifying efficiency in the global branch and bound algorithm [Guoming J.; Bifeng S. (2002)]. This improvement strategy is adopted here and applied to the critical strength branch and bound algorithm. The set composed of the sequence of failure elements corresponding to any failure path is called the "cue set". The set containing the smallest number of failed elements that can lead to failure of the whole structure is called the "minimum cue set". In identifying the main failure mode only the minimum cue set of the failure path needs to be stored. For example, if there are three failure modes corresponding to the failure path ①-②-③, ①-③-② and ②-③ just the minimum cue set ②-③ is stored. Removing the redundant failure modes may simplify the fault tree and reduce the workload.

2.2.3 Improvement 3: Defining the bounding criterion

It is a very common phenomenon that redundant failure modes are repeated during the identification of the failure modes. This phenomenon is more obviously observed with an increasing number of failure elements [Andrzej S.; Nowak, T.C. (2007)]. In the absence of a relevant feedback mechanism and a feedback loop, information contained in the minimum and integrated failure paths identified by preceding failure elements cannot been utilized. To overcome this drawback, two coefficients, the global bounding parameters c_s and the minimum boundary value R_S^* of critical strength of system, are introduced. By including these additional terms the objective of the global bound and branch may be correctly defined. The implementation of these feedback mechanisms, however, is slightly complicated because the parameters c_s and R_S^* need to be introduced and repeatedly amended after identifying every complete (integrated) failure path [Cong. D. (2001)]. However, it may be noted that the critical strengths corresponding to the redundancy failure modes belonging to different failure paths, are approximately equal. In the course of identifying the failure modes of a structure using the stage critical strength branch and bound algorithm, the information in the minimum and integrated failure path, based on the preceding failure path, may directly feed back into the redundant failure mode in which the preceding failure elements are different, and contribute to the selection of the minimum critical strength. A redundant failure mode is removed after completing the selection of the minimum critical strength in a stage. This improvement to the algorithm skips the intermediate steps of the global boundary approach and simplifies the implementation steps. The efficiency of identifying main failure modes is thereby increased.

2.2.4 Schematic of structural system failure mode identification algorithm

The improved approaches to the stage critical strength branch and bound algorithm proposed in this paper have been written into the modelling capabilities of ANSYS using the APDL parametric design language and a MATLAB script. The main flow chart to identify the failure modes for structural system is shown in fig.1.

2.3 Calculation and analysis for the main failure modes of 10 elements truss

The main failure mode of a double layer truss with 10 elements is firstly analyzed as a classical example shown in fig.2. Elements ①-⑩ are made of elastic-plastic material, and the failure forms are tension (or compression). The mean value of the load F is 2kN. The strength R of these elements and the load S are assumed to be normally distributed. The mean of the compressive strength and the tension stress of elements ①-⑥ are -10kN and 20kN, respectively, and the corresponding value of elements ⑥-⑩ are -2.5kN and 5kN, respectively. The coefficient of variation for the resistance is $\delta_R = 0.1$ and that of the load, δ_s, is 0.2.

Assuming bounding parameters $c_1 = 1.0$, $c_2 = 2.0$, $c_3 = 2.0$, the failure modes of the double layer 10 element truss is shown in fig.3 (a). These may be compared with the structural failure modes when the bounding parameters are assumed to be more constrained with $c_1 = 1.0$, $c_2 = 1.2$, $c_3 = 1.2$, as shown in fig. 3(b). From fig. 3(a) and (b) it can be observed that a greater number of secondary failure modes will be displayed if the value of the bounding parameters is high. The failure modes ⑧-④-⑤, ⑧-④-⑨, ⑧-④-③ and ⑧-⑩-③ with higher stage critical strengths are secondary failure modes. The potential impacts of the choice of bounding parameters on

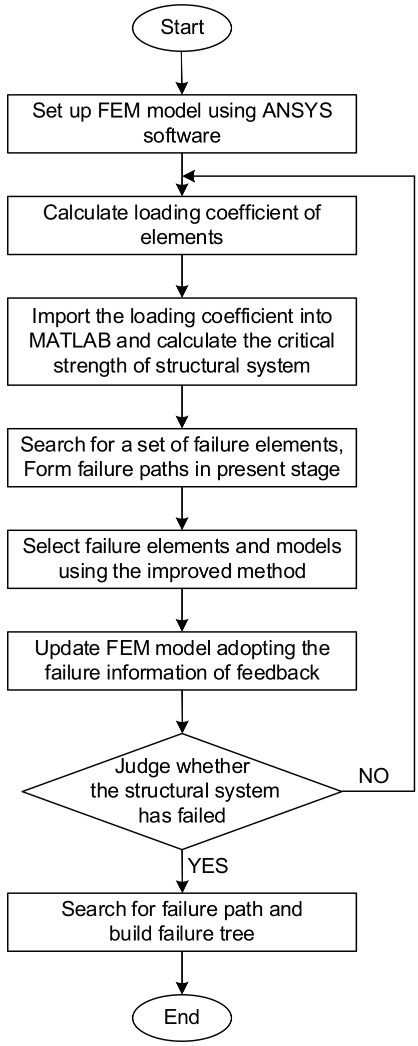


Figure 1: Flow chart of identifying failure modes for structural system

computational efficiency have been noted previously.

Element ⑧ is the first element to fail in stage one of failure process. Based on the calculations of the central safety factor $K_{\min} = 5/2.64 = 1.894$ in the first stage, $\beta_{\min}$ can be calculated from equation (12). The mid-value of β in the range $[\beta_{\min}, 1.29 + 0.86\beta_{\min}]$ can be expressed and it may be approximately regarded as mean of β value in this range. We obtain the central safety factor from equation (13) and the bounded parameters c_k from equation (10). Using the revised estimate for the bounding parameter c_1 outlined in §2.2.1, the influence of the uncertain-

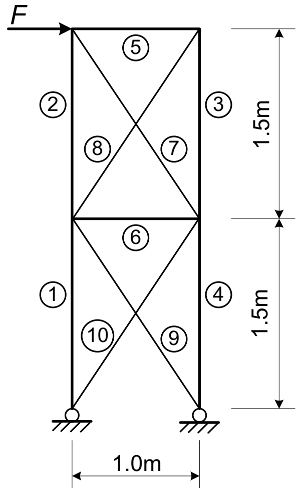
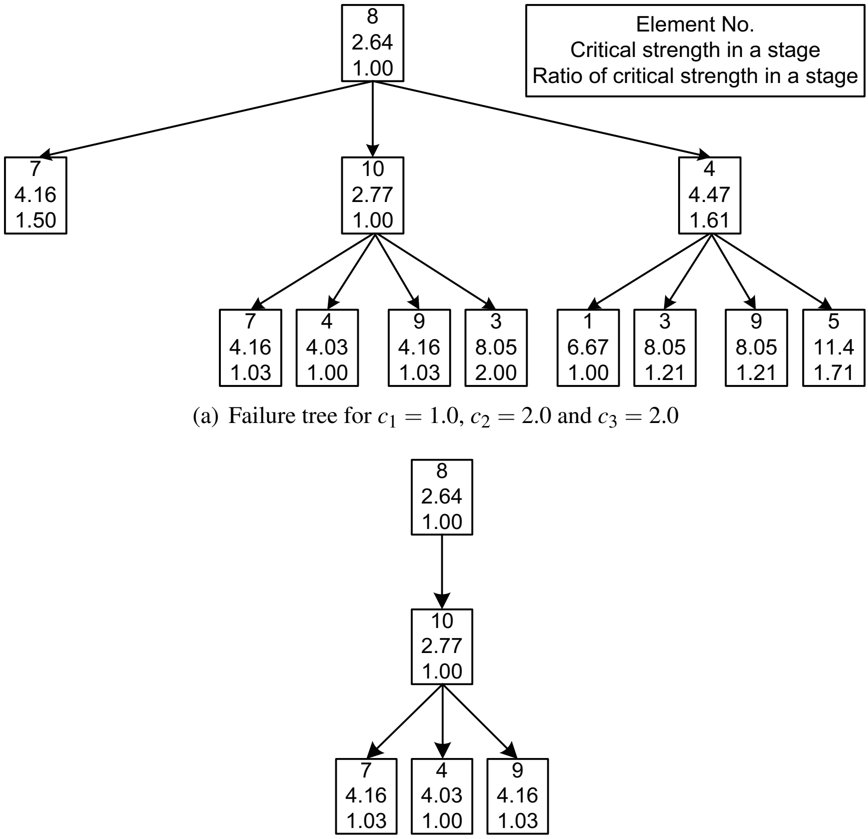


Figure 2: Example double layer truss with 10 elements

ty in the resistance of load effect, δ_R and δ_S, can be quantified as shown in fig.4. Based on this approach, reasoned values of the bounding parameter are significantly dependent on the variable coefficients δ_R and δ_S. When δ_R and δ_S are small, the bounding parameter c_1 is close to 1.2, which is value that conforms to the those based on experience of using the algorithm [Cong. D. (2001)]. When δ_R and δ_S are large, the bounded parameter c_1 may approach a value of 1.95. Calculated values for the bounding parameter c_k are suggested to be in the range [1, 2] [Feng Y.S.; Cong D. (1991)], supporting the present work. The results of the example show that the parameter c_k is close to 1.2 when the coefficients δ_R and δ_S are both small; otherwise, the parameter c_k is greater than 1.2 to ensure the main failure modes are not missed.

Assuming bounding parameter values $c_1 = 1.0$, $c_2 = 2.0$ and $c_3 = 1.5$, the predicted main failure modes for the truss with 10 elements are shown in Fig.5. Fig.5 (a) illustrates the failure tree generated by the unimproved branch and stage critical strength bound algorithm. Those in Fig.5 (b), (c) and (d) are the failure trees generated by using improvement 2, improvement 3 separately and the combination of improvements 2 and 3. Meanwhile, the mean of the critical strength and the reliability index for these failure modes are listed in table 1, and validated by Cong D. (2001). The main failure modes of this truss are ⑧-⑦, ⑧-⑩-④ and ⑧-⑩-⑨. It can be observed from Fig.5 and table 1 that using the proposed improvements the revised branch and bound algorithm demonstrates better performance, can readily remove redundant failure and secondary failure modes in the presence of a large reliability index, and improve the recognition efficiency without omitting the main failure modes of structural system.



(a) Failure tree for $c_1 = 1.0$, $c_2 = 2.0$ and $c_3 = 2.0$

(b) Failure tree for $c_1 = 1.0$, $c_2 = 1.2$ and $c_3 = 1.2$

Figure 3: Qualitative influence of the bounding parameter on failure mode

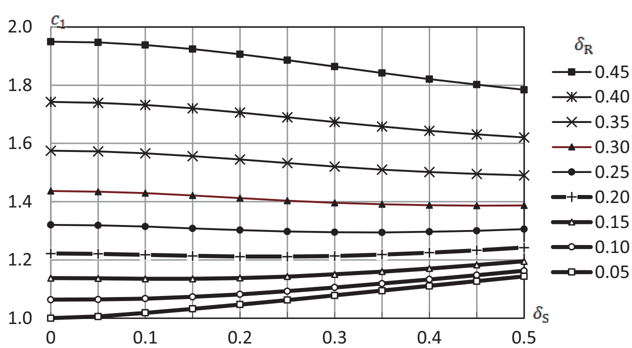
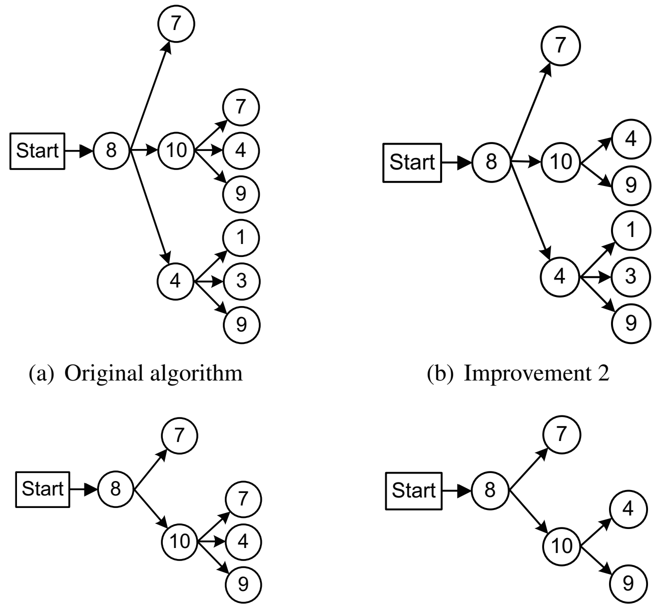


Figure 4: The change curve of the bounded parameter c_1 with δ_R and δ_S



(a) Original algorithm (b) Improvement 2

(c) Improvement 3 (d) Improvements 2 and 3

Figure 5: The failure tree generated by using different algorithm

Table 1: The failure modes of a truss with 10 elements and the relevant reliability index

Failure mode	The order of failure elements	Mean critical trength (kN)	Reliability of index
m	$i-j-k$	$E(R_s)$	β
1	8–7	4.1602	4.2683
2	8–10–7	4.1602	4.2683
3	8–10–4	4.0267	3.8583
4	8–10–9	4.1602	4.2683
5	8–4–1	6.6667	7.3175
6	8–4–3	8.0534	7.6652
7	8–4–9	8.0534	7.6652

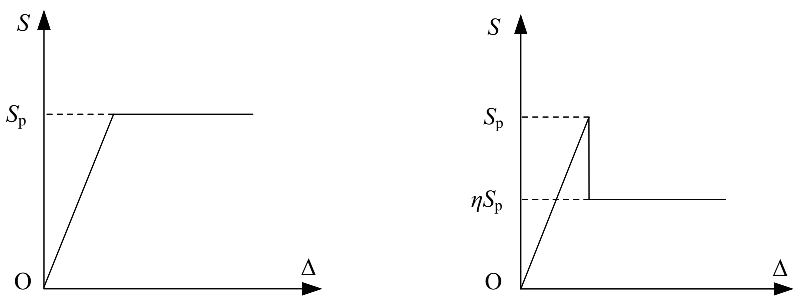
3 Structural system reliability assessment of a transmission tower

3.1 Failure criterion and statistical characteristics of the basic variables

3.1.1 Structural element failure models

The appropriate choice of failure model for the structural elements is the basis on which to establish a comparatively rational failure criterion for the structural system. The primary elements in the structural system of a transmission tower include tie rods and struts that have well known failure modes related to material failure (yielding and rupture) and instability (buckling either locally or globally within a structural element), respectively. In general, the stress-strain curve of an element after failure may be represented as a plateau or a sudden drop to zero depending on the stress state of element and associated failure mode. These two representations of the stress-strain approximations, when applied to strut (representing local or global buckling failure respectively), may underestimate or overestimate the capacity of the structural element upon onset of failure and lead to a deviation from the true failure path. The connections of steel structure clearly influence the reliability of the structural system. However, the stresses in the gusset plates at the connections are very complex, meaning that the connection details are normally strengthened. In order to simplify the reliability calculation, connection is assumed to be unlikely. In endeavouring to simulate the ultimate bearing capacity of a structural system, the relatively rational failure model of an element is to adopt an elastic-plastic stress-strain relationship for all struts and ties. In the case of tension members it is presumed that the stress is maintained at yield strength after the onset of first yield. For compression members, once the yield point has been reached a so-called "half elastic-plastic failure model", where it is supposed that the element remains at a certain reduced capacity after the loss of stability (denoted by the "yield point") and the axial force exceeding the capacity of this element is redistributed to remaining valid (connected and not failed) elements. The relationships between strength or bearing capacity and displacement of these two types of failure modes, which are frequently used in tension and compression elements, are respectively given in Fig.6 (S_p is the bearing capacity of element and η is the reduction factor applied to the bearing capacity when the compression element suffers instability in Fig.6).

The value of the reduction factor of bearing capacity η is mainly related to the slenderness ratio λ of compression element. Given that the slenderness ratio λ of compression element of transmission towers is normally within the range of 75–150, and following the onset of buckling instability, a value for the reduction factor η is generally assumed to be in the range 0.50–0.35 [Deng H.Z.; Wang Z.M. (2000)]. In creating the numerical model of the structural system, the value of η for individual elements of differing slenderness ratios may be determined in range of 0.50–0.35 using linear interpolation based on the slenderness ratio λ. Therefore, $\eta = 1.0$ for an idealised elastic-plastic failure model in tension. For half elastic-plastic failure model in compression, $\eta = 0.5$ when the slenderness ratio $\lambda \leq 75$, $\eta = 0.35$ when $\lambda = 150$. When λ is between two values, η can be calculated by linear interpolation, and when $\lambda > 150$, it is assumed that the element losses all capacity at the onset of instability and that $\eta = 0$.



(a) Elastic-plastic failure model in tension (b) Half elastic-plastic failure model in compression

Figure 6: Failure models of transmission tower elements

3.1.2 Structural system failure criteria

The structural system of a transmission tower is statically indeterminate. The complete structure (the structural system) may not be described to have failed until the failure of a sufficient number of elements has occurred to induce a complete system failure mode. These modes may be in the form of a mechanism resulting in collapse (zero stiffness; an ultimate limit state) or excessive deformation (a type of serviceability limit state). The failure criteria for structural system of transmission tower may be summarised as [Cong. D. (2001)]:

(1) The formation of a structural mechanism;

(2) The bearing capacity of the structural systems reaches a maximum, or its load bearing capacity reduces for the first time;

In this paper the preceding criteria have been applied in identifying failure of a transmission tower as a structural system.

3.1.3 Statistical characteristics of variables

In the calculation of the system reliability of a transmission tower, the material strength, basic wind pressure, and dead load have been assumed to be random variables. The statistical characteristics of these random variables are given in table 2.

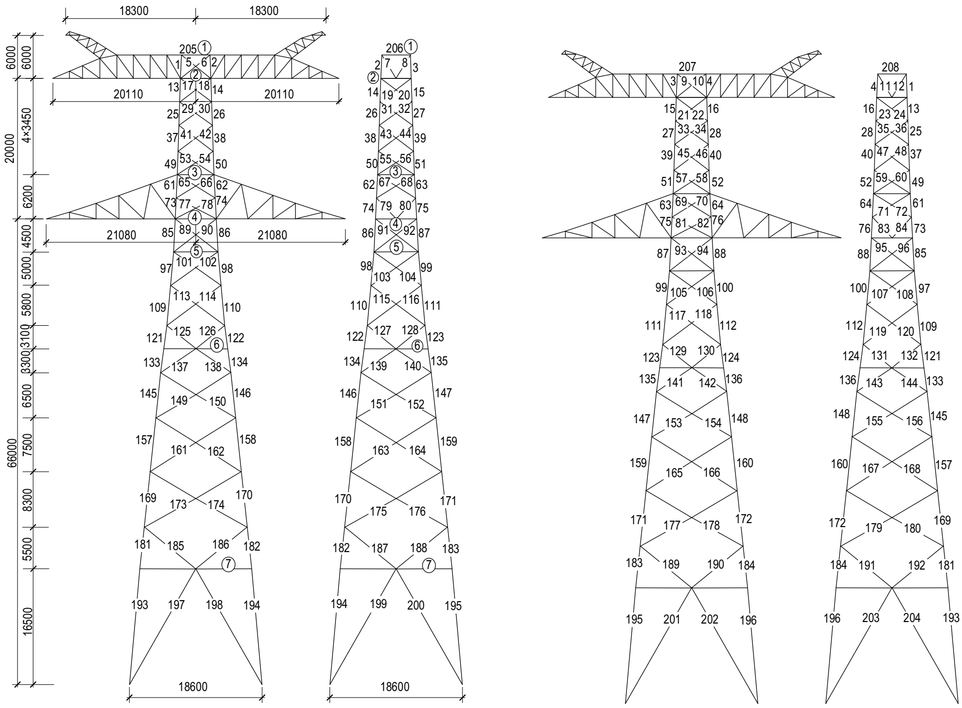
Table 2: Statistical characteristics of random variables

Variable name		$\frac{\text{mean value}}{\text{standard value}}\mu$	coefficient of variation δ	Probabilistic distribution type
Dead load		1.06	0.070	Normal distribution
Wind load		0.908	0.193	Extremum type I distribution
Steel strengths Q420/Q345/Q235	compression	1.121/1.060/1.104	0.108/0.109/0.117	Lognormal distribution
	tension	1.142/1.080/1.126	0.108/0.109/0.117	

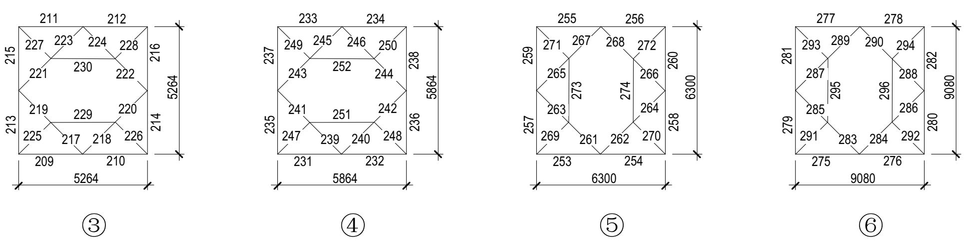
3.2 Failure mode analysis of a suspension straight towers of ± 800kV double circuit

The computational model described in this paper has been applied to the structural system reliability analysis of a ±800kV double circuit suspension straight tower (tangent tower). This is a type of self-supporting tower that stands along straight sections of a transmission route, and its only function is to suspend the wires and not create or regulate tension. The tower's height is 66m (see Fig.7). The mid-span length l_h of this tower, the distance between two mid-points on either side of the tower span, required for calculating the horizontal forces acting on the tower, is 510m. The maximum sag span length l_v, is defined as the horizontal distance between the two sag points of on either side of the tower, used to calculate the vertical forces acting on the tower, is 650m (see Fig.8). Steel Grades Q420, Q345 and Q235, have been assumed for the structural elements of the transmission tower, and material specifications of the tower are listed in the table 3. The basic design climate conditions (assumed for north-western China) are: the recurrence intervals of the basic wind speeds and ice thicknesses for a suspension straight tower are 100 years; the maximum mean wind speed is 27m/s and the benchmark height for measuring the wind speed is 10 metres above ground level; the mean thickness of covering ice is 10mm. Icing of transmission lines is not the controlling case for transmission tower structures located in arid and semi-arid regions of north-western China, where the climate is predominantly dry with extremely low precipitation. In this paper, the case of combined wind and gravity (ice) loads is considered to identify the main failure mode for the more general scenarios. The typical wind direction is assumed to be at 60° degree to the primary axes of the transmission tower. (a) The elevation of the transmission tower (four sides)

For a transmission tower comprising a high number of elements, the number of possible failure paths is huge resulting in an equivalent and infeasible number of non-linear simulations. The primary failure modes high correlativity and lower ultimate bearing capacities have been selected in this paper. The main failure modes of this suspension straight tower under 60° wind load are illustrated in Table 4.



(a) The elevation of the transmission tower (four sides)



(b) Horizontal diaphragm

Figure 7: The simplification model of the transmission tower

The computer simulations demonstrate that the process of the first 30 dominant failure modes just takes CPU (DELL OptiPlex 320 computer environment) 46s.

3.3 Computation of the structural system reliability

3.3.1 Methodology

Following the principles of identifying failure modes using the modified stage critical strength branch and bound algorithm presented in this paper, the structural calculation of the model is gradually changed in line with the development of the

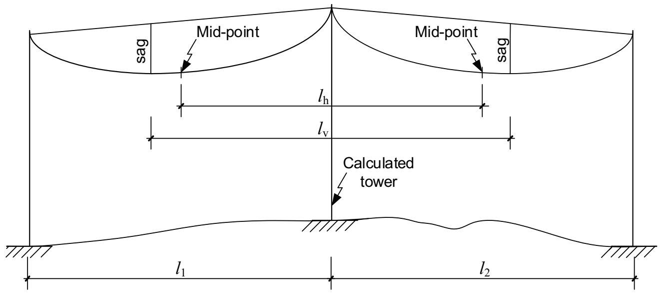


Figure 8: Mid span l_h and maximum sag span l_v of transmission tower

failure path of structural system and the internal force is re-analysed. The failure probability or conditional failure probability of every element is then calculated. The equivalent linear limit state equations and corresponding reliability index of a failure mode (parallel system) can be obtained by using the step-by-step equivalent linear Johnson method [Liu N.; Li T.C. (1994)]. The basic idea of this method is that the number of failed elements in a failure mode is supposed to be n. the equivalent failure boundary for element 1 and 2 can be calculated initially. Aa new equivalent failure boundary for this new state is defined for element 3 can then be calculated, and so on. Finally the equivalent linear failure boundary for the whole failure mode can be obtained. Therefore, the step-by-step equivalent linear Johnson method calculates the equivalent failure boundary of two failure boundaries. For example, suppose two linearized performance function in a parallel system are

$$\left.\begin{aligned} \bar{\boldsymbol{G}}_1(\mathbf{Y}) = \alpha_1^{\mathrm{T}}\mathbf{Y} + \beta_1 = 0 \\ \bar{\boldsymbol{G}}_2(\mathbf{Y}) = \alpha_2^{\mathrm{T}}\mathbf{Y} + \beta_2 = 0 \end{aligned}\right\} \tag{14}$$

and the equivalent linearized performance function is

$$\boldsymbol{G}_E(\mathbf{Y}) = \boldsymbol{\alpha}_{\mathrm{E}}^{\mathrm{T}}\mathbf{Y} + \beta_{\mathrm{E}} \tag{15}$$

where, $\beta_{\mathrm{E}i}$ is the reliability index of equivalent limit state function, $\boldsymbol{\alpha}_{\mathrm{E}}$ is a unit normal vector at the point β_{E} of equivalent performance function.

The equivalent condition is: (1) the reliability β_{E} of the equivalent limit state function equals to the broad reliability $\beta_p = -\Phi^{-1}[P(\bar{\boldsymbol{G}}_1 < 0) \cap P(\bar{\boldsymbol{G}}_2 < 0)]$; (2) the sensitivity of reliability with respect to every random variable is same as before and subsequently

The whole structural system may be seen as a series system composed of several failure modes, with calculated correlation coefficients between the different failure modes. Finally the failure probability of whole structural system is calculated using

Table 3: Elements Corresponding Material Specifications and Steel Grade

Element No.	Material specification	Steel strengths
1–4, 13–16, 25–28, 205–208, 209–212	L140x10B	Q420
5–6, 9–10, 55–56, 59–60, 77–84, 235–238, 275–282	L125x8H	Q345
7–8, 11–12, 213–216, 239–246, 291–294	L90x7H	Q345
17–24, 31–32, 35–36, 43–44, 47–48, 67–68, 71–72, 125–132, 197–204	L125x10H	Q345
29–30, 33–34, 41–42, 45–46, 53–54, 57–58, 253–260, 283–290	L110x8H	Q345
37–40, 49–52	L140x12B	Q420
61–64, 73–76, 85–88	L160x14B	Q420
65–66, 69–70	L100x7H	Q345
89–90, 93–94	L180x14H	Q345
91–92, 95–96, 149–156	L160x10H	Q345
97–100, 109–112	L200x18B	Q420
101–108, 113–120, 137–144	L140x10H	Q345
121–124, 133–136	L200x20B	Q420
145–148	L200x24B	Q420
157–160	2xL180x14B	Q420
161–168, 185–192, 231–234	L160x12H	Q345
169–172, 181–184, 193–196	2xL180x16B	Q420
173–180	L180x12H	Q345
217–224, 269–272	L63x5H	Q345
225–228	L50x4S	Q235
229–230	L45x4S	Q235
247–250	L70x5H	Q345
251–252, 273–274	L80x6H	Q345
261–268, 295–296	L75x5H	Q345

the Ditlevsen narrow bound method [Ditlevsen O. (1979)]. The specific calculation steps are as follows:

(1) Identify failure mode of structural system using the modified stage critical strength branch and bound algorithm, and suppose that the number of a failure mode is m;

(2) Assume the input random variable is $\mathbf{X} = (x_1, x_2, \cdots, x_k)^T$. The number of failure elements in a failure mode is n. Update the structural ANSYS model, making use of the results of the failure path. Calculate the internal forces, and establish

Table 4: Main failure modes of a ±800kV suspension straight tower under 60° wind load

Failure Modes No.	Failure Path
1	49→ 218→ 221→ 268→ 240→ 229→ 54→ 109→ 197→ 87
2	73→ 240→ 262→ 267→ 78→ 102→ 87→ 220→ 221→ 216
3	85→ 90→ 262→ 95→ 267→ 222→ 219→ 240→ 87
4	109→ 286→ 264→ 222→ 219→ 149→ 257→ 267→ 120
5	133→ 268→ 286→ 161→ 222→ 262→ 219→ 246→ 289→ 168
6	197→ 202→ 178

limit state equation of every element in a failure mode.

$$g_j(\mathbf{X}) = 0 \quad (j = 1, 2, \cdots, n) \tag{16}$$

(3) Compute the failure probability or conditional failure probability of every element using the first-order second-moment method; transform the independent random variables X into the standard normal random variables $\mathbf{Y}$ and the limit state equation of the corresponding elements into standard normal space with $\boldsymbol{G}_j(\mathbf{Y}) = 0$. Expand $\boldsymbol{G}_j(\mathbf{Y})$ into a Taylor series at the design point $\mathbf{Y}^*$. Choose the linear term of the Taylor series and obtain the equation of performance function as:

$$\bar{\boldsymbol{G}}_j(\mathbf{Y}) = \alpha_j^{\mathrm{T}}\mathbf{Y} + \beta_j = 0 \quad (j = 1, 2, \cdots, n) \tag{17}$$

where, β_j is the reliability index of failure element j. α_j is a unit normal vector at the point β_j of performance function.

(4) Using the step-by-step equivalent linear Johnson method, obtain the equivalent linear performance function of failure elements in a failure mode:

$$\boldsymbol{G}_{\mathrm{E}i}(\mathbf{Y}) = \boldsymbol{\alpha}_{\mathrm{E}i}^{\mathrm{T}}\mathbf{Y} + \beta_{pi} \quad (i = 1, 2, \cdots, m) \tag{18}$$

where, β_{pi} is the reliability index of failure mode i, $\boldsymbol{\alpha}_{\mathrm{E}i}$ is a unit normal vector at the point β_{pi} of equivalent linear performance function in failure mode i.

(5) Calculate the correlation coefficient between failure modes

$$\rho_{ij} = \boldsymbol{\alpha}_{\mathrm{E}i}^{\mathrm{T}}\boldsymbol{\alpha}_{\mathrm{E}j} \quad (i, j = 1, 2, \cdots, \mathrm{m}) \tag{19}$$

(6) View the failure modes of the whole structure as a series system. Using the Ditlevsen narrow bound method compute the failure probability P_f of structural

system from;

$$P_{f1}+\sum_{i=2}^{m}\max\left(P_{fi}-\sum_{j=1}^{i-1}P_{fi,j},0\right)\leq P_f\leq\sum_{i=1}^{m}P_{fi}-\sum_{i=2}^{m}\max_{j\leq i}\left(P_{fi,j}\right) \tag{20}$$

where, P_{fi} is the failure probability for failure mode i, $P_{fi}=\Phi\left(-\beta_{\mathrm{E}i}\right)$. $P_{fi,j}$ is the joint failure probability for failure mode i and j, $P_{fi,j}=\Phi_2\left(-\beta_{\mathrm{E}i},-\beta_{\mathrm{E}j},\rho_{ij}\right)$. $\Phi_2\left(\cdot\right)$ is a function of a two dimensional standard normal distribution.

3.3.2 Structural system reliability predictions

Under a 60° wind load condition, the equivalent linear performance function and the relevant reliability index corresponding to the six main failure modes illustrated in Table 4 are;

$$\left.\begin{array}{l}G_{\mathrm{E}1}=0.4182Y_{\mathrm{R}}-0.0199Y_{\mathrm{G}}-0.9081Y_{\mathrm{W}}+4.6994\\G_{\mathrm{E}2}=0.3822Y_{\mathrm{R}}-0.0006Y_{\mathrm{G}}-0.9241Y_{\mathrm{W}}+3.8529\\G_{\mathrm{E}3}=0.3759Y_{\mathrm{R}}-0.0052Y_{\mathrm{G}}-0.9266Y_{\mathrm{W}}+3.7951\\G_{\mathrm{E}4}=0.3970Y_{\mathrm{R}}-0.0103Y_{\mathrm{G}}-0.9178Y_{\mathrm{W}}+3.9936\\G_{\mathrm{E}5}=0.4014Y_{\mathrm{R}}-0.0110Y_{\mathrm{G}}-0.9158Y_{\mathrm{W}}+4.4520\\G_{\mathrm{E}6}=0.4162Y_{\mathrm{R}}-0.0229Y_{\mathrm{G}}-0.9090Y_{\mathrm{W}}+3.9555\end{array}\right\} \tag{21}$$

where, Y_{R}, Y_{G}, Y_{W} are the random variables of the structural resistance, dead load and wind load in standard normal space, respectively.

Using these equivalent linear performance functions, (19) and (20) predict the system reliability under the combination of dead load and a 60° wind load to be in the range $3.795\leq\beta\leq3.800$.

4 Discussion

(1) The main failure modes of a suspension straight transmission tower of ±800kV double circuit under dead load and 60° wind load begins with failure of the main element of the tower body, in which, the failure modes No.1, 2, and 3 (see Table 4) are caused by the failure of main element of the tower body near the lower cross arm. This leads to failure of the elements in the horizontal diaphragm ③, ④ and ⑤ (see Fig.7(b)), the near crossed diagonal member, and other major elements of the tower body. In contrast, failure modes No.4 and 5 are mainly generated by failure of the major elements of tower waist, near the crossed diagonal members, and elements of the horizontal diaphragm ⑤ and ⑥ (see Fig.7(b)). Failure mode No.6 is caused by the failure of tower legs and a crossed diagonal member. The main failure modes are mostly distributed in 4 regions: the tower body near the lower cross arm, the tower waist, the tower leg, and the horizontal diaphragm.

There is a large local deformation at the location of the hanging line on the cross arm.

(2) Failure of the tower under wind load chiefly begins with failure of the main elements of the tower body, the elements composing of horizontal diaphragm account for a large proportion of failure elements (see Table 4). The horizontal diaphragm, therefore, is more important to a high transmission tower, and the reasonable design of that can improve the wind resistance of the tower.

(3) The reliability index of a series system mainly depends on the failure mode which has a smaller reliability index. The system reliability index of the studied transmission tower is very close to that of failure mode 3 under a 60° wind load condition (see equation (19)).

(4) The current "*Unified standard for reliability design of building structure* (*GB 50068-2008*)" [GB 50135-2006 (2006)] in China divides the safety level of structural members into level 1, 2 and 3 according to the importance of structure. The target reliability index of a structural member subject to a ductile failure is specified for each level as 3.7, 3.2 and 2.7, respectively. The structural safety level of a ± 800kV transmission tower is set as level 1. The target reliability index of its members should, therefore, be 3.7. The current code, however, does not make any demands on the target reliability index for the structural system. If the specification of the target reliability index for a structure member in current code is to be taken as a guide, the system reliability of this transmission tower under 60 degree wind load can meet the requirements. Eurocode 0 "*Basis of structural design (EN 1990:2002)*" [Eurocode 0 (2002)] also divides the Reliability Consequence (RC) into RC1, RC2 and RC3 associated with the three consequences classes. The recommended minimum reliability index for ultimate limit states associated with different RC is 3.3, 3.8 and 4.3, respectively, for a 50 year reference period. The system reliability of this transmission tower under 60-degree wind load is slightly below the target reliability of RC2.

5 Conclusions

A modified stage critical strength branch and bound algorithm has been proposed and implemented to study the failure modes of a complex skeletal structure in the form of a transmission tower. Furthermore, the structural system reliability of a transmission tower has been computed. The main conclusions of the research presented in this paper are summarized as follows:

(1) A set of three improvements to the critical strength branch and bound algorithm has been proposed. The solution to a test-case example of a 10-bar truss verified these modifications. The improved branch and bound algorithm considers

the influence of the variability of the load and the resistance to the bounding parameters. It has been shown to effectively remove redundant and secondary failure modes with higher reliability indices, and improve the efficiency without omitting the main failure mode of structural system. The test-case formed the basis of extending the application of the method to the analysis of a more complex skeletal structural system in the form of a transmission tower.

(2) A program to efficiently identify main failure modes for structural system of transmission tower has been successfully written with the comprehensive use of MATLAB and ANSYS. The results have been qualitatively validated against similar published work. The speed and accuracy of the program is consistent with the requirements of both scientific research and engineering practice, and can act as a reference.

(3) The numerical simulations of the failure mode of a suspension straight transmission tower of ±800kV double circuit indicate that the most typical failure modes of this type of transmission tower are failures in the tower body near the lower cross arm, the waist of tower, and the tower leg. However, it may be noted that at the location of the suspended transmission line at cross arm, the local deformation is largest under the action of a 60° wind load. In terms of structural design, it is clearly demonstrated and intuitively expected, that if the load capacities of those elements in which the ratio of strength to internal force is relatively low in failure region of the transmission tower are enhanced, the load capacity or the reliability of the whole structural system will be effectively improved, along with the overall structural performance.

(4) In the structural system analysis of a transmission tower, an elastic-plastic failure model may be adopted to represent the behaviour the tension members, and the so-called "half elastic-plastic failure model" may be adopted in the case of compression member. Computational analyses suggest that the proposed failure models may realistically represent the actual stress condition with the elements of a transmission tower.

(5) It is valuable to study the optimization the design of a transmission tower whilst satisfying the system reliability indices, to maximise the performance of the complete structure.

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