

## Texture Segmentation based on Multivariate Generalized Gaussian Mixture Model

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**Abstract:** Texture Analysis is one of the prime considerations for image analysis and processing. Texture segmentation gained lot of importance due to its ready applicability in automation of scene identification and computer vision. Several texture segmentation methods have been developed and analysed with the assumption that the feature vector associated with the texture of the image region is modelled as Gaussian mixture model. Due to the limitations of the Gaussian model being meso kurtic, it may not characterise the texture of all image regions accurately. Hence in this paper, a texture segmentation algorithm is developed and analysed with the assumption that the feature vector of the texture associated with the whole image is characterised by multivariate generalized Gaussian mixture model. The generalized Gaussian mixture model includes several leptokurtic, platykurtic and mesokurtic distributions as particular cases. The model parameters are estimated through EM algorithm. The segmentation algorithm is developed using maximum likelihood under Bayesian framework. The performance of the proposed algorithm is evaluated through segmentation quality metrics and conducting experimentation with a set of 8 sample images taken from Brodatz texture database. A comparative study of the proposed algorithm with that of Gaussian mixture model revealed that the proposed algorithm outstands the existing algorithms.

**Keywords:** Texture; Multivariate generalized gaussian mixture model; EM algorithm; Performance measures.

### 1 Introduction

Image analysis and image retrievals are prerequisites for image processing. In image processing, the scene and texture in the image have a strong one to one cor-

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respondence. The ability of identifying the texture is the basic requisition for the semantic information about the scene. As a result of it, the texture description and segmentation formed a thrust area of computer vision and image processing [Haim, Joseph, and Ian (2006)]. Generally, texture refers to the arrangement of the basic constituents of a material. In digital images, the spatial inter relationships between the arrangement of the image pixels describe the texture of the image.

Several texture analysis techniques have been developed and used for analysing the natural scenes which are consisting of textures surfaces [Haralick, Shanmugam, and Dinstein (1973); Haralick (1979); Du (1990); Weszka, Dyer, and Rosenfield (1976); Lu, Chung, and Chen (1997)]. Among these methods, texture segmentation methods are more useful because of the variation in textural patterns make the difference in coarseness, complexity, shape, direction, content etc. The human perception in comparing textures is highly inaccurate since the texture of the image is covered by the scene. For efficient analysis of images, the texture classification (segmentation) is highly needed.

The texture segmentation methods can be classified in to two categories namely statistical methods and heuristic methods. The statistical methods include model based methods, non parametric methods, classification trees, vector quantisation, Markov random fields etc., [Unser (1995); Weldon and Higgins (1996); Unser (1986); Randen and Husoy (1999); Hossein, Shankar, Allen, Shankar, and Yi (2011); Mihran and Anil (1998)]. Among these methods, model based texture segmentation methods are more important since they capture the information more effectively and accurately. Several model based image segmentation methods have been developed [Pal and Pal (1993); Conrad and Kuldip (2003)]. In all these models, they assumed that the feature vector associated with texture of image is meso kurtic and symmetry. But in many images, the feature vector of the texture may not be meso kurtic even though it is symmetry. Hence, the texture segmentation methods developed based on Gaussian mixture model (GMM) may not serve the purpose accurately due to the limitations on GMM. Therefore, it is needed to model the feature vector of the texture in the image segment using a suitable probability model.

With this motivation, in this paper we develop and analyse texture segmentation algorithm based on multivariate generalized Gaussian mixture model (MGGMM). Here, it is assumed that the feature vector associated with the texture of the image region is characterised by multivariate generalized Gaussian distribution and the whole image is a collection of several image regions. The generalized Gaussian distribution is capable of including several platy kurtic, lepty kurtic and meso kurtic distributions as particular cases for specific values of the shape parameters. It is interesting to note that generalized Gaussian distribution also includes Gaussian

distribution as a particular case.

The rest of the article is organised as follows: Section 2 deals with the feature vector extraction of the texture associated with the image using DCT (Discrete Cosine Transformation) coefficients. Section 3 deals with multivariate generalized Gaussian mixture model and its properties. Section 4 deals with estimation of model parameters using EM algorithm and initialisation of the parameters using moment method of estimation and K-means algorithm. Section 5 is concerned with texture segmentation algorithm under Bayesian frame using maximum likelihood function. Section 6 deals with experimental results and performance evaluation of the developed algorithm using Brodatz texture database and computing the segmentation performance measures. Section 7 deals with comparative study of the proposed algorithm with that of Gaussian mixture model. The last Section 8 is to summarise the results with conclusions and scope for further research in this area.

## **2 Feature vector extraction**

For developing the texture segmentation model, the important consideration is deriving the feature vectors of the texture present in the image. Several techniques have been adopted to extract the feature vectors associated with each texture in the image [Conrad and Kuldip (2003); Mariana (2008)]. Among these techniques, the 2D discrete cosine transformation is simple and more efficient in characterising the features of the texture. This method has been considered as a worldwide standard [JPEG] for image compression [Huang (2005)]. To obtain the feature vectors associated with the image texture, we assume that the image is divided into  $M \times M$  blocks. In each block, the 2D DCT coefficients are computed using the method [(Huang (2005))]. These coefficients are ordered in zigzag pattern (consisting of 16 coefficients) which are sufficient to reflect the amount of information stored in the image. After finding the DCT coefficients, we get the feature vectors of the image as  $\vec{x}_t = [\vec{x}_1, \vec{x}_2, \vec{x}_3, \dots, \vec{x}_N]^T$  consisting of  $N \times 16$  coefficients where  $N$  is number of blocks.

## **3 Multivariate generalized gaussian MixtureModel**

In texture analysis, the entire image texture is considered as a union of several repetitive patterns. In this section, we briefly discuss the probability distribution (model) used for characterizing the feature vector of the texture. After extracting the feature vector of each individual texture it can be modeled by a suitable probability distribution such that the characteristics of the feature vector should match the statistical theoretical characteristics of the distribution. The feature vector characterizing the image is to follow a  $M$ -component mixture distribution. Therefore we develop and

analyze the textures in an image by considering that the feature vectors representing textures follow a M-Component MGGMM model. The joint probability density function (pdf) of the feature vector associated with each individual texture is

$$p(\vec{x}_r/\theta) = \sum_{i=1}^M w_i g_i(\vec{x}_r, \theta) \tag{1}$$

where,  $\vec{x}_r = (x_{rij}), j = 1, 2, \dots, D$ , is a  $D$  dimensional random vector represents the feature vector.

$i = 1, 2, \dots, M$  representing the groups,

$r = 1, 2, \dots, T$  representing the samples.

$\theta$  is a parametric set such that  $\theta = (\mu, \sigma, \beta)$

$w_i$  is the component weight such that  $\sum_{i=1}^M w_i = 1$

and  $g_i(\vec{x}_r, \theta)$  is the probability of  $i^{\text{th}}$  class representing by the feature vectors of the image and the D-dimensional generalized Gaussian distribution (GGD) is of the form [(Allili, Bouguila, and Ziou (2007)].

$$g(\vec{x}_r/\theta) = \prod_{j=1}^D \frac{\beta_j K(\beta_j)}{2\sigma_j} e^{\left\{(-A(\beta_j) \left| \frac{x_j - \mu_j}{\sigma_j} \right|^{\beta_j})\right\}} \tag{2}$$

where,  $\mu_j, \sigma_j, \beta_j$  are location, scale and shape parameters.

Also we have

$$K(\beta) = \frac{\left[ \frac{\Gamma(3/\beta)}{\Gamma(1/\beta)} \right]^{1/2}}{\Gamma(1/\beta)} \quad \text{and} \quad A(\beta) = \left[ \frac{\Gamma(3/\beta)}{\Gamma(1/\beta)} \right]^{\beta/2}$$

with  $\Gamma(\cdot)$  denoting gamma function.

Each parameter  $\beta \geq 0$  controls the shape of GGD.

Expanding and rearranging the terms in (2), we get

$$\begin{aligned} g(\vec{x}_r/\theta) &= \prod_{j=1}^D \frac{\beta_j \left[ \frac{\Gamma(3/\beta_j)}{\Gamma(1/\beta_j)} \right]^{1/2}}{\Gamma(1/\beta_j) 2\sigma_j} e^{\left\{ - \left[ \frac{\Gamma(3/\beta_j)}{\Gamma(1/\beta_j)} \right]^{\beta_j/2} \left| \frac{x_j - \mu_j}{\sigma_j} \right|^{\beta_j} \right\}} \\ &= \prod_{j=1}^D \frac{1}{2\sigma_j \beta_j \frac{\Gamma(1/\beta_j)}{\left[ \frac{\Gamma(3/\beta_j)}{\Gamma(1/\beta_j)} \right]^{-1/2}}} e^{\left\{ - \left[ \frac{x_j - \mu_j}{\sigma_{ij} \left( \sqrt{\frac{\Gamma(1/\beta_j)}{\Gamma(3/\beta_j)}} \right)} \right]^{\beta} \right\}} \end{aligned} \tag{3}$$

$$\text{Let } A(\beta_j, \sigma_j) = \left[ \frac{\Gamma(1/\beta_j)}{\Gamma(3/\beta_j)} \right]^{1/2} \sigma_j$$

Then

$$g(\vec{x}_r/\theta) = \prod_{j=1}^D \frac{1}{\frac{2}{\beta_j} A(\beta_j, \sigma_j) \Gamma(1 + \frac{1}{\beta_j})} e^{-\left| \frac{x_j - \mu_j}{A(\beta_j, \sigma_j)} \right|^{\beta_j}} \tag{4}$$

When  $\beta = 1$ , the corresponding generalized Gaussian corresponds to a Laplacian or double exponential distribution. When  $\beta = 2$ , the corresponding generalized Gaussian corresponds to Gaussian distribution. In limiting cases,  $\beta \rightarrow +\infty$ , Eq.4 converges to a uniform distribution in  $(\mu - \sqrt{3}\sigma, \mu + \sqrt{3}\sigma)$  and when  $\beta \rightarrow 0+$ , the distribution becomes degenerate are in  $x = \mu$

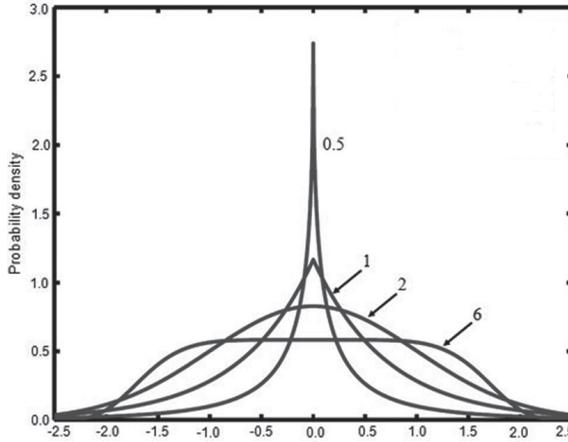


Figure 1: Generalized Gaussian pdf's for different values of shape parameter  $\beta$ .

The mean value of the generalized Gaussian distribution is

$$\begin{aligned} E(x_j) &= \frac{1}{2\Gamma(1 + \frac{1}{\beta_j})A(\beta_j, \sigma_j)} \int_{-\infty}^{\infty} x e^{-\left| \frac{x_j - \mu_j}{A(\beta_j, \sigma_j)} \right|^{\beta_j}} dx \\ &= \mu_{ij} + \frac{1}{2\Gamma(1 + \frac{1}{\beta_j})A(\beta_j, \sigma_j)} \int_{-\infty}^{\infty} (x_j - \mu_j) e^{-\left| \frac{x_j - \mu_j}{A(\beta_j, \sigma_j)} \right|^{\beta_j}} dx \\ &= \mu_{ij} \end{aligned} \tag{5}$$

The GGD is symmetric with respect to  $\mu$ , hence the odd center moments are zero i.e.,  $E|x_{ij} - \mu_{ij}|^t = 0, t = 1, 3, 5, \dots$ . The even central moments can be obtained from absolute center moments and given by

$$E|x_j - \mu_j|^t = \left[ \frac{\sigma_j^2 \Gamma\left(\frac{1}{\beta_j}\right)}{\Gamma\left(\frac{3}{\beta_j}\right)} \right]^{r/2} \frac{\Gamma\left(\frac{t+1}{\beta_j}\right)}{\Gamma\left(\frac{1}{\beta_j}\right)} \tag{6}$$

The variance is given by

$$\text{Var}(x) = E(x - \bar{x})^2 = E(x - \mu)^2 = \sigma^2 \tag{7}$$

The model can have one covariance matrix for a generalized Gaussian density of the class. The covariance matrix  $\Sigma$  can be full or diagonal. In this paper, the diagonal covariance matrix is considered. This choice based on the initial experimental results. Therefore

$$\Sigma_i = \begin{bmatrix} \sigma_{i1}^2 & \cdot & \cdot & \cdot \\ \cdot & \sigma_{i2}^2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \sigma_{iD}^2 \end{bmatrix} \tag{8}$$

As a result of diagonal covariance matrix for the feature vector, the features are independent and the probability density function of the feature vector is

$$g_i(\vec{x}_r / \theta) = \prod_{j=1}^D \frac{\exp\left(-\left|\frac{x_{ij} - \mu_{ij}}{A(\beta_{ij}, \sigma_{ij})}\right|^{\beta_{ij}}\right)}{2\Gamma\left(1 + \frac{1}{\beta_{ij}}\right) A(\beta_{ij}, \sigma_{ij})} \tag{9}$$

$$= \prod_{j=1}^D f_{ij}(x_{rij}) \tag{10}$$

**4 Estimation of model parameters using EM algorithm and initialisation of the parameters**

In this section, we consider estimation of model parameters using EM algorithm that maximizes the likelihood of the model [Mclanchlan and Peel (2000)]. The sample observations (DCT Coefficients)  $[\vec{x}_1, \vec{x}_2, \dots, \vec{x}_r]$  are drawn from image texture which is characterized by the joint probability density function

$$p(\vec{x}_r / \theta) = \sum_{i=1}^M w_i g_i(\vec{x}_r, \theta)$$

where,  $g_i(\vec{x}_r, \theta)$  is given in the Eq.9.

The likelihood function is given by

$$L(\theta) = \prod_{r=1}^T \left[ \sum_{i=1}^M w_i g_i(\vec{x}_r, \theta) \right] = \prod_{r=1}^T \left[ \sum_{i=1}^M w_i \left( \prod_{j=1}^D \frac{\exp\left(-\left|\frac{x_{ij} - \mu_{ij}}{A(\beta_{ij}, \sigma_{ij})}\right|^{\beta_{ij}}\right)}{2\Gamma\left(1 + \frac{1}{\beta_{ij}}\right) A(\beta_{ij}, \sigma_{ij})} \right) \right] \quad (11)$$

This implies

$$\begin{aligned} \log L(\theta) &= \log \prod_{r=1}^T \left[ \sum_{i=1}^M w_i g_i(\vec{x}_r, \theta) \right] = \sum_{r=1}^T \log \left[ \sum_{i=1}^M w_i g_i(\vec{x}_r, \theta) \right] \\ &= \sum_{r=1}^T \log \left[ \sum_{i=1}^M w_i \left( \prod_{j=1}^D \frac{\exp\left(-\left|\frac{x_{ij} - \mu_{ij}}{A(\beta_{ij}, \sigma_{ij})}\right|^{\beta_{ij}}\right)}{2\Gamma\left(1 + \frac{1}{\beta_{ij}}\right) A(\beta_{ij}, \sigma_{ij})} \right) \right] \end{aligned} \quad (12)$$

To find the refined estimates of parameters  $w_i, \mu_{ij}$  and  $\sigma_{ij}$  for  $i = 1, 2, 3, \dots, M; j = 1, 2, \dots, D$ . we maximize the expected value likelihood or log likelihood function. The shape parameter  $\beta_{ij}$  is estimated by procedure given by Shaoquan YU, et al., 2012.

To estimate  $w_i, \mu_{ij}$  and  $\sigma_{ij}$ , we use the EM algorithm which consists of two steps namely Expectation (E) Step and Maximization (M) Step. The first step of EM algorithm is to estimate initial parameters  $w_i, \mu_{ij}$  and  $\sigma_{ij}$  from a given texture image data.

#### 4.1 E-Step

Given the estimates,  $\theta^{(l)} = (\mu_{ij}^{(l)}, \sigma_{ij}^{(l)})$  for  $i = 1, 2, \dots, M; j = 1, 2, 3, \dots, D$ . One can estimate probability density function as

$$p(\vec{x}_r / \theta) = \sum_{i=1}^M w_i g_i(\vec{x}_r, \theta)$$

The conditional probability of any observation  $x_r$  belongs to  $M^{\text{th}}$  class is

$$t_i(\vec{x}_r / \theta^{(l)}) = p((i/x), \theta^{(l)}) = \frac{w_i g_i(\vec{x}_r, \theta^{(l)})}{p_i(\vec{x}_r, \theta^{(l)})} = \frac{w_i g_i(\vec{x}_r, \theta^{(l)})}{\sum_{i=1}^M w_i g_i(\vec{x}_r, \theta)} \quad (13)$$

The expected value of  $L(\theta)$  is

$$Q(\theta, \theta^{(0)}) = E_{\theta^{(l)}} \{ \log L(\theta / \vec{x}_r) \} \tag{14}$$

Following heuristic arguments of Jeff A Bilmes (1997), we get

$$\begin{aligned} Q(\theta, \theta^{(l)}) &= \sum_{r=1}^T \sum_{i=1}^M \left[ \log(w_i \cdot g_i(\vec{x}_r, \theta^{(l)})) \right] t_i(\vec{x}_r, \theta^{(l)}) = \sum_{r=1}^T \sum_{i=1}^M \left[ \log(w_i) t_i(\vec{x}_r, \theta^{(l)}) \right] \\ &+ \sum_{r=1}^T \sum_{i=1}^M \left[ \log \left( \frac{\exp \left( - \left| \frac{x_{ij} - \mu_{ij}}{A(\beta_{ij}, \sigma_{ij})} \right|^{\beta_{ij}} \right)}{\prod_{j=1}^D \frac{2\Gamma \left( 1 + \frac{1}{\beta_{ij}} \right) A(\beta_{ij}, \sigma_{ij})}{\Gamma \left( 1 + \frac{1}{\beta_{ij}} \right) A(\beta_{ij}, \sigma_{ij})}} \right) \right] t_i(\vec{x}_r, \theta^{(l)}) \\ &= \sum_{r=1}^T \sum_{i=1}^M \left[ \log(w_i) t_i(\vec{x}_r, \theta^{(l)}) \right] \\ &+ \sum_{r=1}^T \sum_{i=1}^M \left[ \sum_{j=1}^D \log \left( \frac{\exp \left( - \left| \frac{x_{ij} - \mu_{ij}}{A(\beta_{ij}, \sigma_{ij})} \right|^{\beta_{ij}} \right)}{2\Gamma \left( 1 + \frac{1}{\beta_{ij}} \right) A(\beta_{ij}, \sigma_{ij})} \right) \right] t_i(\vec{x}_r, \theta^{(l)}) \\ &= \sum_{r=1}^T \sum_{i=1}^M \left[ \log(w_i) t_i(\vec{x}_r, \theta^{(l)}) \right] + \sum_{r=1}^T \sum_{i=1}^M \sum_{j=1}^D \log \left[ - \left| \frac{x_{ij} - \mu_{ij}}{A(\beta_{ij}, \sigma_{ij})} \right|^{\beta_{ij}} \right. \\ &\left. - \log 2\Gamma \left( 1 + \frac{1}{\beta_{ij}} \right) A(\beta_{ij}, \sigma_{ij}) \right] t_i(\vec{x}_r, \theta^{(l)}) \tag{15} \end{aligned}$$

#### 4.2 M-Step

To maximize  $Q(\theta, \theta^{(l)})$ , we can maximize the term containing  $w_i$  and containing  $\theta^{(l)}$  independently, since they are not related. To update the component weights  $w_i$ , we maximize the likelihood function such that  $\sum_{i=1}^M w_i = 1$

We construct the first order Lagrange type function as

$$L = \sum_{r=1}^T \sum_{i=1}^M \log(w_i) t_i(\vec{x}_r, \theta^{(l)}) + \gamma \left( \sum_{i=1}^M w_i - 1 \right) \tag{16}$$

Where,  $\gamma$  is Lagrange multiplier and maximizing this Lagrange function with respect to  $w_i$ , we have to differentiate  $L$  with respect to  $w_i$  and equate to zero i.e.,

$$\frac{\partial}{\partial w_i} [L] = 0$$

This implies

$$\frac{\partial}{\partial w_i} \sum_{r=1}^T \sum_{i=1}^M \log(w_i) t_i(\vec{x}_r, \boldsymbol{\theta}^{(l)}) + \frac{\partial}{\partial w_i} \gamma \left( \sum_{i=1}^M w_i^{(l)} - 1 \right) = 0 \quad (17)$$

This implies

$$\sum_{r=1}^T \left[ \frac{1}{w_i} t_i(\vec{x}_r, \boldsymbol{\theta}^{(l)}) - \gamma \right] = 0$$

Therefore,

$$\sum_{r=1}^T \left[ t_i(\vec{x}_r, \boldsymbol{\theta}^{(l)}) \right] = \gamma w_i \quad (18)$$

Summing  $i = 1, 2, 3, \dots, M$ , we have

$$\sum_{i=1}^M \sum_{r=1}^T \left[ t_i(\vec{x}_r, \boldsymbol{\theta}^{(l)}) \right] = \gamma \quad (19)$$

This implies

$$\gamma = T \quad (20)$$

Therefore

$$\sum_{r=1}^T \left[ t_i(\vec{x}_r, \boldsymbol{\theta}^{(l)}) \right] = T w_i \quad (21)$$

This implies

$$w_i^{(l+1)} = \frac{1}{T} \sum_{r=1}^T \left[ t_i(\vec{x}_r, \boldsymbol{\theta}^{(l)}) \right] \quad (22)$$

Hence, updated equations for  $w_i$  is

$$w_i^{(l+1)} = \frac{1}{T} \sum_{r=1}^T \left[ \frac{w_i^{(l)} \cdot g_i(\vec{x}_r, \boldsymbol{\theta}^{(l)})}{\sum_{i=1}^M w_i^{(l)} \cdot g_i(\vec{x}_r, \boldsymbol{\theta}^{(l)})} \right] \quad (23)$$

Where  $\boldsymbol{\theta}^{(l)} = (\mu_{ij}^{(l)}, \sigma_{ij}^{(l)})$  are the estimates at  $i^{\text{th}}$  iteration.

### 4.3 Updating $\mu_{ij}$

For updating  $\mu_{ij}$ , we consider derivative of  $Q(\theta, \theta^{(l)})$  with respect to  $\mu_{ij}$  for  $i = 1, 2, \dots, M, j = 1, 2, \dots, D$  and equate to zero i. e,

$$\frac{\partial}{\partial \mu_{ij}} Q(\theta, \theta^{(l)}) = 0$$

$$\frac{\partial}{\partial \mu_{ij}} \left( \sum_{r=1}^T \sum_{i=1}^M \log(w_i) t_i(\vec{x}_r, \theta^{(l)}) + \sum_{r=1}^T \sum_{i=1}^M g_i(\vec{x}_r, \theta^{(l)}) t_i(\vec{x}_r, \theta^{(l)}) \right) = 0 \quad (24)$$

$$\frac{\partial}{\partial \mu_{ij}} \left( \sum_{r=1}^T \sum_{i=1}^M \log(w_i) t_i(\vec{x}_r, \theta^{(l)}) + \sum_{r=1}^T \sum_{i=1}^M \sum_{j=1}^D \log \left( \frac{\exp \left( - \left| \frac{x_{rij} - \mu_{ij}}{A(\beta_{ij}, \sigma_{ij})} \right|^{\beta_{ij}} \right)}{2\Gamma \left( 1 + \frac{1}{\beta_{ij}} \right) A(\beta_{ij}, \sigma_{ij})} \right) t_i(\vec{x}_r, \theta^{(l)}) \right) = 0$$

Since  $\mu_{ij}$  involves only one element of feature vector, mean  $\mu_{ij}$ , the equation reduces to

$$\frac{\partial}{\partial \mu_{ij}} \sum_{r=1}^T \sum_{i=1}^M \left( -\log 2\Gamma \left( 1 + \frac{1}{\beta_{ij}} \right) - \log A(\beta_{ij}, \sigma_{ij}) \right) t_i(\vec{x}_r, \theta^{(l)}) - \left( \left| \frac{x_{rij} - \mu_{ij}}{A(\beta_{ij}, \sigma_{ij})} \right|^{\beta_{ij}} \right) t_i(\vec{x}_r, \theta^{(l)}) = 0 \quad (25)$$

This implies

$$- \sum_{r=1}^T \frac{\partial}{\partial \mu_{ij}} \left| \frac{x_{rij} - \mu_{ij}}{A(\beta_{ij}, \sigma_{ij})} \right|^{\beta_{ij}} t_i(\vec{x}_r, \theta^{(l)}) = 0 \quad (26)$$

This implies

$$\sum_{r=1}^T t_i(\vec{x}_r, \theta^{(l)}) \text{sign} \left[ \frac{x_{rij} - \mu_{ij}}{A(\beta_{ij}, \sigma_{ij})} \right]^{\beta_{ij}-1} \cdot (x_{rij} - \mu_{ij})^{\beta_{ij}-1} = 0 \text{ for } \beta_{ij} \neq 1 \quad (27)$$

This is non trivial equation, explicit expression for  $\mu_{ij}$  is little complicated. To update  $\mu_{ij}$ , solve equation by using Newton's Raphson method and obtain  $\mu_{ij}^{(l+1)}$ .

This  $\mu_{ij}^{(l+1)}$  provides the refined estimates for  $\mu_{ij}$ . For explicit estimate of  $\mu_{ij}$ , consider some special cases

**Case 1:** The Gaussian case,  $\beta_{ij} = 2$  leads to

$$\sum_{r=1}^T t_i(\vec{x}_r, \theta^{(l)}) \cdot (x_{rij} - \mu_{ij}^{(l)}) = 0 \quad (28)$$

This implies

$$\mu_{ij}^{(l+1)} = \frac{\sum_{r=1}^T t_i(\vec{x}_r, \theta^{(l)}) \cdot (x_{rij})}{\sum_{r=1}^T t_i(\vec{x}_r, \theta^{(l)})} \quad (29)$$

**Case 2:** Consider for  $\beta_{ij} \neq 1$

$$\sum_{r=1}^T t_i(\vec{x}_r, \theta^{(l)}) \text{sign} \left[ \frac{x_{rij} - \mu_{ij}}{A(\beta_{ij}, \sigma_{ij})} \right]^{\beta_{ij}-1} (x_{rij} - \mu_{ij}^{(l)})^{\beta_{ij}-1} = 0$$

This implies that

$$\mu_{ij}^{(l+1)} = \frac{\left( \sum_{r=1}^T (x_{rij}) w_i^{(l)} g_i(\vec{x}_r, \theta^{(l)}) \right)^{\frac{1}{\beta_{ij}-1}}}{\left( \sum_{r=1}^T w_i^{(l)} g_i(\vec{x}_r, \theta^{(l)}) \right)^{\frac{1}{\beta_{ij}-1}}} \quad (30)$$

For general case: we can also develop a general approximation without using numerical method for updating  $\mu_{ij}^{(l+1)}$  by adopting an axiom that of the form  $\mu_{ij}$  which must be a weighted average of data vectors with weights provided by some power of the assignment probabilities of those data vectors, notified in part by symmetry of system, and in part by pragmatism leads to

$$\mu_{ij}^{(l+1)} = \frac{\sum_{r=1}^T t_i(\vec{x}_r, \theta^{(l)})^{A(N, \beta_{ij})} (x_{rij})}{\sum_{r=1}^T t_i(\vec{x}_r, \theta^{(l)})^{A(N, \beta_{ij})}} \quad (31)$$

Where  $A(N, \beta_{ij})$  is some function = 1 for  $\beta_{ij} = 2$  and must be equal to  $\frac{1}{\beta_{ij}-1}$  for  $\beta_{ij} \neq 1$ , in the case of  $N = 2$ , we have also observed that  $A(N, \beta_{ij})$  must be increasing function of  $\beta_{ij}$ .

#### 4.4 Updating $\sigma_{ij}$

For updating  $\sigma_{ij}$ , we consider maximization of  $Q(\theta, \theta^{(l)})$  w.r.t.  $\sigma_{ij}$  for  $i = 1, 2, \dots, M, j = 1, 2, \dots, D$  and so differentiate w.r.t.  $\sigma_{ij}$  and equate to zero i.e.,  $\frac{\partial}{\partial \sigma_{ij}} Q(\theta, \theta^{(l)}) = 0$

$$\frac{\partial}{\partial \sigma_{ij}} \left( \sum_{r=1}^T \sum_{i=1}^M \log(w_i) t_i(\vec{x}_r, \theta^{(l)}) + \sum_{r=1}^T \sum_{i=1}^M g_i(\vec{x}_r, \theta^{(l)}) t_i(\vec{x}_r, \theta^{(l)}) \right) = 0 \quad (32)$$

This implies

$$\frac{\partial}{\partial \sigma_{ij}} \left( \sum_{r=1}^T \sum_{i=1}^M \log(w_i) t_i(\vec{x}_r, \theta^{(l)}) + \sum_{r=1}^T \sum_{i=1}^M \sum_{j=1}^D \log \left( \frac{\exp \left( - \left| \frac{x_{rij} - \mu_{ij}}{A(\beta_{ij}, \sigma_{ij})} \right|^{\beta_{ij}} \right)}{2\Gamma \left( 1 + \frac{1}{\beta_{ij}} \right) A(\beta_{ij}, \sigma_{ij})} \right) t_i(\vec{x}_r, \theta^{(l)}) \right) = 0 \tag{33}$$

Since  $\sigma_{ij}$  involves only one element feature vector, we have

$$\sum_{r=1}^T t_i(\vec{x}_r, \theta^{(l)}) \left[ \frac{\partial}{\partial \sigma_{ij}} \log \sigma_{ij} - \frac{\Gamma \left( \frac{3}{\beta_{ij}} \right)}{\Gamma \left( \frac{1}{\beta_{ij}} \right)} \cdot \frac{\partial}{\partial \sigma_{ij}} (x_{rij} - \mu_{ij})^{\frac{1}{\beta_{ij}-1}} \right] = 0 \tag{34}$$

This implies

$$\sum_{r=1}^T t_i(\vec{x}_r, \theta^{(l)}) \left[ \frac{2 \cdot \Gamma \left( \frac{3}{\beta_{ij}} \right)}{\Gamma \left( \frac{1}{\beta_{ij}} \right)} \cdot |x_{rij} - \mu_{ij}|^{\frac{1}{\beta_{ij}}} - \frac{2}{\beta_{ij}} |\sigma_{ij}|^{\frac{1}{\beta_{ij}}} \right] = 0 \tag{35}$$

Therefore,

$$\sigma_{ij}^{(l+1)} = \left[ \frac{\sum_{r=1}^T t_i(\vec{x}_r, \theta^{(l)}) \left( \frac{\Gamma \left( \frac{3}{\beta_{ij}} \right)}{\beta_{ij} \Gamma \left( \frac{1}{\beta_{ij}} \right)} \right) \cdot |x_{rij} - \mu_{ij}|^{\frac{1}{\beta_{ij}}}}{\sum_{r=1}^T t_i(\vec{x}_r, \theta^{(l)})} \right]^{\frac{1}{\beta_{ij}}} \tag{36}$$

#### 4.5 Initialization of model parameters

The efficiency of the EM algorithm in estimating the parameters is heavily dependent on the number of groups and the initial estimates of the model parameters  $w_i, \mu_{ij}$  and  $\sigma_{ij}$  for  $i = 1, 2, 3, \dots, M; j = 1, 2, \dots, D$ . Usually in EM algorithm, the mixing parameter  $w_i$  and the distribution parameters  $\mu_{ij}$  and  $\sigma_{ij}$  are given with some initial values. A commonly used method in initialization is by drawing a random sample from the entire data. This method can be performed well when the sample size is large, but the computation is heavily increased. When the sample size is small it is likely that some small regions may not be sampled. To overcome this problem, we use K-means algorithm to initialize the multivariate Generalized Gaussian mixture parameters for initialization of the problem. The number of the

mixture components is initially taken for K-means algorithm by the histogram of the texture image. After determining the final value of the groups  $M$ , we obtain the initial estimates of the parameters through sample moments as

$$w_i = 1/M; \sigma_{ij} = \text{Std. Deviation of } M^{\text{th}} \text{ class}; \mu_{ij} = \frac{1}{T} \sum_{r=1}^T x_{rij}$$

Substituting these values as the initial estimates, the refined estimates of the parameters can be obtained using updated equations of the EM Algorithm and simultaneously solving the Eqn's (23), (31) and (36) using MATLAB package.

### 5 Texture segmentation algorithm

Once the texture is considered, the main purpose is to identify the regions of interest. The following algorithm can be adopted for texture segmentation using Multivariate Generalized Gaussian Mixture model.

*Step 1:* Obtain the feature vectors from the texture image using technique presented in feature vector extraction section.

*Step 2:* Divide the samples into  $M$  groups by K-means algorithm.

*Step 3:* Find the mean vector, variance vector,  $\mu_{ij}$  and  $\sigma_{ij}$  for each class of the multivariate data.

*Step 4:* Take  $w_i = 1/M$ , for  $i = 1, 2, 3, \dots, M$  groups.

*Step 5:* Obtain the refined estimates of  $w_i, \mu_{ij}$  and  $\sigma_{ij}$  for each class using the updated equations of the EM algorithm.

*Step 6:* Assign each feature vector into the corresponding  $j^{\text{th}}$  region (segment) according to the maximum likelihood of the  $j^{\text{th}}$  component  $L_j$ .

That is, Feature vector  $\vec{x}_i$  is assigned to the  $j^{\text{th}}$  region for which  $L_j$  is maximum, where,

$$L_j = \max \left\{ \prod_{j=1}^D \frac{\exp \left( - \left| \frac{x_{ij} - \mu_{ij}}{A(\rho_{ij}, \sigma_{ij})} \right|^{\beta_{ij}} \right)}{2\Gamma \left( 1 + \frac{1}{\beta_{ij}} \right) A(\beta_{ij}, \sigma_{ij})} \right\} \quad (37)$$

## 6 Experimental results and performance evaluation of the developed algorithm

To demonstrate the ability of the developed model, texture segmentation is to be performed by using the dataset of textures available in the Brodatz Texture database (<http://sipi.usc.edu/database/database.php?volume=textures>). For each texture image, the histogram is plotted to identify the number of the components in the given texture image. Based on the number of the peaks ( $M$ ), K-means algorithm is employed over the multivariate data of feature vectors to divide in to  $M$  groups. For each group, the initial estimate of the parameters  $w_i$ ,  $\mu_{ij}$  and  $\sigma_{ij}$  are obtained using K-means and moment estimators.

Using these initial estimates, the refined estimates are calculated based on the updated equations obtained through EM Algorithm. With these values, texture segmentation is performed based of the assignment of the data to a particular group for which the likelihood is maximum. Then the segmentation image is drawn based on the application of the developed algorithm.

Figure 2 presents the segmented texture images taken from Brodatz texture database along with the refined estimates of the model parameters.

The performance of the developed image segmentation method is studied by obtaining the image segmentation performance measures namely; Probabilistic Rand Index (PRI), the Variation of Information (VOI) and Global Consistency Error (GCE). The Rand index given by Unnikrishnan et al., (2007) counts the fraction of pairs of pixels whose labeling are consistent between the computed segmentation and the ground truth. This quantitative measure is easily extended to the Probabilistic Rand index (PRI).

The variation of information (VOI) metric given by Meila (2007) is based on relationship between a point and its cluster. It uses mutual information metric and entropy to approximate the distance between two clustering's across the lattice of possible clustering's. It measures the amount of information that is lost or gained in changing from one clustering to another.

The Global Consistency Error (GCE) given by Martin, Fowlkes, Tal, and Malik (2001) measures the extent to which one segmentation map can be viewed as a refinement of segmentation. For a perfect match, every region in one of the segmentations must be identical to, or a refinement (i.e., a subset) of, a region in the other segmentation.

A comparative study of the developed algorithm based on Multivariate Generalized Gaussian mixture model with K-means clustering with the image segmentation algorithms based on Gaussian mixture model with K-means algorithm is performed and the image segmentation performance measures PRI, GCE, VOI are computed.

<b>Description</b>	<b>Image 1</b>		<b>Image 2</b>			<b>Image 3</b>			<b>Image 4</b>	
<b>Original Image Texture</b>										
<b>Histogram of Image</b>										
<b>Initial Value</b>	M=2		M=3			M=3			M=2	
<b>Weights</b>	w <sub>1</sub>	w <sub>2</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>3</sub>	w <sub>1</sub>	w <sub>2</sub>	w <sub>3</sub>	w <sub>1</sub>	w <sub>2</sub>
<b>Refined Estimates</b>	0.475	0.525	0.297	0.169	0.534	0.197	0.397	0.406	0.374	0.626
<b>Segmented Image</b>										
<b>Description</b>	<b>Image 5</b>		<b>Image 6</b>		<b>Image 7</b>		<b>Image 8</b>			
<b>Original Image Texture</b>										
<b>Histogram of Image</b>										
<b>Initial Value</b>	M=2		M=2		M=2		M=4			
<b>Weights</b>	w <sub>1</sub>	w <sub>2</sub>	w <sub>3</sub>	w <sub>4</sub>						
<b>Refined Estimates</b>	0.669	0.331	0.636	0.364	0.644	0.359	0.363	0.281	0.257	0.099
<b>Segmented Image</b>										

Figure 2: Original and Segmented images along with the revised estimates of the parameters.

The image segmentation performance measures namely, PRI, GCE, VOI are computed for all the eight images with respect to the developed model, Generalized Gaussian Mixture Model with  $K$ -means are presented in Table 1.

From Table 1 and Figure 3, its observed that the proposed segmentation algorithm based on multivariate generalized Gaussian mixture model have shown a significant improvement over the earlier texture segmentation method based on Gaussian mixture model.

Table 1: Segmentation Performance Measures of the textured images.

Segmentation Performance Measures				
Description	Model	PRI	GCE	VOI
Image 1	MGGMM	0.821	0.158	0.817
	GMM	0.732	0.222	1.06
Image 2	MGGMM	0.997	0.204	0.236
	GMM	0.659	0.267	1.305
Image 3	MGGMM	0.901	0.344	1.922
	GMM	0.710	0.414	2.123
Image 4	MGGMM	0.633	0.336	1.263
	GMM	0.507	0.413	1.376
Image 5	MGGMM	0.828	0.12	0.719
	GMM	0.694	0.162	0.977
Image 6	MGGMM	0.924	0.072	0.401
	GMM	0.798	0.173	0.809
Image 7	MGGMM	0.99	0.010	0.120
	GMM	0.971	0.028	0.188
Image 8	MGGMM	0.897	0.131	1.741
	GMM	0.663	0.422	2.246

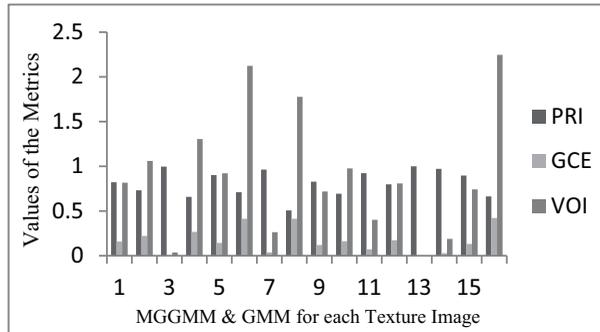


Figure 3: Texture Segmentation Performance Measures for MGGMM and GMM.

## 7 Comparative Study

The developed algorithm performance is evaluated by comparing the algorithm with the Gaussian mixture model. Table 2 presents the miss classification rate of the pixels of the sample using the proposed model and Gaussian mixture model [Gui, Zhang, and Shang (2012)].

From the Table 2, it is observed that the misclassification rate of the classifier with the multivariate generalized Gaussian mixture model is less when compared to that

Table 2: Miss-classification rate of the classifier.

Model	Miss-classification Rate
MGGMM	10%
GMM	15%

of GMM.

The accuracy of the classifier is also studied for the sample images by using confusion matrix for segmented regions and computing the metrics [(David (2011))]. Table 3 shows the values of Accuracy, Sensitivity, Specificity, Precision, F-Measure and G-mean for the segmented regions in the image texture.

Table 3: Comparative study of MGGMM and GMM.

Description	MODEL	Accuracy	Sensitivity (TPR)	1- Specificity (FPR)	Precision	F-Measure	G-mean
Image 1	MGGMM	0.901	0.975	0.125	0.986	0.921	0.918
	GMM	0.841	0.864	0.226	0.683	0.803	0.869
Image 2	MGGMM	0.998	0.997	0.215	1	0.998	0.998
	GMM	0.831	0.854	0.355	0.035	0.942	0.886
Image 3	MGGMM	0.924	0.936	0.286	0.888	0.912	0.925
	GMM	0.749	0.797	0.291	0.115	0.866	0.863
Image 4	MGGMM	0.84	0.760	0.27	0.72	0.76	0.74
	GMM	0.81	0.711	0.33	0.65	0.71	0.68
Image 5	MGGMM	0.905	0.98	0.104	0.606	0.742	0.926
	GMM	0.812	0.957	0.219	0.431	0.603	0.884
Image 6	MGGMM	0.961	1	0.082	0.93	0.964	0.958
	GMM	0.886	0.91	0.100	1	0.865	0.873
Image 7	MGGMM	1	1	0.010	1	1	1
	GMM	0.986	0.91	0.039	0.978	0.989	0.98
Image 8	MGGMM	0.926	0.934	0.085	0.934	0.934	0.924
	GMM	0.355	0.875	0.169	0.418	0.895	0.852

From Table 3, it is observed that the F-measure value for the proposed classifier is more. This indicates that the proposed classifier performs better than that of Gaussian mixture model. Figure 4 shows the ROC curves associated with the proposed classifier and classifier with GMM.

From Figure 4, it is observed that the proposed classifier is having less false detection of the segmented pixels compared to classifier with GMM. The figure also shows that regions can be well identified from the background scene.

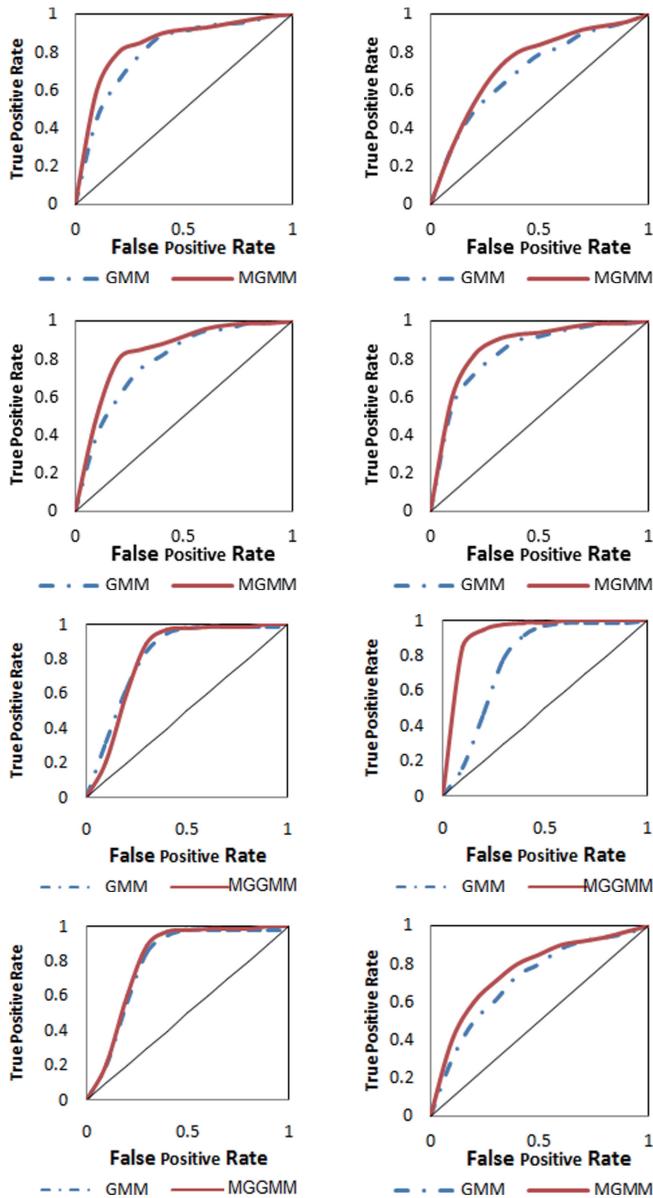


Figure 4: ROC Curves of The GMM and MGGMM Model for different texture images.

## 8 Conclusion

This paper addresses a novel and simple texture segmentation method based on multivariate generalized Gaussian mixture model. Here, the DCT coefficients of

the image are used for extracting the feature vectors. The DCT coefficients utilize the global and local information of the pixels in the image more effectively. Here, the feature vector associated with the texture of the whole image is characterized by zero correlation multivariate generalized Gaussian mixture model. The generalized Gaussian model includes platy, leptokurtic and mesokurtic distributions as particular cases. The updated equations of the EM algorithm for multivariate generalized Gaussian mixture model are derived. The texture segmentation algorithm is developed based on component maximum likelihood under Bayesian frame.

Using 8 images randomly taken from Brodatz texture database, the experimentation is carried for performance evaluation of the proposed method. From the experimentation and computed measures of PRI, GCE and VOI, it is observed that the segmentation obtained through the proposed method is close to the ground truth. A comparative study of the developed model with respect to that of Gaussian mixture model using ROC curves revealed that the performed method works extremely better than the earlier methods.

It is also possible to develop the texture segmentation with a combination of DCT coefficients and local binary patterns (LBP) using multivariate generalized Gaussian mixture model which will be taken up elsewhere. The proposed algorithm is useful for detecting the water bed and animal trajectories in remote sensing images.

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