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On the Discrete-Analytical Solution Method of the Problems Related to the Dynamics of Hydro-Elastic Systems Consisting of a Pre-Strained Moving Elastic Plate, Compressible Viscous Fluid and Rigid Wall

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Abstract: The discrete-analytical solution method is proposed for the solution to problems related to the dynamics of the hydro-elastic system consisting of an axially-moving pre-stressed plate, compressible viscous fluid and rigid wall. The fluid flow caused by the axial movement of the plate and the pre-stresses in the plate are taken into consideration as the initial state of the system under consideration. It is assumed that the additional lineally-located time-harmonic forces act on the plate and these forces cause additional flow field in the fluid and an additional stress-strain state in the plate. The additional stress-strain state in the plate is described by utilizing the equations and relations of the three-dimensional linearized theory of elastic waves in initially stressed elastic bodies. The additional fluid flow field is described with linearized Navier-Stokes equations for compressible viscous fluid. As the fluid flow velocities in the initial state are non-homogeneous, the linearized Navier-Stokes equations have variable coefficients and this situation causes difficulties in obtaining an analytical solution to these equations. The proposed discrete-analytical solution method allows this difficulty to be overcome and for approximate analytical solutions for these types of problems to be obtained. The proposed solution method is examined with respect to concrete problems. Numerical results obtained with the proposed approach are presented and discussed.

Keywords: Discrete-analytical solution method; Compressible viscous fluid; Prestressed elastic plate; Frequency response; Moving plate; Forced vibration.

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1 Introduction

Present levels of aeronautical, astronautical, nuclear, chemical, biological, mechanical and civil engineering require more detailed and exact study of the dynamic problems related to plate - fluid interaction, taking into consideration the viscosity and compressibility of the fluid as well as the description of the motion of the plate within the scope of the exact three-dimensional equations and relations of deformable body mechanics. As usual in most investigations carried out in this field, the fluid is modeled as inviscid and the plate motion is described within the scope of various approximate plate theories. Consequently, such approaches restrict the application area of the obtained results. Now we consider a brief review of the literature.

The first attempt in this field was made by Lamb (1921) in which vibrations of a circular elastic "baffled" plate in contact with still water, which was modeled as inviscid fluid, were studied. It used the so-called "non-dimensional added virtual mass incremental" (NAVMI) method which has also been employed for the solution to plate-fluid interaction problems. According to this method, it is assumed that the modes of vibration of the plate in contact with still water are the same as those in a vacuum, and the natural frequency is determined by the use of the Rayleigh quotient, i.e. natural frequencies of the plate are equated to the ratio between the maximum potential energy of the plate and the sum of the kinetic energies of both the plate and the fluid.

The NAVMI method has also been employed in many related investigations such as in papers by Kwak and Kim (1991), Fu and Price (1987), Zhao and Yu (2012) and in many others listed therein. It should also be noted that there have also been investigations (see, for instance, papers by Tubaldi and Armabili (2013), Charman and Sorokin (2005) and others listed therein) which have been carried out without employing the NAVMI method.

Another aspect of investigations related to plate-fluid interaction regarding wave propagation problems was studied in a paper by Sorokin and Chubinskij (2008) and others listed therein. It should be noted that before this paper the problems of time harmonic linear wave propagation in elastic structure-fluid systems were investigated within the framework of the theory of compressible inviscid fluid. Sorokin and Chubinskij (2008) also first investigated the role of fluid viscosity in wave propagation in the plate-fluid system. However, in this paper and all the papers indicated above, the equations of motion of the plate were written within the scope of approximate plate theories using various types of hypotheses, such as the Kirchhoff hypothesis for plates. Moreover, in the foregoing investigations (except the paper by Zhao and Yu (2012) the initial strains (or stresses) in plates, which can be one of their characteristics, were not taken into account. These two characteristics, namely the use of the exact equations of plate motion and the existence of initial stresses in the plate were taken into consideration in a paper by Bagno, Guz, and Shchuruk (1994) and others, a review of which is given in a survey paper by Bagno and Guz (1997). Note that in these papers, in studying wave propagation in pre-stressed plate+ compressible viscous fluid systems, the motion of the plate was written within the scope of the so-called three-dimensional linearized theory of elastic waves in initially-stressed bodies while the motion of the viscous fluid was written within the scope of the linearized Navier-Stokes equations. Detailed consideration of related results was made in the monograph by Guz (2009).

Until recently, within this framework, there has been no investigation related to the forced vibration of the pre-strained plate+ compressible viscous fluid system. The first attempt in this field was made in a work by Akbarov (2013b) in which the frequency response of the system consisting of the pre-stressed metal elastic plate and a half-plane with compressible viscous fluid was studied. The subsequent step in this field was made in a paper by Akbarov and Ismailov (2014a) which considered the forced vibrations of a system consisting of a pre-stressed highly elastic plate under compressible viscous fluid loading. In this paper it was also assumed that the half-plane which is in contact with the plate is filled with a compressible viscous fluid. Moreover, in another paper by Akbarov and Ismailov (2014b) the foregoing investigations were developed for the case where the plate material is viscoelastic. It was assumed that the viscoelasticity was described by fractional exponential operators.

However, in all the foregoing papers related to the interaction of the plate and compressible viscous fluid it was assumed that the fluid is at rest. At the same time, many cases can exist in which the plate is in contact with the flowing fluid, which, as usual, is non-homogeneous, before the action of external forces, i.e. the velocities of the fluid flow depend on the space coordinates. According to this statement, the linearized Navier-Stokes equations describing the perturbation field in the fluid become equations with variable coefficients. The variability of the coefficients causes serious difficulties in obtaining an analytical solution to these equations. In the present paper we attempt to develop a discrete-analytical solution to these equations which uses an analytical solution method to investigate a class of problems related to the dynamic interaction of the plate with a flowing compressible viscous fluid. The proposed approach is examined for the hydro-elastic system consisting of the axially-moving pre-stressed plate, compressible viscous fluid and rigid wall.

2 Formulation of the problem

Consider a hydro-elastic system consisting of the pre-strained and axially-moving elastic plate, compressible barotropic viscous fluid and rigid wall. We introduce the Cartesian coordinate system $Ox_1x_2x_3$ which is fixed on, and moves with, the plate and we also introduce the Cartesian coordinate system $O_0x_{10}x_{20}x_{30}$ which is associated with the rigid wall (Fig. 1). Considered below is the two-dimensional problem in the plane Ox_1x_2 (or in the plane $O_0x_{10}x_{20}$). Therefore, in Fig. 1 the coordinate axes Ox_3 and O_0x_{30} are not shown and according to Fig. 1, the plate occupies the region $\{|x_1| < \infty, -h < x_2 < 0\}$ and the fluid occupies the region $\{|x_1| < \infty, -h_d - h < x_2 < -h\}$. Thus we assume that the plate moves in the direction of the Ox_1 (or O_0x_{10}) axis with constant velocity V and this movement causes a corresponding flow of the fluid. According to the foregoing assumptions, there exists the following relation between the coordinates x_i and x_{i0}

$$x_1 = x_{10} - Vt, \quad x_2 = x_{20} \tag{1}$$

where t is the time.

According to Fig. 1 and the notation shown therein, we can write the following well-known expression for the fluid-flow velocity caused by the plate's axial movement.

$$v_1^0 = V\frac{x_{20}}{h_d} + V(\frac{h}{h_d} + 1), \quad v_2^0 = 0. \tag{2}$$

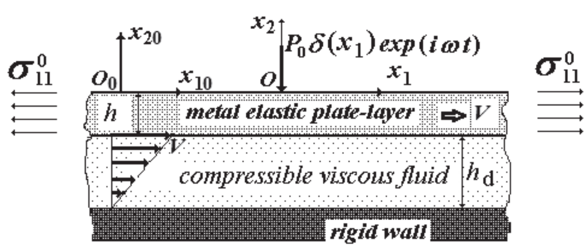


Figure 1: Sketch of the hydro-elastic system under consideration.

At the same time, we assume that the initial stresses in the plate are caused by the uniaxial stretching or compressing along the Ox_1 axis and are determined within the scope of the classical linear theory of elasticity, according to which, $\sigma_{11}^0 \neq 0$ and $\sigma_{ij}^0 = 0$ if $ij \neq 11$.

Thus, within the scope of the foregoing expressions and assumptions we have determined the initial state in the hydro-elastic system under consideration. Now we attempt to investigate the forced vibration of this system with the foregoing initial state caused by the additional lineally-located time-harmonic forces acting on the moving plate, as shown in Fig. 1. We will assume that the amplitudes of the fluid flow velocities caused by the additional time-harmonic force are significantly less than the plate moving velocity V. Moreover, we assume that the amplitudes of the stresses caused by the aforementioned additional force are also significantly less than the absolute value of the initial stress σ_{11}^{0}. Consequently, perturbation of the motion of the hydro-elastic system under consideration can be described within the scope of the linearized equations. Therefore we use the three-dimensional linearized theory of elastic waves in initially stressed elastic bodies (TDLTEWISB) to describe perturbation of the plate motion and use the linearized Navier-Stokes equations for compressible viscous fluid to describe perturbation of the fluid flow.

According to Akbarov (2015) and Guz (2004), the field equations of the TDLTEWISB in the moving system of coordinates Ox_1x_2, are written as follows:

$$\frac{\partial \sigma_{11}}{\partial x_1}+\frac{\partial \sigma_{12}}{\partial x_2}+\sigma_{11}^{0}\frac{\partial^2 u_1}{\partial x_1^2}=\rho\frac{\partial^2 u_1}{\partial t^2}, \quad \frac{\partial \sigma_{12}}{\partial x_1}+\frac{\partial \sigma_{22}}{\partial x_2}+\sigma_{11}^{0}\frac{\partial^2 u_2}{\partial x_1^2}=\rho\frac{\partial^2 u_2}{\partial t^2}.$$

$$\sigma_{11}=(\lambda+2\mu)\varepsilon_{11}+\lambda\varepsilon_{22}, \quad \sigma_{22}=\lambda\varepsilon_{11}+(\lambda+2\mu)\varepsilon_{22}, \quad \sigma_{12}=2\mu\varepsilon_{12},$$

$$\varepsilon_{11}=\frac{\partial u_1}{\partial x_1}, \quad \varepsilon_{22}=\frac{\partial u_2}{\partial x_2}, \quad \varepsilon_{12}=\frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right) \tag{3}$$

In Eq. (3) conventional notation is used.

Now, according to Guz (2009), we write the linearized Navier-Stokes equations for the compressible viscous fluid in the fixed coordinate system $O_0x_{10}x_{20}$ (Fig. 1):

$$\rho_0^{(1)}\frac{\partial v_1}{\partial t}+\rho_0^{(1)}v_1^0(x_{20})\frac{\partial v_1}{\partial x_{10}}-\mu^{(1)}\left(\frac{\partial^2 v_1}{\partial x_{10}^2}+\frac{\partial^2 v_1}{\partial x_{20}^2}\right)$$
$$-(\lambda^{(1)}+\mu^{(1)})\left(\frac{\partial^2 v_1}{\partial x_{10}^2}+\frac{\partial^2 v_2}{\partial x_{10}\partial x_{20}}\right)+\frac{\partial p^{(1)}}{\partial x_{10}}=0,$$

$$\rho_0^{(1)}\frac{\partial v_2}{\partial t}+\rho_0^{(1)}v_1^0(x_{20})\frac{\partial v_2}{\partial x_{10}}-\mu^{(1)}\left(\frac{\partial^2 v_2}{\partial x_{10}^2}+\frac{\partial^2 v_2}{\partial x_{20}^2}\right)$$
$$-(\lambda^{(1)}+\mu^{(1)})\left(\frac{\partial^2 v_1}{\partial x_{10}\partial x_{20}}+\frac{\partial^2 v_2}{\partial x_{20}^2}\right)+\frac{\partial p^{(1)}}{\partial x_{20}}=0,$$

$$\frac{\partial \rho^{(1)}}{\partial t}+\rho_0^{(1)}\left(\frac{\partial v_1}{\partial x_{10}}+\frac{\partial v_2}{\partial x_{20}}\right)+v_1^0(x_{20})\frac{\partial \rho^{(1)}}{\partial x_{10}}=0,$$

$$T_{11}=(-p^{(1)}+\lambda^{(1)}\theta)+2\mu^{(1)}e_{11},$$

$$T_{22}=(-p^{(1)}+\lambda^{(1)}\theta)+2\mu^{(1)}e_{22}, \quad T_{12}=2\mu^{(1)}e_{12},$$

$$\theta = \frac{\partial v_1}{\partial x_{10}} + \frac{\partial v_2}{\partial x_{20}}, \quad e_{11} = \frac{\partial v_1}{\partial x_{10}}, \quad e_{22} = \frac{\partial v_2}{\partial x_{20}},$$
$$e_{12} = \frac{1}{2}\left(\frac{\partial v_1}{\partial x_{20}} + \frac{\partial v_2}{\partial x_{10}}\right), \quad a_0^2 = \frac{\partial p^{(1)}}{\partial \rho^{(1)}}. \tag{4}$$

where$\rho_0^{(1)}$ is the fluid density before perturbation. The other notation used in Eq. (4) is also conventional.

Moreover, it is assumed that the following boundary, contact and impermeability conditions are satisfied:

$$\sigma_{21}(t,x_1,x_2)|_{x_2=0} = 0, \, \sigma_{22}(t,x_1,x_2)|_{x_2=0} = -P_0\delta(x_1)e^{i\omega t}, \tag{5}$$

$$\left.\frac{\partial u_1(t,x_1,x_2)}{\partial t}\right|_{\substack{x_1=x_{10}-Vt \\ x_2=x_{20}=-h}} = v_1(t,x_{10},x_{20})|_{x_2=x_{20}=-h},$$
$$\left.\frac{\partial u_2(t,x_1,x_2)}{\partial t}\right|_{\substack{x_1=x_{10}-Vt \\ x_2=x_{20}=-h}} = v_2(t,x_{10},x_{20})|_{x_2=x_{20}=-h},$$
$$\sigma_{21}(t,x_1,x_2)|_{\substack{x_1=x_{10}-Vt \\ x_2=x_{20}=-h}} = T_{21}(t,x_{10},x_{20})|_{x_2=x_{20}=-h},$$
$$\sigma_{22}(t,x_1,x_2)|_{\substack{x_1=x_{10}-Vt \\ x_2=x_{20}=-h}} = T_{22}(t,x_{10},x_{20})|_{x_2=x_{20}=-h},$$
$$v_1(t,x_{10},x_{20})|_{x_2=x_{20}=-h-h_d} = 0, \quad v_2(t,x_{10},x_{20})|_{x_2=x_{20}=-h-h_d} = 0. \tag{6}$$

This completes formulation of the problem.

3 Method of solution

As noted above, the field equations describing the fluid motion and given in (4) are written for the fixed coordinate system $O_0x_{10}x_{20}$. However, the field equations describing the plate motion and given in (3) are written with respect to the coordinate system Ox_1x_2 which is fixed on the plate's upper plane and moves with the plate with respect to the rigid wall or the coordinate system $O_0x_{10}x_{20}$. First, using the relations (1) and $g(x_{10},x_{20}) = g(x_1+Vt,x_2) = \tilde{g}(x_1,x_2)$, and omitting the over symbol "$\sim$", below we rewrite the field equations in (4) and the contact and impermeability conditions in (6) in the moving coordinate system Ox_1x_2. For this purpose we must replace the derivatives $\partial/\partial t$, $\partial/\partial x_{10}$ and $\partial/\partial x_{20}$ in (4) with $\partial/\partial t - V\partial/\partial x_1$, $\partial/\partial x_1$ and $\partial/\partial x_2$, respectively. As a result of these replacements, we obtain the following equations instead of the equations given in (4):

$$\rho_0^{(1)}\frac{\partial v_1}{\partial t} + \rho_0^{(1)}(v_1^0(x_2) - V)\frac{\partial v_1}{\partial x_1} - \mu^{(1)}\left(\frac{\partial^2 v_1}{\partial x_1^2} + \frac{\partial^2 v_1}{\partial x_2^2}\right)$$

$$-(\lambda^{(1)}+\mu^{(1)})\left(\frac{\partial^2 v_1}{\partial x_1^2}+\frac{\partial^2 v_2}{\partial x_1 \partial x_2}\right)+\frac{\partial p^{(1)}}{\partial x_1}=0,$$

$$\rho_0^{(1)}\frac{\partial v_2}{\partial t}+\rho_0^{(1)}(v_1^0(x_2)-V)\frac{\partial v_2}{\partial x_1}-\mu^{(1)}\left(\frac{\partial^2 v_2}{\partial x_1^2}+\frac{\partial^2 v_2}{\partial x_2^2}\right)$$

$$-(\lambda^{(1)}+\mu^{(1)})\left(\frac{\partial^2 v_1}{\partial x_1 \partial x_2}+\frac{\partial^2 v_2}{\partial x_2^2}\right)+\frac{\partial p^{(1)}}{\partial x_2}=0,$$

$$\frac{\partial \rho^{(1)}}{\partial t}+\rho_0^{(1)}\left(\frac{\partial v_1}{\partial x_1}+\frac{\partial v_2}{\partial x_2}\right)+(v_1^0(x_2)-V)\frac{\partial \rho^{(1)}}{\partial x_1}=0,$$

$$\begin{aligned} &T_{11}=(-p^{(1)}+\lambda^{(1)}\theta)+2\mu^{(1)}e_{11},\\ &T_{22}=(-p^{(1)}+\lambda^{(1)}\theta)+2\mu^{(1)}e_{22}, \quad T_{12}=2\mu^{(1)}e_{12},\\ &\theta=\frac{\partial v_1}{\partial x_1}+\frac{\partial v_2}{\partial x_2}, \quad e_{11}=\frac{\partial v_1}{\partial x_1}, \quad e_{22}=\frac{\partial v_2}{\partial x_2},\\ &e_{12}=\frac{1}{2}\left(\frac{\partial v_1}{\partial x_2}+\frac{\partial v_2}{\partial x_1}\right), \quad a_0^2=\frac{\partial p^{(1)}}{\partial \rho^{(1)}}, \end{aligned} \tag{7}$$

and the following contact and impermeability conditions instead of (6).

$$\begin{aligned} &\left.\frac{\partial u_1(t,x_1,x_2)}{\partial t}\right|_{x_2=-h}=\left.v_1(t,x_1,x_2)\right|_{x_2=-h},\\ &\left.\frac{\partial u_2(t,x_1,x_2)}{\partial t}\right|_{x_2=-h}=\left.v_2(t,x_1,x_2)\right|_{x_2=-h},\\ &\left.\sigma_{21}(t,x_1,x_2)\right|_{x_2=-h}=\left.T_{21}(t,x_1,x_2)\right|_{x_2=-h},\\ &\left.\sigma_{22}(t,x_1,x_2)\right|_{x_2=-h}=\left.T_{22}(t,x_1,x_2)\right|_{x_2=-h},\\ &\left.v_1(t,x_1,x_2)\right|_{x_2=-h-h_d}=0, \quad \left.v_2(t,x_1,x_2)\right|_{x_2=-h-h_d}=0 \end{aligned} \tag{8}$$

In this way, we have all the field equations and relations in the moving system of coordinates Ox_1x_2 and, according to the boundary condition (5), we can represent all sought quantities as $d(t,x_1,x_2)=\bar{d}(x_1,x_2)e^{i\omega t}$ (over-bar will be omitted below) in this coordinate system. Substituting this into the foregoing equations and conditions we obtain the corresponding equations and relations for the amplitudes of the sought values. Moreover, after this procedure we apply the exponential Fourier transformation

$$f_F(s,x_2)=\int_{-\infty}^{+\infty} f(x_1,x_2)e^{-isx_1}dx_1 \tag{9}$$

to the foregoing equations and relations. As a result, we obtain field equations and relations for the Fourier transformations of the sought values. The equations and relations related to the plate are obtained from (3) and (9) as follows:

$$is\sigma_{11F}+\frac{d\sigma_{12F}}{dx_2}+(\rho\omega^2-s^2\sigma_{11}^0)u_{1F}=0,\quad is\sigma_{12F}+\frac{d\sigma_{22F}}{dx_2}+(\rho\omega^2-s^2\sigma_{11}^0)u_{2F}=0$$

$$\sigma_{11F}=(\lambda+2\mu)\varepsilon_{11F}+\lambda\varepsilon_{22F},\quad \sigma_{22F}=\lambda\varepsilon_{11F}+(\lambda+2\mu)\varepsilon_{22F},\quad \sigma_{12F}=2\mu\varepsilon_{12F},$$

$$\varepsilon_{11F}=isu_{1F},\quad \varepsilon_{22F}=\frac{du_{2F}}{dx_2},\quad \varepsilon_{12F}=\frac{1}{2}\left(\frac{du_{1F}}{dx_2}+isu_{2F}\right). \tag{10}$$

The corresponding equations and relations related to the fluid are obtained from (7) and (9) as follows:

$$\rho_0^{(1)}i\omega v_{1F}+\rho_0^{(1)}V_1^0(x_2)\ is\ v_{1F}-\mu^{(1)}\left(-s^2v_{1F}+\frac{d^2v_{1F}}{dx_2^2}\right)$$
$$-(\lambda^{(1)}+\mu^{(1)})\left(-s^2v_{1F}+is\frac{dv_{2F}}{dx_2}\right)+isp_F^{(1)}=0,$$
$$\rho_0^{(1)}i\omega v_{2F}+\rho_0^{(1)}V_1^0(x_2)isv_{2F}-\mu^{(1)}\left(-s^2v_{2F}+\frac{d^2v_{2F}}{dx_2^2}\right)$$
$$-(\lambda^{(1)}+\mu^{(1)})\left(is\frac{dv_{1F}}{dx_2}+\frac{d^2v_{2F}}{dx_2^2}\right)+\frac{dp_F^{(1)}}{dx_2}=0,$$
$$i\omega\rho_F^{(1)}+\rho_0^{(1)}\left(isv_{1F}+\frac{dv_{2F}}{dx_2}\right)+V_1^0(x_2)is\rho_F^{(1)}=0,$$

$$V_1^0(x_2)=v_1^0(x_2)-V,$$
$$T_{11F}=(-p_F^{(1)}+\lambda^{(1)}\theta_F)+2\mu^{(1)}e_{11F},$$
$$T_{22F}=(-p_F^{(1)}+\lambda^{(1)}\theta_F)+2\mu^{(1)}e_{22F},\quad T_{12F}=2\mu^{(1)}e_{12F},$$
$$\theta_F=isv_{1F}+\frac{dv_{2F}}{dx_2},\quad e_{11F}=isv_{1F},\quad e_{22F}=\frac{dv_{2F}}{dx_2},$$
$$e_{12F}=\frac{1}{2}\left(\frac{dv_{1F}}{dx_2}+isv_{2F}\right),\quad a_0^2=\frac{\partial p^{(1)}}{\partial\rho^{(1)}}, \tag{11}$$

Now we consider determination of the solutions to the system of equations (10) and (11). It should be noted that finding the analytical solution to the system of e-quations (10) is not difficult. So, after some mathematical manipulations we obtain from (10) the following equations for u_{1F} and u_{2F}:

$$Au_{1F}-B\frac{du_{2F}}{dx_2}+C\frac{d^2u_{1F}}{dx_2^2}=0,\quad Du_{2F}+B\frac{du_{1F}}{dx_2}+G\frac{d^2u_{2F}}{dx_2^2}=0, \tag{12}$$

where

$$A = X^2 - s^2\omega_{1111}, \quad B = s(\omega_{1122} + \omega_{2121}), \quad C = \omega_{2112},$$
$$D = X^2 - s^2\omega_{1221}, \quad G = \omega_{2222}, X^2 = \omega^2 h^2/c_2^2, \quad c_2 = \sqrt{\mu/\rho}. \tag{13}$$

Introducing the notation

$$A_0 = \frac{AG + B^2 + CD}{CG}, \quad B_0 = \frac{BD}{CG},$$
$$k_1 = \sqrt{-\frac{A_0}{2} + \sqrt{\frac{A_0^2}{4} - B_0}}, \quad k_2 = \sqrt{-\frac{A_0}{2} - \sqrt{\frac{A_0^2}{4} - B_0}}, \tag{14}$$

we can write the solution of the equation (12) as follows:

$$u_{2F} = Z_1 e^{k_1 x_2} + Z_2 e^{-k_1 x_2} + Z_3 e^{k_2 x_2} + Z_4 e^{-k_2 x_2},$$
$$u_{1F} = Z_1 a_1 e^{k_1 x_2} + Z_2 a_2 e^{-k_1 x_2} + Z_3 a_3 e^{k_2 x_2} + Z_4 a_4 e^{-k_2 x_2}, \tag{15}$$

where

$$a_1 = \frac{-D - Gk_1^2}{Bk_1^2}, \quad a_2 = -a_1, \quad a_3 = \frac{-D - Gk_2^2}{Bk_2^2}, \quad a_4 = -a_3. \tag{16}$$

However, to find the analytical solution to the system of equations (11) is not so simple because this system contains the variable coefficient $V_1^0(x_2)$. Therefore, in the present work we attempt to employ the discrete-analytical solution method to solve this system of equations, the essence of which is as follows:

The strip $S = [-h_d - h \le x_2 \le -h]$ which is filled with the fluid is divided into the following M sub-strips

$$S_k = \left[-k\frac{h_d}{M} - h \le x_2 \le -(k-1)\frac{h_d}{M} - h\right], \quad k = 1, 2, \ldots, M, \quad S = \sum_{k=1}^{M} S_k \tag{17}$$

and it is assumed that in each of these strips, the function $V_1^0(x_2)$ is constant and equal to

$$V_1^{0(k)} = V_1^0(x_2)\big|_{x_2 = -h-(2k-1)h_d/(2M)}. \tag{18}$$

Taking the relations (17) and (18) into consideration, we suppose that the system of equations in (11) are satisfied separately within each strip S_k so that within each strip we obtain the following system of equations with constant coefficients:

$$\rho_0^{(1)} i\omega v_{1F}^{(k)} + \rho_0^{(1)} V_1^{0(k)} \ is \ v_{1F}^{(k)} - \mu^{(1)}\left(-s^2 v_{1F}^{(k)} + \frac{d^2 v_{1F}^{(k)}}{dx_2^2}\right)$$

$$
\begin{aligned}
&-(\lambda^{(1)}+\mu^{(1)})\left(is\frac{dv_{2F}^{(k)}}{dx_2}-s^2v_{1F}^{(k)}\right)+isp_{Fk}^{(1)}=0,\\
&\rho_0^{(1)}i\omega v_{2F}^{(k)}+\rho_0^{(1)}V_1^{0(k)}isv_{2F}^{(k)}-\mu^{(1)}\left(\frac{d^2v_{2F}^{(k)}}{dx_2^2}-s^2v_{2F}^{(k)}\right)\\
&-(\lambda^{(1)}+\mu^{(1)})\left(is\frac{dv_{1F}^{(k)}}{dx_2}+\frac{d^2v_{2F}^{(k)}}{dx_2^2}\right)+\frac{dp_{Fk}^{(1)}}{dx_2}=0,\\
&i\omega\rho_{Fk}^{(1)}+\rho_0^{(1)}\left(isv_{1F}^{(k)}+\frac{dv_{2F}^{(k)}}{dx_2}\right)+V_1^{0(k)}is\rho_{Fk}^{(1)}=0,\\
&T_{11F}^{(k)}=(-p_{Fk}^{(1)}+\lambda^{(1)}\theta_F^{(k)})+2\mu^{(1)}e_{11F}^{(k)},\\
&T_{22F}^{(k)}=(-p_{Fk}^{(1)}+\lambda^{(1)}\theta_F^{(k)})+2\mu^{(1)}e_{22F}^{(k)},\quad T_{12F}^{(k)}=2\mu^{(1)}e_{12F}^{(k)},\\
&\theta_F^{(k)}=isv_{1F}^{(k)}+\frac{dv_{2F}^{(k)}}{dx_2},\quad e_{11F}^{(k)}=isv_{11F}^{(k)},\quad e_{22F}^{(k)}=\frac{dv_{2F}^{(k)}}{dx_2},\\
&e_{12F}^{(k)}=\frac{1}{2}\left(\frac{dv_{1F}^{(k)}}{dx_2}+isv_{2F}^{(k)}\right),\quad \text{for}\quad 1\le k\le M.
\end{aligned}
\tag{19}
$$

In (19), through the upper and lower index k, the corresponding quantities belonging to the $k-th$ sub-strip are shown. In this way, we obtain the system of equations (19) with constant coefficients instead of the system of equations (11) with variable coefficients. Moreover we assume that on the upper plane of the plate, on the interface between the strip S_1 and plate, on the interfaces between the strips $S_1,\ldots,S_M$ (17) and on the interface between the strip S_M and rigid wall, the following conditions are satisfied:

$$
\begin{aligned}
&\sigma_{21F}(s,x_2)|_{x_2=0}=0,\quad \sigma_{22F}(s,x_2)|_{x_2=0}=-P_0,\\
&i\omega u_{1F}(s,x_2)|_{x_2=-h}=v_{1F}^{(1)}(s,x_2)\Big|_{x_2=-h},\quad i\omega u_{2F}(s,x_2)|_{x_2=-h}=v_{1F}^{(2)}(s,x_2)\Big|_{x_2=-h},\\
&\sigma_{21F}(s,x_2)|_{x_2=-h}=T_{21F}^{(1)}(s,x_2)\Big|_{x_2=-h},\quad \sigma_{22F}(s,x_2)|_{x_2=-h}=T_{22F}^{(1)}(s,x_2)\Big|_{x_2=-h},\\
&\left\{v_{1F}^{(k-1)}(s,x_2),v_{2F}^{(k-1)}(s,x_2),T_{21F}^{(k-1)}(s,x_2),T_{22F}^{(k-1)}(s,x_2)\right\}\Big|_{x_2=-h-(k-1)h_d/M}=\\
&\left\{v_{1F}^{(k-1)}(s,x_2),v_{2F}^{(k)}(s,x_2),T_{21F}^{(k)}(s,x_2),T_{22F}^{(k)}(s,x_2)\right\}\Big|_{x_2=-h-(k-1)h_d/M},\quad 2\le k\le M\\
&v_{1F}^{(M)}(s,x_2)\Big|_{x_2=-h-h_d/M}=0,\quad v_{2F}^{(M)}(s,x_2)\Big|_{x_2=-h-h_d/M}=0
\end{aligned}
\tag{20}
$$

Now we attempt to find the solution to the system of equations in (19). According to Guz (2009), the solution to the system of equations related to each k-th system of equations in (19) is reduced to finding the two potentials $\varphi_F^{(k)}$ and $\psi_F^{(k)}$ which are determined from the following equations:

$$\left[\left(1+\frac{\lambda^{(1)}+2\mu^{(1)}}{a_0^2\rho_0^{(1)}}i(\omega+sV_1^{(k)})\right)\Delta_F+\frac{(\omega+sV_1^{(k)})^2}{a_0^2}\right]\varphi_F^{(k)}=0,$$

$$\left(\nu^{(1)}\Delta_F-i(\omega+sV_1^{(k)})\right)\psi_F^{(k)}=0, \quad \Delta_F=-s^2+\frac{\partial^2}{\partial x_2^2}, \tag{21}$$

where $\nu^{(1)}$ is the kinematic viscosity, i.e. $\nu^{(1)}=\mu^{(1)}/\rho_0^{(1)}$.
The Fourier transformations of the velocities $v_{1F}^{(k)}$ and $v_{2F}^{(k)}$, and the pressure $p_{Fk}^{(1)}$ are expressed by the potentials $\varphi_F^{(k)}$ and $\psi_F^{(k)}$ as below.

$$v_{1F}^{(k)}=is\varphi_F^{(k)}+\frac{d\psi_F^{(k)}}{dx_2}, \quad v_2^{(k)}=\frac{d\varphi_F^{(k)}}{dx_2}-is\psi_F^{(k)},$$

$$p_{Fk}^{(1)}=\rho_0^{(1)}\left(\frac{\lambda^{(1)}+2\mu^{(1)}}{\rho_0^{(1)}}\Delta_F-i(\omega+sV_1^{(k)})\right)\varphi_F^{(k)}. \tag{22}$$

We consider determination of $\varphi_F^{(k)}$ and $\psi_F^{(k)}$ from the equation (21). Introducing the notation

$$\varphi_F=(\omega+sV_1^{(k)})h^2\tilde{\varphi}_F, \quad \psi_F=(\omega+sV_1^{(k)})h^2\tilde{\psi}_F \tag{23}$$

according to Guz (2009) and assuming that $\lambda^{(1)}=-2\mu^{(1)}/3$ it can be written from (21) that

$$\frac{d^2\tilde{\varphi}_F^{(k)}}{dx_2^2}+\left(\frac{\Omega_1^2}{1+i4\Omega_1^2/(3N_w^2)}-s^2\right)\tilde{\varphi}_F^{(k)}=0, \quad \frac{d^2\tilde{\psi}_F^{(k)}}{dx_2^2}-\left(s^2+iN_w^2\right)\tilde{\psi}_F^{(k)}=0, \tag{24}$$

where

$$\Omega_1=\frac{\omega h}{a_0}+\frac{hsV_1^{(k)}}{a_0}=\Omega_{10}+\Omega_{1s}, \quad \Omega_{10}=\frac{\omega h}{a_0}, \quad \Omega_{1s}=\frac{hsV_1^{(k)}}{a_0},$$

$$N_w^2=\frac{\omega h^2}{\nu^{(1)}}+\frac{sV_1^{(k)}h^2}{\nu^{(1)}}=N_{w0}^2+N_{ws}^2. \quad N_{w0}^2=\frac{\omega h^2}{\nu^{(1)}}, N_{ws}^2=\frac{sV_1^{(k)}h^2}{\nu^{(1)}}. \tag{25}$$

The dimensionless parameter N_w in (25) can be taken as the parameter which characterizes the influence of the fluid viscosity on the mechanical behavior of the system while the dimensionless frequency Ω_1 in (25) can be taken as the parameter which characterizes the influence of the compressibility of the fluid on the mechanical behavior of the system.

Thus, the solutions to the equations in (24) are found as follows:

$$\tilde{\varphi}_F^{(k)} = Z_5^{(k)} e^{\delta_1 x_2} + Z_7^{(k)} e^{-\delta_1 x_2}, \quad \tilde{\psi}_F^{(k)} = Z_6^{(k)} e^{\gamma_1 x_2} + Z_8^{(k)} e^{-\gamma_1 x_2}, \tag{26}$$

where

$$\delta_1 = \sqrt{s^2 - \frac{\Omega_1^2}{1 + i4\Omega_1^2/(3N_w^2)}}, \quad \gamma_1 = \sqrt{s^2 + iN_w^2}. \tag{27}$$

Using (26), (23) and (22) we obtain the following expressions for the Fourier transformations of the fluid velocities:

$$v_{1F}^{(k)} = (\omega + sV_1^{(k)})h\left[-Z_5^{(k)} s e^{\delta_1 x_2} - Z_7^{(k)} s e^{-\delta_1 x_2} + Z_6^{(k)} e^{\gamma_1 x_2} + Z_8^{(k)} e^{-\gamma_1 x_2}\right],$$
$$v_{2F}^{(k)} = (\omega + sV_1^{(k)})h\left[Z_5^{(k)} \delta_1 e^{\delta_1 x_2} - Z_7^{(k)} \delta_1 e^{-\delta_1 x_2} - Z_6^{(k)} s e^{\gamma_1 x_2} - Z_8^{(k)} s e^{-\gamma_1 x_2}\right]. \tag{28}$$

Substituting the expressions (28) into the relations for the stresses given in (19) we obtain the expressions for the stresses in each $k-th$ layer. It follows from the solutions (15) and (28) that they contain the unknown constants Z_1, Z_2, Z_3, Z_4, $Z_5^{(k)}$, $Z_6^{(k)}$, $Z_7^{(k)}$ and $Z_8^{(k)}$ $(k = 1, 2, \ldots, M)$ for which we obtain a closed system of linear algebraic equations from the conditions given in (20). In this way we completely determine the Fourier transformation of the sought quantities, after which these quantities are found from the inverse transformation.

$$\left\{\sigma_{22}, \sigma_{11}, u_1, u_2, T_{22}^{(k)}, T_{11}^{(k)}, T_{12}^{(k)}, v_1^{(k)}, v_2^{(k)}\right\} =$$
$$\frac{1}{2\pi} Re\left\{e^{i\omega t}\int_{-\infty}^{+\infty}\left[\sigma_{22F}, \sigma_{11F}, u_{1F}, u_{2F}, T_{22F}^{(k)}, T_{11F}^{(k)}, T_{12F}^{(k)}, v_{1F}^{(k)}, v_{2F}^{(k)}\right] e^{isx_1} ds\right\} \tag{29}$$

It should be noted that under the foregoing solution procedure the number M is determined from the convergence requirement of the numerical results obtained from the calculation of the integrals in (29).

Note that the solution approach developed here was also used in a paper by Akbarov (2006) for investigation of the frequency response of a strip made of functionally-graded material and resting on a rigid foundation. The results obtained in that paper were also discussed in the monograph by Akbarov (2015) and a paper by Akbarov (2013a). Therefore the solution method proposed above can also be considered as a development of the method presented in Akbarov's (2006) paper for the considered type of hydro-elastic problems. Now we attempt to employ the proposed solution method to obtain numerical results for concrete selected problem parameters.

4 Numerical results and discussions

We assume that the material of the plate-layer is Steel with mechanical constants: $\mu = 79 \times 10^9 Pa$, $\lambda = 94.4 \times 10^9 Pa$ and density $\rho = 1160 kg/m^3$ (Guz and Makhort (2000), Guz (2004)), but the material of the fluid is Glycerin with viscosity coefficient $\mu^{(1)} = 1,393 kg/(m \cdot s)$, density $\rho = 1260 kg/m^3$ and sound speed $a_0 = 1927 m/s$ (Guz (2009)). We also introduce the notation $c_2 = \sqrt{\mu/\rho}$ which is the shear wave propagation velocity in the plate material. After selection of these materials, the dimensionless parameters such as Ω_1 and N_w in (25) and dimensionless parameter M_ω $(= \mu^{(1)}\omega/\mu)$ which arises in contact conditions between the S_1 fluid sub-strip and the plate, can be determined through the three quantities: h (the thickness of the plate-layer), h_d (the thickness of the fluid strip) and ω (the frequency of the time-harmonic external forces). At the same time, it should be noted that the main parameter of the present investigation is the plate moving velocity V in the initial state. As a result of this moving velocity, all solution difficulties and new mechanical effects appear, as discussed below. Therefore, in all the numerical investigations the focus is on the influence of the moving velocity V on the frequency response of the hydro-elastic system under consideration. For simplicity, we consider the case where the initial stress in the plate is absent, i.e. we consider only the case where $\sigma_{11}^0 = 0$.

Before discussion of the numerical results we note that under calculation procedures, the improper integral $\int_{-\infty}^{+\infty} f(s) e^{isx_1} ds$ in (29) is replaced by the corresponding definite integrals $\int_{-S_1^*}^{+S_1^*} f(s) e^{isx_1} ds$. The values of S_1^* are determined from the convergence requirement of the numerical results. Note that under calculation of the integral $\int_{-S_1^*}^{+S_1^*} f(s) e^{isx_1} ds$, the integration interval $[-S_1^*; +S_1^*]$ is further divided into a certain number of shorter intervals, which are used in the Gauss integration algorithm. The values of the integrated expressions at the sample points are calculated through the equations (20) and expressions (15), (10), (28) and (19). All procedures were performed automatically with the PC programs constructed by the authors in MATLAB.

Thus, we begin discussion of the numerical results with consideration of the convergence of the calculation algorithm.

4.1 Convergence of the numerical algorithm

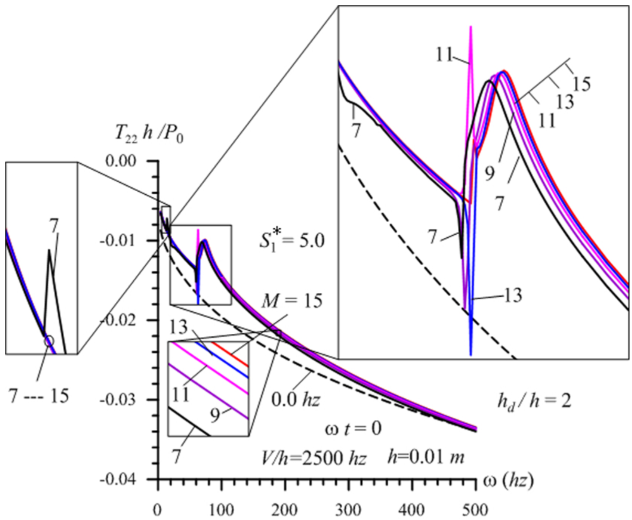
Under numerical investigation we assume that $0 \leq V/h \leq 2500 (1/s)$ and $h = 0.01 m$, according to which $0 \leq V \leq 90 km/h$ and this change diapason of the plate-moving velocity is quite real. Moreover, we assume that $4 hs \leq \omega \leq 500 hs$.

The convergence of the numerical results will be tested with respect to the three parameters: M which shows the number of fluid sub-strips, S_1 and the number of the shorter intervals (denoted by N) to which the integration interval $[-S_1^*; +S_1^*]$ is divided. Note that the convergence with respect to S_1^* and N will be discussed below. Now we consider the numerical results illustrating their convergence with respect to the sub-strip number M. These results relate to the frequency response of the normal stress and velocity which arise on the interface plane between the plate and fluid. We recall that this stress and velocity are caused by the action of the additional time-harmonic force (Fig. 1). Thus, we consider the graphs given in Figs. 2 and 3 which are constructed in the cases where $h_d/h = 2$ and 6, respectively and illustrate the frequency response of $T_{22}h/P_0$ (Figs. 2a and 3a) and $v_2\mu h/(P_0c_2)$ (Figs. 2b and 3b). Note that these graphs are obtained for various values of the number M in the case where $V/h = 2500hz$, $h = 0.01m$, $\omega t = 0$ and $x_1/h = 0$ under $S_1^* = 5$ and $N = 2000$.

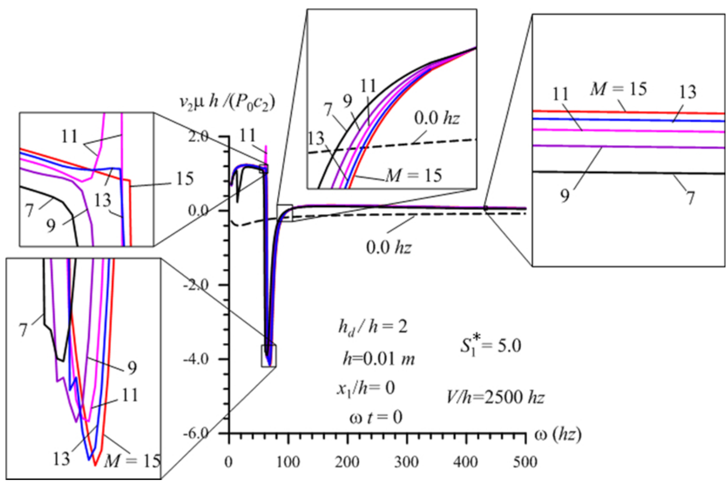
It follows from the numerical results given in Figs. 2 and 3, (and others which are not given here) that for the considered range of the problem parameters the difference between the results obtained in the cases where $M = 13$ and $M = 15$ is less than 10^{-5}. Therefore, we can conclude that for the cases considered in the present paper, the case where $M = 15$ is sufficient to obtain guaranteed numerical results in the sense of their convergence with respect to the number M. According to this conclusion, all the numerical results, which will be discussed below, have been obtained in the case where $M = 15$. At the same time, it should be noted that the final value of the number M depends significantly on the plate moving velocity, i.e. on the ratio V/h and it is evident that the final value of the number M must increase with V/h.

At the same time, it follows from the graphs that the results approach a certain limit with the number M and this means that the proposed approach is justified for the problem under consideration. Similar results are also obtained for the other problem parameters. Moreover, these results (and others which are not given here) show that as a result of the plate moving there exists such a value of the frequency under which a jump arises in the values of the studied quantities. We will call this frequency a "critical frequency" (denoted by ω_{cr}) and provide some analysis below.

Now we analyze the convergence of the numerical results with respect to the number N. The results obtained for various possible values of the problem parameters show that the very disadvantaged cases in the convergence sense arise under low frequencies of the external force and under small values of the ratio h_d/h. Moreover, these disadvantages become more considerable in the cases where the frequency of the external force is near to the critical frequency. Therefore, for illustration of this convergence, we consider the case where $4hz \leq \omega \leq 100hz$ and $h_d/h = 2$, and analyze the graphs given in Fig. 4a which illustrate the frequency response of the dimensionless stress $T_{22}h/P_0$ obtained for various values of the



(a)



(b)

Figure 2: Convergence of the numerical results with the number of fluid sub-layers in the case where $h_d/h = 2$: frequency response of the dimensionless stress $T_{22}h/P_0$ (a) and dimensionless velocity $v_2\mu h/(P_0c_2)$ (b).

number N in the case where $S_1^* = 5$ and $V/h = 2500\,hz$ under $\omega t = 0$ and $x_1/h = 0$. It follows from Fig. 4a that the values of the stress approach a certain asymptote with the number N. In other words, the numerical results obtained for the studied quantities approach a certain limit with the number N. In addition, after a certain

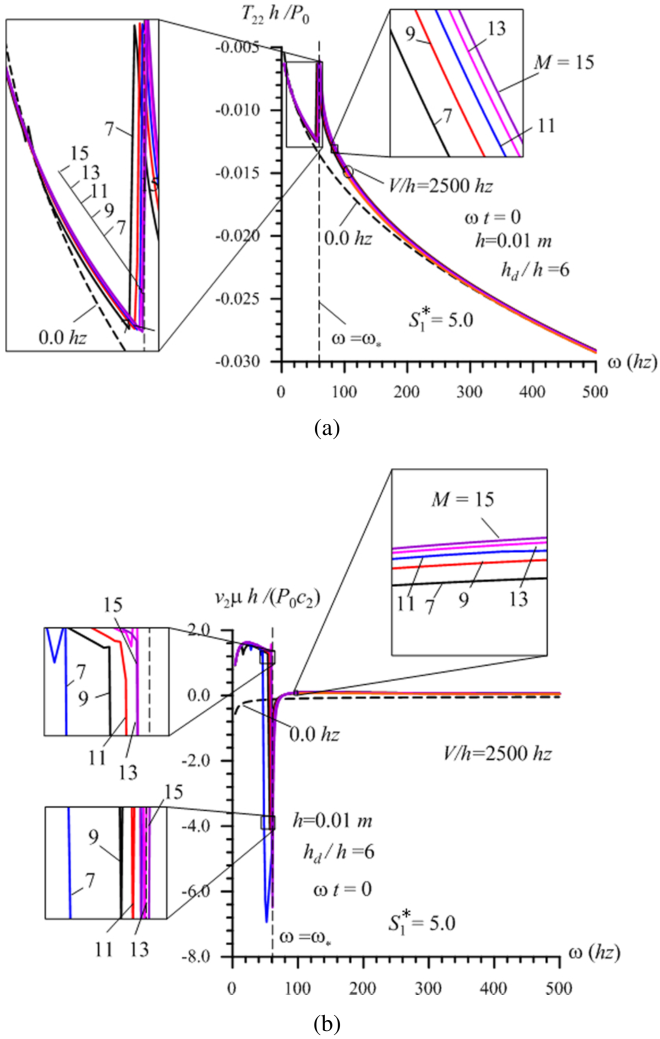


Figure 3: Convergence of the numerical results with the number of fluid sub-layers in the case where $h_d/h = 6$: frequency response of the dimensionless stress $T_{22}h/P_0$ (a) and dimensionless velocity $v_2\mu h/(P_0c_2)$ (b).

value of N (denoted by N^*) the numerical results obtained for the various $N > N^*$ coincide with each other with accuracy of $10^{-5} - 10^{-6}$. It should be noted that the value of N^* depends not only on the frequency, but also on the other problem parameters and mainly on h and h_d/h. For instance, for the case under consideration it can be taken that $N^* = 2000$.

Consider also the graphs which illustrate the convergence of the numerical results

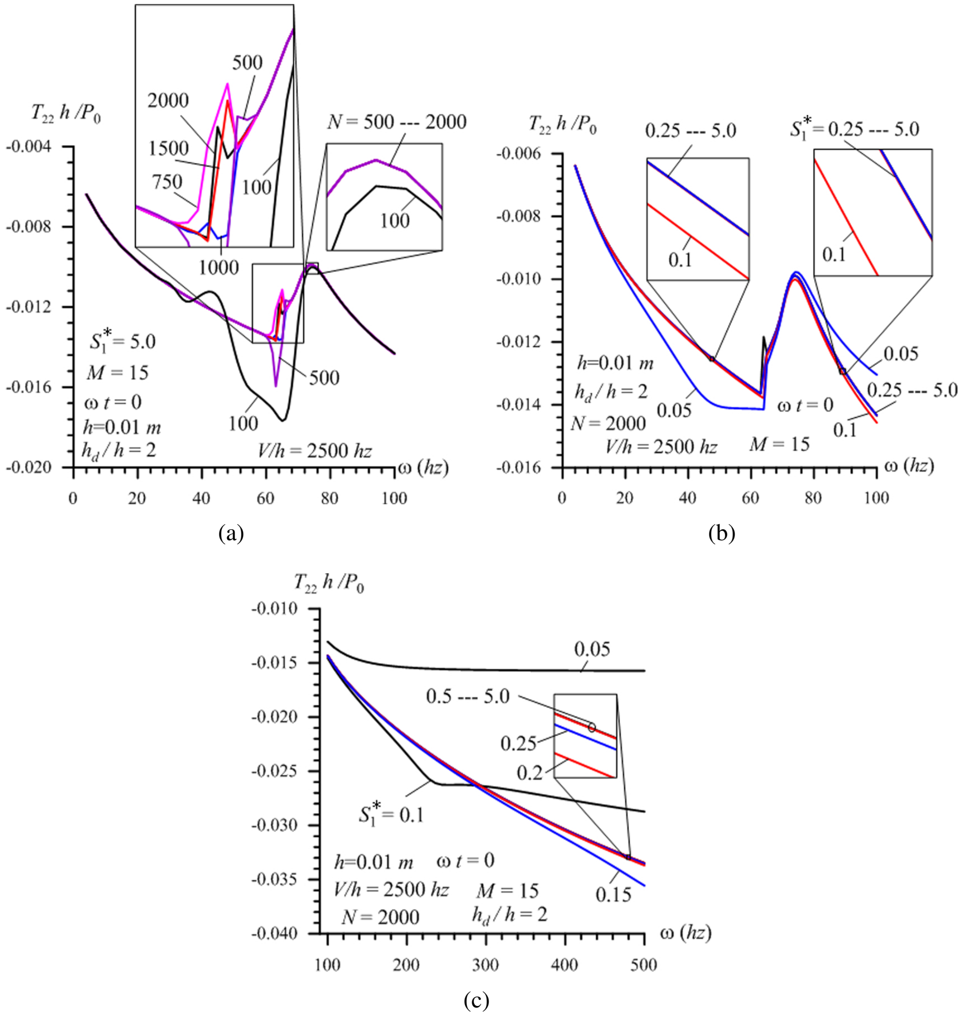


Figure 4: Convergence of the numerical results with respect to the number N (a) and with respect to the integration interval S_1^* in the cases where $4\,hz \leq \omega \leq 100\,hz$ (b) and $100\,hz \leq \omega \leq 500\,hz$ (c).

with respect to the integration interval, i.e. with respect to the values of S_1^*. These graphs are given in Figs. 4b and 4c which are constructed for the dimensionless stress $T_{22}h/P_0$ in the cases where $4\,hz \leq \omega \leq 100\,hz$ and $100\,hz \leq \omega \leq 500\,hz$, respectively. It is assumed that $N = 2000$ and the problem parameters have the same values as those selected for the previous graphs. It follows from these results that the numerical results approach a certain asymptote with S_1^* and the frequency ω. The convergence of the numerical results with respect to S_1^* requires an increase in the values of S_1^*. In obtaining the numerical results all the foregoing characteristics related to the convergence of the numerical results are taken into consideration and it is established that the case where $S_1^* = 5$ and $N = 2000$ is sufficient to obtain accurate numerical results. Thus, we assume that $S_1^* = 5$ and $N = 2000$. At the same time, it should be noted that the foregoing results on the convergence of the numerical results can also be used as validation of the algorithm and programs which were composed by the authors. Unfortunately, we have not found any related results of other authors for comparison with the present ones. Therefore validation of the present results can be proven with the convergence of the numerical results and with their agreement to mechanical considerations.

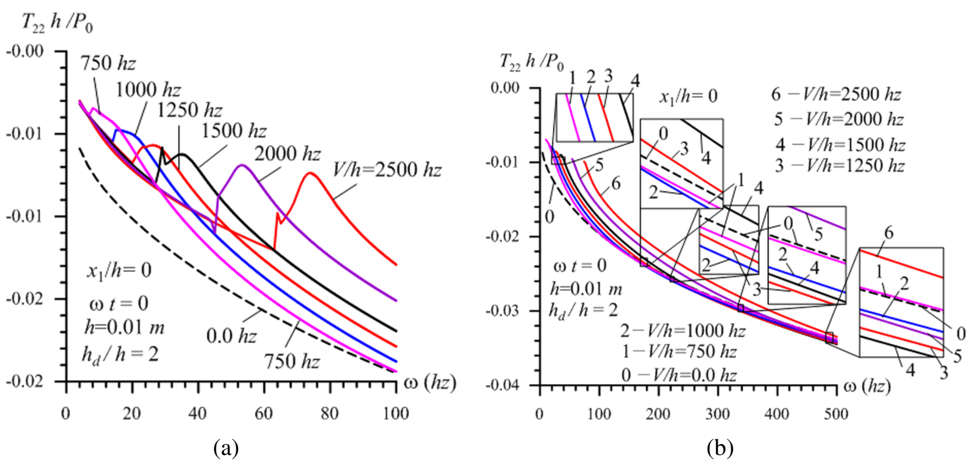
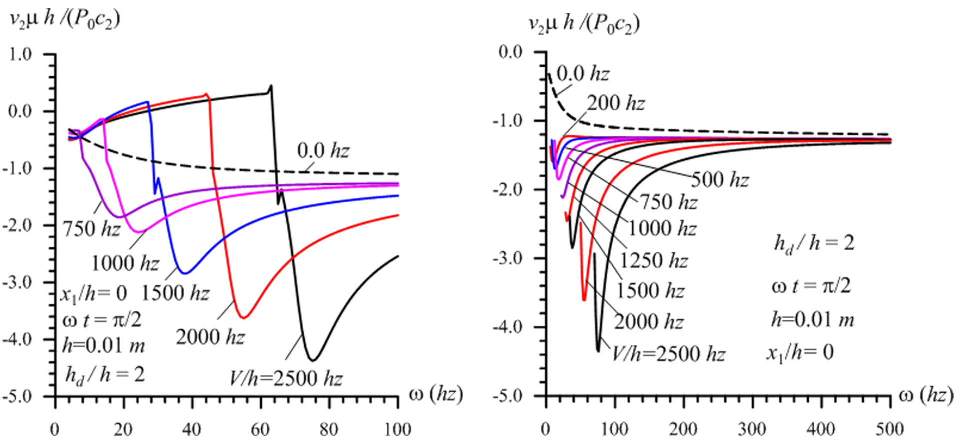


Figure 5: The influence of the plate moving velocity on the frequency response of the dimensionless stress $T_{22}h/P_0$ in the cases where $4\,hz \leq \omega \leq 100\,hz$ (a) and $\omega_{cr} < \omega' \leq \omega \leq 500\,hz$ (b).

4.2 Some results related to the influence of the plate moving velocity on the frequency response of the stress and velocities

Although detailed analysis of the numerical results will be the subject of further papers by the authors, here we illustrate the usefulness of the proposed solution approach. For this purpose we consider the influence of the plate moving velocity V on the frequency response of the normal stress and velocities arising on the interface plane between the plate and fluid. The graphs given in Figs. 5, 6 and 7 show the dependence among the dimensionless stress $T_{22}h/P_0$ (Fig. 5 under $\omega t = 0$), the dimensionless velocity $v_2\mu h/(P_0c_2)$ (Fig. 6 under $\omega t = \pi/2$), the dimensionless velocity $v_1\mu h/(P_0c_2)$ (Fig. 7 under $\omega t = \pi/2$) and the frequency ω which are constructed for various values of V/h in the case where $h_d/h = 2$, and $x_1/h = 0$.

Note that for a clear illustration of the influence of the moving plate velocity on the values of the critical frequency as well as on the values of the studied quantities the graphs related to the cases where $4\,hz \leq \omega \leq 100$ and $\omega_{cr} < \omega' \leq \omega \leq 500\,hz$ are presented separately by letters a and b, respectively. Here the values of ω' vary according to V/h and these values can be easily determined from the foregoing figures. Moreover, note that in these figures the dashed lines show the frequency response of the corresponding quantity in the case where the plate in the initial state is at rest, i.e. the case where $V/h = 0$.

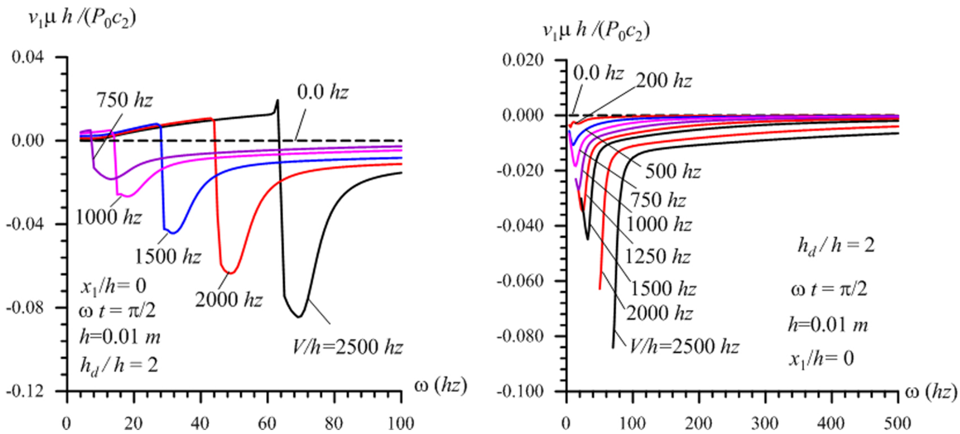


(a) (b)

Figure 6: The influence of the plate moving velocity on the frequency response of the dimensionless velocity $v_2\mu h/(P_0c_2)$ in the cases where $4\,hz \leq \omega \leq 100\,hz$ (a) and $\omega_{cr} < \omega' \leq \omega \leq 500\,hz$ (b).

Thus, it follows from the results that the values of ω_{cr} decrease with decreasing plate axial-moving velocity V/h. Moreover the graphs show that there exists a certain value of the frequency (denoted by ω^*) before which (i.e. for $\omega' < \omega < \omega^*$) the plate moving velocity in the initial state causes a decrease (an increase) in the absolute values of the dimensionless stress $T_{22}h/P_0$ (of the dimensionless velocities $v_2\mu h/(P_0c_2)$ and $v_1\mu h/(P_0c_2)$) with respect to the corresponding ones obtained in the case where $V/h = 0$. However, after this frequency (i.e. in the cases where $\omega > \omega^*$) the plate moving velocity causes the absolute values of $T_{22}h/P_0$ with respect to those obtained in the case where $V/h = 0$ to increase slightly.

To demonstrate the influence of the fluid viscosity on the values of the studied quantities, graphs of the frequency response of the dimensionless stress $T_{22}h/P_0$ in the case where the fluid is modelled as inviscid are given in Fig. 8. Comparison of these graphs with the corresponding ones given in Fig. 5 shows that the influence



(a) (b)

Figure 7: The influence of the plate moving velocity on the frequency response of the dimensionless velocity $v_1\mu h/(P_0c_2)$ in the cases where $4\,hz \leq \omega \leq 100\,hz$ (a) and $\omega_{cr} < \omega' \leq \omega \leq 500\,hz$ (b).

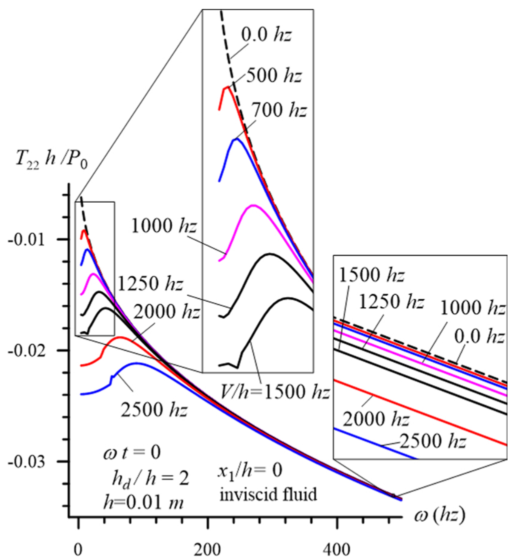
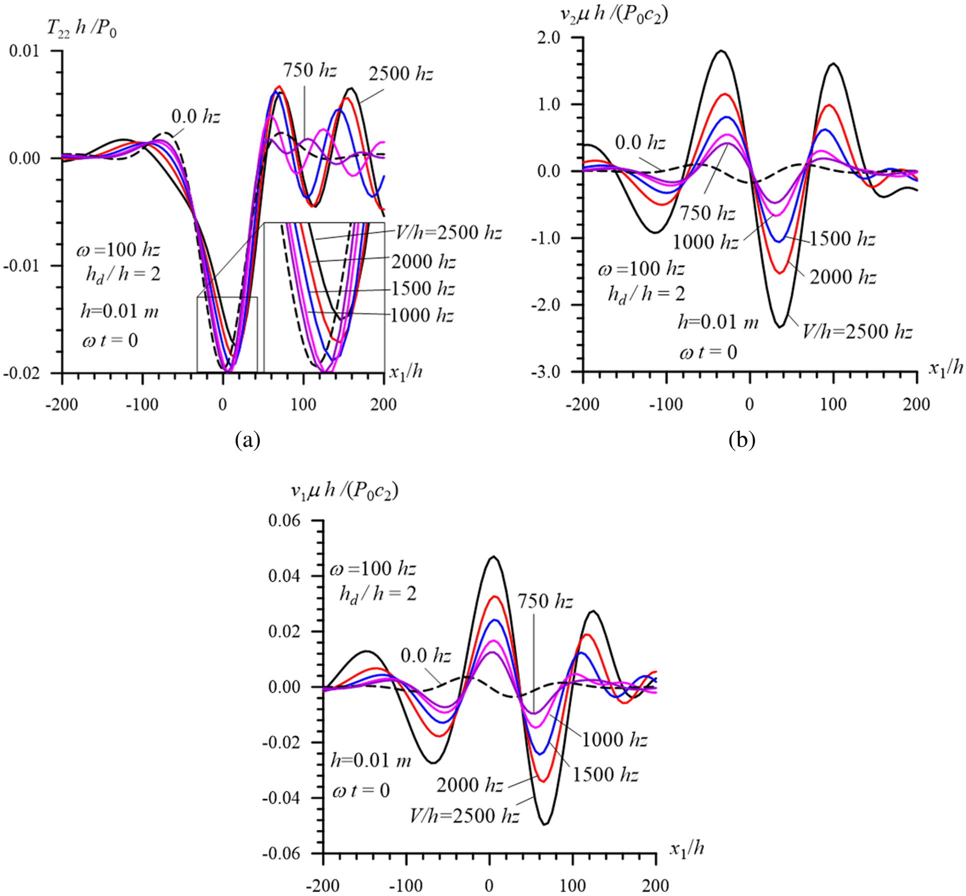


Figure 8: The influence of the plate moving velocity on the frequency response of the dimensionless stress $T_{22}h/P_0$ in the case where the fluid is modeled as inviscid.

of the fluid viscosity on the studied frequency response has important significance, not only qualitatively but also quantitatively. We recall that in the case where the plate is in contact with the inviscid fluid, the plate moving in the initial state does not cause the fluid flow. Consequently, the inviscid fluid model is not adequate for



(c)

Figure 9: The influence of the plate moving velocity on the distribution of $T_{22}h/P_0$ (a), $v_2\mu h/(P_0c_2)$ (b) and $v_1\mu h/(P_0c_2)$) (c) with respect to x_1/h.

mathematical modelling of the type of problems studied. This result shows again the significance of the proposed approach for solution to the dynamic problems related to hydro-elastic systems containing a viscous fluid.

Another characteristic of the influence of the plate's axially-moving velocity on the plate-fluid interaction under consideration is illustrated in the case where $V/h = 0$, i.e. in the case where the plate is at rest in the initial state and the distribution of the stresses and velocities caused by the additional time harmonic forces with respect to the moving coordinate x_1/h is symmetric or asymmetric with respect to the point $x_1/h = 0$. However, as can be predicted according to expressions and equations (21)–(25), in the case where $V/h > 0$ this distribution becomes nonsymmetric or non-asymmetric with respect to $x_1/h = 0$, i.e. with respect to the point at which the external additional time-harmonic force acts. The results that prove this conclusion are illustrated in the graphs in Fig. 9 which show the distribution for the dimensionless stress $T_{22}h/P_0$ (Fig. 9a) and velocities $v_2\mu h/(P_0c_2)$ (Fig. 9b) and $v_1\mu h/(P_0c_2)$ (Fig. 9c) in the case where $\omega = 100hz$ and $\omega t = 0$.

This completes consideration of the numerical results obtained by employing the proposed approach for the problem under consideration. More detailed analysis of these and other numerical results will be the subject of further investigations by the authors.

5 Conclusions

In the present paper the discrete-analytical solution method has been developed for the solution to problems related to the dynamics of hydro-elastic systems consisting of a compressible viscous fluid which has non-homogeneous laminar flow, an initially-stressed axially-moving elastic plate and rigid wall. It is assumed that the non-homogeneous flow of the fluid is caused by the plate moving and that on the plate an additional lineally-located time-harmonic force acts with respect to the forces causing the initial stresses in the plate. This additional force causes an additional fluid flow field. The additional stress-strain field in the plate is described by utilizing the three-dimensional linearized theory of elastic waves in initially stressed bodies and the additional fluid flow field is described by utilizing the linearized Navier-Stokes equations for compressible viscous fluids. As a result of the heterogeneity of the fluid flow in the initial state the linearized Navier-Stokes equations have coefficients which are variable with respect to the coordinates. This situation causes difficulties in obtaining an analytical solution to the problem under consideration. To overcome this difficulty, we propose the discrete-analytical solution method, the essence of which is as follows. The fluid layer which has non-homogeneous laminar flow is replaced with a certain number of sub-layers each of which has homogeneous flow. For each sub-layer the linearized Navier-Stokes equations are written and the corresponding contact conditions between the sub-layers are satisfied. In this way the solution to the boundary-value problem for the linearized Navier-Stokes equations with variable coefficients is reduced to the series of the corresponding boundary-value problems for the linearized Navier-Stokes equations with constant coefficients which in turn allows an analytical solution to be obtained. The applicability of the proposed approach is illustrated with the solution of the concrete problem considered in the present paper. The numerical convergence of the developed solution method is also discussed with respect to the concrete problems under consideration. Moreover, some numerical results are given and discussed, which illustrate the influence of the heterogeneity of the fluid flow in the initial state on the stress and flow field in the hydro-elastic system caused by the additional time-harmonic force.

It should be noted that the proposed approach can also be developed and applied for more complicated initial fluid flow cases for more complex hydro-elastic systems. These cases as well as more detailed investigations and analyses of the numerical results related to the problem considered in the present paper, will be the subject of further works by the authors.

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