Copyright © 2017 Tech Science Press CMES, vol.113, no.4, pp.475-495, 2017

Uncertainty Analysis Method of Casing Extrusion Load for Ultra-Deep Wells

Meng Li[1], Kanhua Su[1], Zijian Li[2], Dongjie Li[3] and Lifu Wan[1]

Abstract: With the consideration of the randomness of complex geologic parameters for ultra-deep wells, an uncertainty analysis method is presented for the extrusion load on casing in ultra-deep wells through complex formation at a certain confidence level. Based on the extrusion load model for casing in ultra-deep wells and the prerequisite of integrity of formation-cement ring-casing, the probability and statistics theory are introduced and the sensitivity analysis on the uncertainty of extrusion load on casing is conducted. The distribution types of each formation parameters are determined statistically. The distribution type and distribution function of the extrusion load on casing are derived. Then, the uncertainty analysis of the extrusion load on casing is carried out on several ultra-deep wells in Shanqian block as case study. Several conclusions are made regarding to the field trial result. The randomness of formation elasticity modulus and formation Poisson's ratio are the main influence factors. The equivalent density profile of extrusion load on casing in ultra-deep wells is a confidence interval with a certain confidence level, rather than a single curve; the higher the confidence level is, the larger the bandwidth of the confidence interval of equivalent density profile becomes, and the larger the range of uncertainty interval becomes. Compared with the result of uncertainty analysis, an error exists in the result of traditional single valued calculation method. The error varies with different casing program and can be either positive or negative. The application of uncertainty analysis of extrusion load on casing provides proof for the accurate determination of casing collapse safety factor. Thus, the over engineering design or under engineering design as a result of tradition casing design will be avoided.

Keywords: Ultra-deep well, formation parameters, uncertainty, extrusion load, confidence interval.

1 Introduction

Uncertainty exist in extrusion load on the casing in oil and gas wells is induced by the randomness of formation parameters. The uncertainty of extrusion load on the casing increases in deep wells and ultra-deep wells, where the geology structure is complex and the lithology varies significantly [Long, Li and Guan (2013)]. Numerical value of

[1] School of Petroleum, Chongqing University of Science and Technology, Chongqing, 401331, China.

[2] Memorial University of Newfoundland, St. John's, A1C5S7, Canada.

[3] Engineering Technology Research Institute, PetroChina Huabei Oilfield Company, Renqiu, 062552, China.

formation parameters changes in different location, even in different depth of the same formation. Due to the uncertainty of casing extrusion load in ultra-deep well, the risk of casing collapse accident is higher during drilling. According to incomplete survey [Li (2008); Liao, Guan and Feng (2009); Wan, Li and Guan (2012)], casing collapse has occurred in well Dabei 6, well Keshen 2, well Tubei 4, well Keshen 5, well Dabei 203, well Wubo1, well Dongqiu 5, well Chegu 2, well Quele 1, well Xin 101, etc. In reality, the actual value of extrusion load on casing lies in a confidence interval rather than beinga deterministic single value, due to its uncertainty.

Recently, the main stream calculation methods for the prediction of extrusion load on casing are all single deterministic methods [Li and Yin (2006); Pattilo, Last and Asbill (2003); El-Sayed and Khalaf (1992)]. For the first type of the calculation method, it is assumed that the effective extrusion stress is evenly distributed on the outer surface of the casing. Then, the extrusion load is calculated as a uniformly distributed stress. Pattilo (2003) calculated the non-uniform extrusion load as equivalent uniform extrusion stress in his model. Liao (2010) further developed the equivalent uniform extrusion load model for casing with the consideration of the wear condition of the inner surface of casing. For the second type of the calculation method, based on the uniform distribution load model for casing, the in-situ stress and formation creep are taken into consideration and then the non-uniform load are calculated. El-Sayed et al. (1989) presented the calculation model for abnormal damage of casing under the non-uniform extrusion stress. For different casing type and loading feature combination, prediction accuracy for casing damage is calculated correspondingly. Du et al. (2007) presented the mechanical model for casing under non-uniform extrusion stress. The distribution of bending moment, shear stress, axial stress, radius stress and displacement on the casing wall are analyzed quantitively. For the third type of calculation model, considering part of the formation parameters, finite element method (FEM) analysis is applied to calculate the non-uniform extrusion stress; then the safety condition of the casing is analyzed. Shi (2008) established three-dimension finite element method (FEM) for casing extrusion based on the mudstone creep under non-uniform extrusion stress. Wan (2012) carried out finite element method (FEM) analysis on casing extrusion for the extrusion load of different types of casing under different in-situ stresses. However, due to the complexity of the geology structure and the randomness of formation parameters in ultra-deep wells, the prediction methods above for the extrusion load on casing in deep wells and ultra-deep wells are not satisfying.

In order to solve the uncertainty of the extrusion load on the casing in deep wells and ultra-deep wells induced by the randomness of formation parameters and complex geology structure, based on Lame equations, with the consideration of the integrity of formation-cement ring- casing system, an extrusion load prediction model is established for casing in deep wells and ultra-deep wells [Li (2008); Li and Yin (2006)], which is of reference significance to the design of casing and well structure for deep wells and ultra-deep wells.

2 Extrusion load model for casing in ultra-deep wells

2.1 Determination of equivalent density of overlying strata

For wells already drilled, of which the density logging data is available, the equivalent density of overlying strata can be calculated using the discrete integral formula [Fan, Ye and Ji (2011)]:

$$G_o = \frac{\rho_o h_o + \sum_{i=1}^{n} \rho_{bi} \Delta h}{h_o + \sum_{i=1}^{n} \Delta h} \tag{1}$$

For the wells undrilled, the calculation of the equivalent density of overlying strata should be carried out based on the density logging data of adjacent wells.

Where, G_o is the equivalent density of overlying strata, g/cm^3. ρ_o, h_0 are the average density and thickness of the strata without available density logging data up to the surface. ρ_{bi} is the density of a certain formation, g/cm^3. Δh is distance for two adjacent logging data points, m.

For the well interval near surface with no available density logging data, the equivalent density of overlying strata can be obtained from the continuous fitting function:

$$G_0 = A + Bh - Ce^{-Dh} \tag{2}$$

Where, *h* is the depth, m. *A*, *B*, *C*, *D* are fitting coefficients, dimensionless.

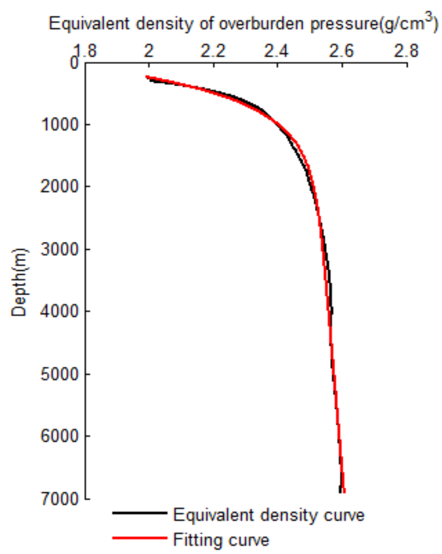


Figure 1: Equivalent density profile of overburden pressure for GN-203 well

2.2 Establishment of extrusion load for casing in ultra-deep wells

Because of the complexity of the geology structure and randomness of formation parameters for ultra-deep wells, an error exists in the convention extrusion load calculation model the result. In addition, the unloading effect of well cured cement ring to the casing also influence the accuracy of the calculation result [Wang, Li and Yang (2008); Klever and Tamano (2004); Yang and Chen (2006); Aasen, Bernt and Aadnoy (2007)]. Thus, based on Lame equations and equivalent non-uniform extrusion stress model for formation-casing system, with the consideration of the integrity of formation-cement ring-casing system, the extrusion load prediction model is established for casing in ultra-deep wells.

$$p=2\frac{E_c\left(1-m^2\right)\left(1+\upsilon_s\right)\left(1-\upsilon_s\right)\sigma+E_s\left(1+\upsilon_c\right)\left(1-\upsilon_c\right)m^2q}{E_c\left(1-m^2\right)\left(1+\upsilon_s\right)+E_s\left(1+\upsilon_c\right)\left(1-2\upsilon_c+m^2\right)} \tag{3}$$

$$\sigma=\sqrt{\sigma_H\cdot\sigma_h} \tag{4}$$

$$\begin{cases}\sigma_H=\left(\dfrac{\upsilon_s}{1-\upsilon_s}+\beta\right)\left(\sigma_v-\alpha P_p\right)+\alpha P_p\\ \sigma_h=\left(\dfrac{\upsilon_s}{1-\upsilon_s}+\gamma\right)\left(\sigma_v-\alpha P_p\right)+\alpha P_p\end{cases} \tag{5}$$

$$q=-0.00981\left(1-k_m\right)\rho h \tag{6}$$

Let

$$\begin{cases}k_1=E_c\left(1-m^2\right)\left(1+\upsilon_s\right)\left(1-\upsilon_s\right)\sigma\\ k_2=\left(1+\upsilon_c\right)\left(1-\upsilon_c\right)m^2q\\ k_3=E_c\left(1-m^2\right)\left(1+\upsilon_s\right)\\ k_4=\left(1+\upsilon_c\right)\left(1-2\upsilon_c+m^2\right)\end{cases} \tag{7}$$

Then, k_1, k_3 are functions of υ_s, k_2, k_4 are constants. Substitute Eq. (7) into Eq. (3):

$$p=2\frac{k_1+E_sk_2}{k_3+E_sk_4} \tag{8}$$

Where, σ_v is vertical principal stress, MPa, $\sigma_v=\frac{G_ogh}{10^3}$; p is the extrusion load on casing, MPa; σ is the equivalent in-situ stress, MPa; E_c is the casing elastic modulus, GPa; υ_c is the Poisson's ratio of casing, dimensionless; m is the ratio between the inner diameter and the outer diameter of the casing, dimensionless; E_s is the elastic modulus of formation, GPa; υ_s is the Poisson's ratio of formation, dimensionless; q is the stress on the inner well of casing, MPa; ρ is the density of drilling fluid, g/cm^3; k_m is the cementing drawdown coefficient, dimensionless.

3 Uncertainty analysis of extrusion load on casing in ultra-deep wells

3.1 Analysis of parameters' randomness of extrusion load on casing in ultra-deep wells

(1) Comparison of coefficient of variation for extrusion load.

The randomness of parameters can be evaluated by coefficient of variation, which is defined as:

$$COV = \frac{\sigma}{\mu} \tag{9}$$

Where, *COV* is the coefficient of variation for a random variable, dimensionless; σ is the standard deviation of the random variant; μ is the mean value of the random variant.

According to the statistical result of 2000 different steel grade casings from 5 manufacturers [Adams, Parfitts and Reeves (1993)] and addition research [Prasongsit, Paolo and Jerome (2001)], case study is carried out using the ultra-deep well data in Shanqian block of Western Oilfield of China. The COVs of influence parameters of extrusion load on casing are obtained statistically as shown in Figure 2. According to the result, it is noted that the COV of formation elastic modulus and formation Poisson's ratio is much larger than the COV of casing elastic modulus, casing Poisson' ratio, drilling fluid density, casing size or any other parameters. It is concluded that the randomness of formation elastic modulus and formation Poisson's ratio are the main influence factors on the uncertainty of extrusion load on casing.

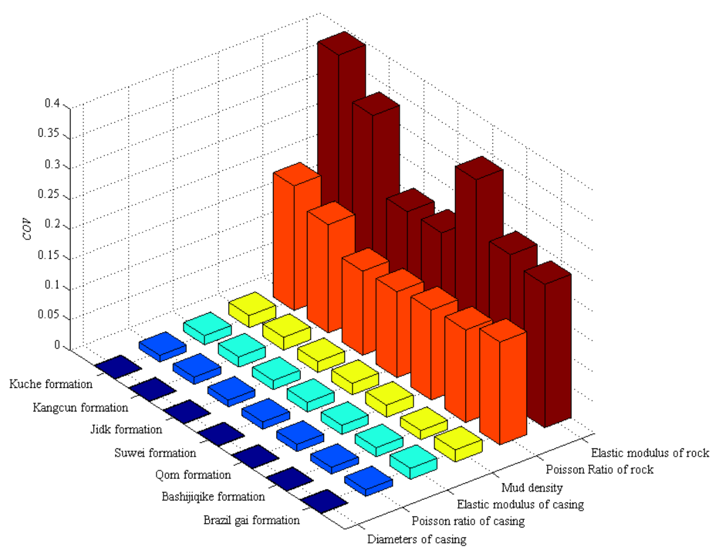


Figure 2: Comparison of variation coefficients of main parameters of extrusion load in ultra deep wells

(2) Determination of the distribution types of formation elastic modulus and formation Poisson's ratio.

Formation elastic modulus and formation Poisson's ratio of each formation are analyzed statically as shown in the histograms in Figure 3 and Figure 4. Non-linear fitting is conducted using Eqs. (10-12) [Sun and Chen (2003)]. The fitting result is shown in the fitting curve in Figure 3 and Figure 4.

Normal distribution:

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \tag{10}$$

Where, μ is the mean value; σ is the standard deviation.

Lognormal distribution:

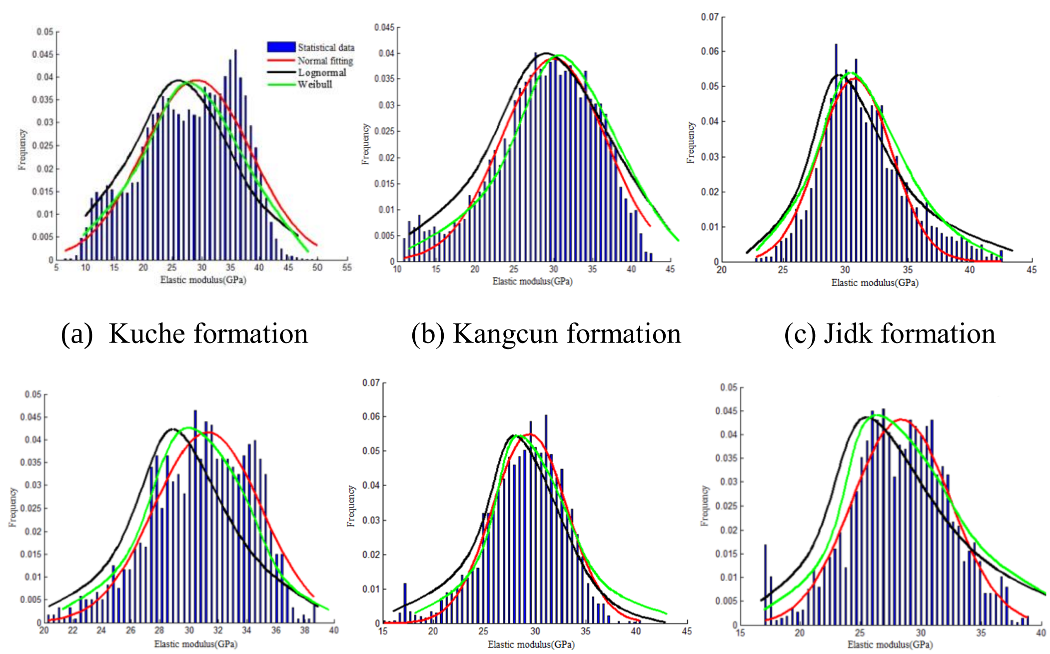
$$f(x)=\frac{1}{\sqrt{2\pi}\sigma x}\exp\left[-\frac{(\ln x-\mu)^2}{2\sigma^2}\right] \tag{11}$$

Where, $e^{(\mu+\sigma^2/2)}$ is the mean value; $\sqrt{e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)}$ is the standard deviation.

Weibull distribution:

$$f(x)=\left[\frac{b}{\theta-x_0}\left(\frac{x-x_0}{\theta-x_0}\right)^{b-1}\right]\cdot\exp\left[\left(\frac{x-x_0}{\theta-x_0}\right)^{b}\right] \tag{12}$$

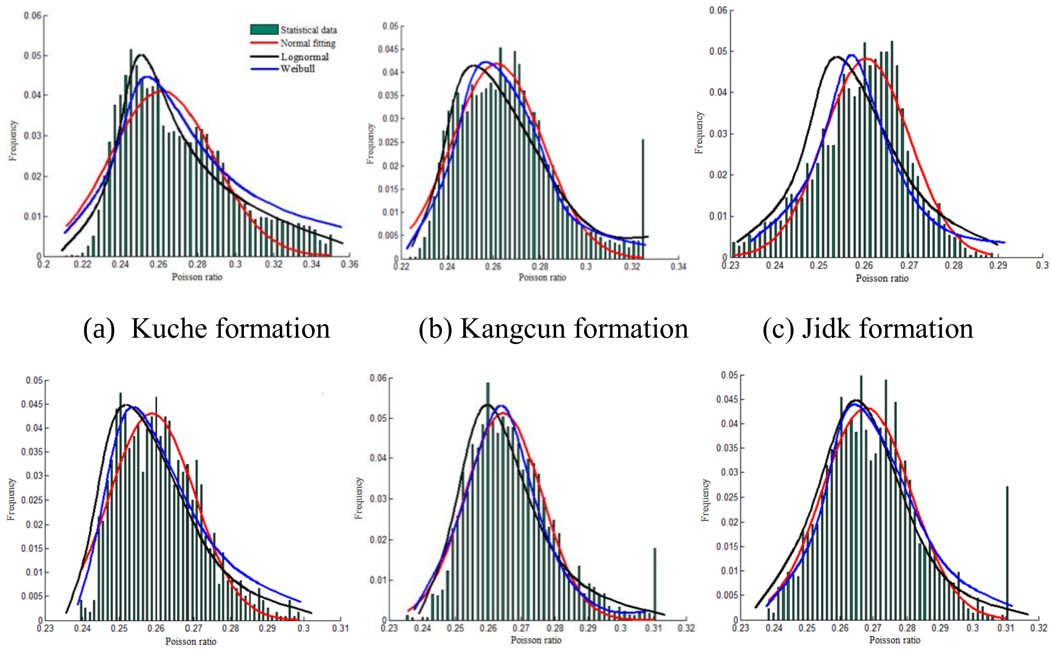
Where, x_0 is the displacement parameter; b is the shape factor; θ is the scale parameter.



(a) Kuche formation (b) Kangcun formation (c) Jidk formation

(d) Suwei formation (e) Qom formation (f) Bashijiqike formation

Figure 3: Elastic modulus statistics and fitting distribution results in each layer



(a) Kuche formation (b) Kangcun formation (c) Jidk formation

(d) Suwei formation (e) Qom formation (f) Bashijiqike formation

Figure 4: Statistics and fitting distribution results of Poisson's ratio in each layer

Correlation tests between statistic results and fitting curves are carried out [Xiao (2002)], then the average correlation coefficients of formation elastic modulus and formation Poisson's ratio with respect to these three distribution types mentioned above are obtained. The result is shown in Table 1.

Table 1: Average correlation coefficient

Parameters	Normal	Lognormal	Weibull
Elastic modulus of sediments	0.962	0.836	0.851
Poisson's ratio of sediments	0.921	0.842	0.857

It is noted that the average correlation coefficients of normal distribution of formation elastic modulus and formation Poisson's ratio are 0.962 and 0.921 respectively, which shows that significant strong correlations exist between statistic results and normal distribution. In addition, the average correlation coefficients of normal distribution are obviously larger than those of lognormal distribution and Weibull distribution, as shown in Table 1. Thus, normal distribution is the best distribution type for the statistic result of formation elastic modulus and formation Poisson's ratio.

3.2 Determination of the distribution type of extrusion load on the casing in ultra-deep wells

Formation elastic modulus and formation Poisson's ratio, which are the main influence factors of extrusion load on the casing, are regarded as two-dimensional variables. Suppose that the extrusion load *p* is expressed by *Z*, formation elastic modulus E_s is expressed by *X* and formation Poisson's ratio υ_s is expressed by *Y*. Then, their random values are represented as *z, x* and *y*. Then, Eq. (8) can be transformed to:

$$z = g(x, y) = \frac{2(k_1 + xk_2)}{k_3 + xk_4} \tag{13}$$

Known from Eq. (7), k_1, k_3 are functions of *y*. Then:

$$x = g^{-1}(z, y) = s(z, y) = \frac{2k_1 - zk_3}{zk_4 - 2k_2} \tag{14}$$

Take the partial derivative of $s(z,y)$ with respect to *z*:

$$\frac{\partial s}{\partial z} = \frac{-k_3(zk_4 - 2k_2) - k_4(2k_1 - zk_3)}{(zk_4 - 2k_2)^2} = \frac{2(k_2k_3 - k_1k_4)}{(zk_4 - 2k_2)^2} \tag{15}$$

Let $f_{X,Y}(x,y)$ be the probability density function of (*X, Y*). Since *X* and *Y* are independent on each other, thus [Sun and Chen (2003)]:

$$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y) \tag{16}$$

Where, $f_X(x)$ and $f_Y(y)$ are probability density functions of random variable *X* and *Y* respectively.

According to probability theory [Xiao (2002)], the probability density function of *z* can be expressed as:

$$f_Z(z) = \int_{-\infty}^{+\infty} f_{X,Y}(g^{-1}, y)\frac{\partial g^{-1}}{\partial z}dy = \int_{-\infty}^{+\infty} f_X\left[s(z,y)\right] f_Y(y)\frac{\partial s}{\partial z}dy \tag{17}$$

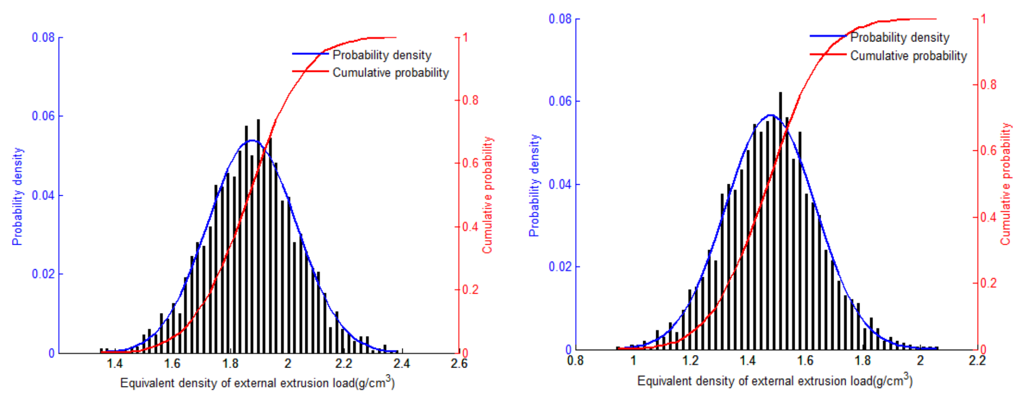
Given that formation elastic modulus E_s and formation Poisson's ratio υ_s obey normal distribution, distribution of $f_X(x)$ and $f_Y(y)$ can be expressed using Eq. (10). Therefore, the probability density function of the extrusion load $f_Z(z)$ on the casing in ultra-deep wells can be obtained by substituting Eq. (10) and Eqs. (14-16) into Eq. (17):

$$f_Z(z) = \frac{1}{\pi\sigma_x\sigma_y}\int_{-\infty}^{+\infty}\frac{(k_2k_3 - k_1k_4)}{(zk_4 - 2k_2)^2}\exp\left[-\frac{\left(\frac{2k_1 - zk_3}{zk_4 - 2k_2} - \mu_x\right)^2}{2\sigma_x^2} - \frac{(y - \mu_y)^2}{2\sigma_y^2}\right]dy \tag{18}$$

Make the integration of Eq. (18), the cumulative probability function of extrusion load on casing in ultra-deep wells can be obtained:

$$F_Z(z)=\frac{1}{\pi\sigma_x\sigma_y}\int_{-\infty}^{+\infty}\int_{-\infty}^{\frac{2(k_1+xk_2)}{k_3+xk_4}}\frac{(k_2k_3-k_1k_4)}{(zk_4-2k_2)^2}\exp\left[-\frac{\left(\frac{2k_1-zk_3}{zk_4-2k_2}-\mu_x\right)^2}{2\sigma_x^2}-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right]dydz \tag{19}$$

Known from Eq. (18) that the uncertainty of extrusion load on casing in ultra-deep wells obeys normal distribution. For a given cumulative probability, the correlating extrusion load can be calculated using Eq. (19). Taking GN-101 well in Shanqian block as an example, probability density and cumulative probability of extrusion load are calculated and plotted in Figure 5.



(a) In 3290m (b) In 5290m

Figure 5: Equivalent density distribution of external collapse load

3.3 Analysis on the confidence interval of extrusion load on casing in ultra-deep wells

The cumulative probability distribution function $F_{Z,h_i}(z)$ of extrusion load on casing at the well depth of h_i is obtained by using Eq. (19).

$$F_{Z,H}(z)=\left\{F_{Z,h_1}(z),F_{Z,h_2}(z),F_{Z,h_3}(z),\cdots,F_{Z,h_n}(z)\right\} \tag{20}$$

For the cumulative probability of *u*, *v* (*v*>*u*), the extrusion loads correlating with set element $F_{Z,h_1}(z)$, $F_{Z,h_2}(z)$, $F_{Z,h_3}(z)$,…, $F_{Z,h_n}(z)$ are calculated as:

$$\begin{aligned}[p]_u&=\left\{[p]_{h_1,u},[p]_{h_2,u},[p]_{h_3,u},\cdots,[p]_{h_n,u}\right\}\\ [p]_v&=\left\{[p]_{h_1,v},[p]_{h_2,v},[p]_{h_3,v},\cdots,[p]_{h_n,v}\right\}\end{aligned} \tag{21}$$

Where, respectively, $[p]_{h_i,u}$, $[p]_{h_i,v}$ are the extrusion load on casing with respect to the cumulative probability *u*, *v* at the depth of h_i.

Thus, the distribution interval at the casing extrusion load confidence level of

(v-u)×100% is $\left[[p]_{h_i,u},[p]_{h_i,v}\right]$.

4 Case study

Taking Shanqian block in Western oilfield, China as an example, the uncertainty of extrusion load on casing in ultra-deep wells are analyzed. The data is selected from three wells. In this block, the stratigraphic sequence from the surface downward is Kuqa formation, Kangcun formation, Jidk formation, Suwei formation, Qom formation, Bashijiqike formation and Brazil gai formation. Firstly, equivalent density profile of overlying strata is constructed. Then, formation elasticity modulus and formation Poisson's ratio are analyzed statically and the distribution types and distribution parameters' characterization values are obtained, as shown in Table 2. Using Eq. (19), which is the cumulative probability function of extrusion load on casing, the equivalent density of extrusion load is calculated along the wellbore downward with respect to the cumulative probability of 5%, 95%, 35% and 65%. The distribution intervals of equivalent density of extrusion load corresponding to the confidence levels of 90% and 30% are obtained respectively. The results are compared with the result of traditional calculation method.

Table 2: Statistical results of randomness of main formation parameters in different formations in Shanqian block

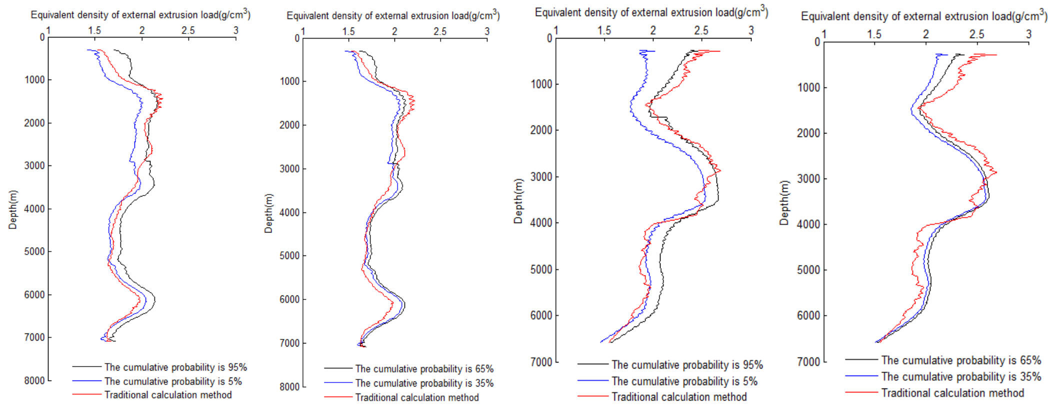
Formation	Elastic modulus			Poisson's ratio		
	Mean value (GPa)	Standard deviation (GPa)	Coefficient of variation	Mean value	Standard deviation	Coefficient of variation
Kuche	28.028	8.253	0.294	0.270	0.028	0.106
Kangcun	28.860	6.673	0.231	0.266	0.021	0.079
Jidk	31.444	3.432	0.109	0.259	0.010	0.039
Suwei	30.855	3.452	0.111	0.261	0.011	0.042
Qom	28.889	3.952	0.136	0.267	0.013	0.049
Bashijiqike	28.169	4.200	0.149	0.269	0.014	0.053
Brazil gai	26.464	3.627	0.137	0.255	0.018	0.070

Table 3: Casing program and safety situation in GN-3 well

Well No.	Casing Program	Casing size (mm)	Casing depth(m)	Casing grade	Wall thickness (mm)	Remarks
GN-3	Surface casing	508.0	0~308	J55	12.7	Safe
	Technical casing 1	339.7	0~3891	TP110V	12.19	Deformed
	Technical casing 2	244.5	3773~4750	BG110TT	11.99	Deformed
		250.8	4750~5184	TP140V	15.88	Safe
		244.5	5184~6219.5	BG110TT	11.99	Failed
		250.8	6219.5~6555	TP140V	12.19	Failed
		244.5①	0~3250	TP140V	11.99	Safe
		244.5①	3250~3773	BG110TT	11.99	Safe
	Production casing 1	177.8	6617~6845	TP140V	14.8	Safe
		177.8②	0~6616	TP140V	12.65	Safe
	Production casing 2	127.0	6332~6918	BG140	9.5	Safe
GN-203	Surface casing	508	0~276.86	J-55	12.70	Safe
	Technical casing 1	339.7	0~3898.15	TP110V	12.19	Collapse
	Technical casing 2	273.0	5250~5696.17	TP110V	15.88	Deformed
		250.8	4729.76~5250.80	TN110HS	12.19	Deformed
		244.5①	0~4729.76	BG140	11.99	Deformed
	Production casing	177.8	0~6097.87	NKHC140	12.65	Safe

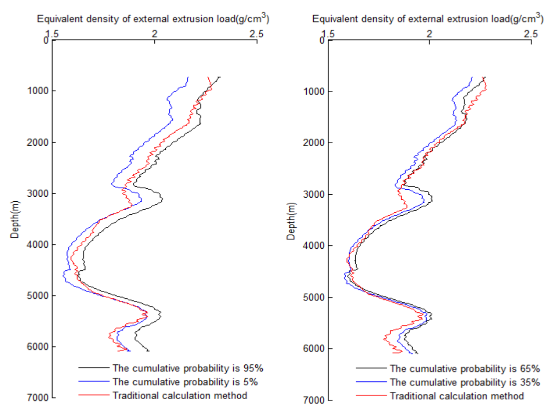
GN-101	Surface casing	339.7	0~805.81	P110	12.19	Safe
	Technical casing 1	250.8	4341.95~5100.00	TN110HC	15.88	Safe
		244.5	0.00~4341.95	SM110TTB	11.99	Collapse
	Liner 1	206.4	4897.07~5690.00	TP140V	16.00	Safe
	Technical casing 2	177.8①	0.00~4897.07	NKHC140	12.65	Safe
	Liner 2	139.7	5253.86~5790.00	TP110V	12.09	Deformed

Note: Technical casing 1 is the first layer of technical casing, Technical casing 2 is the second layer of technical casing, Production casing 1is the first layer of production casing, Production casing 2 is the second layer of production casing, ① or ② is tie back casing.



(I) (II) (I) (II)

(a) Well GN-3 (b) Well GN-203



(I) (II)

(c) Well GN-101

Figure 6: Equivalent density profile of casing extrusion load with different confidence levels

Note: (I) is the distribution of the extrusion load interval with a confidence level of 90%, (II) is the distribution of the extrusion load interval with a confidence level of 30%

According to the statistical probability results of formation elasticity modulus and formation Poisson's ratio in Table 2, for three wells of GN-3, GN-203 and GN101, the casing extrusion load confidence intervals correlating to the confidence levels of 90% and 30% are obtained by using Eq. (18), (19) and (20), as shown in Figure 6. Unlike the traditional method result of casing extrusion load, which is a single curve, for which the extrusion load on the casing for a certain depth is a deterministic value, the extrusion load on the casing for a certain depth obtained by new uncertainty analysis varies in a range. It is because that the variation of formation parameters of each formation along the ultra-deep well is considered in the calculation of casing extrusion load confidence interval with a certain confidence level. For practical design of drilling engineers, the variation range of casing extrusion load is of more significance than a single deterministic value obtained by using the traditional method. If the single deterministic casing extrusion load value is higher than the actual variation range of casing extrusion load, higher grade of casing will be selected. The over-engineering will induce an increase in cost. Otherwise, if the single deterministic casing extrusion load value is lower than the actual variation range of casing extrusion load, lower grade of casing will be selected. The under-engineering will increase the risk of casing collapse. Thus, the casing extrusion load variation range obtained by using the new method is of more significance in the design, by decreasing the accident risk and the cost in casing.

It is seen from Figure 6 (a)(I) that in the well section from depth 1200 m to 2800 m, the deterministic value of casing extrusion load obtained using the traditional single valued method appears to be a little higher than or close to the equivalent density corresponding to the cumulative probability of 95%. Known from Figure 6 (a)(II) that in the well section from depth 1200 m to 2800 m, the deterministic value of casing extrusion load obtained using the tradition single valued method is higher by 0.17 g/cm^3 on average than the equivalent density corresponding to the cumulative probability of 65%. This suggests that the steel grade of the casing is higher than enough and an over-engineering and a waste of cost exist. In the well section from depth 5200 m to 7000 m, the deterministic value of casing extrusion load obtained using the traditional single valued method is lower by 0.12 g/cm^3 and 0.18 g/cm^3 on average than the equivalent density corresponding to the cumulative probability of 5% and 35% respectively. This suggests that the steel grade of the casing is lower than enough and an under-engineering and an increase in risk of casing collapse exist.

Known from Figure 6 (b) that in the well section from depth 0m to 1500 m, the deterministic value of casing extrusion load obtained using the traditional single valued method is higher by 0.1 g/cm^3-0.2 g/cm^3 and 0.1 g/cm^3-0.4 g/cm^3 than the equivalent density corresponding to the cumulative probability of 95% and 65% respectively. This suggests that the steel grade of the casing is higher than enough and an over-engineering and a waste of cost exist. In the well sections from depth 3100 m to 3600 m and from depth 4000 m to 6000 m, the deterministic value of casing extrusion load obtained using the tradition single valued method is lower by 0.12 g/cm^3 and 0.2 g/cm^3 on average than the equivalent density corresponding to the cumulative probability of 5% and 35%. This suggests that the steel grade of the casing is lower than enough and an under-engineering and an increase in risk of casing collapse exist.

Known from Figure 6 (c) that in the well section from depth 0 m to 2800 m, the deterministic value of casing extrusion load obtained using the traditional single valued method lies in the equivalent density ranges corresponding to the cumulative probability of [5%, 95%] and [35%, 65%]. This suggests that the steel grade of the casing is selected reasonably. In the well sections from depth 3000 m to 3500 m and from depth 5200 m to 6200 m, the deterministic value of casing extrusion load obtained using the tradition single valued method is lower by 0.1 g/cm^3-0.12 g/cm^3 and 0.1 g/cm^3-0.2 g/cm^3 than the equivalent density corresponding to the cumulative probability of 5% and 35%. This suggests that the steel grade of the casing is lower than enough and an under-engineering and an increase in risk of casing collapse exist.

Comparing Figure 6 (I) and Figure 6 (II), it is noted that at a certain well depth, the higher the confidence level is, the larger the confidence interval and the window of casing extrusion load equivalent density are. Otherwise, at a certain well depth, the lower the confidence level is, the smaller the confidence interval and the window of casing extrusion load equivalent density are. However, if the confidence interval is too large, the selected casing steel grade will be too high, increasing the cost. In this situation, the confidence interval can be evaluated and adjusted to be smaller according to the practical geology condition of adjacent wells which are already drilled. In addition, referring to the field drilling experience of other wells in this block, formation parameters can be corrected by omitting the highly deviated data. Then, the random distribution range of formation parameters will be narrowed. Thus, a smaller distribution range of the extrusion load on casing is obtained at a relatively high confidence level. However, even at the same confidence level, the window of equivalent density of extrusion load on casing varies with the well depth. It is because that the randomness level of formation parameters is different at different depth.

Applying the same method, the equivalent density of extrusion load on casing is calculated for more than 30 wells in this block, where casing collapse accidents happened. The extrusion loads of each program of casing obtained using the traditional method is checked whether they lie in the 90% confidence interval obtained by the new uncertainty analysis method. Then the ratio of the number of wells of which the traditional result lies in the interval at the confidence level of 90% obtained by the new method to the total well number is calculated, as shown in Figure 7. The safety factors obtained using the traditional method are compared with those obtained using the new uncertainty analysis. The result is shown in Figure 8. The number of failure predicted using the traditional method and the number of failure predicted using the new uncertainty analysis are obtained and their ratio is also calculated, as shown in Figure 9.

$$S_c = \frac{p_{cs}}{p_{ce}} \tag{21}$$

$$S_{cp} = S_c \times \frac{p_{ce}}{\left[p_{c5\%}, p_{c95\%}\right]} \tag{22}$$

$$S_{cp5\%} = S_c \times \frac{p_{ce}}{p_{c5\%}} \tag{23}$$

$$S_{cp95\%} = S_c \times \frac{p_{ce}}{p_{c95\%}} \tag{24}$$

Where, S_c is the average safety factor for casing under extrusion load condition obtained by the tradition method, dimensionless; S_{cp} is the corrected average safety factor in the 90% confidence interval, dimensionless; $S_{cp5\%}$ is the corrected average safety factor at the 5% confidence level, which is the lower limit of the 90% confidence interval, dimensionless; $S_{cp95\%}$ is the corrected average safety factor at the 95% confidence level, which is the upper limit of the 90% confidence interval, dimensionless; p_{cs} is the collapse strength of the casing calculated using traditional method, MPa; p_{ce} is the extrusion load on the casing calculated using traditional method, MPa; $p_{c5\%}$ is the extrusion load on the casing calculated using new uncertainty analysis method at the 5% confidence level, which is the lower limit of the 90% confidence interval, MPa; $p_{c95\%}$ is the extrusion load on the casing calculated using new uncertainty analysis method at the 95% confidence level, which is the upper limit of the 90% confidence interval, MPa;

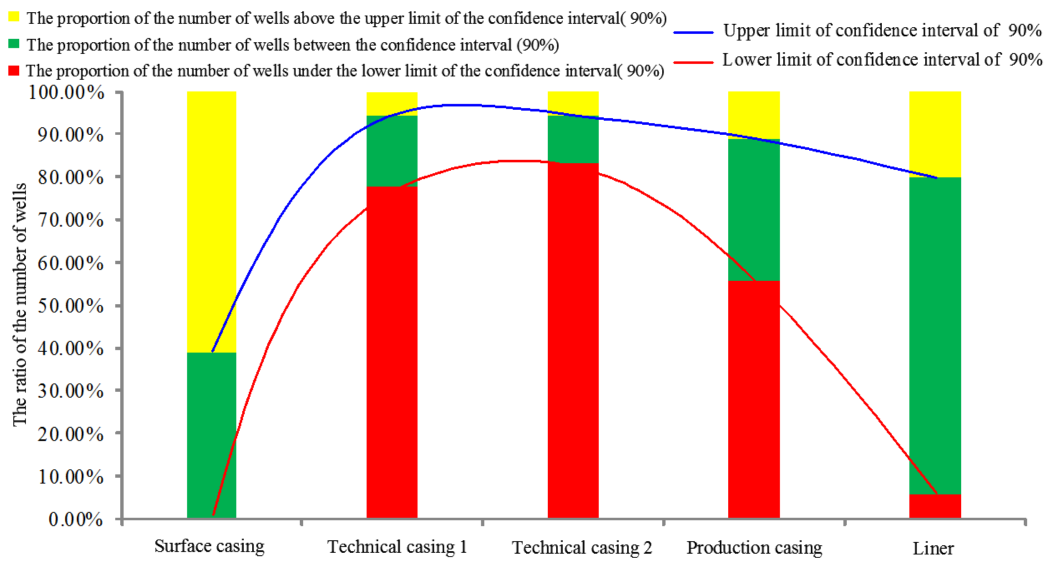
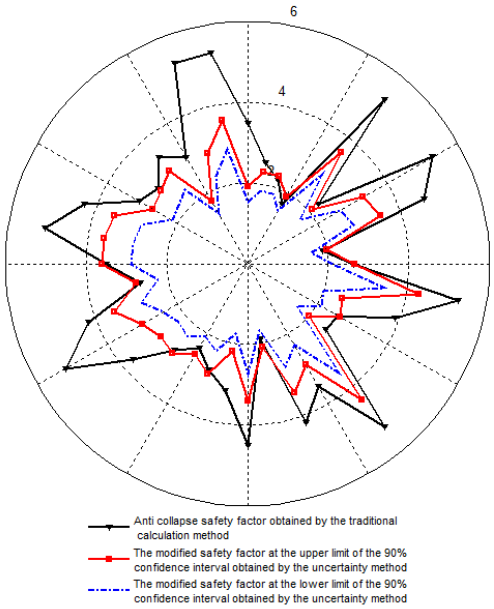
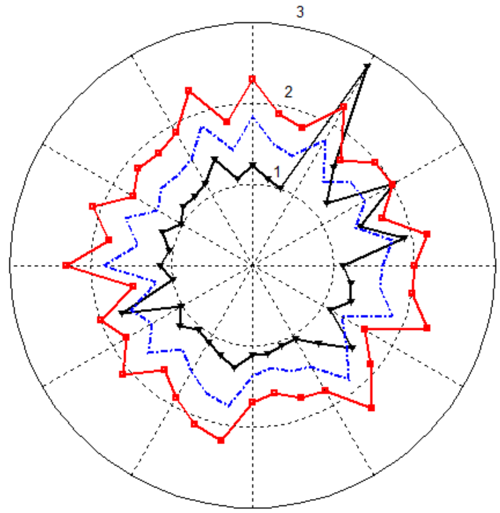


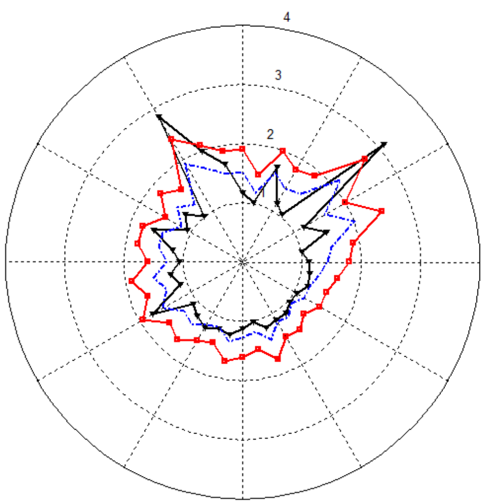
Figure 7: The proportion of the number of well at which the extrusion loads on casing obtained using the traditional method lie in and outside of the confidence interval at the confidence level of 90% obtained by the new uncertainty analysis method



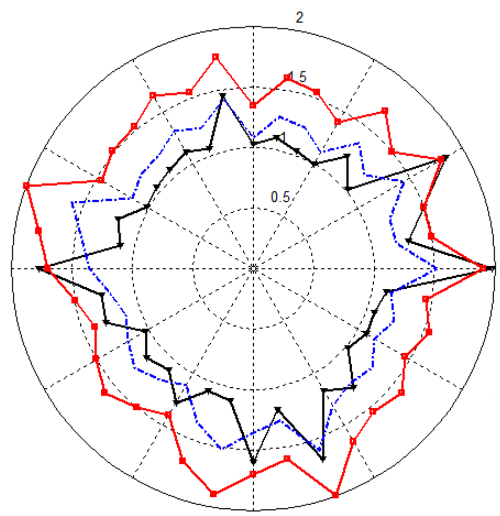
(a) Surface casing



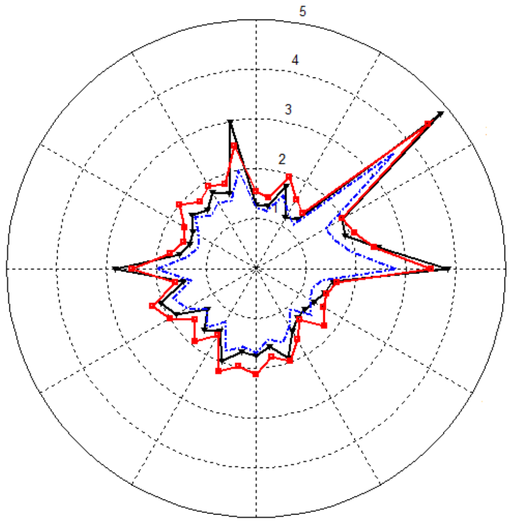
(b) Technical casing 1



(b) Technical casing 2



(d) Production casing



(e) Liner

Figure 8: Comparison of casing collapse safety factor

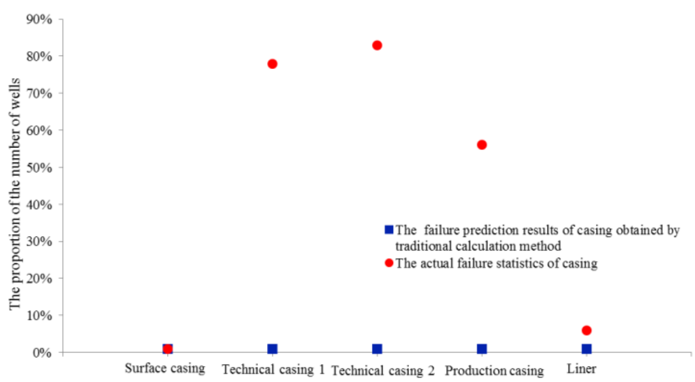


Figure 9: Proportion of casing failure wells for each casing program

According to the statistical results, regarding to surface casing, the extrusion load on the casing calculated using traditional method for all wells is larger than the extrusion load on the casing calculated using new uncertainty analysis method at the lower limit of the 90% confidence interval $p_{c5\%}$. In addition, for 62% of all the wells, p_{ce} is larger than the $p_{c95\%}$, as shown in Figure 7. S_c, $S_{cp5\%}$ and $S_{cp95\%}$ are calculated using Eq. (22), (23) and (24). The result shows that for most wells, the safety factor calculated using the traditional method is larger than the safety factor obtain using new uncertainty analysis at both the upper limit and the lower limit of the 90% interval, as shown in Figure 8 (a).

Based on uncertainty analysis method, it is concluded that S_c value is always larger than required, which induces an over-engineering and an increase in cost. It is proved by the well history that no surface casing collapse accident took place, as shown in Figure 9.

Regarding to technical casing 1, technical casing 2 and production casing, as shown in Figure 7, in portions of 77.8%, 83.3% and 55.6% of all the wells, $p_{ce} \le p_{c5\%}$; while $p_{c5\%} < p_{ce} < p_{c95\%}$ in portions of 16.7%, 11.1% and 33.3% of all the wells and $p_{ce} \ge p_{c95\%}$ only in portions of 5.6%, 5.6% and 11.1% of all the wells. To sum up, the extrusion load on these 3 types of casing calculated using the traditional method is always lower than that calculated using the new uncertainty analysis method. The calculation result of S_c, $S_{cp5\%}$ and $S_{cp95\%}$ shows that S_c is always lower than $S_{cp5\%}$ and $S_{cp95\%}$, as shown in Figure 8 (b), Figure 8 (c) and Figure 8 (d). Based on the uncertainty analysis method, it is concluded that S_c is always lower than the value required, inducing an under-engineering and an increase in risk of casing accident for technical casing 1, technical casing 2 and production casing. Using traditional method, the predicted casing collapse accident number is zero. However, according to the well history, that the portions of wells where casing collapse accidents took place in technical casing 1, technical casing 2 and production casing are 80%, 82.8% and 56% respectively, as shown in Figure 9. Thus, a better accuracy of uncertainty analysis method than the traditional method is proved.

Regarding to the liner, as shown in Figure 7, in a portion of 74.4% wells, $p_{c5\%} < p_{ce} < p_{c95\%}$;in a portion of 20% wells, $p_{ce} \ge p_{c95\%}$;while only in a portion of 5.6% wells, $p_{ce} \le p_{c5\%}$. In other words, the extrusion load on liner calculated using traditional method always lies in the 90% confidence interval obtained using uncertainty analysis. According to Eqs. (22-24), for most of the wells, S_c lies in the range of [$S_{5\%}$, $S_{95\%}$], as shown in Figure 8 (e), suggesting that the design of liners is safe for most wells. The number of liner collapse accidents is predicted to be zero using tradition method, and only three collapse accident took place on liner according to the well history, as shown in Figure 9. Thus, a better accuracy of new uncertainty analysis method than the traditional method is proved.

In summary, based on traditional calculation method for casing extrusion load, the design of surface casing is over-engineered, while the design of technical casing and production casing is under-engineered, of which the safety is questionable. For the design based on new uncertainty analysis method, surface casing of lower steel grade is favorable, which decreases the casing cost; however, higher steel grade will be selected for technical casing and production casing to ensure the safety against collapse. Compared with the traditional casing extrusion load calculation method, of which the result of extrusion load of a certain depth is a deterministic single value, the variation of extrusion load with depth is taken into consideration in new uncertainty analysis method. For deep wells and ultra-deep wells, new uncertainty analysis method better describes the real load condition of the casing. The variation range of extrusion load on casing obtained using new uncertainty analysis method is more reliable as a reference for the determination of the casing safety factor against extrusion than the single valued result of traditional method.

Then, by the application of a better safety factor value, waste of over-engineering is avoided and the safety of casing is better ensured.

5 Conclusion

(1) With consideration of the randomness of formation parameters, which influences the uncertainty of the extrusion load on the casing in ultra-deep wells, a new uncertainty analysis method is presented for the extrusion in ultra-deep wells.

(2) Because of the uncertainty of the extrusion load on casing in complex geology structure, the value of the extrusion load at a certain depth should be described as a confidence interval at a certain confidence level, rather than a single curve. The description of confidence interval is of more significance in the characterization of the in-situ stress condition in complex geology structure.

(3) As the increase of the confidence level, the reliability of the extrusion load on casing in ultra-deep wells increases, while the range of uncertainty increases as well. Otherwise, reliability and range of uncertainty decrease with the confidence level. Thus, it is recommended that the collection of precise geology information is of great significance because it is the proof for the correction of formation data by omitting the highly deviated data. Then, the uncertainty range of the extrusion load on casing will be decreased. The increase in the accuracy of determination of the extrusion load on casing is advantageous to a more reasonable design of well structure, the guarantee of safety and decrease of cost.

(4) According to the analysis on several wells in Shanqian block, it is concluded that the new uncertainty analysis provides a more reasonable prediction result of extrusion load on casing in ultra-deep wells than traditional single valued method. It is proved that the new uncertainty analysis method is more reliable as a reference for the design of well structure and the selection of casing in deep wells and ultra-deep wells.

Acknowledgements: This work was supported by the National Science and Technology Special Project of China [2016ZX05022-002], the Chongqing Research Program of Basic Research and Frontier Technology [CSTC2015jcyjA90021], and the Scientific and Technological Research Program of Chongqing Municipal Education Commission [KJ1501302], and the Academician Led Special Project of Chongqing Science and Technology Commission [CSTC2017zdcy-yszxX0009]. This research is also supported by other projects (Grant numbers: 2017ZX05009003-007, 2016D-5007-0308, 51374266, 51774063).

References

Aasen, J. A.; Bernt, S.; Aadnoy. (2007): Three-dimensional well tubular design improves margins in critical wells. *Journal of Petroleum Science and Engineering*, vol. 56, no. 4, pp. 232-240.

Adams, A. J.; Parfitts, H. L.; Reeves, T. B.; Thorogood, J. L. (1993): *Casing system risk analysis using structural reliability*, *SPE/ IADC* 25693.

El-Sayed, A. H.; Khalaf, F. (1992): Resistance of cemented concentric casing strings under non-uniform loading. *SPE Drilling Engineering,* vol. 7*,* no. 1, pp. 59-64.

Fan, H. H.; Ye, Zhi.; Ji, R. Y. (2011): Investigation on three-dimensional overburden pressure calculation method. Chinese *Journal of Rock Mechanics and Engineering*, vol. 30, no. 2, pp. 3879-3883.

Klever, F. J.; Tamano, T. (2004): *A new OCTG strength equation for collapse undercombined loads*. SPE 90904.

Liao, H. L.; Guan, Z. C.; Feng, G. T. (2009): Casing failure mechanism and strength design considerations for deep and ultra-deep wells. *Oil Drilling & Production Technology*, vol. 31, no. 2, pp. 1-4.

Long, G.; Li M.; Guan, Z. C. (2013): Evaluation method of casing safety and reliability in deep wells. *Petroleum Drilling Techniques*, vol. 41, no. 4, pp. 48-53.

LI, M. (2008): *Analysis on Distribution of Load Uncertainty and Safety and Reliability of Casing in Ultra-Deep Well*. Qingdao: China University of Petroleum (East China).

Li, Z. M.; Yin, Y. Q. (2006): *Casing external loading calculation and its mechanics foundation in oil wells*. Beijing, Petroleum Industry Press.

Pattilo, P. D.; Last, N. C.; Asbill, W. T. (2003): *Effect of nonuniform loading on conventional casing collapse resistance*. SPE 79871.

Prasongsit, C.; Paolo, G.; Jerome, S. (2001): *Structural reliability: assessing the condition and reliability of casing in compacting reservoirs*. IPTC 14297.

Steve, H. (1999): *Pre-drill overburden estimation*. [S.l.]: [s.n.].

Sun, Z.; Chen, L. (2003): *Practical mechanical reliability design theory and method*. Beijing: Science Press.

Wan, L. F.; Li, G. S.; Guan, Z. C. (2012): A numerical analysis of the actual external collapse load on deep-well casing in its strength design. China *Petroleum Machinery,* vol. 40, no. 4, pp. 1-4.

Wang, Y. F.; Li, J. Q.; Yang, X. H. (2008): Stress Distribution in Casing-Cement-Stratum. *Petroleum Drilling Techniques*, vol. 36, no. 5, pp. 8-12.

Xiao, X. N. (2002): *Probability theory and mathematical statistics*. Beijing: Peking University.

Yang, H. L.; Chen, M.; Jin, Y. (2006): Analysis of casing equivalent collapse resistance in creep formations. *Journal of China University of Petroleum*, vol. 30, no. 4, pp. 95-98.