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# Grey Wolf Optimizer to Real Power Dispatch with Non-Linear Constraints

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**Abstract:** A new and efficient Grey Wolf Optimization (GWO) algorithm is implemented to solve real power economic dispatch (RPED) problems in this paper. The nonlinear RPED problem is one the most important and fundamental optimization problem which reduces the total cost in generating real power without violating the constraints. Conventional methods can solve the ELD problem with good solution quality with assumptions assigned to fuel cost curves without which these methods lead to suboptimal or infeasible solutions. The behavior of grey wolves which is mimicked in the GWO algorithm are leadership hierarchy and hunting mechanism. The leadership hierarchy is simulated using four types of grey wolves. In addition, searching, encircling and attacking of prey are the social behaviors implemented in the hunting mechanism. The GWO algorithm has been applied to solve convex RPED problems considering the all possible constraints. The results obtained from GWO algorithm are compared with other state-of-the-art algorithms available in the recent literatures. It is found that the GWO algorithm is able to provide better solution quality in terms of cost, convergence and robustness for the considered ELD problems.

**Keywords:** Grey wolf optimization (GWO), constraints, power generation dispatch, evolutionary computation, computational complexity, algorithms.

## 1 Introduction

Real power economic dispatch (RPED) is one of the most important non - linear problem to be solved in the modern power system. The objective of the RPED problem is to allocate optimal real power generation to the existing thermal units without violating the constraints in the system. Conventional methods like lambda-iteration method, and so on are used to solve traditional RPED problem with assumptions many assumptions [Park, Lee, Shin et al. (2005); Sayah and Hamouda (2013)].

However, in practical, the nonlinearities and discontinuities like valve point loading, ramp rate limits and so on represent RPED problem as a non-smooth or non-convex optimization

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problem which makes it difficult for the traditional methods to obtain the global optimum [Park, Lee, Shin et al. (2005); Sayah and Hamouda (2013)]. Moreover considerable number of researchers has shown interest in developing an efficient algorithm in solving the RPED problem with nonlinearities [Mandal, Roy and Mandal (2014)]. Though the conventional methods have advantages like few control parameters and less computational time, it fails to reach global optima for the ELD problems with large dimensional and discrete search space [Nguyen and Vo (2015)].

According to No Free Lunch (NFL) theorem, there exist no meta heuristic optimization algorithm which is applicable in solving all real world optimization problems [Mirjalili, Mirjalili and Lewis (2014); Basu (2014)]. The development of numerous meta heuristic algorithms by various researchers around the world over the past two decades has successfully solved the ELD problem with superior convergence characteristics, high solution quality and robustness, eliminating most of the difficulties of classical methods [Mandal, Roy and Mandal (2014); Basu (2015)].

Grey Wolf Optimization (GWO) algorithm, a recent swarm intelligence algorithm is proposed to solve the non-convex optimization problem [Mirjalili, Mirjalili and Lewis (2014)]. The leadership and hunting behaviors of grey wolves in nature is incorporated in the algorithm and has superior exploration and exploitation ability. In solving real world problems, the GWO algorithm has the capability of providing higher quality solutions and good computational efficiency with few parameters and ease of implementation [Mandal, Roy and Mandal (2014); Mirjalili, Mirjalili and Lewis (2014)]. These properties have motivated few researchers to implement the GWO algorithm in solving problems like combined heat and power dispatch [Mandal, Roy and Mandal (2014)], hyper spectral band selection [Medjaheda, Ait Saadib, Benyettoua et al. (2015)], load frequency control [Guha, Roy and Banerjee (2015)], optimal reactive power dispatch [Sulaimana, Mustaffab, Mohameda et al. (2015)], power system stabilizer design [Shakarami and Faraji Davoudkhani (2015)], MPPT design [Mohanty, Subudhi and Ray (2016)], flow shop scheduling [Komakia and Kayvanfar (2015)], attribute reduction [Emarya, Yamany, Hassaniena et al. (2015)], feature selection [Emary, Zawbaa and Hassaniena (2015)], parameter estimation [Song, Tang, Zhao et al. (2015)] and automatic generation control [Sharma and Saikia (2015)]. In this paper, GWO algorithm is implemented to solve the RPED problem to validate its effectiveness over other meta heuristic algorithms. The simulation results show that this algorithm performs better than the other algorithms in terms of solution quality, convergence efficiency and robustness.

Section 2 describes the formulation of ELD problem with constraints like ramp rate limits and so on. The detailed description of GWO algorithm is discussed in Section 3. Section 4 describes the implementation of GWO to the complex RPED problem. The numerical results and discussion of the GWO algorithm for different test systems are presented in Section 5 and conclusion is drawn in Section 6.

## 2 Formulation of the ELD problem

Minimization of the total cost in producing real power in a power system without violating constraints is the main aim of RPED [Sahoo, Dash, Prusty et al. (2015)]. In this paper, RPED problem without valve point loading is considered.

***2.1 RPED problem with smooth cost function***

The objective of the RPED problem with smooth cost function is given by

$$\text{Minimize } F_T = \sum_{q=1}^{N} F_q\left(P_q\right) \tag{1}$$

where $F_T$ represents total fuel cost of all the thermal units present in the system ($/hr), $N$ is the total number of thermal units existing in the system and $F_q\left(P_q\right)$ represents fuel cost of the $q^{th}$ thermal unit ($/hr) and $P_q$ represents power generated by the $q^{th}$ thermal unit (MW).

In general, the fuel cost function $F_q\left(P_q\right)$ for $q^{th}$ thermal unit is expressed in quadratic polynomial as

$$F_q\left(P_q\right) = a_q P_q^2 + b_q P_q + c_q \tag{2}$$

where $a_q, b_q$ and $c_q$ are the cost coefficients of $q^{th}$ thermal unit.

The different practical constraints to which the above minimization problem is subjected are power balance or demand constraint, generator output limits, prohibited operating zones and ramp rate limits.

***2.2 Power balance or demand constraint***

The sum of individual power generated from each thermal unit existing in the system must be equal to the sum of transmission loss and total demand of the system which is represented as

$$\sum_{q=1}^{N} P_q = D + P_{Loss} \tag{3}$$

where $D$ is the total demand of the system (MW) and $P_{Loss}$ is the transmission loss of the system (MW).

The transmission loss of the system $P_{Loss}$ which is a function of power generated by each unit is given by

$$P_{Loss} = \sum_{q=1}^{N}\sum_{r=1}^{N} P_r B_{qr} P_r + \sum_{q=1}^{N} B_{0q} P_q + B_{00} \tag{4}$$

where $B_{qr}, B_{0q}$ and $B_{00}$ are the loss coefficients or $B-$coefficients.

***2.3 Real power generating limits***

The power generated $P_q$ from each thermal unit must lie within its permissible limits which is represented as

$$P_{q,\min} \le P_q \le P_{q,\max} \tag{5}$$

where $P_{q,\min}$ and $P_{q,\max}$ are the minimum and maximum generation of the $q^{th}$ generator (MW) respectively.

### *2.4 Ramp rate limit*

Theoretically in RPED problems it is assumed that the output from thermal units is adjusted linearly. But in practical, this assumption is not plausible as the operating limits of each generators are restricted by their corresponding up-rate limit $UR_q$ , down-rate limit $DR_q$ and previous hour generations $P_q^0$. Hence, for $q^{th}$ generating unit,

$$P_q - P_q^0 \le UR_q \text{ , if } P_q > P_q^0 \tag{6}$$

$$P_q^0 - P_q \le DR_q \text{, if } P_q < P_q^0 \tag{7}$$

Therefore, using the ramp rate limits the real power generating constraints given in Eq. (5) can be modified as

$$Max\left(P_{q,\min}, P_q^0 - DR_q\right) \le P_q \le Min\left(P_{q,\max}, P_q^0 + UR_q\right) \tag{8}$$

### *2.5 Prohibited operating zones*

In the characteristic curve of the thermal units, due to some non-linear behavior existing in shaft bearing or faults in the machines or its associated auxiliary equipment, some thermal unit might have prohibited operating zones which is to be avoided. The input-output characteristics of a generator with POZ is shown in Fig. 1 [Subbaraj, Rengaraj and Salivahanan (2009)]

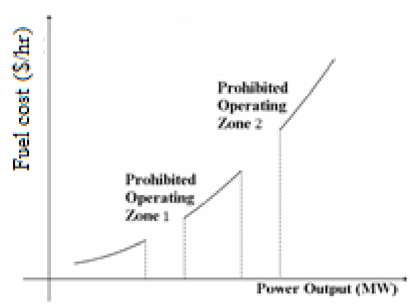



**Figure 1:** Characteristic curve of thermal unit with POZs

Therefore, the operating constraint of the $q^{th}$ unit with POZ is

$P_q \notin \left[P_{qL}^k, P_{qU}^k\right]$

i.e., $P_q < P_{qL}^k$ and $P_q < P_{qU}^k$ (9)

where $k = 1,2,..., z_j$ is the index of POZ of $q^{th}$ thermal unit, $z_j$ is the total number of POZ exist for $q^{th}$ generator and $P_{qL}^k$ and $P_{qU}^k$ are the lower and upper limit of $k^{th}$ POZ of the $q^{th}$ thermal unit (MW) respectively

## 3 Grey Wolf Optimization (GWO) algorithm

GWO is a very recent optimization algorithm inspired by gray wolves and is developed in 2014 [Mirjalili, Mirjalili and Lewis (2014)]. The algorithm imitates the hunting and the social hierarchy behaviors of grey wolves. In addition, to the advantages of meta heuristic algorithms the GWO algorithm requires no specific input parameters to be initialized. Also, the GWO algorithm is straight forward, free from computational complexity and can be easily implemented in any programming languages [Guha, Roy and Banerjee (2015)]. The interesting fact of grey wolves is that it possesses social dominant hierarchy as shown in Fig. 2 and this hierarchy is used in GWO. The leader wolf or alpha $(\alpha)$ wolf takes decisions like hunting, searching, time to wake and so on. The beta $(\beta)$ wolf supports alpha $(\alpha)$ wolf in decision making and the delta $(\delta)$ wolf follows the alpha $(\alpha)$ and beta $(\beta)$ wolves. The wolves which do not come under these category are called as omega $(\omega)$ wolves and are used basically as a scapegoat [Medjaheda, Ait Saadib, Benyettoua et al. (2015); Sharma and Saikia (2015)].

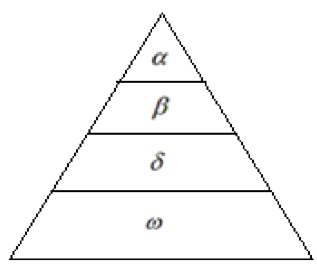



**Figure 2:** Hierarchy of grey wolves

In addition, the group hunting another social behavior is considered in the algorithm. The three stages by which the grey wolf attacks the prey are explained in Muro et al. [Muro, Escobedo, Spector et al. (2011)] and is modeled as follows.

### *3.1 Modeling of GWO*

#### *3.1.1 Social hierarchy*

For modeling the GWO algorithm, the wolves are classified based on the fitness value of the problem. The best solution is considered as $\alpha$ wolf, followed by $\beta$, $\delta$ and $\omega$ wolves.

#### *3.1.2 Encircling of prey*

The encircling behavior of grey wolf around the prey is modeled mathematically using Eq. (10) and Eq. (11). Using these equations, a grey wolf updates its position within solution space around the prey.

$$\vec{C} = \left| \vec{B} . \vec{Y_P}(t) - \vec{Y}(t) \right| \tag{10}$$

$$\vec{Y}(t+1) = \vec{Y_P}(t) - \vec{A} . \vec{C} \tag{11}$$

Where the current iteration in the problem is represented as $t$, $\vec{Y_P}(t)$ is the position of the prey, $\vec{Y}(t)$ indicates the position of grey wolf at $t$, $\vec{Y}(t+1)$ indicates the position of grey wolf at $t+1$ and $\vec{A}$ and $\vec{B}$ are the coefficient vectors which are computed using Eq. (12) and Eq. (13) respectively.

$$\vec{A} = 2\,\vec{a}\,.rand - \vec{a} \tag{12}$$

$$\vec{B} = 2.rand \tag{13}$$

Where $\vec{a}$ decreases from 2 to 0 linearly as the iteration increases and $rand$ is the random vectors between $[0, 1]$ such that $A$ gets values within $[-a, a]$.

#### *3.1.3 Hunting*

Though all the grey wolves can recognize the prey's location, $\alpha, \beta$ and $\delta$ grey wolves have more knowledge about the location. Therefore, the positions of these wolves are saved and force the other wolves to update their position using Eq. (14) through Eq. (16).

$$\vec{C_\alpha} = \left| \vec{B_1} . \vec{Y_\alpha}(t) - \vec{Y}(t) \right|, \vec{C_\beta} = \left| \vec{B_2} . \vec{Y_\beta}(t) - \vec{Y}(t) \right|, \ \vec{C_\delta} = \left| \vec{B_3} . \vec{Y_\delta}(t) - \vec{Y}(t) \right| \tag{14}$$

$$\vec{Y_1}(t) = \vec{Y_\alpha}(t) - \vec{A_1} . \vec{C_\alpha}, \ \vec{Y_2}(t) = \vec{Y_\beta}(t) - \vec{A_2} . \vec{C_\beta}, \ \vec{Y}(t) = \vec{Y_\delta}(t) - \vec{A_3} . \vec{C_\delta} \tag{15}$$

$$\vec{Y}(t+1) = \frac{\vec{Y_1}(t) + \vec{Y_2}(t) + \vec{Y_3}(t)}{3} \tag{16}$$

Where $\vec{Y}_{\alpha}(t)$, $\vec{Y}_{\beta}(t)$ and $\vec{Y}_{\delta}(t)$ are the position of first, second and third best fitness value, $\vec{C}_{\alpha}, \vec{C}_{\beta}$ and $\vec{C}_{\delta}$ is determined using Eq. (10) , $\vec{A}_i$ and $\vec{B}_i$ are determined using Eqs. (12) and (13) , $\vec{Y}_i(t)$, $i=1,2$ and 3 are the updated position of $\vec{Y}(t)$ based on position of alpha, beta and grey wolves respectively.

*3.1.4 Attacking prey (Exploitation)*

During this phase, $\vec{a}$ value gets reduced which reduces the fluctuation of $\vec{A}$. Since $\vec{A}$ is a vector whose value is in the range of $[-1,1]$, the position of grey wolf will be towards the position of prey in the next generation.

*3.1.5 Searching the prey (Exploration)*

The $\alpha, \beta$ and $\delta$ wolves diverges to search the prey and then converges to attack it. All other grey wolves search the prey with respect to above three wolves. This process of searching the prey emphasizes the exploration capability of grey wolves to search globally. Figure 3 represents the flowchart of GWO algorithm.

## 4 Implementation of GWO algorithm to RPED (GWO-RPED)

The implementation of GWO algorithm to solve RPED complex problem is described as follows:

Step 1: For the chosen test system, read the input data to compute the total fuel cost of the system.

Step 2: Initialization of GWO parameters i.e. population size $N$ and select the stopping criteria.

Step 3: Select the number of design variables, $D$ and initialize the design variables i.e. the real power outputs for each generating units in the chosen system. In accordance to the population size, the design variable is generated randomly using Eq. (17).

$$P_{rq} = P_{q,\min} + rand(1) \times \left(P_{q,\max} - P_{q,\min}\right) \tag{17}$$

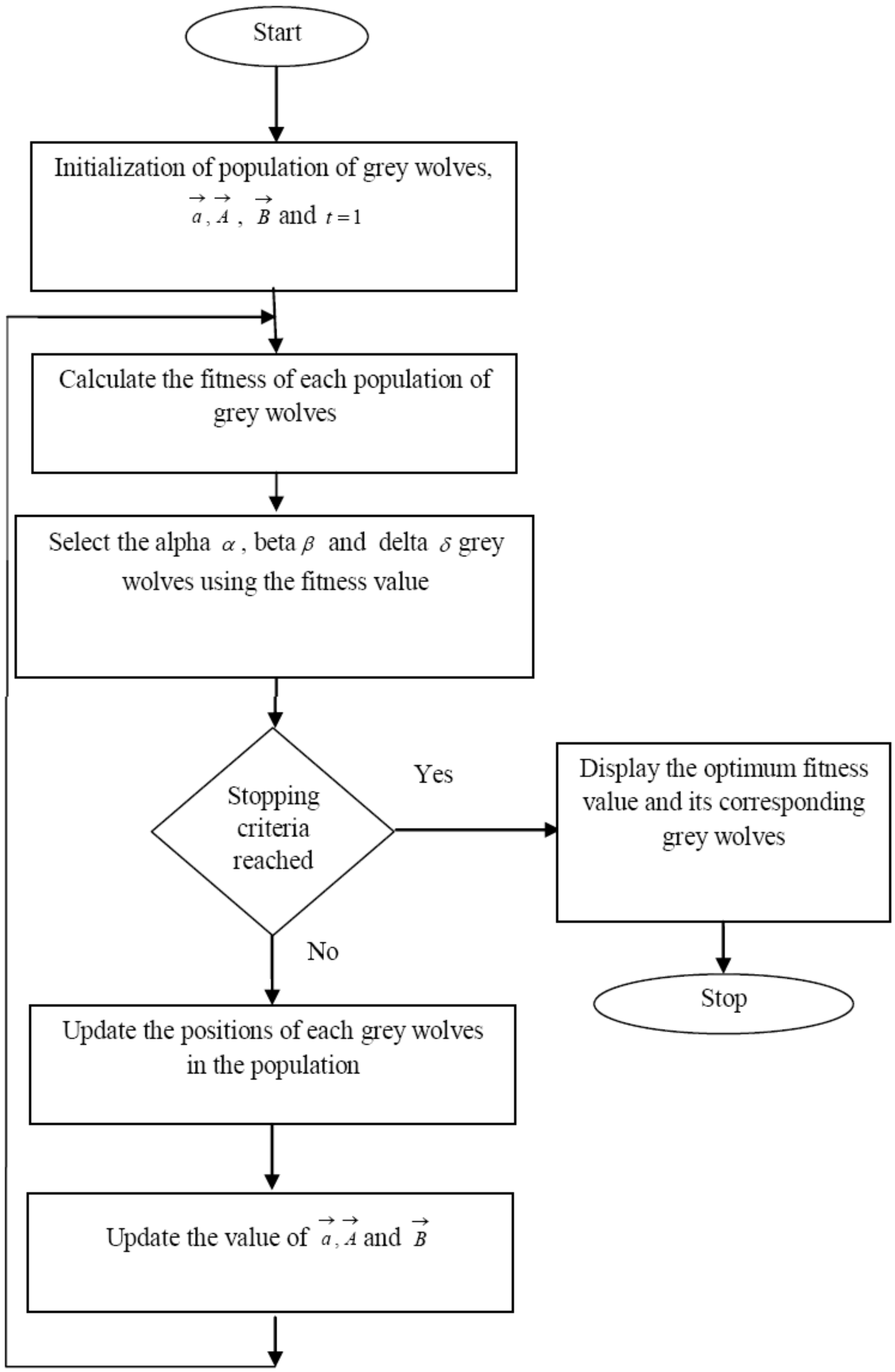



**Figure 3:** Flowchart of GWO

where $q = 1,2,..., N$ , $r = 1,2,..., D$

Therefore, the matrix of $D \times N$ is initialized using Eq. (17).

Step 4: The fitness of each population is calculated using $F_T$. After sorting the fitness value in descending order, the minimal fitness value is saved as alpha $(\alpha)$, next minimal as beta $(\beta)$ and third minimal as delta $(\delta)$ grey wolves as given in Eq. (18).

$$F_\alpha = F_T(N), F_\beta = F_T(N-1) \text{ and } F_\delta = F_T(N-2) \tag{18}$$

Step 5: The individual population corresponding to $F_\alpha$, $F_\beta$ and $F_\delta$ are saved as $\vec{P_\alpha}(t)$, $\vec{P_\beta}(t)$ and $\vec{P_\delta}(t)$ respectively.

Step 6: Determine $\vec{A}_i$ and $\vec{B}_i$ using Eqs. (12) and (13).

Step 7: Update the position of each grey wolf in the population using Eqs. (14)-(16).

Step 8: Select the termination criterion

Step 4 to step 7 will be repeated till the termination criteria is reached by the algorithm.

## 5 Results and discussions

In this section, the performance of the algorithm in solving various complex RPED problems with 6, 15, 20 and 40 thermal unit is discussed. The different constraints considered for these test systems are ramp rate limits, POZ and individual generator limits. The GWO algorithm for different test system has been implemented in MATLAB 2013a on Intel (R) Core (TM) i7-3517U CPU 2.40GHz with 8G-RAM. Simulation results obtained are compared with the results reported in the recent literatures in terms of solution quality.

### *5.1 RPED problem with POZ and transmission line loss*

For RPED problem with POZ and transmission line loss characteristics, the GWO algorithm has been implemented on (i) 6 generating unit and (ii) 15 generating unit for comparison. The performance of GWO is compared with the results obtained in the recent literatures

#### *5.1.1 Test system 1: 6 unit system*

Initially, the GWO algorithm is applied to a small test system comprising of 6 generating unit with load demand of 1263 MW which is referred as SYS1. The system coefficients and loss coefficients are listed in Gaing [Gaing (2003)]. The transmission loss, ramp rate limits and POZ are considered for this test system. All the six generating units have two sets of prohibited operating zones. The minimum fuel cost reported in recent literature [Mandal, Roy and Mandal (2013)] is 15,443.06 \$/hr. The best fuel cost obtained by GWO algorithm is 15,443 \$/hr. The result obtained using GWO indicates that the algorithm attains the global solution with reasonable computational time. The optimal power generation and its corresponding minimum cost obtained using GWO algorithm is given in Tab. 1.

The comparison of statistical data of GWO algorithm with the results obtained using different algorithm is given in Tab. 2. The results presented in Tab. 2 suggest that GWO algorithm has the capability of attaining global minimum value for the ELD problems. To move further, the GWO algorithm is applied to large sized problems to assess the efficiency of the algorithm.

**Table 1:** Optimal power for SYS1 using GWO

| Unit | Real power output in MW | Unit | Real power output in MW |
|---|---|---|---|
| P1 | 447.1631 | P4 | 138.368 |
| P2 | 173.5742 | P5 | 165.5965 |
| P3 | 263.4559 | P6 | 87.2987 |
| Total power generation in MW | | | 1275.456 |
| Power loss in MW | | 12.4563 | |
| Total fuel cost in $/hr | | | 15443 |

*5.1.2 Test system 2: 15 thermal unit system*

Here, 15 thermal unit system is considered to demonstrate goodness of the GWO algorithm in solving this convex RPED problems including all the constraints and is referred as SYS2 in this paper. The system coefficients and loss coefficients for SYS2 are listed in Gaing [Gaing (2003)]. The transmission loss, ramp rate limits and POZ are considered.

**Table 2:** Comparison of various algorithms with GWO for SYS1

| Algorithm | **Total fuel cost in $/hr** | | |
|---|---|---|---|
| | **Minimum** | **Average** | **Maximum** |
| IPSO [Yu and Li (2016)] | 15,449.88 | NA | NA |
| MPSO-TVAC [Abdullah, Baskar, Rahim et al. (2013)] | 15,449.91 | 15,450.17 | 15,451.57 |
| New MPSO [Moradi-Dalvand, Mohammadi-Ivatloo, Najafi et al. (2015)] | 15,447.00 | 15,447.00 | 15,455.00 |
| SGA [Moradi-Dalvand, Mohammadi-Ivatloo, Najafi et al. (2015)] | 15,447.00 | 15,450.00 | 15,470.00 |
| BFO [Yu and Li (2016)] | 15,443.85 | NA | NA |
| BBO [Moradi-Dalvand, Mohammadi-Ivatloo, Najafi et al. (2015)] | 15,443.09 | 15,443.09 | 15,443.09 |
| KHA-IV [Mandal, Roy and Mandal (2014)] | 15,443.07 | 15,443.19 | 15,443.33 |
| HCRO-DE [Yu and Li (2016)] | 15,443.07 | NA | NA |
| IPSO-TVAC [Yu and Li (2016)] | 15,443.06 | NA | NA |
| GWO | 15,443.00 | 15,443.05 | 15,443.37 |

*NA - Not available

The total demand is 2630 MW. Thermal units 2, 5, 6 and 12 have prohibited operating zones. The best fuel cost reported in Basu [Basu (2016)] is 32,548.17 $/hr. The best fuel cost obtained by GWO algorithm for SYS2 is 32,548.13 $/hr. The optimal power generation obtained using GWO algorithm is given in Tab. 3. The comparison of statistical results obtained using GWO algorithm and other algorithms are summarized in Tab. 4.

**Table 3:** Optimal power for SYS2 using GWO

| Unit | Real power output in MW | Unit | Real power output in MW | Unit | Real power output in MW |
|---|---|---|---|---|---|
| P1 | 455 | P6 | 460 | P11 | 77.0107 |
| P2 | 455 | P7 | 465 | P12 | 80 |
| P3 | 130 | P8 | 60 | P13 | 25 |
| P4 | 130 | P9 | 25 | P14 | 15 |
| P5 | 235.8869 | P10 | 29 | P15 | 15 |
| Total power generation in MW | | 2656.90 | | | |
| Power loss in MW | | 26.90 | | | |
| Total fuel cost in $/hr | | 32,548.138 | | | |

It can be inferred from Tab. 3 and Tab. 4 that the best result has been obtained using GWO algorithm without violating any system constraints.

**Table 4:** Comparison of various algorithms with GWO for SYS2

| Algorithm | **Total fuel cost in $/hr** | | |
|---|---|---|---|
| | **Minimum** | **Average** | **Maximum** |
| SARGA [Subbaraj, Rengaraj and Salivahanan (2009)] | 32,709.63 | 32,730.79 | 32,829.23 |
| MPSO-TVAC [Abdullah, Baskar, Rahim et al. (2013)] | 32,704.47 | 32,705.00 | 33,728.99 |
| RTO [Sayah and Hamouda (2013)] | 32,701.81 | NA | NA |
| SSA [Yu and Li (2016)] | 32,662.51 | NA | NA |
| DEPSO [Basu (2014)] | 32,588.81 | 32,588.99 | 32,588.99 |
| DPD [Parouha and Das (2016)] | 32,548.58 | 32,556.67 | 32,564.40 |
| KGMO [Basu (2016)] | 32,548.17 | 32,548.21 | 32,548.37 |
| GWO | 32,548.13 | 32,548.14 | 32,548.15 |

*5.1.3 Test system 3: 20 generating unit system-ELD problem with transmission losses*

For this variant of ELD, a test system with 20 thermal unit is adopted for evaluating using GWO algorithm and is referred as SYS3. In this system, POZ is not considered. The demand to be met by SYS3 is 2500 MW. The system data and the transmission loss coefficients are considered from Su et al. [Su and Lin (2000)]. The authors in Moradi-Dalvand et al. [Moradi-Dalvand, Mohammadi-Ivatloo, Najafi et al. (2015)] have reached a optimum value of 62,456.63 $/hr. The exploration and exploitation of GWO algorithm has converged the system to reach a better optimum value of 62,454.27 $/hr without violating any system constraints. Tab. 5 provides the optimal power for each unit in the test system obtained using GWO.

**Table 5:** Optimal power for SYS3 using GWO

| Unit | Real power output in MW | Unit | Real power output in MW |
|---|---|---|---|
| P1 | 505.0337 | P11 | 152.2849 |
| P2 | 166.8719 | P12 | 289.7256 |
| P3 | 110.7706 | P13 | 117.3326 |
| P4 | 110.029 | P14 | 43.8318 |
| P5 | 118.6765 | P15 | 111.9407 |
| P6 | 75.2081 | P16 | 36.3903 |
| P7 | 112.0637 | P17 | 65.0679 |
| P8 | 109.4335 | P18 | 93.581 |
| P9 | 110.9587 | P19 | 94.4718 |
| P10 | 112.6017 | P20 | 56.7383 |
| Total power generation in MW | | 2593.012 | |
| Power loss in MW | | 93.012 | |
| Total fuel cost in $/hr | | 62454.27 | |

The results obtained using GWO algorithm for SYS3 is compared with the previously obtained results using various algorithms and is summarized in Tab. 6. The statistical data for these algorithms are obtained from Moradi-Dalvand et al. [Moradi-Dalvand, Mohammadi-Ivatloo, Najafi et al. (2015)]. The authors Moradi-Dalvand et al. [Moradi-Dalvand, Mohammadi-Ivatloo, Najafi et al. (2015)] suggest that the reported results in algorithms SOA, CGPSO and CMSFLA do not satisfy the system constraints.

**Table 6:** Comparison of various algorithms with GWO for SYS3

| Algorithm | **Total fuel cost in $/hr** | | |
|---|---|---|---|
| | **Minimum** | **Average** | **Maximum** |
| CGPSO [Moradi-Dalvand, Mohammadi-Ivatloo, Najafi et al. (2015)] | 59,804.05 | 61,171.84 | 63,184.63 |
| CMSFLA [Moradi-Dalvand, Mohammadi-Ivatloo, Najafi et al. (2015)] | 60,412.88 | 60,412.88 | 60,412.92 |
| EHSA [Vanitha (2012)] | 62,456.75 | NA | NA |
| EBBO [Vanitha (2012)] | 62,456.65 | NA | NA |
| GSO [Moradi-Dalvand, Mohammadi-Ivatloo, Najafi et al. (2015) (2015)] | 62,456.63 | 62,456.63 | 62,456.63 |
| CQGSO [Moradi-Dalvand, Mohammadi-Ivatloo, Najafi et al. (2015)] | 62,456.63 | 62,456.63 | 62,456.63 |
| GWO | 62,454.27 | 62,454.29 | 62,455.39 |

### *5.1.4 Test system 4: 40 thermal unit system-RPED problem with POZ*

A complex system with 40 thermal unit with POZ is considered here and is referred as SYS4. The transmission line losses are neglected. The total demand for SYS4 is 7000 MW. The test system data is available in Chen et al. [Chen and Chang (1995)]. The optimal generation schedule for the test system using GWO algorithm is presented in Tab. 7. The minimum fuel cost achieved by GWO is 99722.99 $/hr. In the recent literature, the minimum fuel cost achieved for the 40 unit system is 100767.68 $/hr in Balamurugan et al. [Balamurugan and Subramanian (2008)].

**Table 7:** Optimal power for SYS4 using GWO

| Unit | Real power output in MW | Unit | Real power output in MW |
|---|---|---|---|
| P1 | 40.7439 | P21 | 456.6604 |
| P2 | 60 | P22 | 459.999 |
| P3 | 140.0025 | P23 | 460 |
| P4 | 24 | P24 | 460 |
| P5 | 26 | P25 | 459.999 |
| P6 | 115 | P26 | 460 |
| P7 | 110.1324 | P27 | 460 |
| P8 | 217 | P28 | 10 |
| P9 | 265 | P29 | 10 |
| P10 | 130 | P30 | 10 |
| P11 | 204.999 | P31 | 20.1001 |
| P12 | 205 | P32 | 20 |
| P13 | 125 | P33 | 20 |
| P14 | 132 | P34 | 20 |
| P15 | 125.1357 | P35 | 18 |
| P16 | 125 | P36 | 18 |
| P17 | 125 | P37 | 20 |
| P18 | 456.6654 | P38 | 25 |
| P19 | 458.9178 | P39 | 25 |
| P20 | 456.6604 | P40 | 25 |
| Total power generation in MW | | | 7000 |
| Total fuel cost in $/hr | | | 99722.99 |

Tab. 8 summarizes the statistical cost achieved by various algorithms for 40 unit system with prohibited operating zone over the decade. It can be observed from Tab. 7 that the GWO algorithm results in a better solution when compared to others and it reveals the capability of algorithm to produce the global optimal cost from a large solution space which has large local optima.

**Table 8:** Comparison of various algorithms with GWO for SYS4

| Algorithm | Fuel cost in $/hr | | |
|---|---|---|---|
| | Minimum | Average | Maximum |
| TPNN [Subbaraj, Rengaraj, Salivahanan et al. (2010)] | 105,236.00 | 106,756.48 | 107,894.59 |
| PSO [Subbaraj, Rengaraj, Salivahanan et al. (2010)] | 101,436.25 | 102,978.56 | 103,568.24 |
| PAPSO [Subbaraj, Rengaraj, Salivahanan et al. (2010)] | 101,246.26 | 101,956.37 | 102,568.84 |
| PSO-MSAF [Subbaraj, Rengaraj, Salivahanan et al. (2010)] | 101,209.99 | 101,584.63 | 101,954.68 |
| SARGA [Subbaraj, Rengaraj, Salivahanan et al. (2010)] | 101,403.99 | NA | NA |
| TSARGA [Subbaraj, Rengaraj, Salivahanan et al. (2010)] | 101,226.51 | NA | NA |
| PSARGA-MAS [Subbaraj, Rengaraj, Salivahanan et al. (2010)] | 101,187.45 | NA | NA |
| IDP [Balamurugan and Subramanian (2008)] | 100,767.68 | NA | NA |
| GWO | 99,722.99 | 99,723.22 | 99,728.81 |

### *5.2 Result analysis*

#### *5.2.1 Parameter selection*

According to many research experts, the efficiency of stochastic search algorithms (such as GA, PSO, DE, etc.) depends on user defined parameters. Using parameter tuning, testing and evaluating different combinations of parameters, the optimal parameter values of an algorithm are obtained for a specific test system [Barisal and Prusty (2015); Amjady and Sharifzadeh (2010)]. In GWO, the parameter which affects the convergence and search capability of the algorithm is the number of grey wolf population. An optimal choice of population size is necessary as the other values makes an algorithm slow, computationally inefficient and leads to local minima than to the global minima. The optimal population size directly depends on problem dimension and complexity to achieve optimum value for the problem [Chaturvedia, Panditb and Srivastava (2009); Roy, Roy and Chakrabartic (2013)]. The numerical values presented in Tabs. 2, 4, 6 and 8 summarizes that GWO algorithm provides the optimal fuel cost when compared to the recent literatures. In addition, the performance of GWO algorithm is demonstrated by executing 50 test runs for different population sizes and the obtained solutions are presented in Tab. 9. The population size selected for different test systems is indicated in Tab. 9.

**Table 9:** Effect of population size on different test systems

| *N* | Total fuel cost in $/hr | | | | Number of hits to global solution |
|---|---|---|---|---|---|
| | **Minimum** | **Maximum** | **Average** | **Standard deviation** | |
| | | | **SYS1: 6-unit system** | | |
| 10 | 15,449.15 | 15,453.86 | 15,449.92 | 0.2356 | 18 |
| 20 | 15,445.14 | 15,444.52 | 15,443.15 | 0.0403 | 44 |
| 30 | 15,443.01 | 15,443.35 | 15,443.09 | 0.0286 | 48 |
| 40 | 15,443.08 | 15,443.14 | 15,443.10 | 0.0034 | 49 |
| 50 | 15,443.08 | 15,443.10 | 15,443.09 | 4.61E-7 | 49 |
| | | | **SYS2: 15-unit, system** | | |
| 20 | 32,548.383 | 32,548.65 | 32,548.41 | 0.105 | 42 |
| 30 | 32,548.138 | 32,548.23 | 32,548.14 | 0.0021 | 48 |
| 40 | 32,548.136 | 32,548.17 | 32,548.13 | 0.0020 | 49 |
| 50 | 32,548.133 | 32,548.15 | 32,548.13 | 0.0019 | 49 |
| 60 | 32,548.133 | 32,548.14 | 32,548.13 | 0.0025 | 49 |
| | | | **SYS3: 20 Unit system** | | |
| 40 | 62,462.91 | 62,517.56 | 62,485.81 | 1.379 | 25 |
| 60 | 62,459.50 | 62,507.22 | 62,472.09 | 1.010 | 40 |
| 80 | 62,454.27 | 62,484.28 | 62,466.19 | 0.787 | 45 |
| 100 | 62,454.25 | 62,479.20 | 62,462.68 | 0.621 | 46 |
| 120 | 62,454.62 | 62,491.26 | 62,460.42 | 0.700 | 45 |
| | | | **SYS 4: 40-unit system** | | |
| 100 | 99,760.31 | 99,777.73 | 99,761.21 | 10.29 | 15 |
| 120 | 99,754.71 | 99,767.45 | 99,756.01 | 6.9580 | 30 |
| 140 | 99,743.01 | 99,748.41 | 99,745.26 | 1.5487 | 42 |
| 160 | 99,722.99 | 99,725.48 | 99,724.29 | 0.7578 | 44 |
| 180 | 99,722.91 | 99,724.32 | 99,723.80 | 0.3903 | 46 |

### *5.2.2 Convergence characteristics*

The convergence characteristic of GWO algorithm for SYS1,SYS2, SYS3 and SYS4 discussed in the previous sections is presented in Fig. 4. The search agents in GWO explores solution space and determine the optimal solution quickly and since it has good search mechanism, the algorithm attains the optimal solution within 100 iterations for small test system and within an admissible number of iterations for large test systems. The GWO algorithm attains the global solution due to the proper selection of number of grey wolf population for a particular test system. This proper tuning of parameters steers the GWO algorithm for global optimal solution making it more compatible for RPED problems. Premature convergence of the GWO algorithm is avoided by proper tuning of grey wolf population. Also, it is to be noted that it determines the optimal global solution for RPED problems with large decision varaibles.

*5.2.3 Robustness*

The minimum cost achieved by GWO algorithm for different test system is given in Tabs. 1, 3, 5 and 7 and it is least when compared with other well known algorithm. Tabs. 2, 4, 6 and 8 compares the minimum cost achieved by GWO algorithm with the other algorithm listed in the literature which emphasizes the better solution quality of the GWO algorithm. In GWO, a stochastic simulation technique, initialization of grey wolf is performed using random numbers.

Due to this inherent randomness involved in the algorithm, performance of the algorithm cannot be judged through a single trial. Hence, the performance and strength of GWO algorithm is evaluated through number of test runs. Many test runs with different initial population values were performed to test the robustness or the consistency level of the GWO algorithm.

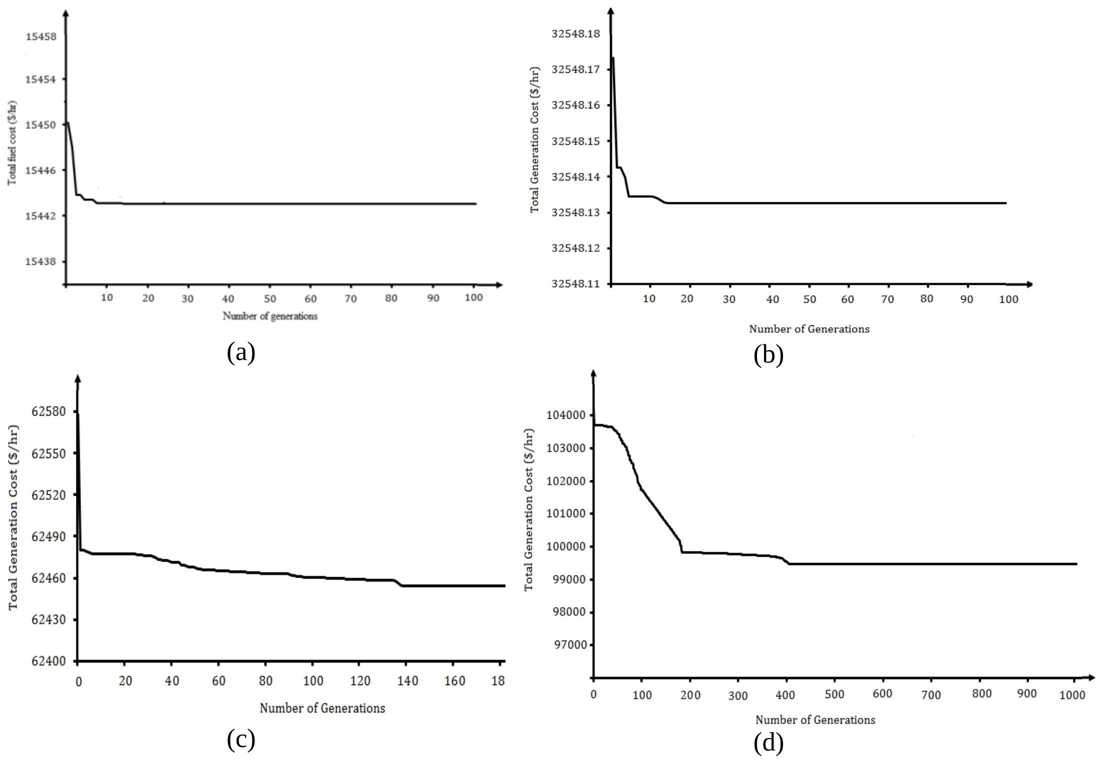



**Figure 4:** Convergence characteristics of (a) SYS1 (b) SYS2 (c) SYS3 and (d) SYS4

The minimum cost attained by the GWO algorithm for different test systems for 100 different trials is shown in Fig. 5. It can be inferred from Fig. 5 that GWO algorithm has the capability of achieving the minimum cost more consistently. The general converging characteristic of GWO algorithm for the entire test systems discussed is shown in Fig. 6. It is observed that after a certain number of generations the difference between the maximum fuel cost and minimum fuel cost is almost the same which gives the strength of GWO algorithm in solving complex, non-convex type problems.

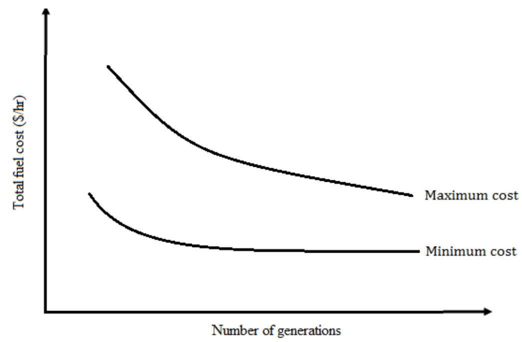
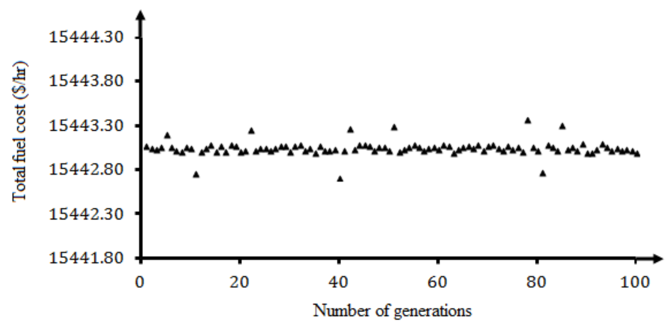
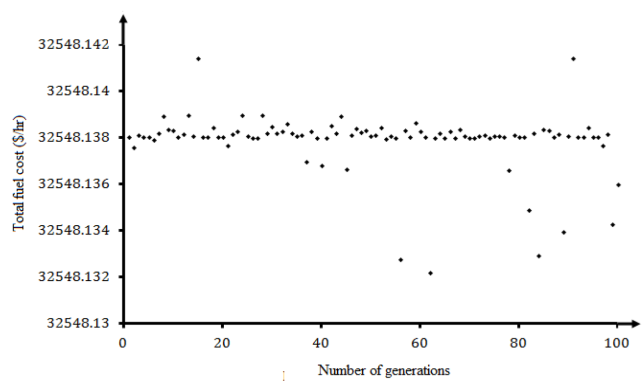



**Figure 6:** General convergence characteristics of GWO

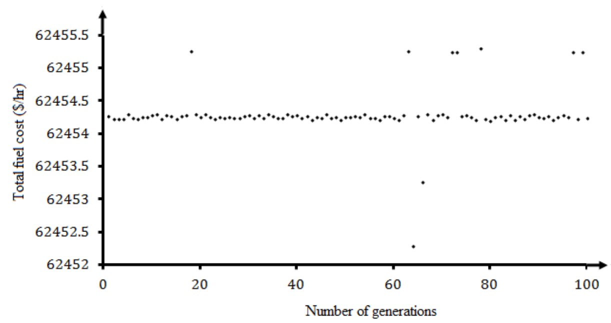
(a)




(b)



(c)




(d)

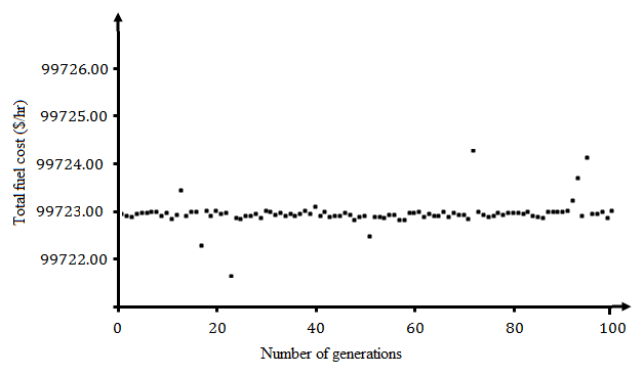



**Figure 5:** Robustness of (a) SYS1 (b) SYS2 (c) SYS3 and (d) SYS4

*5.2.4 Superiority of GWO algorithm*

The statistical results presented in Tabs. 2, 4, 6 and 8 show that GWO outperforms many algorithms. The mathematical formulation of GWO, reveals that total generations is allotted to exploration and exploitation equally. This type of allocation promotes the exploration of the solution space which leads in determining the diverse solution space during algorithm process. Also, the parameter $B$ which randomly forces the search agents to take random steps towards the optimum solution which is very helpful in resolving the local optima problem.

These adaptive parameters which smoothly balance the solution space between the exploration and exploitation is the main reason for the success of GWO algorithm. In addition, in GWO in every generation the best three solutions are saved which also guides the search agents to exploit the most promising regions of solution space. These are the reasons which assist the GWO algorithm to provide good exploration, exploitation, local optima avoidance and fast convergence simultaneously. These performances of GWO algorithm on the ELD problems with and without valve-point loading with or without all the constraints conveys that the GWO algorithm can be successfully applied to different types of practical power system optimization problems in future

## 6 Conclusion

This paper presents a novel GWO algorithm in solving non-convex and convex ELD problems. The effectiveness, feasibility and robustness of the GWO algorithm has been investigated on six ELD problems including the system with 6, 15, 20 and 40 generating units. Tests were carried on systems with different kinds of constraints. The simulation results show that the proposed algorithm has succeeded in achieving minimum fuel cost and the statistical results were compared with recent results reported in the literature. The success of GWO algorithm on the test system illustrates the efficiency and robustness of the GWO algorithm in solving ELD problems. In future, a study can be taken on dynamic ELD problems and the multi-objective ELD problems considering environmental impact and to implement the successful GWO algorithm to it.

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