

DOI: 10.32604/cmes.2023.028314

ARTICLE





# Migration Algorithm: A New Human-Based Metaheuristic Approach for Solving Optimization Problems

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## ABSTRACT

This paper introduces a new metaheuristic algorithm called Migration Algorithm (MA), which is helpful in solving optimization problems. The fundamental inspiration of MA is the process of human migration, which aims to improve job, educational, economic, and living conditions, and so on. The mathematical modeling of the proposed MA is presented in two phases to empower the proposed approach in exploration and exploitation during the search process. In the exploration phase, the algorithm population is updated based on the simulation of choosing the migration destination among the available options. In the exploitation phase, the algorithm population is updated based on the efforts of individuals in the migration destination to adapt to the new environment and improve their conditions. MA's performance is evaluated on fifty-two standard benchmark functions consisting of unimodal and multimodal types and the CEC 2017 test suite. In addition, MA's results are compared with the performance of twelve well-known metaheuristic algorithms. The optimization results show the proposed MA approach's high ability to balance exploration and exploitation to achieve suitable solutions for optimization problems. The analysis and comparison of the simulation results show that MA has provided superior performance against competitor algorithms in most benchmark functions. Also, the implementation of MA on four engineering design problems indicates the effective capability of the proposed approach in handling optimization tasks in real-world applications.

## **KEYWORDS**

Optimization; metaheuristic; migration; human-based algorithm; exploration; exploitation

## 1 Introduction

Optimization problems are a type of problems that have more than one feasible solution. Thus, the optimization process consists of finding the best feasible solution among all the available solutions [1]. Numerous optimization problems in science, engineering, and industry, and real-world applications have become more complex as science and technology advance. Therefore, solving these problems requires effective optimization tools [2]. Problem-solving techniques in optimization studies are classified into two groups: deterministic and stochastic approaches [3].

Deterministic approaches in gradient-based and non-gradient-based categories have good efficiency in solving linear, convex, differentiable, continuous, low-dimensional, and simple problems [4]. However, as optimization problems become more complex, deterministic approaches lose their



efficiency and, by getting stuck in local optima, cannot provide suitable solutions. Meanwhile, many existing and emerging optimization problems in science and real-world applications are nonlinear, non-convex, non-differentiable, non-continuous, high-dimensional, and complex in nature. The difficulties of deterministic approaches, on the one hand, and the increasing complexity of optimization problems, on the other hand, have led researchers to develop stochastic approaches to deal with these problems [5,6].

Metaheuristic algorithms are one of the most effective stochastic approaches that can solve optimization problems based on random search in the problem-solving space using random operators and trial and error processes [7]. The optimization process in metaheuristic algorithms is such that first, several candidate solutions are initialized under the name of the algorithm population. Then, in a repetition-based process, these initial solutions are improved based on algorithm update steps. Finally, the best solution obtained during the iterations of the algorithm is presented as the solution to the problem [8]. Advantages such as simplicity of concepts, easy implementation, no need for derivative process, efficiency in complex, high dimensions, and NP-hard problems, and efficiency in unknown and discrete search spaces have led to the popularity of metaheuristic algorithms among researchers [9].

Metaheuristic algorithms must be able to accurately search the problem-solving space at global and local levels to achieve optimal solutions [10]. Global search with the concept of discovery leads to the ability of the algorithm to comprehensively scan the problem-solving space and prevent the algorithm from getting stuck in local optima. Local search with the concept of exploitation enables the algorithm to converge to possible better solutions near the discovered solutions. In addition to having high power in exploration and exploitation, the primary key to the success of metaheuristic algorithms in optimization is balancing exploration and exploitation during the search process. Due to the nature of random search in metaheuristic algorithms, there is no guarantee that metaheuristic algorithms will provide global optimal. However, the solutions obtained from these methods are acceptable as quasi-optimal solutions due to their proximity to the global optimal [11].

Since the search process and updating steps in metaheuristic algorithms differ, implementing metaheuristic algorithms on a similar optimization problem provides different solutions. Therefore, in comparing the performance of several metaheuristic algorithms, the more effectively an algorithm provides the search process, it will converge to a better solution and be superior to other algorithms. The desire to achieve more effective solutions for optimization problems has led to the design of numerous metaheuristic algorithms. These algorithms are employed in various optimization applications in science, such as energy [12–15], protection [16], energy carriers [17,18], and electrical engineering [19–24].

The main research question is, considering that countless metaheuristic algorithms have been designed so far, whether there is still a need for newer metaheuristic algorithms. In response to this question, the No Free Lunch (NFL) theorem [25] explains that the effective performance of an algorithm in solving a set of optimization problems is not a guarantee of providing the same performance of that algorithm in other optimization problems. Hence, a successful algorithm in solving some optimization problems may even fail in solving another optimization problem. Based on the NFL theorem concept, no specific metaheuristic algorithm is the best optimizer for all optimization problems. The NFL theorem encourages researchers to be able to provide more effective solutions to optimization problems by designing newer algorithms. The NFL theorem also motivates the authors of this paper to introduce and design a new metaheuristic algorithm to handle optimization tasks in science and engineering.

The aspects of innovation and novelty of this paper are in the design of a new metaheuristic algorithm called the Migration Algorithm (MA), which has applications in solving optimization problems. The contributions of this article are as follows:

- The fundamental inspiration of MA is the strategies of choosing the migration destination and adapting to the new environment in the migration process, which are mathematically modeled in two phases of exploration and exploitation.
- The performance of MA in optimization applications is evaluated on fifty-two standard benchmark functions and is compared with twelve well-known metaheuristic algorithms.
- The effectiveness of the proposed MA approach in real-world applications is evaluated on four engineering design problems.

The rest of the paper is organized, so the literature review is presented in Section 2. Then the proposed MA approach is introduced and modeled in Section 3. Simulation studies and results are presented in Section 4. The efficiency of MA in handling real-world applications is evaluated in Section 5. Finally, conclusions and suggestions for future works are provided in Section 6.

### 2 Literature Review

Metaheuristic algorithms are inspired by various natural phenomena, living organisms' behaviors, biological sciences, laws of physics, rules of games, human activities, etc. Based on the design idea, metaheuristic algorithms are classified into five groups: swarm-based, evolutionary-based, physics-based, game-based, and human-based approaches.

Swarm-based metaheuristic algorithms are designed based on the simulation of swarm behaviors of living organisms such as animals, insects, birds, aquatic animals, plants, etc., in nature. Particle Swarm Optimization (PSO) [26], Ant Colony Optimization (ACO) [27], Artificial Bee Colony (ABC) [28], and Firefly Algorithm (FA) [29] are among the most widely used crowd-based approaches. Design of PSO is based on the movement of flocks of birds or fish and their strategy in searching for food. ACO is derived from the ant colony's ability to identify the shortest path between the food source and the nest. ABC is proposed based on modeling the hierarchical activities of honeybees in the colony to access food resources. FA is inspired by the behavior of fireflies in attracting prey and the opposite sex by using their luminous ability to produce flashing light based on the biological phenomenon of bioluminescence. Grey Wolf Optimization (GWO) is a swarm-based method based on simulating gray wolves' hierarchical strategy during hunting [30]. Some other swarm-based algorithms are Penicillium Reproduction Algorithm (PRA) [31], Dandelion Algorithm (DA) [32], Pelican Optimization Algorithm (POA) [33], Emperor Penguin Optimizer (EPO) [34], Marine Predators Algorithm (MPA) [35], Rat Swarm Optimization (RSO) [36], Mutated Leader Algorithm (MLA) [37], Reptile Search Algorithm (RSA) [38], Cat and Mouse Based Optimizer (CMBO) [39], Donkey Theorem Optimization (DTO) [40], All Member Based Optimizer (AMBO) [41], Group Mean-Based Optimizer (GMBO) [42], Tunicate Swarm Algorithm (TSA) [43], Two Stage Optimization (TSO) [44], White Shark Optimizer (WSO) [45], and African Vultures Optimization Algorithm (AVOA) [46].

Evolutionary-based metaheuristic algorithms are developed inspired by the concepts of biological, genetics sciences, and natural selection. Genetic Algorithm (GA) [47] and Differential Evolution (DE) [48] are the most famous evolutionary-based algorithms that have been widely used in solving various optimization problems. The design of these algorithms has been inspired by the reproduction process, the concepts of survival of the fittest, Darwin's evolutionary theory, and the use of random selection, crossover, and mutation operators. Some other evolutionary-based metaheuristic algorithms are

Biogeography-based Optimizer (BBO) [49], Artificial Immune System (AIS) [50], Evolution Strategy (ES) [51], Cultural Algorithm (CA) [52], and Genetic Programming (GP) [53].

Physics-based metaheuristic algorithms are introduced based on modeling phenomena, laws, forces, and physics concepts. Simulated Annealing (SA) is one of the most famous physics-based algorithms developed based on the simulation of the physical process of metal annealing. In this physical process, the metal is first melted under heat and then slowly cooled so that the crystals are perfectly formed [54]. Physical forces have been sources of inspiration in designing algorithms such as the Gravitational Search Algorithm (GSA) based on gravitational force [55] and Momentum Search Algorithm (MSA) based on impulse force [56]. Cosmological concepts are employed in the design of algorithms such as the Galaxy-Based Search Algorithm (GbSA) [57], Multi-Verse Optimizer (MVO) [58], and Black Hole (BH) [59]. Some other physics-based algorithms are Small World Optimization Algorithm (SWOA) [60], Magnetic Optimization Algorithm (MOA) [61], Ray Optimization (RO) [62] algorithm, and Artificial Chemical Reaction Optimization Algorithm (ACROA) [63].

Game-based metaheuristic algorithms are proposed based on simulating the rules of different individual and group games, as well as the strategies and behaviors of people influencing games such as players, referees, and coaches. Holding matches in different sports has been the source of inspiration for designing algorithms, such as Volleyball Premier League (VPL) [64] based on the simulation of the volleyball league and Football Game Based Optimizer (FGBO) [65] based on the simulation of the football league. The main idea in the design of the Orientation Search algorithm (OSA) has been the players' efforts to change the direction of movement on the playing field based on the direction determined by the reference [66].

Human behaviors, activities, interactions, and communication in individual and social life inspire human-based metaheuristic algorithms. Teaching-Learning Based Optimization (TLBO) is one of the most widely used human-based approaches, which is designed based on modeling students' interactions with each other and students with the teacher in the classroom learning environment [67]. People's effort to improve society by following the leader of that society has been used in the design of the Following Optimization Algorithm (FOA) [68]. The process of learning a foreign language by people by referring to language schools is employed in the design of Language Education Optimization (LEO) [69], and Election Based Optimization Algorithm (EBOA) [70] mimics the voting process to select the leader. Some other human-based metaheuristic algorithms are Multimodal Nomad Algorithm (MNA) [71], Archery Algorithm (AA) [72], War Strategy Optimization (WSO) [73], and Brain Storm Optimization (BSO) [74].

A short description of the algorithms mentioned in this paper is presented in Table 1.

Class	Algorithm	Main idea
	Particle Swarm Optimization (PSO)	Swarming movement of flocks of birds and fish
	Ant Colony Optimization (ACO)	The ability of ant colony to discover the shortest path
	Artificial Bee Colony (ABC)	Activities of honey bees colony in finding food sources

 Table 1: Parameter values for the competitor algorithms

# Table 1 (continued)

Class	Algorithm	Main idea
	Grey Wolf Optimization (GWO)	Hierarchical strategy of gray wolves during hunting.
	Penicillium Reproduction Algorithm (PRA) Dandelion Algorithm (DA) Pelican Optimization Algorithm (POA) Emperor Penguin Optimizer (EPO)	Behavior of penicillium reproduction. Biological intelligence of dandelion seeding. Natural behavior of pelicans during hunting. Huddling behavior of emperor penguins in
	Marine Predators Algorithm (MPA)	nature. Interaction between the prey and predator in
		the ocean.
SB	Rat Swarm Optimization (RSO)	Social and hunting conduct of a group of rats.
	Mutated Leader Algorithm (MLA)	Concepts of mutated leader.
	Reptile Search Algorithm (RSA)	Encirclement and hunt mechanisms of crocodiles.
	Cat and Mouse Based Optimizer (CMBO)	The interactions between the cat and the mouse and the escape of the mouse towards the haven.
	Donkey Theorem Optimization (DTO)	Concepts of donkey theorem and choosing the shortest path to food.
	All Member Based Optimizer (AMBO)	Participation of all members of the swarm in updating the algorithm population.
	Group Mean-Based Optimizer (GMBO)	Using composite members based on averaging from two good and bad groups of a swarm.
	Tunicate Swarm Algorithm (TSA)	Jet propulsion and swarm behaviors of tunicates during the navigation
	Two Stage Optimization (TSO)	Two-step update of each population member based on selected members from the good and bad groups
	White Shark Optimizer (WSO)	The great white shark's exceptional hearing and sense of smell are used for navigation and foraging.
	African Vultures Optimization Algorithm (AVOA)	African vultures' foraging and navigation behaviors.
	Genetic Algorithm (GA)	Darwinian evolution theory.
	Differential Evolution (DE)	Natural phenomenon of evolution.
	Biogeography-Based Optimizer (BBO)	Biogeographic concepts.
EB	Artificial Immune System (AIS)	The defense mechanism of the human body against microbes and diseases.
	Evolution Strategy (ES)	Darwinian evolution theory.
	Cultural Algorithm (CA)	Cultural-social evolution.
	Genetic Programming (GP)	The biological model of evolution.

 Table 1 (continued)

Class	Algorithm	Main idea
	Simulated Annealing (SA) Gravitational Search Algorithm (GSA) Momentum Search Algorithm (MSA)	Metal annealing process. Gravity law. Momentum law and Newton's laws of
PB	Galaxy-Based Search Algorithm (GbSA) Multi-Verse Optimizer (MVO)	motion. Spiral arm of spiral galaxies. Concepts in cosmology: white hole, black hole, and wormhole
	Black Hole (BH) Small World Optimization Algorithm (SWOA)	Black hole phenomena. Mechanism of the small-world phenomenon.
	Magnetic Optimization Algorithm (MOA) Ray Optimization (RO) Artificial Chemical Reaction Optimization Algorithm (ACROA)	Magnetic field theory. Snell's light refraction law. Types and occurring of chemical reactions.
	Volleyball Premier League (VPL)	Football league and holding matches in the
GB	Football Game Based Optimizer (FGBO)	Volleyball league and behavior of players and coaches during the game.
	Orientation Search algorithm (OSA)	The game of orientation and movement of players in the direction determined by the referee.
	Teaching-Learning Based Optimization (TLBO) Following Optimization Algorithm (FOA)	Interactions between teachers and students in the classroom learning environment. Relationships between members and the leader of a community
	Language Education Optimization (LEO)	The process of teaching and learning a foreign language.
	Election Based Optimization Algorithm (EBOA)	The process of voting and holding elections.
HB	Multimodal Nomad Algorithm (MNA)	Migratory behavior of the nomadic tribes on Mongolia grassland.
	Archery Algorithm (AA)	The archer's effort in throwing the arrow toward the panel.
	War Strategy Optimization (WSO)	Strategic movement of army troops during the war.
	Brain Storm Optimization (BSO)	Human brainstorming process.

Based on the best knowledge obtained from the literature review, no metaheuristic algorithm has been designed based on the modeling of the human migration process. Meanwhile, the migration process is an intelligent human activity with extraordinary potential as a new metaheuristic algorithm design. In order to address this research gap, in this paper, a new human-based metaheuristic algorithm

is designed based on the mathematical modeling of the human migration process, which is discussed in the next section.

### **3** Migration Algorithm

In this section, the proposed Migration Algorithm (MA) is introduced, and its mathematical model is presented.

### 3.1 Inspiration

Human migration refers to the movement of people from one place to another for work or life [75]. People usually migrate to escape adverse circumstances, such as poverty, disease, political issues, food shortages, natural disasters, war, unemployment, and lack of security. In addition, the favorable conditions of the migration destination, such as more health facilities, better education, more income, better housing, political freedoms, and a better atmosphere, are the other reasons for people to migrate [76]. Based on their own needs and criteria, people choose the final destination among the immigration options. After moving to the migration destination, people try to adapt to the new environment.

When people decide to migrate, based on their expectations and conditions, they will have different candidates for the migration destination. After analyzing each destination's conditions, they choose their final migration destination. Then they immigrate. When a person is in a new situation, they try to adapt themself to the new conditions in the immigration destination. The steps of the immigration process are considered as follows:

- A person checks different destinations for immigration.
- From among the candidate destinations, a person finally chooses the immigration destination based on examining their benefits and conditions.
- The person migrates to the chosen destination.
- In a new situation, a person tries to adapt to the conditions of the new society.

Among humans' activities in the migration process, the two strategies of (i) choosing and moving to the migration destination and (ii) trying to adapt to the new environment at the migration destination are more significant. Mathematical modeling of these two strategies of the migration process is employed in the design of the proposed MA approach.

#### 3.2 Initialization

The proposed MA approach is a population-based metaheuristic algorithm that can solve optimization problems by using the search power of this population in the problem-solving space in a repetition-based process. The members of the MA population are people in different life situations who are trying to improve their situation by migrating to new places. Each population member, according to its position in the search space, determines the values for the decision variables of the problem. Therefore, each member of the population is a candidate solution to the problem, which is mathematically modeled using a vector. Together, these members form the population of the algorithm, which can be represented from a mathematical point of view using a matrix according to Eq. (1). At the beginning of the MA execution, the initial position of the population members in the search space is initialized using Eq. (2).

$$X = \begin{bmatrix} X_{1} \\ \vdots \\ X_{i} \\ \vdots \\ X_{N} \end{bmatrix}_{N \times m} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,j} & \cdots & x_{1,m} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i,1} & \cdots & x_{i,j} & \cdots & x_{i,m} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{N,1} & \cdots & x_{N,j} & \cdots & x_{N,m} \end{bmatrix}_{N \times m}$$
(1)

$$X_i: x_{i,j} = lb_j + r \cdot (ub_j - lb_j), i = 1, 2, \dots, N, j = 1, 2, \dots, m,$$
(2)

where X is the population matrix of the proposed MA, N is the number of population members, m is the number of decision variables,  $X_i$  is the *i*th candidate solution,  $x_{i,j}$  is its *j*th variable, r is a random number in the interval [0, 1],  $lb_j$  is the lower bound, and  $ub_j$  is the upper bound of the *j*th decision variable.

Considering that each population member is a candidate solution for the problem, a value for the objective function of the problem is evaluated corresponding to each population member. The set of calculated values for the objective function can be represented using a vector according to Eq. (3).

	$\left\lceil F_{1}\right\rceil$		$\left\lceil F(X_1) \right\rceil$				
	÷		:				
F =	$F_i$	=	$\int F(X_i)$	,			(3)
	:						
	$\lfloor F_N \rfloor$	$N \times 1$	$[F(X_N)]$	N×1			

where F is the vector of values of the objective function and  $F_i$  is the value of the objective function for the *i*th candidate solution.

The calculated values for the objective function are suitable for the qualitative assessment of candidate solutions. Therefore, the best value calculated for the objective function corresponds to the best member of the population. Similarly, the worst value calculated for the objective function corresponds to the worst member of the population. Considering that the position of the population members in the search space is updated in each iteration, the best member must also be updated. At the end of the algorithm execution, the best member of the population is available as a solution to the problem.

In the design of the proposed MA approach, the position of the population members is updated based on the simulation of the human migration process in two phases, which are explained below. In the following, mathematical equations of the proposed MA are presented to minimize the objective functions of optimization problems. It should be noted that MA can also be used for maximization problems, but the relevant equations would have to be rewritten for these problems.

#### 3.3 Phase 1: Choosing and Moving to the Migration Destination (Exploration Phase)

One of the most important actions in the migration process is choosing the migration destination among all the available options. When people migrate, they choose a destination based on their criteria and move there. The simulation of this behavior is employed in the design of the first phase of population update in the proposed MA approach. Modeling this strategy leads to significant changes in the population position of the algorithm, which leads to increased algorithm ability in global search and exploration. In the MA design, for each population member, the position of other population members (with the better fitness function value) is considered a set of possible destinations for its migration. This set of candidate destinations for each member is determined using Eq. (4). Then, the migration destination MD is randomly chosen for each member of the population among the options selected in this set. Then, the proposed new position for each member is calculated based on the displacement towards the migration destination using Eq. (5). Finally, if the value of the objective function is improved in the new position, the proposed position is accepted as the position of the corresponding member according to Eq. (6).

$$CD_i = \{X_k, F_k < F_i \text{ and } k \in \{1, 2, ..., N\}\}, \text{ where } i = 1, 2, ..., N,$$
 (4)

$$x_{i,i}^{P_1} = x_{i,j} + r_{i,j} \cdot (MD_{i,j} - I_{i,j} \cdot x_{i,j}),$$
(5)

$$X_i = \begin{cases} X_i^{P_1}, & F_i^{P_1} \le F_i, \\ X_i, & else, \end{cases}$$
(6)

where  $CD_i$  is the set of all possible candidate destinations for the *i*th member,  $X_k$  is the *k*th row of X matrix which has better objective function value than the *i*th member,  $MD_i$  is the migration destination for *i*th member,  $MD_{ij}$  is its *j*th dimension,  $X_i^{P1}$  is the new position calculated for the *i*th population member based on first phase of the proposed MA,  $x_{ij}^{P1}$  is its *j*th dimension,  $F_i^{P1}$  is its objective function value,  $r_{ij}$  are random numbers from the interval [0, 1], and  $I_{ij}$  are numbers which are randomly selected as 1 or 2. The random numbers used in MA design create a random nature in changing the position of the algorithm are used in many metaheuristic algorithms. These random numbers can cause random changes in the population members in the search space and improve the optimization operation.

### 3.4 Phase 2: Adaptation to the New Environment in the Migration Destination (Exploitation Phase)

After a person migrates and enters a new environment and society, they try to adapt themself to the new conditions. In this activity, a person achieves better performance and social needs. This human activity in the migration process is employed in the second phase of population update in the proposed MA algorithm. Modeling this behavior leads to small changes in the position of the population members, which leads to an increase in the ability of local search and exploitation in the proposed MA approach. To simulate people's efforts in adapting to the new environment, for each member of the population, a proposed random position near the same member is generated using Eq. (7). This proposed position, if it leads to the improvement of the objective function value, replaces the position of the corresponding member according to Eq. (8).

$$x_{ij}^{P2} = x_{ij} + (1 - 2r_{ij}) \cdot \frac{ub_j - lb_j}{t}$$
(7)

$$X_{i} = \begin{cases} X_{i}^{P2}, & F_{i}^{P2} \leq F_{i} \\ X_{i}, & else \end{cases}$$

$$(8)$$

where  $X_i^{P2}$  is the new position calculated for the *i*th population member based on second phase of the proposed MA,  $x_{i,j}^{P2}$  is its *j*th dimension,  $F_i^{P2}$  is its objective function value,  $r_{i,j}$  are random numbers from the interval [0, 1], and *t* is the iteration counter.

### 3.5 Repetitions Process, Flowchart, and Pseudocode of MA

After updating all population members based on the first and second phases of the proposed MA approach, the first iteration of the algorithm is completed. After that, with the new values calculated for the position of the population members and the objective function, the algorithm enters the next

iteration. The population update process is repeated until the last iteration of the algorithm based on Eqs. (4) to (8). In each iteration, the best member of the population is updated as the best-obtained solution until that iteration. After completing the implementation of the algorithm, the best candidate solution saved during the iterations of the algorithm is presented as a solution to the problem. The implementation steps of the proposed MA approach are shown in the form of a flowchart in Fig. 1 and the form of pseudocode in Algorithm 1.



Figure 1: Flowchart of the proposed MA

# Algorithm 1: Pseudocode of the proposed MA

# Start MA.

- 1. Input the optimization problem information.
- 2. Set the number of iterations T and the number of members of the population N.
- 3. Generate the initial population at random based on Eq. (2).

Algorithm 1. (Continued	A	lgorithm	1: (	(Continued)	)
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- 4. Evaluate the initial population.
- 5. For t = 1: *T*
- 6. For i = 1: N
- 7. **Phase 1:** *Choosing and Moving to the Migration Destination.*
- 8. Determine candidate destinations for migration based on Eq. (4).
- 9. Calculate new position of population member based on Eq. (5).
- 10. Update the *i*th population member using Eq. (6).
- 11. **Phase 2:** Adaptation to the New Environment in the Migration Destination.
- 12. Calculate a new position of a population member based on Eq. (7)
- 13. Update the *i*th population member using Eq. (8).
- 14. end
- 15. Save the best proposed solution so far.
- 16. end
- 17. Output the best obtained proposed solution.

```
End MA.
```

### **4** Simulation Studies

In this section, the performance of the proposed MA approach in optimization tasks is evaluated. For this purpose, a set of fifty-two standard benchmark functions consisting of unimodal, highdimensional multimodal, and fixed-dimensional multimodal types [77] and also the CEC 2017 test suite [78] are employed. The details of these functions are specified in the appendix and in Tables A1 to A4. The results obtained from MA have been compared with the performance of twelve wellknown metaheuristic algorithms: GA, PSO, GSA, GWO, MVO, WOA, TSA, MPA, AVOA, WSO, and RSA. The adjusted values for the control parameters are specified in Table 2. Optimization results are reported using six statistical indicators: mean, best, worst, standard deviation, median, and rank. It should be noted that the mean value is chosen as the ranking criterion of metaheuristic algorithms.

Algorithm	Parameter	Value
GA		
	Туре	Real coded.
	Selection	Roulette wheel (Proportionate).
	Crossover	Whole arithmetic (Probability $= 0.8$ ,
		$\alpha \in [-0.5, 1.5]$ ).
	Mutation	Gaussian (Probability $= 0.05$ ).
PSO		
	Topology	Fully connected.
	Cognitive and social constant	$(C_1, C_2) = (2, 2)$ .
	Inertia weight	Linear reduction from 0.9 to 0.1
	Velocity limit	10% of the dimension range.

	Table 2:	Parameter	values	for the	competitor	alg	orithr	ns
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Table 2 (co	ontinued)	
Algorithm	Parameter	Value
GSA		
	Alpha, $G_0$ , $R_{norm}$ , $R_{power}$	20, 100, 2, 1
TLBO		
	$T_F$ : the teaching factor	$T_F = \text{round}[(1 + rand)].$
	random number <i>rand</i>	<i>rand</i> is a random number from the interval [0, 1].
GWO		
	Convergence parameter ( <i>a</i> )	<i>a</i> : Linear reduction from 2 to 0.
MVO		
	Wormhole existence probability (WEP)	Min(WEP) = 0.2 and $Max(WEP) = 1$ .
	Exploitation accuracy over the iterations ( <i>p</i> )	p = 6.
WOA		
	Convergence parameter <i>a</i>	<i>a</i> : Linear reduction from 2 to 0.
	Parameters r and l	<i>r</i> is a random vector in [0, 1],
		<i>l</i> is a random number in $[-1, 1]$ .
TSA		
	$P_{min}$ and $P_{max}$	1,4
	$c_1, c_2, c_3$	random numbers lie in the range [0, 1].
MPA		
	Constant number	P = 0.5
	Random vector	<i>R</i> is a vector of uniform random numbers
		from [0, 1].
	Fish Aggregating Devices (FADs)	FADs = 0.2
	Binary vector	U = 0  or  1.
RSA		
	Sensitive parameter	$\beta = 0.01.$
	Sensitive parameter	$\alpha = 0.1.$
	Evolutionary Sense (ES)	ES are randomly decreasing values between 2 and $-2$ .
AVOA		
	$L_1, L_2$	$(L_1, L_2) = (0.8, 0.2).$
	W	w = 2.5.
	$P_1, P_2, P_3$	$(P_1, P_2, P_3) = (0.6, 0.4, 0.6).$
WSO		
	$F_{min}$ and $F_{max}$	$(F_{\min}, F_{\max}) = (0.07, 0.75)$ .
	$\tau, a_0, a_1, a_2$	$(\tau, a_0, a_1, a_2) = (4.125, 6.25, 100, 0.0005).$

Seven benchmark functions, F1 to F7, have been selected from the unimodal type to evaluate the ability to exploit metaheuristic algorithms. The optimization results of unimodal functions F1 to F7 using MA and competitor algorithms are reported in Table 3. Based on the optimization results, MA, with high exploitation ability in optimizing functions F1, F2, F3, F4, F5, and F6, has converged to the global optimal. In the optimization of the F7 function, MA is the first best optimizer. The analysis of the optimization results shows that MA has provided superior performance in the optimization of unimodal functions F1 to F7 by providing high power in exploitation and local search compared to competitor algorithms.

# 4.2 Evaluation of High-Dimensional Multimodal Objective Functions

Six benchmark functions F8 to F13, have been selected from the high-dimensional multimodal type to evaluate the exploration ability of metaheuristic algorithms. The implementation results of MA and competitor algorithms on functions F8 to F13 are presented in Table 4. The optimization results show that MA, with high exploration ability, in optimizing F9 and F11 functions, has converged to the global optimal by accurately identifying the main optimal area in the search space. Also, MA is the first best optimizer for solving functions F8, F10, F12, and F13. Based on the analysis of the simulation results, it is concluded that the proposed MA approach with high exploration ability and optimal global search has provided superior performance in the optimization of high-dimensional multimodal functions compared to competitor algorithms.

# 4.3 Evaluation of Fixed-Dimensional Multimodal Objective Functions

Ten benchmark functions, F14 to F23, are selected from the fixed-dimension multimodal to evaluate the ability of metaheuristic algorithms to balance exploration and exploitation during the search process. The results of employing MA and competitor algorithms are reported in Table 5. The results show that MA is the first best optimizer for functions F14, F15, F21, F22, and F23. Furthermore, in solving functions F16 to F20, although MA has similar conditions with some competitor algorithms in the mean criterion, by providing better results for the std index, it has provided a more effective performance in optimizing these functions. What is clear from the analysis of the simulation results is that the proposed MA approach, with a high ability to balance exploration and exploitation, has provided superior performance in the optimization of fixed-dimension multimodal functions compared to competitor algorithms.

F		MA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
	Mean	0	65.90797	0	0	1.92E-49	4.65E-47	1.4E-151	0.149636	1.77E-59	2.53E-74	1.33E-16	0.100957	30.50201
	Best	0	5.295156	0	0	3.81E-52	1.44E-50	9.3E-171	0.105509	1.49E-61	5.87E-77	5.36E-17	0.000487	17.92696
F1	Worst	0	238.9103	0	0	1.66E-48	3.3E-46	2.7E-150	0.201297	7.72E-59	2.6E-73	3.74E-16	1.397744	56.92799
	Std	0	52.77091	0	0	3.93E-49	1E-46	6E-151	0.027758	2.14E-59	6.16E-74	7.15E-17	0.31078	10.46286
	Median	0	45.41997	0	0	4.16E-50	4.27E-48	2.2E-159	0.150528	1.08E-59	1.7E-75	1.13E-16	0.00972	28.19897
	Rank	1	11	1	1	5	6	2	9	4	3	7	8	10
	Mean	0	2.13984	1.1E-276	0	6.97E-28	2.11E-28	2.5E-105	0.259174	1.35E-34	6.76E-39	5.49E-08	0.895505	2.788395
	Best	0	0.662477	1.3E-306	0	1.84E-29	2.03E-30	7.9E-118	0.160075	4.87E-36	8.82E-40	3.49E-08	0.045282	1.745356
F2	Worst	0	7.445497	2.2E-275	0	4.71E-27	1.82E-27	2.8E-104	0.36451	7.91E-34	2.44E-38	1.23E-07	2.493315	3.806556
	Std	0	1.774278	0	0	1.09E-27	5.29E-28	6.9E-105	0.062992	1.96E-34	5.58E-39	1.87E-08	0.722723	0.544788
	Median	0	1.530461	6.5E-290	0	3.51E-28	1.97E-29	3.4E-108	0.268348	6.5E-35	4.98E-39	5.13E-08	0.584164	2.741555
	Rank	1	11	2	1	7	6	3	9	5	4	8	10	12
	Mean	0	1786.31	0	0	2.51E-12	1.18E-10	19959.22	15.97333	2.17E-14	3.84E-24	475.4998	388.1315	2168.983
	Best	0	1040.447	0	0	6.19E-19	1.37E-21	2064.881	5.974275	2.36E-19	2.2E-29	245.9638	21.76826	1424.187
F3	Worst	0	3543.114	0	0	1.44E-11	1.95E-09	34688.44	48.93977	4.05E-13	3.61E-23	1186.317	1025.393	3458.935
	Std	0	627.7929	0	0	4.38E-12	4.36E-10	8557.149	10.76486	9.02E-14	1.08E-23	220.2836	288.4306	639.6914
	Median	0	1558.29	0	0	1.83E-13	1.08E-13	20324.26	11.87926	4.66E-16	4.04E-26	400.3348	293.0444	2100.7
	Rank	1	9	1	1	4	5	11	6	3	2	8	7	10
	Mean	0	17.29599	3.2E-265	0	2.98E-19	0.004423	51.82134	0.547118	1.23E-14	1.84E-30	1.235881	6.279883	2.829395
	Best	0	11.91482	0	0	3.02E-20	9.65E-05	0.904572	0.265926	6.55E-16	5.81E-32	9.9E-09	2.290268	2.216469
F4	Worst	0	23.83573	4.5E-264	0	9.61E-19	0.035828	91.70973	0.963047	5.74E-14	8.12E-30	4.927695	13.36024	3.992738
	Std	0	2.887421	0	0	2.29E-19	0.007944	29.61469	0.192208	1.46E-14	2.4E-30	1.387146	2.502379	0.466936
	Median	0	17.77269	2E-282	0	2.59E-19	0.00147	55.42445	0.531045	6.35E-15	6.53E-31	0.906948	5.882471	2.783478
	Rank	1	11	2	1	4	6	12	7	5	3	8	10	9
	Mean	0	10799.4	1.43E-05	12.99862	23.32398	28.47735	27.3097	96.22156	26.58159	26.78794	44.0499	4611.934	595.3854
	Best	0	1347.31	1.39E-06	8.7E-29	22.80862	25.67105	26.72206	27.63173	25.56655	25.58869	25.8846	26.28099	228.808
F5	Worst	0	92715.88	5.91E-05	28.99021	24.04927	28.89167	28.73536	377.9041	27.15605	28.75268	167.2442	90077.28	2257.058
	Std	0	20068.39	1.45E-05	14.74463	0.388633	0.78813	0.577718	101.4641	0.52633	0.936343	44.3234	20116.61	424.9867
	Median	0	5609.695	9.38E-06	1.22E-28	23.29493	28.82258	27.08683	30.01805	26.23168	26.32785	26.34642	86.09804	475.573
	Rank	1	13	2	3	4	8	7	10	5	6	9	12	11
	Mean	0	100.9068	4.98E-08	6.457884	1.81E-09	3.681907	0.081573	0.151003	0.660849	1.261405	1.05E-16	0.063446	34.14746
	Best	0	16.953	7.11E-09	3.663258	8.08E-10	2.552812	0.010521	0.079233	0.246729	0.233121	5.52E-17	1.9E-06	15.61244
F6	Worst	0	382.4943	1.36E-07	7.250003	4.8E-09	4.787676	0.326748	0.25011	1.252278	2.164793	1.81E-16	0.541731	62.76702
	Std	0	95.47735	3.29E-08	1.027942	9.36E-10	0.693358	0.10162	0.047381	0.306609	0.497225	3.71E-17	0.148563	13.54999
	Median	0	69.57653	4.61E-08	6.884954	1.6E-09	3.795995	0.031607	0.160156	0.727316	1.217425	9.48E-17	0.002057	31.68218
	Rank	1	13	4	11	3	10	6	7	8	9	2	5	12
													(	Continued)

**Table 3:** Evaluation results of unimodal objective functions

Tał	ole 3 (con	tinued)												
F		MA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
	Mean	2.54E-05	9E-05	6.25E-05	3.01E-05	0.000547	0.004343	0.001278	0.011614	0.000831	0.00153	0.052809	0.184141	0.010589
	Best	2.35E-06	1.06E-05	8.55E-07	2.43E-06	0.000111	0.001493	2.02E-05	0.003971	0.000182	9.01E-05	0.014124	0.069017	0.003032
F7	Worst	6.89E-05	0.000339	0.000261	0.000133	0.000899	0.009973	0.005399	0.022569	0.001957	0.002947	0.095575	0.411351	0.021939
	Std	1.98E-05	8.95E-05	7.33E-05	3.45E-05	0.000215	0.002341	0.001445	0.005034	0.000467	0.00088	0.024958	0.079015	0.004819
	Median	1.83E-05	6.38E-05	4.01E-05	1.54E-05	0.000533	0.003721	0.000818	0.011315	0.000845	0.001506	0.051832	0.177731	0.010178
	Rank	1	4	3	2	5	9	7	11	6	8	12	13	10
Sum	rank	7	72	15	20	32	50	48	59	36	35	54	65	74
Mea	n rank	1	10.28571	2.142857	2.857143	4.571429	7.142857	6.857143	8.428571	5.142857	5	7.714286	9.285714	10.57143
Tota	l ranking	1	12	2	3	4	8	7	10	6	5	9	11	13

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F		MA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
F8	Mean Best Worst	-12498.6 -12622.8 -11936.3 190,3992	-7051.29 -9000.4 -6082.61 734.4512	-12470.7 -12569.5 -11896.8 195,8589	-5436.21 -5656.07 -4909.67	-9687.45 -10475.5 -9090.67 370.2257	-6139.18 -7319.04 -4369.92 729.8809	-11065.1 -12569.5 -7740.1 1735.097	-7832.95 -9188.24 -6879.62 728.446	-6079.64 -6863.36 -5047.96 481.8826	-5598.39 -7028.14 -4549.98 609.125	-2781.26 -3974.43 -2148.32 495.5458	-6547.41 -8244.17 -4988.99 748.5216	-8421.5 -9681.18 -7028.99
	Median	-12577.8	-6972.73	-12569.5	-5491.24	-9719.53	-6097.61	-12040.8	-7710.82	-6072.83	-5613.67	-2692.95	-6693.09	-8399.11
	Rank	1	7	2	12	4	9	3	6	10	11	13	8	5
F9	Mean	0	24.63015	0	0	0	173.1242	0	97.82972	1.71E-14	0	28.50556	67.7144	54.68123
	Best	0	14.61964	0	0	0	89.74484	0	52.78684	0	0	13.92943	39.79836	23.23239
	Worst	0	45.95061	0	0	0	288.1844	0	149.2806	1.14E-13	0	48.75295	114.5621	76.90086
	Std	0	8.618138	0	0	0	51.00724	0	25.19698	3.25E-14	0	9.166096	18.84112	13.80758
	Median	0	22.68872	0	0	0	166.6755	0	97.08297	0	0	26.3664	65.06856	52.61443
	Rank	1	3	1	1	1	8	1	7	2	1	4	6	5
F10	Mean	8.88E-16	5.291383	8.88E-16	8.88E-16	4.26E-15	1.242493	4.09E-15	0.577899	1.67E-14	4.44E-15	8.21E-09	2.727233	3.5751
	Best	8.88E-16	3.38294	8.88E-16	8.88E-16	8.88E-16	7.99E-15	8.88E-16	0.1006	7.99E-15	4.44E-15	4.66E-09	1.693449	2.881962
	Worst	8.88E-16	8.198706	8.88E-16	8.88E-16	4.44E-15	3.373453	7.99E-15	2.515189	2.22E-14	4.44E-15	1.45E-08	5.057072	4.641967
	Std	0	1.221469	0	0	7.94E-16	1.569506	2.28E-15	0.677185	3.55E-15	0	2.34E-09	0.857798	0.396644
	Median	8.88E-16	5.179479	8.88E-16	8.88E-16	4.44E-15	2.22E-14	4.44E-15	0.194315	1.51E-14	4.44E-15	7.72E-09	2.733921	3.62958
	Rank	1	11	1	1	3	8	2	7	5	4	6	9	10
F11	Mean Best Worst Std Median Rank	0 0 0 0 1	1.716157 1.103877 3.284729 0.542611 1.600984 7	0 0 0 0 0 1	0 0 0 0 0 1	0 0 0 0 0 1	0.008843 0 0.020547 0.006293 0.008994 3	0 0 0 0 0 1	0.399675 0.254148 0.535986 0.081857 0.416518 5	0.00134 0 0.018824 0.004484 0 2	0 0 0 0 0 1	7.208015 2.995643 12.63778 2.720906 7.31113 8	0.185266 0.002367 0.875849 0.228487 0.122356 4	1.473471 1.288095 1.725859 0.123868 1.447709 6
F12	Mean	1.57E-32	3.269703	2.58E-09	1.317616	2.04E-10	5.792791	0.020096	0.914642	0.039878	0.071329	0.210037	1.501058	0.274894
	Best	1.57E-32	0.953135	4.03E-10	0.769179	5.19E-11	1.036858	0.001227	0.000999	0.012562	0.02411	4.75E-19	0.000107	0.060841
	Worst	1.57E-32	7.388687	7.83E-09	1.645905	3.81E-10	14.13599	0.136901	3.848045	0.086783	0.135135	0.931771	5.21922	0.650842
	Std	2.81E-48	1.829413	1.65E-09	0.303868	9.61E-11	3.880435	0.040002	1.196737	0.021332	0.02095	0.307418	1.285627	0.138648
	Median	1.57E-32	2.891986	2.39E-09	1.389398	2.05E-10	4.304904	0.005783	0.42028	0.03791	0.06869	0.080199	1.285267	0.264424
	Rank	1	12	3	10	2	13	4	9	5	6	7	11	8
F13	Mean	1.35E-32	3599.682	1E-08	3.13E-31	0.002498	2.716891	0.214604	0.032775	0.513821	1.101997	0.056661	3.607621	2.707835
	Best	1.35E-32	13.7976	1.15E-09	6.53E-32	9.95E-10	2.012451	0.037203	0.006442	4.69E-05	0.588492	4.66E-18	0.009572	1.291959
	Worst	1.35E-32	62161.32	3.81E-08	5.43E-31	0.025313	3.713937	0.700345	0.091627	0.95012	1.541205	0.958375	12.58563	3.940231
	Std	2.81E-48	13853.99	8.78E-09	2.25E-31	0.006343	0.55753	0.183521	0.024787	0.25783	0.23137	0.213649	3.031014	0.754476
	Median	1.35E-32	44.23045	6.52E-09	4E-31	2.82E-09	2.53517	0.165798	0.023634	0.517151	1.114617	1.78E-17	3.305798	2.867222
	Rank	1	13	3	2	4	11	7	5	8	9	6	12	10

**Table 4:** Evaluation results of high-dimensional multimodal objective functions

Table 4 (con	Fable 4 (continued)														
F	MA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA		
Sum rank	6	53	11	27	15	52	18	39	32	32	44	50	44		
Mean rank	1	8.833333	1.833333	4.5	2.5	8.666667	3	6.5	5.333333	5.333333	7.333333	8.333333	7.333333		
Total ranking	1	11	2	5	3	10	4	7	6	6	8	9	8		

F		MA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
F14	Mean	0.998004	1.097407	1.097209	3.107269	1.009791	8.646875	2.569752	0.998004	3.695176	0.998005	3.561315	3.595793	1.048667
	Best	0.998004	0.998004	0.998004	0.998036	0.998004	1.992031	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004
	Worst	0.998004	1.992031	2.982105	12.67051	1.233486	15.50382	10.76318	0.998004	10.76318	0.998019	11.86988	12.67051	1.992037
	Std	0	0.305955	0.443659	3.056678	0.052652	5.051266	2.946269	5.66E-12	3.730984	3.27E-06	2.754069	3.787922	0.222066
	Median	0.998004	0.998004	0.998004	2.225114	0.998004	11.71684	0.998004	0.998004	2.982105	0.998004	2.891705	1.992031	0.998004
	Rank	1	7	6	9	4	13	8	2	12	3	10	11	5
F15	Mean	0.000307	0.001357	0.000356	0.001123	0.001207	0.016426	0.000809	0.002647	0.003365	0.000594	0.002352	0.002499	0.015388
	Best	0.000307	0.000307	0.000307	0.000711	0.000309	0.000308	0.000311	0.000308	0.000307	0.000311	0.000887	0.000307	0.000782
	Worst	0.000307	0.020363	0.000732	0.00288	0.001674	0.110282	0.002252	0.020363	0.020363	0.001249	0.006959	0.020363	0.066917
	Std	2.54E-19	0.004478	0.000101	0.000468	0.000547	0.03002	0.000491	0.006065	0.007329	0.000401	0.001368	0.006126	0.016221
	Median	0.000307	0.000307	0.000311	0.001022	0.0016	0.000871	0.000686	0.000681	0.000308	0.000325	0.002169	0.000307	0.014273
	Rank	1	7	2	5	6	13	4	10	11	3	8	9	12
F16	Mean	-1.03163	-1.03163	-1.03163	-1.02941	-1.02929	-1.03005	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163
	Best	-1.03163	-1.03163	-1.03163	-1.03161	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163
	Worst	-1.03163	-1.03163	-1.03163	-1	-1.00093	-1	-1.03163	-1.03163	-1.03163	-1.03162	-1.03163	-1.03163	-1.03161
	Std	1.84E-16	2.64E-07	1.02E-16	0.006998	0.006906	0.007073	4.05E-11	5.49E-08	8.6E-09	1.68E-06	1.02E-16	1.14E-16	4.78E-06
	Median	-1.03163	-1.03163	-1.03163	-1.03129	-1.0316	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163
	Rank	1	5	1	9	10	8	2	4	3	7	1	1	6
F17	Mean	0.397887	0.397887	0.397887	0.410593	0.398401	0.397924	0.397888	0.397887	0.397888	0.39796	0.397887	0.744637	0.466023
	Best	0.397887	0.397887	0.397887	0.398539	0.397887	0.39789	0.397887	0.397887	0.397887	0.397891	0.397887	0.397887	0.397887
	Worst	0.397887	0.397889	0.397887	0.485262	0.401154	0.398206	0.39789	0.397888	0.39789	0.398172	0.397887	2.791184	1.75218
	Std	0	4.03E-07	0	0.019449	0.000956	6.83E-05	7.27E-07	6.64E-08	8.87E-07	6.77E-05	0	0.709302	0.302731
	Median	0.397887	0.397887	0.397887	0.403776	0.397974	0.397907	0.397888	0.397887	0.397888	0.397948	0.397887	0.397887	0.397905
	Rank	1	3	1	9	8	6	4	2	5	7	1	11	10
F18	Mean Best Worst Std Median Rank	3 3 1.17E-15 3 1	3 3 5.85E-16 3 1	3.000001 3 3.000007 1.88E-06 3 5	5.77479 3 31.31493 8.513419 3.00004 9	6.161661 3.013933 30.00128 6.359267 3.563655 10	11.50179 3.000001 92.03536 26.20036 3.000008 12	3.000026 3 3.000156 4.26E-05 3.000002 8	3 3.000002 4.46E-07 3 4	3.000013 3 3.000063 1.46E-05 3.00001 7	3.000001 3 3.000006 1.73E-06 3 6	3 3 3.57E-15 3 3	3 3 3E-15 3 2	7.302903 3 34.94955 10.54375 3.00117 11
F19	Mean Best Worst Std Median Rank	-3.86278 -3.86278 -3.86278 2.28E-15 -3.86278 1	-3.86278 -3.86278 -3.86278 2.28E-15 -3.86278 1	-3.86278 -3.86278 -3.86278 1.94E-11 -3.86278 2	-3.83693 -3.85894 -3.77916 0.022957 -3.8442 9	-3.72483 -3.86278 -3.2931 0.137426 -3.72574 10	-3.86238 -3.86278 -3.85597 0.00151 -3.86273 5	-3.86042 -3.86276 -3.8549 0.002902 -3.86188 8	-3.86278 -3.86278 -3.86278 2.15E-07 -3.86278 3	-3.86126 -3.86278 -3.85496 0.00261 -3.86276 7	$\begin{array}{r} -3.86168 \\ -3.86269 \\ -3.85488 \\ 0.00232 \\ -3.8624 \\ 6 \end{array}$	-3.86278 -3.86278 -3.86278 1.97E-15 -3.86278 1	-3.86278 -3.86278 -3.86278 2.06E-15 -3.86278 1	-3.86262 -3.86278 -3.86183 0.000295 -3.86278 4
F20	Mean	-3.322	-3.30416	-3.26849	-2.76525	-2.53258	-3.25505	-3.24989	-3.27434	-3.25903	-3.24275	-3.322	-3.26462	-3.2283
	Best	-3.322	-3.322	-3.322	-3.06927	-3.22483	-3.32157	-3.32199	-3.32199	-3.32199	-3.31585	-3.322	-3.322	-3.32163
	Worst	-3.322	-3.2031	-3.2031	-1.67014	-1.78365	-3.08946	-3.08934	-3.20229	-3.08401	-3.01379	-3.322	-3.13764	-2.99723
	Std	4.44E-16	0.043556	0.060685	0.312143	0.336978	0.071193	0.083878	0.059884	0.076101	0.080187	3.81E-16	0.074972	0.078203

**Table 5:** Evaluation results of fixed-dimensional multimodal objective functions

Tal	ole 5 (con	tinued)												
F		MA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
	Median	-3.322	-3.322	-3.322	-2.83538	-2.58954	-3.26107	-3.3181	-3.32199	-3.32199	-3.29176	-3.322	-3.322	-3.23661
	Rank	1	2	4	11	12	7	8	3	6	9	1	5	10
	Mean	-10.1532	-8.40651	-10.1532	-5.0552	-7.55876	-5.9252	-9.38543	-8.8855	-9.39035	-6.85273	-7.19413	-5.62381	-6.26023
	Best	-10.1532	-10.1532	-10.1532	-5.0552	-10.1515	-10.1295	-10.1531	-10.1532	-10.1532	-9.41523	-10.1532	-10.1532	-9.73855
F21	Worst	-10.1532	-2.68286	-10.1532	-5.0552	-5.0552	-2.60302	-5.0551	-5.05518	-5.05519	-3.24272	-2.68286	-2.63047	-2.38578
	Std	2.08E-15	3.143351	1.31E-14	3.39E-07	2.052467	3.235554	1.866342	2.252712	1.861953	2.077471	3.457706	2.883857	2.711083
	Median	-10.1532	-10.1532	-10.1532	-5.0552	-7.90122	-4.99925	-10.1511	-10.1531	-10.1527	-7.31397	-10.1532	-5.10077	-7.06069
	Rank	1	6	2	13	7	11	4	5	3	9	8	12	10
	Mean	-10.4029	-10.0204	-10.4029	-5.08767	-8.0897	-6.8844	-8.10852	-8.4347	-10.4024	-7.94981	-10.1293	-6.38293	-7.37187
	Best	-10.4029	-10.4029	-10.4029	-5.08767	-10.4005	-10.3392	-10.4029	-10.4029	-10.4028	-10.0628	-10.4029	-10.4029	-9.9828
F22	Worst	-10.4029	-2.75193	-10.4029	-5.08767	-5.08767	-1.83282	-1.83745	-2.76589	-10.4015	-4.04839	-4.92953	-2.75193	-2.67682
	Std	3.51E-15	1.710817	2.94E-14	1.07E-06	2.092584	3.509365	3.051708	2.796754	0.000408	1.673445	1.223892	3.469587	1.916626
	Median	-10.4029	-10.4029	-10.4029	-5.08767	-9.04577	-7.4911	-10.3981	-10.4029	-10.4026	-8.38543	-10.4029	-5.10825	-7.86313
	Rank	1	5	2	13	8	11	7	6	3	9	4	12	10
	Mean	-10.5364	-10.5364	-10.5364	-5.12847	-9.15341	-7.41502	-8.58345	-9.46185	-10.5359	-8.08614	-10.2874	-6.42082	-6.36016
	Best	-10.5364	-10.5364	-10.5364	-5.12848	-10.4492	-10.4805	-10.5362	-10.5364	-10.5363	-9.69083	-10.5364	-10.5364	-10.1845
F23	Worst	-10.5364	-10.5364	-10.5364	-5.12847	-5.12848	-2.42013	-1.67654	-5.12847	-10.5352	-4.2682	-5.55587	-2.42173	-2.38229
	Std	2.76E-15	3.78E-15	8.2E-15	2.14E-06	1.473974	3.472863	3.262135	2.204851	0.000322	1.660896	1.113682	3.847923	2.608634
	Median	-10.5364	-10.5364	-10.5364	-5.12847	-9.54713	-10.2904	-10.5339	-10.5363	-10.536	-8.67926	-10.5364	-3.83543	-6.88826
	Rank	1	2	3	13	7	10	8	6	4	9	5	11	12
Sum	rank	10	39	28	100	82	96	61	45	61	68	42	75	90
Mea	n rank	1	3.9	2.8	10	8.2	9.6	6.1	4.5	6.1	6.8	4.2	7.5	9
Tota	l ranking	1	3	2	12	9	11	6	5	6	7	4	8	10

The performance of MA and competitor algorithms in optimizing functions F1 to F23 is presented in the form of boxplot diagrams in Fig. 2. Also, the effectiveness of MA and competitor algorithms in obtaining the first rank of the best optimizer and solving the set of unimodal functions, highdimensional multimodal, and fixed-dimensional multimodal is presented in the form of the bar graphs in Fig. 3. These charts visually show that MA has been ranked as the first best optimizer in 100% of the objective functions in the mentioned sets.



Figure 2: (Continued)



Figure 2: Boxplots of MA and competitor algorithms performances on the F1 to F23



Figure 3: Bar graph of MA and competitor algorithms performances on the F1 to F23

## 4.4 Evaluation of the CEC 2017 Test Suite

In this subsection, the performance of the proposed MA approach has been tested in optimizing the CEC 2017 test suite benchmark functions. The CEC 2017 test suite has thirty benchmark functions C17-F1 to C17-F30. The C17-F2 function is not considered in the simulation studies due to its unstable behavior. The optimization results of this test suite using MA and competitor algorithms are reported in Table 6. Based on the optimization results, MA is the first best optimizer for functions C17-F1, C17-F3 to C17-F6, C17-F8 to C17-F21, and C17-F23 to C17-F30. Analysis of the simulation results shows that the proposed MA algorithm, by providing better results in most functions, has delivered superior performance in optimization of the CEC 2017 test suite compared to competitor algorithms. The effectiveness of the proposed MA approach and competitor algorithms in solving the CEC 2017 test suite is presented in Fig. 4. The visual analysis of these graphs shows that MA has been ranked as the first-best optimizer in 92% of the functions of this test suite.

		14		1001	DCA	ND4	TC 4	work		CIVO	TIDDO	004	DCO	<u></u>
C17-F1	Avg Std Rank	MA 100 1.7E-05 1	5448.254 4872.753 5	AVOA 2609.297 2078.791 4	RSA 8.53E+09 2.28E+09 12	MPA 1.22E+10 2.81E+09 13	15A 1.41E+09 1.94E+09 11	WOA 4027220 1941308 8	7773.088 3491.036 6	1644467 3136967 7	71474843 20225343 10	GSA 1018.466 903.4716 3	516.7751 605.5315 2	GA 21102276 6453371 9
C17-F3	Avg	300	718.4464	309.6546	16366.7	10620.6	6486.396	3493.403	300.6303	1020.12	845.603	12712.89	708.8861	32974.59
	Std	8.88E-11	593.2109	14.04486	1278.807	507.1204	3969.481	3779.991	0.027016	992.2169	84.21148	3684.697	806.7733	12565.83
	Rank	1	5	3	12	10	9	8	2	7	6	11	4	13
C17-F4	Avg	400	404.5963	411.0693	762.0799	1329.035	598.4949	478.3435	405.7807	418.2254	419.9729	406.4805	406.4603	417.8162
	Std	6.61E-08	3.056472	10.79775	263.8552	260.0972	161.9093	70.38746	0.559332	19.0207	11.57954	0.9126	4.934397	4.118445
	Rank	1	2	6	12	13	11	10	3	8	9	5	4	7
C17-F5	Avg	510.9445	516.3407	556.8287	569.8813	590.7058	562.6056	538.9635	515.9566	511.7988	538.6003	554.3365	523.6805	533.9824
	Std	3.589474	6.817304	28.42219	8.645956	21.09107	13.27814	7.949781	5.022443	0.30121	1.748629	11.64052	6.144633	13.77461
	Rank	1	4	10	12	13	11	8	3	2	7	9	5	6
C17-F6	Avg	600.0006	602.5057	629.0535	650.2351	647.7861	630.4929	633.9744	601.8596	604.2087	609.1517	626.5182	616.7245	611.1823
	Std	0.000106	0.869028	9.282709	2.195644	5.418509	17.90023	8.881193	0.487208	3.716772	2.660889	4.948045	15.72331	2.492215
	Rank	1	3	9	13	12	10	11	2	4	5	8	7	6
C17-F7	Avg	722.5537	717.7378	773.3652	804.9964	807.5509	816.9533	788.5726	733.3692	741.8852	760.3543	718.5491	739.805	737.0977
	Std	2.754451	4.802381	25.70196	1.837584	16.78011	48.47718	20.97234	8.726856	15.06105	8.166037	3.027647	21.55877	8.601992
	Rank	3	1	9	11	12	13	10	4	7	8	2	6	5
C17-F8	Avg	807.9597	808.3296	831.0099	862.4313	849.9782	854.3809	847.4582	822.0418	814.6341	827.6841	828.7668	826.0252	822.4269
	Std	1.794737	2.829137	8.062428	7.203307	8.365379	8.9807	6.166724	9.009571	4.138677	7.95629	1.864517	9.93155	6.406589
	Rank	1	2	9	13	11	12	10	4	3	7	8	6	5
C17-F9	Avg	900	935.0944	1034.168	1530.759	1668.499	1466.187	1574.528	902.0567	918.6234	936.4737	901.8	903.2044	908.5793
	Std	3.38E-08	43.21674	41.81114	162.3298	152.9708	377.5842	231.1458	0.286841	32.73898	25.70718	0	1.694174	2.118464
	Rank	1	7	9	11	13	10	12	3	6	8	2	4	5
C17-F10	Avg	1379.646	1448.599	2216.291	2797.641	2615.554	2007.697	1826.678	1718.811	1799.504	1915.856	2785.793	2305.845	1624.998
	Std	211.5795	185.4203	266.0006	191.6339	143.9072	335.6777	493.448	208.9451	364.2204	67.76721	383.299	477.6513	239.1756
	Rank	1	2	9	13	11	8	6	4	5	7	12	10	3
C17-F11	Avg	1101.505	1126.453	1139.788	5337.953	1468.933	2471.929	1195.477	1143.356	1138.476	1141.471	1124.515	1133.854	3420.189
	Std	1.269139	9.351585	9.209205	3721.554	120.0295	2242.55	27.35466	15.64004	10.6613	11.27831	1.087726	21.44905	4168.907
	Rank	1	3	6	13	10	11	9	8	5	7	2	4	12
C17-F12	Avg	1264.785	7468	1876420	4.2E+08	3.61E+08	3106602	3697769	548252.1	1753782	3059993	541145	2073325	787895.9
	Std	70.77641	3951.574	2825571	2.36E+08	2.42E+08	3940521	3837263	398111.2	2888611	1950729	234214.8	4042504	1175629
	Rank	1	2	7	13	12	10	11	4	6	9	3	8	5
C17-F13	Avg	1305.286	1410.749	9335.84	48808051	158023.6	10852.69	11372.2	8952.713	7393.659	7835.741	12260.65	4030.027	17635.36
	Std	3.253005	91.8257	4844.05	34687926	159470.4	4122.611	7246.567	11787.44	3628.673	3028.571	3224.931	2861.429	14292.38
	Rank	1	2	7	13	12	8	9	6	4	5	10	3	11
C17-F14	Avg	1404.229	1421.576	4099.722	4180.239	1532.07	3454.497	3952.096	1455.049	4865.523	1557.597	5499.921	4834.248	5142.65
	Std	3.247945	11.66315	3639.562	2144.977	18.78965	2202.283	1760.147	13.23297	550.3441	53.49639	2325.243	2269.872	2341.673
	Rank	1	2	8	9	4	6	7	3	11	5	13	10	12
													((	Continued)

Table 6	(continued)

		,												
		MA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBBO	GSA	PSO	GA
C17-F15	Avg	1500.466	1535.669	5500.962	18950.77	9998.102	8581.481	6278.955	2104.386	4307.278	1795.41	16812.02	7748.302	3175.22
	Std	0.306769	15.43699	4281.135	7065.287	3448.643	8305.207	3561.047	708.5334	2121.008	62.23526	5117.109	7257.003	2658.747
	Rank	1	2	7	13	11	10	8	4	6	3	12	9	5
C17-F16	Avg	1601.334	1683.232	1841.462	2172.515	2073.709	1981.934	1849.222	1766.712	1782.375	1706.716	2242.427	1991.839	1795.945
	Std	0.862582	91.62543	144.9967	136.225	96.36129	220.7645	87.522	54.42363	184.3148	62.85258	175.7021	151.383	122.9042
	Rank	1	2	7	12	11	9	8	4	5	3	13	10	6
C17-F17	Avg	1720.654	1752.82	1758.844	1885.033	1846.92	1937.379	1815.276	1781.881	1812.56	1764.681	1800.573	1769.714	1757.453
	Std	1.727903	13.60695	26.56794	19.82602	59.27428	181.4092	44.08472	49.43639	78.18535	16.08293	87.91088	29.44371	5.50491
	Rank	1	2	4	12	11	13	10	7	9	5	8	6	3
C17-F18	Avg	1800.479	1826.811	15340.91	23615012	65596574	30607.68	8020.974	21984.93	23524.13	36562.87	16400.59	15682.72	10677.83
	Std	0.05863	14.06095	13427.33	35424891	71910858	22596.6	6123.919	3569.621	16918.14	26288.33	6379.593	12880.49	4442.434
	Rank	1	2	5	12	13	10	3	8	9	11	7	6	4
C17-F19	Avg	1900.702	1913.454	12351.64	441147.9	5849.714	6886.935	197278.1	2194.503	4819.379	2141.324	35035.32	8300.611	7838.698
	Std	0.427842	4.482875	12447.67	671766.4	4145.302	5888.936	366611.8	481.8209	4737.382	117.2737	12800.73	6128.356	5499.438
	Rank	1	2	10	13	6	7	12	4	5	3	11	9	8
C17-F20	Avg	2019.37	2033.366	2129.624	2240.741	2274.791	2182.601	2232.391	2041.174	2082.864	2107.236	2378.645	2157.594	2062.76
	Std	2.038897	18.03211	74.19837	41.57804	71.14648	110.5259	53.72554	23.65441	62.35375	59.39264	117.8283	32.63644	24.84904
	Rank	1	2	7	11	12	9	10	3	5	6	13	8	4
C17-F21	Avg	2200	2291.547	2276.781	2293.98	2388.661	2357.99	2321.637	2297.953	2321.928	2308.287	2367.167	2306.049	2280.548
	Std	1.53E-05	55.34638	78.81185	63.88033	10.38995	14.80519	49.37911	60.32195	3.591627	65.14029	11.75605	64.26354	68.08475
	Rank	1	4	2	5	13	11	9	6	10	8	12	7	3
C17-F22	Avg	2300.224	2314.831	2303.985	3237.317	2903.671	2511.896	2294.845	2308.597	2314.282	2323.166	2304.7	2692.041	2322.973
	Std	0.269337	2.109835	17.6283	278.36	330.9787	157.9215	23.87328	1.378707	11.26983	6.558206	0.197455	457.4206	2.687915
	Rank	2	7	3	13	12	10	1	5	6	9	4	11	8
C17-F23	Avg	2609.635	2645.705	2634.449	2722.862	2723.433	2718.165	2650.951	2632.754	2632.614	2638.827	2744.996	2645.45	2664.474
	Std	1.438651	31.83799	16.747	24.70995	25.64097	43.81511	12.68692	9.625373	7.650694	7.34203	13.70838	11.61482	10.10757
	Rank	1	7	4	11	12	10	8	3	2	5	13	6	9
C17-F24	Avg	2525.171	2754.51	2785.302	2883.228	2864.007	2735.114	2770.687	2760.304	2743.654	2773.51	2584.442	2731.396	2663.883
	Std	49.73738	12.37697	25.86773	38.34163	65.95106	139.95	8.17377	16.69422	4.298292	5.820181	156.9782	151.954	140.8002
	Rank	1	7	11	13	12	5	9	8	6	10	2	4	3
C17-F25	Avg	2823.318	2930.355	2930.764	3334.08	3601.275	3072.219	2949.671	2927.476	2952.861	2976.754	2949.306	2931.062	2959.25
	Std	147.0641	28.35548	29.67163	18.93201	167.8605	137.7503	37.03455	26.73677	8.759469	39.27112	0.035844	24.68336	4.992824
	Rank	1	3	4	12	13	11	7	2	8	10	6	5	9
C17-F26	Avg	2850.001	2980.052	3102.972	4220.038	4353.33	4228.957	3647.311	3157.326	3150.046	2966.358	3502.279	2932.257	3061.582
	Std	57.04117	37.36052	168.5553	277.532	215.2719	514.5373	527.3119	496.7817	482.2802	29.75685	805.1704	96.24618	125.6092
	Rank	1	4	6	11	13	12	10	8	7	3	9	2	5
C17-F27	Avg	3089.072	3109.342	3109.977	3166.764	3159.247	3204.307	3137.73	3097.685	3122.9	3100.388	3242.941	3142.068	3136.073
	Std	0.149314	5.469064	1.052491	15.78007	23.80812	75.64033	47.71376	2.375189	37.85936	1.927928	23.26766	30.23941	8.668241
	Rank	1	4	5	11	10	12	8	2	6	3	13	9	7
C17-F28	Avg	3100	3223.421	3340.534	3749.113	3791.321	3395.719	3286.578	3355.456	3345.938	3359.479	3489.174	3255.45	3405.515
	Std	5.84E-05	119.8243	154.348	155.3563	95.98989	115.4105	94.5982	88.76547	71.58004	117.4031	24.00222	170.6543	161.0559
	Rank	1	2	5	12	13	9	4	7	6	8	11	3	10
C17-F29	Avg	3146.525	3165.086	3256.773	3447.096	3431.013	3309.431	3446.593	3228.585	3208.791	3223.425	3502.994	3254.785	3230.251
	Std	9.568595	10.40137	71.65968	169.8153	67.29951	77.04566	152.4582	116.6436	62.75773	19.0736	263.8243	35.43839	36.41003
	Rank	1	2	8	12	10	9	11	5	3	4	13	7	6
C17-F30	Avg	3400.543	5068.011	1169501	11422827	9796336	6556000	450834.7	739073.4	766163.8	397371.5	1737204	562674.6	2889564
	Std	8.742004	1555.344	580777.1	6852472	7455823	7292446	475610.2	839365.2	737649	703017.7	2083052	737836.8	2687823
	Rank	1	2	8	13	12	11	4	6	7	3	9	5	10
Sum rank		32	94	192	343	330	288	241	134	175	187	244	180	199
Mean rank		1.103448	3.241379	6.62069	11.82759	11.37931	9.931034	8.310345	4.62069	6.034483	6.448276	8.413793	6.206897	6.862069
Total rank		1	2	7	13	12	11	9	3	4	6	10	5	8



Figure 4: Bar graph of MA and competitor algorithms performances on the CEC 2017 test suite

### 4.5 Statistical Analysis

In this subsection, statistical analysis is presented on the performance of MA and competitor algorithms to determine whether the superiority of the proposed approach is significant from a statistical point of view. Wilcoxon sign-rank test [79] statistical analysis is employed for this purpose. Wilcoxon sign-rank test is a non-parametric test used to determine the significant difference between the averages of two data samples. In this test, the presence or absence of a substantial difference is determined using an index called "*p*-value".

The results of implementing Weil's statistical analysis on the performance of MA compared to each of the competing algorithms are presented in Table 7. Based on the obtained results, in cases where the p-value is less than 0.05, the proposed MA approach has a significant statistical advantage compared to the corresponding competing algorithm.

Compared algorithms	Unimodal	High-multimodal	Fixed-multimodal	CEC 2017 test suite
MA vs. WSO	1.85E-24	1.97E-21	3.68E-06	6.44E-17
MA vs. AVOA	3.02E-11	4.99E-05	2.56E-21	5.92E-21
MA vs. RSA	4.25E-07	1.63E-11	1.44E-34	1.97E-21
MA vs. MPA	1.01E-24	1.04E-14	2.09E-34	1.97E-21
MA vs. TSA	1.01E-24	1.31E-20	1.44E-34	1.97E-21
MA vs. WOA	2.44E-24	6.13E-11	1.44E-34	5.64E-21
MA vs. MVO	1.01E-24	1.97E-21	1.44E-34	4.98E-21
MA vs. GWO	1.01E-24	5.34E-16	1.44E-34	4.85E-21
MA vs. TLBO	1.01E-24	6.98E-15	1.44E-34	1.97E-21
MA vs. GSA	1.01E-24	1.97E-21	4.64E-13	3.86E-19
MA vs. PSO	1.01E-24	1.97E-21	3.92E-17	7.58E-20
MA vs. GA	1.01E-24	1.97E-21	1.44E-34	2.02E-21

Table 7: Obtained results from the Wilcoxon sum-rank test

### 5 Application of MA to Real-World Problems

In this section, the effectiveness of the proposed MA approach in solving real-world applications is evaluated. For this purpose, MA is employed in the optimization of four engineering design problems, including tension/compression spring (TCS) design, welded beam (WB) design, speed reducer (SR) design, and pressure vessel (PV) design. The full description and mathematical model of these problems are provided for TCS in [80], WB in [80], SR in [81,82], and PV in [83].

The results of implementing the proposed MA approach and competing algorithms on these four engineering problems are reported in Table 8. The optimization results show that the proposed MA approach has provided the optimal solution for the TCS problem with the values of the design variables equal to (0.051689, 0.356718, 11.28897) and the value of the corresponding objective function is equal to 2996.348. MA has presented the optimal design of the WB problem with optimal values of the design variables equal to (0.20573, 3.470489, 9.036624, 0.20573) and the value of the corresponding objective function equal to 5882.901. In optimizing the SR problem, the proposed MA approach has provided the optimal design with the optimal values of the design variables equal to (3.5, 0.7, 17, 7.3, 7.8, 3.350215, 5.286683) and the value of the corresponding objective function equal to 1.724852. In dealing with the PV problem, the MA has provided the optimal design with the found values of the design variables equal to (0.778027, 0.384579, 40.31228, 200) and the value of the corresponding objective function equal to 0.012665. The analysis of the simulation results shows that the proposed MA approach by providing better outcomes for statistical indicators and more suitable designs for engineering problems has delivered superior performance compared to competitor algorithms. The simulation results show that the proposed MA approach has effective performance in real-world handling applications. The efficiency of MA and competitor algorithms in dealing with engineering design problems is drawn as bar graphs in Fig. 5. The visual analysis of these graphs indicates that MA was the first best optimizer in 100% of the investigated engineering problems (including four problems).

DP		MA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
	Mean	2996.348	2996.351	3001.892	3254.038	2996.348	3032.837	3276.675	3035.441	3005.299	5.44E+13	3561.954	1.45E+14	9.81E+13
	Best	2996.348	2996.348	2996.351	3103.959	2996.348	3014.196	3009.707	3005.881	2999.371	4672.036	3296.523	4948.149	4462.424
TCS	Worst	2996.348	2996.369	3009.676	3363.873	2996.348	3051.492	4667.37	3071.27	3012.814	2.49E+14	4250.992	7.02E+14	6.95E+14
	Std	9.35E-13	0.004769	4.33969	64.50182	8.82E-06	9.086276	451.4213	17.96723	3.663097	6.56E+13	235.7865	2E+14	1.57E+14
	Median	2996.348	2996.349	3001.809	3243.146	2996.348	3032.381	3143.825	3036.836	3005.224	2.83E+13	3522.305	4.11E+13	5.47E+13
	Rank	1	3	4	8	2	6	9	7	5	11	10	13	12
	Mean	5882.901	5882.914	6278.229	10968.96	5882.901	6247.711	7903.704	6520.619	6078.76	30630.33	22626.22	45671.21	34486.36
	Best	5882.901	5882.901	5882.909	6663.599	5882.901	5911.724	6392.426	5931.347	5889.819	14279.64	6845.673	15910.56	13646.15
WB	Worst	5882.901	5883.162	7316.027	20408.23	5882.901	7377.195	10589.99	7269.453	7177.259	46757.48	48860.27	96299.35	62432.29
	Std	1.87E-12	0.058514	414.0676	3052.863	3.22E-05	431.0263	1310.97	369.9664	375.1106	9473.362	11116.72	22564.51	11511.76
	Median	5882.901	5882.901	6199.966	10557.51	5882.901	5990.947	7409.241	6492.134	5907.476	29767.04	21605.73	38252.71	33157.34
	Rank	1	3	6	9	2	5	8	7	4	11	10	13	12
	Mean	1.724852	1.724852	1.747098	2.319169	1.724852	1.744186	2.463605	1.746994	1.727296	2.79E+13	2.364869	7.46E+13	6.22E+12
	Best	1.724852	1.724852	1.724899	1.937455	1.724852	1.733362	1.798388	1.729715	1.725573	2.00194	1.774802	2.756985	2.647148
SR	Worst	1.724852	1.724852	1.805932	4.009434	1.724852	1.751246	4.554507	1.781098	1.731518	4.71E+14	2.667656	9.04E+14	1.21E+14
	Std	6.85E-16	1.09E - 10	0.024366	0.43966	2.59E-08	0.005629	0.806302	0.014574	0.001752	1.05E+14	0.219963	2.14E+14	2.7E+13
	Median	1.724852	1.724852	1.738136	2.226703	1.724852	1.745158	2.06409	1.743206	1.726559	5.103654	2.377888	5.414863	5.296107
	Rank	1	2	7	8	3	5	10	6	4	12	9	13	11
	Mean	0.012665	0.012666	0.013018	0.017829	0.012665	0.012935	0.013487	0.017115	0.012722	0.018439	0.020159	3.97E+13	0.024713
	Best	0.012665	0.012665	0.012667	0.01307	0.012665	0.012717	0.01269	0.012915	0.01269	0.017845	0.01432	0.017773	0.018482
PV	Worst	0.012665	0.012671	0.014139	0.093677	0.012665	0.013343	0.015486	0.01809	0.012743	0.019051	0.025478	3.97E+14	0.034116
	Std	9.77E-19	1.33E-06	0.000415	0.018046	3.37E-09	0.000156	0.000945	0.001561	1.19E-05	0.000362	0.003595	1.23E+14	0.004044
	Median	0.012665	0.012665	0.012856	0.013267	0.012665	0.012943	0.013139	0.017804	0.012726	0.018378	0.019806	0.017773	0.023866
														C (* 1)

**Table 8:** Evaluation results of real-world applications

Table 8 (c	Table 8 (continued)												
DP	MA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
Rank	1	3	6	9	2	5	7	8	4	10	11	13	12
Sum rank	4	11	23	34	9	21	34	28	17	44	40	52	47
Mean rank	1	2.75	5.75	8.5	2.25	5.25	8.5	7	4.25	11	10	13	11.75
Total ranking	1	3	6	8	2	5	8	7	4	10	9	12	11



Figure 5: Bar graph of MA and competitor algorithms performances on the engineering problems

### 6 Conclusion and Future Works

In this paper, a new human-based metaheuristic algorithm called Migration Algorithm (MA) was introduced to solve optimization problems in various sciences. Human activities in the migration process are the fundamental inspiration in MA design. The proposed approach was mathematically modeled based on the simulation of two strategies of choosing the migration destination and adapting to the new environment in two phases of exploration and exploitation. Fifty-two standard benchmark functions including unimodal, multimodal, and the CEC 2017 test suite were employed to evaluate MA performance in solving optimization problems. The optimization results showed that the proposed MA approach with high ability in exploration and exploitation has a favorable performance in optimization. The quality of MA was compared with the performance of twelve well-known metaheuristic algorithms. The results obtained from solving the unimodal functions showed that MA had provided high efficiency in 100% of the functions of this set by winning the first rank. The findings obtained from optimizing unimodal functions showed that MA is highly capable of exploitation and local search. The results of solving high-dimensional multimodal functions indicated the 100% efficiency of the proposed approach and the high capability of MA in exploration and global search. The results of solving fixed-dimensional multimodal functions show 100% efficiency of the proposed approach in getting the rank of the first best optimizer compared to competitor algorithms. The findings obtained from solving the CEC 2017 test suite showed that MA was ranked the first-best optimizer in 92% of the functions of this test suite. Also, the Wilcoxon sign-rank statistical analysis showed that the superiority of the proposed MA approach against competitor algorithms is significant from a statistical point of view. The simulation results showed that the proposed approach with a high ability to balance exploration and exploitation has a superior and far more competitive performance against the compared algorithms. Moreover, the implementation of MA on four engineering design problems indicated the effective performance of the proposed approach in handling real-world applications.

Introducing the proposed MA approach enables several research topics for further studies. One of the most special research potentials for future works is the design of binary and multi-objective versions of the proposed approach. Employing MA in optimization problems in different sciences as well as optimization tasks in real-world applications are other research suggestions for future work.

Acknowledgement: The authors thank Dušan Bednařík from the University of Hradec Kralove for our fruitful and informative discussions.

**Funding Statement:** The research was supported by the Project of Excellence PřF UHK No. 2210/2023–2024, University of Hradec Kralove, Czech Republic.

**Conflicts of Interest:** The authors declare that they have no conflicts of interest to report regarding the present study.

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# **Appendix A. Objective Functions**

The information of the objective functions used in the simulation section is specified in Tables A1 to A4.

$F_1(x) = \sum_{i=1}^m x_i^2$	$[-100, 100]^m$
$F_{2}(x) = \sum_{i=1}^{m}  x_{i}  + \prod_{i=1}^{m}  x_{i} $	$[-10, 10]^m$
$F_{3}(x) = \sum_{i=1}^{m} \left(\sum_{j=1}^{i} x_{i}\right)^{2}$	[-100, 100] <sup>m</sup>
$F_4(x) = max\{ x_i , 1 \le i \le m\}$	$[-100, 100]^m$
$F_{5}(x) = \sum_{i=1}^{m-1} \left[ 100 \left( x_{i+1} - x_{i}^{2} \right)^{2} + \left( x_{i} - 1 \right)^{2} \right) \right]$	$[-30, 30]^m$
$F_{6}(x) = \sum_{i=1}^{m} \lfloor x_{i} + 0.5 \rfloor^{2}$	$[-100, 100]^m$
$F_{7}(x) = \sum_{i=1}^{m} ix_{i}^{4} + random(0, 1)$	[-1.28, 1.28] <sup>m</sup>

**Table A1:** Unimodal objective functions

$$\begin{aligned} \overline{F_{8}(x)} &= \sum_{i=1}^{m} -x_{i} \sin\left(\sqrt{|x_{i}|}\right) & [-500, 500]^{m} \\ \overline{F_{9}(x)} &= \sum_{i=1}^{m} \left[x_{i}^{2} - 10 \cos\left(2\pi x_{i}\right) + 10\right] & [-5.12, 5.12]^{m} \\ \overline{F_{10}(x)} &= -20 \exp\left(-0.2 \sqrt{\frac{1}{m}} \sum_{i=1}^{m} x_{i}^{2}\right) - \exp\left(\frac{1}{m} \sum_{i=1}^{m} \cos\left(2\pi x_{i}\right)\right) + 20 + e & [-32, 32]^{m} \\ \overline{F_{10}(x)} &= \frac{1}{4000} \sum_{i=1}^{m} x_{i}^{2} - \prod_{i=1}^{m} \cos\left(\frac{x_{i}}{\sqrt{i}}\right) + 1 & [-600, 600]^{m} \\ \overline{F_{12}(x)} &= \frac{\pi}{m} \left\{ 10 \sin\left(\pi y_{1}\right) + \sum_{i=1}^{m} (y_{i} - 1)^{2} \left[1 + 10 \sin^{2}\left(\pi y_{i+1}\right)\right] + (y_{n} - 1)^{2} \right\} & [-50, 50]^{m} \\ &+ \sum_{i=1}^{m} u(x_{i}, 10, 100, 4) & \\ \text{where } y_{i} &= 1 + \frac{1 + x_{i}}{4}, u(x_{i}, a, i, n) = \left\{ k \left(x_{i} - a\right)^{n}, x_{i} > -a; \\ 0, -a \leq x_{i} \leq a; \\ k \left(-x_{i} - a\right)^{n}, x_{i} < -a, \\ \hline F_{13}(x) &= 0.1 \left\{ \sin^{2}\left(3\pi x_{1}\right) + \sum_{i=1}^{m} (x_{i} - 1)^{2} \left[1 + \sin^{2}\left(3\pi x_{i} + 1\right)\right] & [-50, 50]^{m} \\ &+ (x_{n} - 1)^{2} \left[1 + \sin^{2}\left(2\pi x_{m}\right)\right] \right\} + \sum_{i=1}^{m} u(x_{i}, 5, 100, 4) \end{aligned}$$

# Table A3: Fixed-dimensional objective functions

$\overline{F_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^{2} (x_i - a_{ij})^6}\right)^{-1}}$	$[-65.53, 65.53]^2$
$\overline{F_{15}(x)} = \sum_{i=1}^{11} \left[ a_i - \frac{x_1 \left( b_i^2 + b_i x_2 \right)}{b_i^2 + b_i x_3 + x_4} \right]^2$	$[-5,5]^4$
$\overline{F_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4}$	$[-5, 5]^2$
$\overline{F_{17}(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos x_1 + 10}$	[-5,10]× [0,15]
	(Continued)

$\overline{F_{18}(x) = \left[1 + (x_1 + x_2 + 1)^2 \left(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2\right)\right]}$	$[-5,5]^2$
$\times \left[ 30 + (2x_1 - 3x_2)^2 \left( 18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2 \right) \right]$	
$F_{19}(x) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{3} a_{ij} \left(x_j - P_{ij}\right)^2\right)$	$[0,1]^3$
$\overline{F_{20}(x) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{6} a_{ij} (x_j - P_{ij})^2\right)}$	$[0, 1]^6$
$\overline{F_{21}(x) = -\sum_{i=1}^{5} \left[ (X - a_i) (X - a_i)^T + 6c_i \right]^{-1}}$	$[0, 10]^4$
$F_{22}(x) = -\sum_{i=1}^{7} \left[ (X - a_i) (X - a_i)^T + 6c_i \right]^{-1}$	$[0, 10]^4$
$\overline{F_{23}(x) = -\sum_{i=1}^{10} \left[ (X - a_i) (X - a_i)^T + 6c_i \right]^{-1}}$	$[0, 10]^4$

|--|

	Functions	$f_{min}$
C1	Shifted and Rotated Bent Cigar Function	100
C2	Shifted and Rotated Sum of Different Power Function	200
C3	Shifted and Rotated Zakharov Function	300
C4	Shifted and Rotated Rosenbrock's Function	400
C5	Shifted and Rotated Rastrigin's Function	500
C6	Shifted and Rotated Expanded Scaffer's Function	600
C7	Shifted and Rotated Lunacek Bi_Rastrigin Function	700
C8	Shifted and Rotated Non-Continuous Rastrigin's Function	800
C9	Shifted and Rotated Levy Function	900
C10	Shifted and Rotated Schwefel's Function	1000
C11	Hybrid Function 1 ( $N = 3$ )	1100
C12	Hybrid Function 2 ( $N = 3$ )	1200
C13	Hybrid Function 3 ( $N = 3$ )	1300
C14	Hybrid Function 4 ( $N = 4$ )	1400
C15	Hybrid Function 5 ( $N = 4$ )	1500
C16	Hybrid Function 6 ( $N = 4$ )	1600
C17	Hybrid Function 6 ( $N = 5$ )	1700
C18	Hybrid Function 6 ( $N = 5$ )	1800

Table A4 (continued)			
	Functions	$f_{min}$	
C19	Hybrid Function 6 ( $N = 5$ )	1900	
C20	Hybrid Function 6 $(N = 6)$	2000	
C21	Composition Function 1 ( $N = 3$ )	2100	
C22	Composition Function 2 ( $N = 3$ )	2200	
C23	Composition Function 3 $(N = 4)$	2300	
C24	Composition Function 4 $(N = 4)$	2400	
C25	Composition Function 5 $(N = 5)$	2500	
C26	Composition Function 6 ( $N = 5$ )	2600	
C27	Composition Function 7 $(N = 6)$	2700	
C28	Composition Function 8 $(N = 6)$	2800	
C29	Composition Function 9 ( $N = 3$ )	2900	
C30	Composition Function $10 (N = 3)$	3000	