# Emerging Trends in Social Networking Systems and Generation Gap with Neutrosophic Crisp Soft Mapping 

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#### Abstract

This paper aims to introduce the novel concept of neutrosophic crisp soft set (NCSS), including various types of neutrosophic crisp soft sets (NCSSs) and their fundamental operations. We define NCS-mapping and its inverse NCS-mapping between two NCS-classes. We develop a robust mathematical modeling with the help of NCSmapping to analyze the emerging trends in social networking systems (SNSs) for our various generations. We investigate the advantages, disadvantages, and natural aspects of SNSs for five generations. With the changing of the generations, it is analyzed that emerging trends and the benefits of SNSs are increasing day by day. The suggested modeling with NCS-mapping is applicable in solving various decision-making problems.


## KEYWORDS

Neutrosophic crisp soft set; operations of NCSSs; NCS-mapping; inverse NCS-mapping; social networking systems; decision-making

## 1 Introduction

Fuzzy set theory [1] and soft set theory [2] are independent abstraction of classical set theory to address uncertainties by using membership functions and parameterizations, respectively. These theories have been widely used for solving real-world problems. The researchers extended the fuzzy set theory to develop various new models such as intuitionistic fuzzy sets [3], interval-valued intuitionistic fuzzy sets [4-7], Pythagorean fuzzy sets [8,9], q-rung orthopair fuzzy sets [10], bipolar fuzzy sets [11,12], and m-polar fuzzy sets [13].

These models have been developed by using the membership function and non-membership function to cover the limitations of membership and non-membership grades. In the case of a bipolar fuzzy set, the membership grades are categorized into positive and negative grades to discuss the counter property of objects. Smarandache [14,15] introduced the idea of the neutrosophic set (NSs), which assign three independent grades (truth-membership, indeterminacy, and falsity). Wang et al. [16] introduced single-valued neutrosophic sets as a consequence of NSs. A neutrosophic set is more general than various existing fuzzy sets. Some neutrosophic concepts have been explored by Salama et al. [17,18]. They presented a novel idea of the neutrosophic crisp set (NCS) and neutrosophic crisp relations with the construction of neutrosophic crisp topological spaces. Salama et al. [19] established the novel concepts about NCS theory and developed some applications of this theory in diverse areas. Karaaslan [20] investigated some properties of the neutrosophic soft set (NSS) and presented interesting applications in decision-making problems. Maji [21] presented some significant results on NSSs. Kharal et al. [22] presented several results on the mappings of soft classes. Wardowski [23] presented innovative results on soft mappings and their fixed points.

Information measures (similarity measures, distance measures, inclusion measures, and entropy) proposed by Peng et al. [24]. They developed algorithms and their applications towards medical diagnosis, pattern recognition, and clustering analysis. Akram et al. [25] suggested extension of competition graphs like k-competition graphs and p-competition complex fuzzy graphs under complex fuzzy environment and their application in ecosystem. Akram et al. [26] suggested an implementation of SVN soft hypergraphs on human nervous system. Zhan et al. [27] proposed new decision-making method based on bipolar neutrosophic information. Feng et al. [28] proposed MADM application by using new score function for ranking of alternatives with generalized orthopair fuzzy membership grades. Akram [29] initiated the concept of BFS graphs and Akram et al. [30] suggested a hybrid decision-making framework by using aggregation operators under a complex spherical fuzzy prioritization approach. Alghamdi et al. [31] proposed some MCDM methods in a bipolar fuzzy environment. Yang et al. [32] investigated certain aspects of single valued neutrosophic relations. Yue [33] proposed the idea of interval intuitionistic fuzzy set based bilateral matching decision-making for knowledge innovation management considering matching willingness.

Naeeem et al. [34] proposed the idea of Pythagorean m-polar fuzzy sets with new directions and applications. Thao et al. [35] proposed similarity measures based on new entropy by using singlevalued neutrosophic sets with application to select supplier material. Ulucay et al. [36] introduced novel similarity measures of bipolar neutrosophic sets and their application to MCDM. Hashim et al. [37] developed new kinds of similarity measures and their applications for neutrosophic bipolar fuzzy information. They suggested a practical application to hope foundation for planning to build a children hospital under uncertain neutrosophic bipolar fuzzy environment. Garg et al. [38] proposed the idea of hybrid weighted aggregation operators under neutrosophic set information. They suggested an interesting application to MCDM for ranking of feasible objects. Karaaslan et al. [39] developed certain operations on single-valued neutrosophic matrices. They proposed a new method for finding an optimal alternative and ranking of feasible alternatives. Hashmi et al. [40] introduced the idea of m -polar neutrosophic sets and m-polar neutrosophic topology. They proposed novel algorithms for MCDM in clustering analysis and medical diagnosis. Zhang et al. [41] suggested the idea of interval neutrosophic sets and their applications in MCDM. Ye [42] developed a novel MCDM approach for uncertain single-value neutrosophic information and related correlation coefficient. Ye [43] suggested simplified neutrosophic aggregation operators for MCDM and ranking of feasible alternatives in the universe of discourse. Feng et al. [44] proposed certain properties of soft sets, fuzzy sets and rough sets. They suggested soft computing approaches in terms of the possible fusion of fuzzy sets, soft sets, and
rough sets. They proposed some significant results related to the extension of Pawlak approximation space, equivalence relation, and granular computing. Pamucar [45] proposed a robust approach for interval grey normalized weighted geometric Dombi Bonferroni mean aggregation operators by using or uncertain MCDM problems. Chen et al. [46] proposed an efficiency-based interval type-2 fuzzy MCGDM technique for makeshift hospital selection. Riaz et al. [47] proposed an innovative bipolar fuzzy sine trigonometric aggregation operators and SIR method for medical tourism supply chain. Hanif et al. [48] suggested the notion of linear Diophantine fuzzy graphs with new decision-making approach. Saeed et al. [49] developed an application of neutrosophic hypersoft mapping to diagnose hepatitis and propose appropriate treatment. Riaz et al. [50] proposed images and inverse images of mpolar neutrosophic soft mappings and their application to mental disorders and multiple personality disorder. Riaz et al. [51] proposed images and inverse images of bipolar fuzzy soft mappings and their application to bipolar disorders. Latreche et al. [52] introduced the concept of single valued neutrosophic mappings defined by single valued neutrosophic relations with applications.

Neutrosophy is a branch of philosophy that investigates the scope, nature, and origin of neutralities, as well as their interactions with various conceptual spectra. Neutrosophy forms the base of neutrosophic science involving the area such as neutrosophic logic, neutrosophic sets, and neutrosophic probability, etc. These have been successfully applied for modeling uncertainties in decision-making problems. Neutrosophic crisp set (NCS) and neutrosophic crisp soft set (NCSS) are strong modeling tools to address vagueness and uncertainties in decision-making problems.

The first objective of this paper is to introduce the novel concept of neutrosophic crisp soft set (NCSS) including various types of neutrosophic crisp soft sets (NCSSs) and their fundamental operations. The second objective is to define the idea of NCS-mapping to investigate images and inverse images of NCS-classes. The third objective is to develop a robust mathematical modeling with the help of NCS-mapping. The fourth objective is to analyze the emerging trends in social networking systems (SNSs) for our various generations. The fifth objective is to derive valid conclusions regarding the advantages, disadvantages, and natural aspects of SNSs for five generations.

The layout of this paper is designed as follows: In Section 2, we first present the notion of NCSS and then we talk about some of its types. We introduce some operations on NCSSs for the dissimilar types of NCSSs. In Section 3, we establish NCS-mapping and inverse NCS-mapping by merging the ideas of soft mapping and NC mapping, then extend the definition with the help of examples. In Section 4, we discuss and construct a model on emerging trends in social networking system (SNS) via NCS-mapping for different generations. In Section 5, we briefly discuss our concepts about this paper and the construction of the articles with the comparison of neutrosophic, fuzzy and soft notions and their hybrid structures. Lastly, judgments are drawn in Section 6.

Some abbreviations used in this paper are as follows:
IFS Intuitionistic fuzzy set
GIFS Generalized intuitionistic fuzzy set
GNS Generalized neutrosophic set
NC Neutrosophic crisp
NSS Neutrosophic soft set
NCS Neutrosophic crisp soft
NCSS Neutrosophic crisp soft set
SNS Social networking system

SNWs Social networking websites
FNS Fuzzy neutrosophic soft

## 2 Neutrosophic Crisp Soft Sets

In this section, we introduce the notion of neutrosophic crisp soft set (NCSS), different types NCSSs, operations on NCSSs with the help of some illustrations. First, we recall the definition of soft set.

Definition 2.1. [2] Let $X$ be the universal set, $P(X)$ be the power set of $X$, and $R$ be the set of decision variables with $A \subseteq R$, then the pair $(F, A)$ is called a soft set over $X$, where $F$ is a set-valued mapping given by $F: A \rightarrow P(X)$. A soft set can be written as
$(F, A)=\{(\delta, F(\delta)): \delta \in A\}$.
Definition 2.2. [18,19] Let $X$ be a non-empty fixed sample space. A neutrosophic crisp set (NC-set) $\mathfrak{N}$ is written as
$\mathfrak{N}=\left\langle\mathfrak{N}_{1}, \mathfrak{N}_{2}, \mathfrak{N}_{3}\right\rangle$
where $\mathfrak{N}_{1}, \mathfrak{N}_{2}$ and $\mathfrak{N}_{3}$ are subsets of $X$.
There are three types of NC-set defined as:

1. NC-set with Type-1 if $\mathfrak{N}_{1} \cap \mathfrak{N}_{2}=\phi, \mathfrak{N}_{2} \cap \mathfrak{N}_{3}=\phi$ and $\mathfrak{N}_{1} \cap \mathfrak{N}_{3}=\phi$.
2. NC-set with Type-2 if $\mathfrak{N}_{1} \cap \mathfrak{N}_{2}=\phi, \mathfrak{N}_{2} \cap \mathfrak{N}_{3}=\phi, \mathfrak{N}_{1} \cap \mathfrak{N}_{3}=\phi$ and $\mathfrak{N}_{1} \cup \mathfrak{N}_{2} \cup \mathfrak{N}_{3}=X$.
3. NC-set with Type-3 if $\mathfrak{N}_{1} \cap \mathfrak{N}_{2} \cap \mathfrak{N}_{3}=\phi$ and $\mathfrak{N}_{1} \cup \mathfrak{N}_{2} \cup \mathfrak{N}_{3}=X$.

Definition 2.3. Let $X$ be an arbitrary non-empty fixed sample space and let $P$ be the set of decision variables. Then, for the soft set or absolute soft set $X_{P}$ (it may be simply an arbitrary soft set or may be an absolute soft set) the neutrosophic crisp soft set (NCSS) can be written as
$N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$
where $\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right)$ and $\left(N_{3}, G_{3}\right)$ be the soft subsets of $X_{P}$ and $G_{1}, G_{2}, G_{3} \subseteq P$.
There are three types of NCSS defined as follows:

1. NCSS with Type-1, if $\left(N_{1}, G_{1}\right) \widetilde{\cap}\left(N_{2}, G_{2}\right)=\widetilde{\phi},\left(N_{2}, G_{2}\right) \widetilde{\cap}\left(N_{3}, G_{3}\right)=\widetilde{\phi}$ and $\left(N_{1}, G_{1}\right) \widetilde{\cap}\left(N_{3}, G_{3}\right)=\widetilde{\phi}$.
2. NCSS with Type-2, if $\left(N_{1}, G_{1}\right) \widetilde{\cap}\left(N_{2}, G_{2}\right)=\widetilde{\phi},\left(N_{2}, G_{2}\right) \widetilde{\cap}\left(N_{3}, G_{3}\right)=\widetilde{\phi},\left(N_{1}, G_{1}\right) \widetilde{\cap}\left(N_{3}, G_{3}\right)=\widetilde{\phi}$ and $\left(N_{1}, G_{1}\right) \widetilde{\cup}\left(N_{2}, G_{2}\right) \widetilde{\cup}\left(N_{3}, G_{3}\right)=X_{P}$.
3. NCSS with Type-3, if $\left(N_{1}, G_{1}\right) \widetilde{\cap}\left(N_{2}, G_{2}\right) \widetilde{\cap}\left(N_{3}, G_{3}\right)=\widetilde{\phi}$ and $\left(N_{1}, G_{1}\right) \widetilde{\cup}\left(N_{2}, G_{2}\right) \widetilde{\cup}\left(N_{3}, G_{3}\right)=X_{P}$.

Example 2.1. Let $X$ be the set of non-negative integers and $P=\left\{\xi_{1}, \xi_{2}, \xi_{3}, \xi_{4}\right\}$ be the set of decision variables, where
$\xi_{1}=$ Even numbers,
$\xi_{2}=$ Odd numbers,
$\xi_{3}=$ Numbers divisible by 2 ,
$\xi_{4}=$ Negative numbers.

Consider $G_{1}=\left\{\xi_{1}, \xi_{3}\right\}, G_{2}=\left\{\xi_{2}\right\}, G_{3}=\left\{\xi_{4}\right\} \subseteq P$, then the corresponding soft subsets are $\left(N_{1}, G_{1}\right)=\left\{\left(\xi_{1},\{2 w: w \in X\}\right),\left(\xi_{3},\{2 w: w \in X\}\right)\right\}$
$\left(N_{2}, G_{2}\right)=\left\{\left(\xi_{2},\{2 w+1: w \in X\}\right)\right\}$
$\left(N_{3}, G_{3}\right)=\left\{\left(\xi_{4}, \phi\right)\right\}$
where $X=\{0,1,2,3, \ldots\}$ the set of whole numbers. So, the type 1 NCSS can be written as
$N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$.
If $X_{P}=\left\{\left(\xi_{1},\{2 w: w \in W\}\right),\left(\xi_{2},\{2 w+1: w \in X\}\right),\left(\xi_{3},\{2 w: w \in X\}\right),\left(\xi_{4}, \phi\right)\right\}$ then $N$ is also of Type-2 and Type-3. The neutrosophic crisp soft set N can be represented in Table 1 as follows.

Table 1: Neutrosophic crisp soft set N

| N | $\xi_{1}$ | $\xi_{2}$ | $\xi_{3}$ | $\xi_{4}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\left(N_{1}, G_{1}\right)$ | $N_{1}\left(\xi_{1}\right)$ | $\phi$ | $N_{1}\left(\xi_{3}\right)$ | $\phi$ |
| $\left(N_{2}, G_{2}\right)$ | $\phi$ | $N_{2}\left(\xi_{2}\right)$ | $\phi$ | $\phi$ |
| $\left(N_{3}, G_{3}\right)$ | $\phi$ | $\phi$ | $\phi$ | $N_{3}\left(\xi_{4}\right)=\phi$ |

where $N_{1}\left(\xi_{1}\right)=N_{1}\left(\xi_{3}\right)=\{2 w: w \in X\}$ and $N_{2}\left(\xi_{2}\right)=\{2 w+1: w \in X\}$.
Definition 2.4. Let $X_{P}$ and $X_{\phi}$ be the absolute and empty soft sets respectively, on the universe of discourse $X$ and the set of decision variables $P$. Then, empty NCSS is defined in many ways as

Type-1: $\Phi_{N}=\left\langle X_{\phi}, X_{\phi}, X_{P}\right\rangle$, Type-2: $\Phi_{N}=\left\langle X_{\phi}, X_{P}, X_{P}\right\rangle$,
Type-3: $\Phi_{N}=\left\langle X_{\phi}, X_{P}, X_{\phi}\right\rangle$, Type-4: $\Phi_{N}=\left\langle X_{\phi}, X_{\phi}, X_{\phi}\right\rangle$.
Definition 2.5. Let $X_{P}$ and $X_{\phi}$ be the absolute and empty soft sets respectively, on the universe of discourse $X$ and the set of decision variables $P$. Then, absolute NCSS denoted by $X_{N}$ is defined by different ways as follows:

Type-1: $X_{N}=\left\langle X_{P}, X_{\phi}, X_{\phi}\right\rangle$, Type-2: $X_{N}=\left\langle X_{P}, X_{P}, X_{\phi}\right\rangle$,
Type-3: $X_{N}=\left\langle X_{P}, X_{\phi}, X_{P}\right\rangle$, Type-4: $X_{N}=\left\langle X_{P}, X_{P}, X_{P}\right\rangle$.
Definition 2.6. Let $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$ be the NCSS on $X$ and $P$. Then, the complement of $N$ denoted by $N^{c}$ is defined by different ways as follows:

Type-1: $N^{c}=\left\langle\left(N_{1}, G_{1}\right)^{c},\left(N_{2}, G_{2}\right)^{c},\left(N_{3}, G_{3}\right)^{c}\right\rangle$
Type-2: $N^{c}=\left\langle\left(N_{3}, G_{3}\right),\left(N_{2}, G_{2}\right),\left(N_{1}, G_{1}\right)\right\rangle$
Type-3: $N^{c}=\left\langle\left(N_{3}, G_{3}\right),\left(N_{2}, G_{2}\right)^{c},\left(N_{1}, G_{1}\right)\right\rangle$
where $\left(N_{1}, G_{1}\right)^{c},\left(N_{2}, G_{2}\right)^{c}$ and $\left(N_{3}, G_{3}\right)^{c}$ are the complements of the soft subsets of the given NCSS.
Definition 2.7. Let $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$ and $M=\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle$ be two NCSSs on $X$ and P. Then, definition of subsets are given as:

Type-1: $N \subseteq M \Leftrightarrow\left(N_{1}, G_{1}\right) \widetilde{\subseteq}\left(M_{1}, H_{1}\right),\left(N_{2}, G_{2}\right) \widetilde{\subseteq}\left(M_{2}, H_{2}\right)$ and $\left(N_{3}, G_{3}\right) \widetilde{\cong}\left(M_{3}, H_{3}\right)$
Type-2: $N \subseteq M \Leftrightarrow\left(N_{1}, G_{1}\right) \widetilde{\subseteq}\left(M_{1}, H_{1}\right),\left(N_{2}, G_{2}\right) \widetilde{\cong}\left(M_{2}, H_{2}\right)$ and $\left(N_{3}, G_{3}\right) \widetilde{\cong}\left(M_{3}, H_{3}\right)$

Proposition 2.1. If $N$ is any NCSS, then the following statements hold:

1. $\Phi_{N} \subseteq N, \Phi_{N} \subseteq \Phi_{N}$,
2. $N \subseteq X_{N}, X_{N} \subseteq X_{N}$.

Proof. The proof is obvious. Therefore, it is omitted.
Definition 2.8. Let $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$ and $M=\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle$ be two NCSSs over $X$ and $P$. Then, union of these sets can be defined in two ways as

Type-1: $N \cup M=\left\langle\left(N_{1}, G_{1}\right) \widetilde{\cup}\left(M_{1}, H_{1}\right),\left(N_{2}, G_{2}\right) \widetilde{\cup}\left(M_{2}, H_{2}\right),\left(N_{3}, G_{3}\right) \widetilde{\cap}\left(M_{3}, H_{3}\right)\right\rangle$
Type-2: $N \cup M=\left\langle\left(N_{1}, G_{1}\right) \widetilde{\cup}\left(M_{1}, H_{1}\right),\left(N_{2}, G_{2}\right) \widetilde{\cap}\left(M_{2}, H_{2}\right),\left(N_{3}, G_{3}\right) \widetilde{\cap}\left(M_{3}, H_{3}\right)\right\rangle$.
where $\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right),\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right)$ and $\left(M_{3}, H_{3}\right)$ are soft subsets of $X_{P}$.
Definition 2.9. Let $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$ and $M=\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle$ be two NCSSs over $X$ and $P$. Then, intersection of these sets can be defined in two ways as

Type-1: $N \cap M=\left\langle\left(N_{1}, G_{1}\right) \widetilde{\cap}\left(M_{1}, H_{1}\right),\left(N_{2}, G_{2}\right) \widetilde{\cap}\left(M_{2}, H_{2}\right),\left(N_{3}, G_{3}\right) \widetilde{\cup}\left(M_{3}, H_{3}\right)\right\rangle$
Type-2: $N \cap M=\left\langle\left(N_{1}, G_{1}\right) \widetilde{\cap}\left(M_{1}, H_{1}\right),\left(N_{2}, G_{2}\right) \widetilde{\cup}\left(M_{2}, H_{2}\right),\left(N_{3}, G_{3}\right) \widetilde{\cup}\left(M_{3}, H_{3}\right)\right\rangle$.
where $\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right),\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right)$ and $\left(M_{3}, H_{3}\right)$ are soft subsets of $X_{P}$.
Proposition 2.2. If $M$ and $N$ are two NCSSs over $X$ and $P$, then the following statements hold:

1. $(N \cap M)^{c}=N^{c} \cup M^{c}$
2. $(N \cup M)^{c}=N^{c} \cap M^{c}$

Proof. The proof is obvious. Therefore, it is omitted.
Remark 2.1. We can prove different basic laws for union and intersection of NCSSs and can generalize them for arbitrary family of NCSS.

Example 2.2. Let $X=\mathbb{Z}$ and $P=\left\{\xi_{1}, \xi_{2}, \xi_{3}, \xi_{4}, \xi_{5}, \xi_{6}, \xi_{7}\right\}$ be the set of decision variables where,
$\xi_{1}=$ Even numbers,
$\xi_{2}=$ Odd numbers,
$\xi_{3}=$ Numbers divisible by 2 ,
$\xi_{4}=$ non terminating decimals,
$\xi_{5}=$ non negative integers,
$\xi_{6}=$ non positive integers,
$\xi_{7}=$ additive identity.
Let $(X, P)$ be the soft set over $X$ and $P$ given as:
$(X, P)=\left\{\left(\xi_{1},\{ \pm 2 w: w \in Z\}\right),\left(\xi_{2},\{ \pm(2 w+1): w \in Z\}\right),\left(\xi_{3},\{ \pm 2 w: w \in Z\}\right),\left(\xi_{4}, \phi\right),\left(\xi_{5},\{w: w \in\right.\right.$ $\left.\left.\left.\mathbb{Z}^{+} /\{0\}\right\}\right),\left(\xi_{6},\left\{w: w \in \mathbb{Z}^{-} /\{0\}\right\}\right),\left(\xi_{7},\{0\}\right)\right\}$. Now consider two NCSSs of Type-1 $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$ and $M=\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle$ where,
$\left(N_{1}, G_{1}\right)=\left\{\left(\xi_{1},\{ \pm 2 w: w \in Z\}\right),\left(\xi_{3},\{ \pm 2 w: w \in Z\}\right)\right\}$,
$\left(N_{2}, G_{2}\right)=\left\{\left(\xi_{2},\{ \pm(2 w+1): w \in Z\}\right)\right\}$,
$\left(N_{3}, G_{3}\right)=\left\{\left(\xi_{4}, \phi\right)\right\}$, where $G_{1}=\left\{\xi_{1}, \xi_{3}\right\}, G_{2}=\left\{\xi_{2}\right\}, G_{3}=\left\{\xi_{4}\right\}$,
$\left(M_{1}, H_{1}\right)=\left\{\left(\xi_{5},\left\{w: w \in \mathbb{Z}^{+} /\{0\}\right\}\right)\right\}$,
$\left(M_{2}, H_{2}\right)=\left\{\left(\xi_{6},\left\{w: w \in \mathbb{Z}^{-} /\{0\}\right\}\right)\right\}$,
$\left(M_{3}, H_{3}\right)=\left\{\left(\xi_{7},\{0\}\right)\right\}$, where $H_{1}=\left\{\xi_{5}\right\}, H_{2}=\left\{\xi_{6}\right\}, H_{3}=\left\{\xi_{7}\right\}$. The neutrosophic crisp soft set $N$ and $M$ are expressed in Tables 2 and 3, respectively.

Table 2: Neutrosophic crisp soft set $N$

| N | $\xi_{1}$ | $\xi_{2}$ | $\xi_{3}$ | $\xi_{4}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\left(N_{1}, G_{1}\right)$ | $N_{1}\left(\xi_{1}\right)$ | $\phi$ | $N_{1}\left(\xi_{3}\right)$ | $\phi$ |
| $\left(N_{2}, G_{2}\right)$ | $\phi$ | $N_{2}\left(\xi_{2}\right)$ | $\phi$ | $\phi$ |
| $\left(N_{3}, G_{3}\right)$ | $\phi$ | $\phi$ | $\phi$ | $N_{3}\left(\xi_{4}\right)=\phi$ |

Table 3: Neutrosophic crisp soft set $M$

| M | $\xi_{5}$ | $\xi_{6}$ | $\xi_{7}$ |
| :--- | :--- | :--- | :--- |
| $\left(M_{1}, H_{1}\right)$ | $M_{1}\left(\xi_{5}\right)$ | $\phi$ | $\phi$ |
| $\left(M_{2}, H_{2}\right)$ | $\phi$ | $M_{2}\left(\xi_{6}\right)$ | $\phi$ |
| $\left(M_{3}, H_{3}\right)$ | $\phi$ | $\phi$ | $M_{3}\left(\xi_{7}\right)$ |

## Union of NCSSs:

Type-1: $L=N \cup M=\left\langle\left(N_{1}, G_{1}\right) \widetilde{\cup}\left(M_{1}, H_{1}\right),\left(N_{2}, G_{2}\right) \widetilde{\cup}\left(M_{2}, H_{2}\right),\left(N_{3}, G_{3}\right) \widetilde{\cap}\left(M_{3}, H_{3}\right)\right\rangle$
$L=\left\langle\left(L_{1}, J_{1}\right),\left(L_{2}, J_{2}\right),\left(L_{3}, J_{3}\right)\right\rangle$ where,
$\left(L_{1}, J_{1}\right)=\left\{\left(\xi_{1}, N_{1}\left(\xi_{1}\right)\right),\left(\xi_{3}, N_{1}\left(\xi_{3}\right)\right),\left(\xi_{5}, M_{1}\left(\xi_{5}\right)\right)\right\}$,
$\left(L_{2}, J_{2}\right)=\left\{\left(\xi_{2}, N_{2}\left(\xi_{2}\right)\right),\left(\xi_{6}, M_{2}\left(\xi_{6}\right)\right)\right\}$,
$\left(L_{3}, J_{3}\right)=\widetilde{\phi}$
Type-2: $N \cup M=\left\langle\left(N_{1}, G_{1}\right) \widetilde{\cup}\left(M_{1}, H_{1}\right),\left(N_{2}, G_{2}\right) \widetilde{\cap}\left(M_{2}, H_{2}\right),\left(N_{3}, G_{3}\right) \widetilde{\cap}\left(M_{3}, H_{3}\right)\right\rangle$.
$O=\left\langle\left(O_{1}, K_{1}\right),\left(O_{2}, K_{2}\right),\left(O_{3}, K_{3}\right)\right\rangle$ where,
$\left(O_{1}, K_{1}\right)=\left\{\left(\xi_{1}, N_{1}\left(\xi_{1}\right)\right),\left(\xi_{3}, N_{1}\left(\xi_{3}\right)\right),\left(\xi_{5}, M_{1}\left(\xi_{5}\right)\right)\right\}$,
$\left(O_{2}, K_{2}\right)=\widetilde{\phi}$,
$\left(O_{3}, K_{3}\right)=\widetilde{\phi}$. This shows that $L$ and $O$ are NCSSs of Type-1.
By using Definition 2.8, the neutrosophic crisp soft set $L$ and $O$ can be demonstrated in Tables 4 and 5, respectively.

Table 4: Neutrosophic crisp soft set $L=N \cup M$ (Type-1)

| L | $\xi_{1}$ | $\xi_{2}$ | $\xi_{3}$ | $\xi_{4}$ | $\xi_{5}$ | $\xi_{6}$ | $\xi_{7}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\left(L_{1}, J_{1}\right)$ | $L_{1}\left(\xi_{1}\right)$ | $\phi$ | $L_{1}\left(\xi_{3}\right)$ | $\phi$ | $L_{1}\left(\xi_{5}\right)$ | $\phi$ | $\phi$ |
| $\left(L_{2}, J_{2}\right)$ | $\phi$ | $L_{2}\left(\xi_{2}\right)$ | $\phi$ | $\phi$ | $\phi$ | $L_{2}\left(\xi_{6}\right)$ | $\phi$ |
| $\left(L_{3}, J_{3}\right)$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ |

Table 5: Neutrosophic crisp soft set $O=N \cup M$ (Type-2)

| O | $\xi_{1}$ | $\xi_{2}$ | $\xi_{3}$ | $\xi_{4}$ | $\xi_{5}$ | $\xi_{6}$ | $\xi_{7}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\left(O_{1}, K_{1}\right)$ | $O_{1}\left(\xi_{1}\right)$ | $\phi$ | $O_{1}\left(\xi_{3}\right)$ | $\phi$ | $O_{1}\left(\xi_{5}\right)$ | $\phi$ | $\phi$ |
| $\left(O_{2}, K_{2}\right)$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ |
| $\left(O_{3}, K_{3}\right)$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ |

## Intersection of NCSSs:

By using Definition 2.9, the intersection of $N$ and $M$ can be expressed in Tables 6 and 7, respectively.

Table 6: Neutrosophic crisp soft set $U=N \cap M$ (Type-1)

| L | $\xi_{1}$ | $\xi_{2}$ | $\xi_{3}$ | $\xi_{4}$ | $\xi_{5}$ | $\xi_{6}$ | $\xi_{7}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\left(L_{1}, J_{1}\right)$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ |
| $\left(L_{2}, J_{2}\right)$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ |
| $\left(L_{3}, J_{3}\right)$ | $\phi$ | $\phi$ | $\phi$ | $L_{3}\left(\xi_{4}\right)=\phi$ | $\phi$ | $\phi$ | $L_{3}\left(\xi_{7}\right)$ |

Table 7: Neutrosophic crisp soft set $V=N \cap M$ (Type-2)

| O | $\xi_{1}$ | $\xi_{2}$ | $\xi_{3}$ | $\xi_{4}$ | $\xi_{5}$ | $\xi_{6}$ | $\xi_{7}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\left(O_{1}, K_{1}\right)$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ | $\phi$ |
| $\left(O_{2}, K_{2}\right)$ | $\phi$ | $L_{2}\left(\xi_{2}\right)$ | $\phi$ | $\phi$ | $\phi$ | $L_{2}\left(\xi_{6}\right)$ | $\phi$ |
| $\left(O_{3}, K_{3}\right)$ | $\phi$ | $\phi$ | $\phi$ | $L_{3}\left(\xi_{4}\right)=\phi$ | $\phi$ | $\phi$ | $L_{3}\left(\xi_{7}\right)$ |

## Example 2.3.

Let $X=\{a, b, c, d, e, f\}$ be the fixed sample space and $P=\left\{\xi_{1}, \xi_{2}, \xi_{3}\right\}$ be the collection of decision variables. Then, soft set $X_{P}$ and be written as
$X_{P}=\left\{\left(\xi_{1}, X\right),\left(\xi_{2}, X\right),\left(\xi_{3}, X\right)\right\}$.
Now, we construct NCSSs of different types and see their complement according to Definition 2.6. (I) Let $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$ be an NCSS of Type-1 where,
(I) $\left(N_{1}, G_{1}\right)=\left\{\left(\xi_{1},\{a\}\right)\right\}$,
$\left(N_{2}, G_{2}\right)=\left\{\left(\xi_{2},\{c\}\right)\right\}$ and
$\left(N_{3}, G_{3}\right)=\left\{\left(\xi_{3},\{e\}\right)\right\}$. This is neither an NCSS of Type-2 nor Type-3.
(II) Let $M=\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle$ be an NCSS of Type-1, Type-2 and Type-3 where, $\left(M_{1}, H_{1}\right)=\left\{\left(\xi_{1}, X\right)\right\}$, $\left(M_{2}, H_{2}\right)=\left\{\left(\xi_{2}, X\right)\right\}$, and $\left(M_{3}, H_{3}\right)=\left\{\left(\xi_{3}, X\right)\right\}$.
(III) Let $O=\left\langle\left(O_{1}, K_{1}\right),\left(O_{2}, K_{2}\right),\left(O_{3}, K_{3}\right)\right\rangle$ be an NCSS of Type-3 where,
$\left(O_{1}, K_{1}\right)=\left\{\left(\xi_{1}, X\right),\left(\xi_{2},\{c\}\right)\right\}$,
$\left(O_{2}, K_{2}\right)=\left\{\left(\xi_{1},\{a, b\}\right),\left(\xi_{2}, X\right),\left(\xi_{3},\{f\}\right)\right\}$ and
$\left(O_{3}, K_{3}\right)=\left\{\left(\xi_{2},\{d\}\right),\left(\xi_{3}, X\right)\right\}$. This is not a NCSS of Type-1, 2.
(IV) Let $L=\left\langle\left(L_{1}, J_{1}\right),\left(L_{2}, J_{2}\right),\left(L_{3}, J_{3}\right)\right\rangle$ be a NCSS where,
$\left(L_{1}, J_{1}\right)=\left\{\left(\xi_{1},\{a, b\}\right),\left(\xi_{2},\{c\}\right),\left(\xi_{3},\{f\}\right)\right\}$,
$\left(L_{2}, J_{2}\right)=\left\{\left(\xi_{1},\{a\}\right),\left(\xi_{2},\{c, d\}\right),\left(\xi_{3},\{f\}\right)\right\}$ and
$\left(L_{3}, J_{3}\right)=\left\{\left(\xi_{1},\{a\}\right),\left(\xi_{2},\{c\}\right),\left(\xi_{3},\{f\}\right)\right\}$. This is neither an NCSS of Type-1 nor Type-2.

## Complement:

(I) The complement of $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$ with different types follow as

Type-1: $N^{c}=\left\langle\left(N_{1}, G_{1}\right)^{c},\left(N_{2}, G_{2}\right)^{c},\left(N_{3}, G_{3}\right)^{c}\right\rangle$ where
$\left(N_{1}, G_{1}\right)^{c}=\left\{\left(\xi_{1},\{b, c, d, e, f\}\right)\right\},\left(N_{2}, G_{2}\right)^{c}=\left\{\left(\xi_{2},\{a, b, d, e, f\}\right)\right\}$ and $\left(N_{3}, G_{3}\right)^{c}=\left\{\left(\xi_{3},\{a, b, c, d, f\}\right)\right\}$
is an NCSS but not of Type-1, Type-2, and Type-3.
Type-2: $N^{c}=\left\langle\left(N_{3}, G_{3}\right),\left(N_{2}, G_{2}\right),\left(N_{1}, G_{1}\right)\right\rangle$ is an Type-1 NCSS but not of Type-2 and Type-3.
Type-3: $N^{c}=\left\langle\left(N_{3}, G_{3}\right),\left(N_{2}, G_{2}\right)^{c},\left(N_{1}, G_{1}\right)\right\rangle$ is an Type-1 NCSS but not of Type-2 and Type-3.
(II) The complement of $M=\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle$ with different types is follow as:

Type-1: $M^{c}=\left\langle\left(M_{1}, H_{1}\right)^{c},\left(M_{2}, H_{2}\right)^{c},\left(M_{3}, H_{3}\right)^{c}\right\rangle$ where
$\left(M_{1}, H_{1}\right)^{c}=\left\{\xi_{1}, \phi\right\},\left(M_{2}, H_{2}\right)^{c}=\left\{\xi_{2}, \phi\right\}$ and $\left(M_{3}, H_{3}\right)^{c}=\left\{\xi_{3}, \phi\right\}$. This is NCSS Type-4 empty set.
Type-2: $M^{c}=\left\langle\left(M_{3}, H_{3}\right),\left(M_{2}, H_{2}\right),\left(M_{1}, H_{1}\right)\right\rangle$ is NCSS of Type-1, Type-2, Type-3.
Type-3: $M^{c}=\left\langle\left(M_{3}, H_{3}\right),\left(M_{2}, H_{2}\right)^{c},\left(M_{1}, H_{1}\right)\right\rangle$ is NCSS of Type-1 but not of Type-2 and Type-3.
(III) The complement of $O=\left\langle\left(O_{1}, K_{1}\right),\left(O_{2}, K_{2}\right),\left(O_{3}, K_{3}\right)\right\rangle$ with different types is as

Type-1: $O^{c}=\left\langle\left(O_{1}, K_{1}\right)^{c},\left(O_{2}, K_{2}\right)^{c},\left(O_{3}, K_{3}\right)^{c}\right\rangle$ where
$\left(O_{1}, K_{1}\right)^{c}=\left\{\left(\xi_{1}, \phi\right),\left(\xi_{2},\{a, b, d, e, f\}\right)\right\},\left(O_{2}, K_{2}\right)^{c}=\left\{\left(\xi_{1},\{c, d, e, f\}\right),\left(\xi_{2}, \phi\right),\left(\xi_{3},\{a, b, c, d, e\}\right)\right\}$ and $\left(O_{3}, K_{3}\right)^{c}=\left\{\left(\xi_{2},\{a, b, c, e, f\}\right),\left(\xi_{3}, \phi\right)\right\}$ is NCSS but not of Type-1, Type-2, Type-3.
Type-2: $O^{c}=\left\langle\left(O_{3}, K_{3}\right),\left(O_{2}, K_{2}\right),\left(O_{1}, K_{1}\right)\right\rangle$ is NCSS of Type-3 but not of Type-1 and Type-2.
Type-3: $O^{c}=\left\langle\left(O_{3}, K_{3}\right),\left(O_{2}, K_{2}\right)^{c},\left(O_{1}, K_{1}\right)\right\rangle$ is NCSS but not of Type-1, Type-2, Type-3.
(IV) The complement of $L=\left\langle\left(L_{1}, J_{1}\right),\left(L_{2}, J_{2}\right),\left(L_{3}, J_{3}\right)\right\rangle$ with different types is calculated as

Type-1: $L^{c}=\left\langle\left(L_{1}, J_{1}\right)^{c},\left(L_{2}, J_{2}\right)^{c},\left(L_{3}, J_{3}\right)^{c}\right\rangle$ where
$\left(L_{1}, J_{1}\right)^{c}=\left\{\left(\xi_{1},\{c, d, e, f\}\right),\left(\xi_{2},\{a, b, d, e, f\}\right),\left(\xi_{3},\{a, b, c, d, e\}\right)\right\}$,
$\left(L_{2}, J_{2}\right)^{c}=\left\{\left(\xi_{1},\{b, c, d, e, f\}\right),\left(\xi_{2},\{a, b, e, f\}\right),\left(\xi_{3},\{a, b, c, d, e\}\right)\right\}$ and
$\left(L_{3}, J_{3}\right)^{c}=\left\{\left(\xi_{1},\{b, c, d, e, f\}\right),\left(\xi_{2},\{a, b, d, e, f\}\right),\left(\xi_{3},\{a, b, c, d, e\}\right)\right\}$ is NCSS but not of Type-1,
Type-2, Type-3.
Type-2: $L^{c}=\left\langle\left(L_{3}, J_{3}\right),\left(L_{2}, J_{2}\right),\left(L_{1}, J_{1}\right)\right\rangle$ is NCSS but not of Type-1, Type-2, Type-3.
Type-3: $L^{c}=\left\langle\left(L_{3}, J_{3}\right),\left(L_{2}, J_{2}\right)^{c},\left(L_{1}, J_{1}\right)\right\rangle$ is NCSS but not of Type-1, Type-2, Type-3.
(1) Every NCSS of Type-1, Type-2 and Type-3 are NCSS.
(2) Every NCSS of Type-1 may not be NCSS Type-2 and Type-3.
(3) Every NCSS of Type-2 may not be NCSS Type-1 and Type-3.
(4) Every NCSS of Type-3 may not be NCSS Type-1 and Type-2.
(5) Every soft set is NCSS.

The relationship between different types of NCSSs and soft set are shown in Fig. 1.
Definition 2.10. Let $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$ be an NCSS on $X$ and $P$. Then, $N_{P}=$ $\left\langle N_{\xi_{1}}, N_{\xi_{2}}, N_{\xi_{3}}\right\rangle, \xi_{1} \neq \xi_{2} \neq \xi_{3}$ is called NCSS point, where $N_{\xi_{1}}, N_{\xi_{2}}$ and $N_{\xi_{3}}$ are soft points or singleton soft sets on sample space $X$ and on the set of decision variables $P$. An NCS point $N_{P}$ belongs to NCSS $N$ if


Figure 1: Relationship between different types of NCSSs and soft set
Type-1: $N_{\xi_{1}} \widetilde{\subseteq}\left(N_{1}, G_{1}\right), N_{\xi_{2}} \widetilde{\subseteq}\left(N_{2}, G_{2}\right), N_{\xi_{3}} \check{\cong}\left(N_{3}, G_{3}\right)$
Type-2: $N_{\xi_{1}} \widetilde{\simeq}\left(N_{1}, G_{1}\right), N_{\xi_{2}} \widetilde{\cong}\left(N_{2}, G_{2}\right), N_{\xi_{3}} \widetilde{\cong}\left(N_{3}, G_{3}\right)$
Theorem 2.1. Let $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$ and $M=\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle$ be two NCSSs on X and P and let $N$ subset $M$. Then, if $N_{P} \in N, N_{P} \in M$ for any NCSS point $N_{P}$ in $X_{P}$.

Proof. Let $N \subseteq M$ and $N_{P} \in N$. Then, by Definition 2.8 we have
Type-1: $N_{\xi_{1}} \widetilde{\subseteq}\left(N_{1}, G_{1}\right) \widetilde{\subseteq}\left(M_{1}, H_{1}\right), N_{\xi_{2}} \widetilde{\subseteq}\left(N_{2}, G_{2}\right) \widetilde{\subseteq}\left(M_{2}, H_{2}\right), N_{\xi_{3}} \widetilde{\cong}\left(N_{3}, G_{3}\right) \widetilde{\cong}\left(M_{3}, H_{3}\right)$.
Type-2: $N_{\xi_{1}} \widetilde{\subseteq}\left(N_{1}, G_{1}\right) \widetilde{\subseteq}\left(M_{1}, H_{1}\right), N_{\xi_{2}} \widetilde{\cong}\left(N_{2}, G_{2}\right) \cong\left(M_{2}, H_{2}\right), N_{\xi_{3}} \cong\left(N_{3}, G_{3}\right) \cong\left(M_{3}, H_{3}\right)$.
This shows that $N_{P} \in M$.
Proposition 2.3.
Let $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$ be an NCSS on $X$ and $P$. Then, $N=\bigcup\left\{N_{P}: N_{P} \in N\right\}$.
Proof. Proof is obvious.

## 3 NCS-Mapping

In this section, we establish NCS-mapping and inverse NCS-mapping by merging the ideas of soft mapping and NC mapping, then explain the definition with the help of examples.

Definition 3.1. Suppose that $N_{C}$ is the collection of all NCSSs on $X$ and $P$ and $M_{C}$ be the class of all NCSSs on $Y$ and $R$. If $\varnothing: X \rightarrow Y$ and $\eta: P \rightarrow R$ are the mappings on the fixed sample spaces $X$ and $Y$ and on the set of decision variables $P$ and $R$, then for the two NCSSs $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle \in N_{C}$ and $M=\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle \in M_{C}$, we can define a mapping $\partial_{\eta}: N_{C} \rightarrow M_{C}$ such that image of
$\partial_{\eta}(N)=M$ and can be as
$\partial_{\eta}\left(N_{1}, G_{1}\right)=\left(M_{1}, H_{1}\right), \partial_{\eta}\left(N_{2}, G_{2}\right)=\left(M_{2}, H_{2}\right)$ and $\partial_{\eta}\left(N_{3}, G_{3}\right)=\left(M_{3}, H_{3}\right)$ and these images can be found by using the mapping of soft sets defined as
$\partial_{\eta}\left(N_{1}, G_{1}\right)(r)=\left\{\begin{array}{ll}\partial\left(\bigcup_{\alpha \in \eta^{-1}(r) \cap G_{1}}\right. \\ \phi & \left.N_{1}(\alpha)\right) ; \\ \text { otherwise }\end{array} \quad\right.$ if, $\eta^{-1}(r) \cap G_{1} \neq \phi$
for $r \in H_{1} \subseteq R$.
$\partial_{\eta}\left(N_{2}, G_{2}\right)(r)=\left\{\begin{array}{l}\partial\left(\bigcup_{\alpha \in \eta^{-1}(r) \cap G_{2}}^{\bigcup} N_{2}(\alpha)\right) ; \quad \text { if, } \eta^{-1}(r) \cap G_{2} \neq \phi \\ \phi \quad \text { otherwise }\end{array}\right.$
for $r \in H_{2} \subseteq R$, and
$\partial_{\eta}\left(N_{3}, G_{3}\right)(r)= \begin{cases}\partial\left(\underset{\alpha \in \eta^{-1}(r) \cap G_{3}}{\bigcup} N_{3}(\alpha)\right) ; & \text { if, } \eta^{-1}(r) \cap G_{3} \neq \phi \\ \phi \quad \text { otherwise }\end{cases}$
for $r \in H_{3} \subseteq R$.
Definition 3.2. Let $\partial_{\eta}: N_{C} \rightarrow M_{C}$ be a mapping as given in above definition and $M=$ $\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle$ be an NCSS in $M_{C}$, where $H_{1}, H_{2}, H_{3} \in R$. Then, inverse image $\partial_{\eta}^{-1}(M)=N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$ is an NCSS in $N_{C}$, calculated as
$\partial_{\eta}^{-1}\left(M_{1}, H_{1}\right)(\wp)= \begin{cases}\partial^{-1}\left(M_{1}(\eta(\wp))\right) ; & \text { if, } \eta(\wp) \in H_{1} \\ \phi \quad \text { otherwise }\end{cases}$
for $\wp \in G_{1} \subseteq P$,
$\partial_{\eta}^{-1}\left(M_{2}, H_{2}\right)(\wp)= \begin{cases}\partial^{-1}\left(M_{2}(\eta(\wp))\right) ; & \text { if, } \eta(\wp) \in H_{2} \\ \phi & \text { otherwise }\end{cases}$
for $\wp \in G_{2} \subseteq P$, and
$\partial_{\eta}^{-1}\left(M_{3}, H_{3}\right)(\wp)= \begin{cases}\partial^{-1}\left(M_{3}(\eta(\wp))\right) ; & \text { if, } \eta(\wp) \in H_{3} \\ \phi & \text { otherwise }\end{cases}$
for $\wp \in G_{3} \subseteq P$.
Example 3.1.
Let $X=\left\{\psi_{1}, \psi_{2}, \psi_{3}\right\}$ and $Y=\left\{\pi_{1}, \pi_{2}, \pi_{3}\right\}$ be the sets of fixed sample space, $P=\left\{\wp_{1}, \wp_{2}, \wp_{3}\right\}$, $R=\left\{r_{1}, r_{2}, r_{3}\right\}$ be the sets of decision variables. Then, we define a mapping $\partial_{\eta}: N_{C} \rightarrow M_{C}$, where $N_{C}$ is the collection of all NCSSs on $X$ and $P$ and $M_{C}$ is the class of all NCSSs on $Y$ and $R$ and $\varnothing: X \rightarrow Y$ and $\eta: P \rightarrow R$ are defined as

$$
\eta\left(\wp_{1}\right)=r_{1}, \eta\left(\wp_{2}\right)=r_{1}, \eta\left(\wp_{3}\right)=r_{3}, \partial\left(\psi_{1}\right)=\pi_{1}, \partial\left(\psi_{2}\right)=\pi_{2}, \partial\left(\psi_{3}\right)=\pi_{2} .
$$

Let $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle \in N_{C}$, where
$\left(N_{1}, G_{1}\right)=\left\{\left(\wp_{1},\left\{\psi_{1}\right\}\right)\right\}$,
$\left(N_{2}, G_{2}\right)=\left\{\left(\wp_{2},\left\{\psi_{2}\right\}\right)\right\}$,
$\left(N_{3}, G_{3}\right)=\left\{\left(\wp_{3},\left\{\psi_{3}\right\}\right)\right\}$ is an NCSS of Type-1. Then, we can find the image of $N$ by using Definition 3.1 as

$$
\begin{aligned}
& \partial_{\eta}\left(N_{1}, G_{1}\right)\left(r_{1}\right)=\partial\left(\bigcup_{\alpha \in \eta^{-1}\left(r_{1}\right) \cap G_{1}} N_{1}(\alpha)\right) \\
& =\varnothing\left(N_{1}\left(\wp_{1}\right)\right) \\
& =\partial\left(\left\{\psi_{1}\right\}\right) \\
& =\left\{\pi_{1}\right\} \\
& \partial_{\eta}\left(N_{1}, G_{1}\right)\left(r_{2}\right)=\phi, \partial_{\eta}\left(N_{1}, G_{1}\right)\left(r_{3}\right)=\phi . \\
& \partial_{\eta}\left(N_{2}, G_{2}\right)\left(r_{1}\right)=\partial\left(\bigcup_{\alpha \in \eta^{-1}\left(r_{1}\right) \cap G_{2}} N_{2}(\alpha)\right) \\
& =\partial\left(N_{2}\left(\wp_{2}\right)\right) \\
& =\partial\left(\left\{\psi_{2}\right\}\right) \\
& =\left\{\pi_{2}\right\} \\
& \partial_{\eta}\left(N_{2}, G_{2}\right)\left(r_{2}\right)=\phi, \partial_{\eta}\left(N_{2}, G_{2}\right)\left(r_{3}\right)=\phi . \partial_{\eta}\left(N_{3}, G_{3}\right)\left(r_{1}\right)=\phi, \partial_{\eta}\left(N_{3}, G_{3}\right)\left(r_{2}\right)=\phi . \\
& \partial_{\eta}\left(N_{3}, G_{3}\right)\left(r_{3}\right)=\varnothing\left(\bigcup_{\alpha \in \eta^{-1}\left(r_{3}\right) \cap G_{3}} N_{3}(\alpha)\right) \\
& =\partial\left(N_{3}\left(\wp_{3}\right)\right) \\
& =\partial\left(\left\{\psi_{3}\right\}\right) \\
& =\left\{\pi_{2}\right\}
\end{aligned}
$$

Then, $\partial_{\eta}(N)=M$ is an NCSS and can be written as
$\partial_{\eta}(N)=M=\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle$
where $\left(M_{1}, H_{1}\right)=\left\{\left(r_{1},\left\{\pi_{1}\right\}\right)\right\},\left(M_{2}, H_{2}\right)=\left\{\left(r_{1},\left\{\pi_{2}\right\}\right)\right\}$ and $\left(M_{3}, H_{3}\right)=\left\{\left(r_{3},\left\{\pi_{2}\right\}\right)\right\}$ are soft sets on $Y$ and $R$.

## Inverse Mapping:

By using Definition 3.2, we find the inverse of above NCSS $M$ as
$\partial_{\eta}^{-1}\left(M_{1}, H_{1}\right)\left(\wp_{1}\right)=\partial^{-1}\left(M_{1}\left(\eta\left(\wp_{1}\right)\right)\right)$

$$
\begin{aligned}
& =\partial^{-1}\left(M_{1}\left(r_{1}\right)\right) \\
& =\partial^{-1}\left(\left\{\pi_{1}\right\}\right) \\
& =\left\{\psi_{1}\right\}
\end{aligned}
$$

$$
\begin{aligned}
\partial_{\eta}^{-1}\left(M_{1}, H_{1}\right)\left(\wp_{2}\right) & =ð^{-1}\left(M_{1}\left(\eta\left(\wp_{2}\right)\right)\right) \\
& =ð^{-1}\left(M_{1}\left(r_{1}\right)\right) \\
& =ð^{-1}\left(\left\{\pi_{1}\right\}\right) \\
& =\left\{\psi_{1}\right\}
\end{aligned}
$$

$$
\begin{aligned}
\partial_{\eta}^{-1}\left(M_{2}, H_{2}\right)\left(\wp_{1}\right) & =ð^{-1}\left(M_{2}\left(\eta\left(\wp_{1}\right)\right)\right) \\
& =ð^{-1}\left(M_{2}\left(r_{1}\right)\right) \\
& =ð^{-1}\left(\left\{\pi_{2}\right\}\right) \\
& =\left\{\psi_{2}, \psi_{3}\right\}
\end{aligned}
$$

$$
\begin{aligned}
\partial_{\eta}^{-1}\left(M_{2}, H_{2}\right)\left(\wp_{2}\right) & =\partial^{-1}\left(M_{2}\left(\eta\left(\wp_{2}\right)\right)\right) \\
& =ð^{-1}\left(M_{2}\left(r_{1}\right)\right) \\
& =ð^{-1}\left(\left\{\pi_{2}\right\}\right) \\
& =\left\{\psi_{2}, \psi_{3}\right\}
\end{aligned}
$$

$\partial_{\eta}^{-1}\left(M_{3}, H_{3}\right)\left(\wp_{3}\right)=\partial^{-1}\left(M_{3}\left(\eta\left(\wp_{3}\right)\right)\right)$

$$
=\partial^{-1}\left(M_{3}\left(r_{3}\right)\right)
$$

$$
=\partial^{-1}\left(\left\{\pi_{2}\right\}\right)
$$

$$
=\left\{\psi_{2}, \psi_{3}\right\}
$$

and $\check{\partial}_{\eta}^{-1}\left(M_{1}, H_{1}\right)\left(\wp_{2}\right)=\partial_{\eta}^{-1}\left(M_{1}, H_{1}\right)\left(\wp_{3}\right)=\partial_{\eta}^{-1}\left(M_{2}, H_{2}\right)\left(\wp_{3}\right)=\partial_{\eta}^{-1}\left(M_{3}, H_{3}\right)\left(\wp_{1}\right)=\partial_{\eta}^{-1}\left(M_{3}, H_{3}\right)$ $\left(\wp_{2}\right)=\phi$.

This implies that $\delta_{\eta}^{-1}(M)=L=\left\langle\left(L_{1}, J_{1}\right),\left(L_{2}, J_{2}\right),\left(L_{3}, J_{3}\right)\right\rangle$ is NCSS of Type-1, where
$\left(L_{1}, J_{1}\right)=\left\{\left(\wp_{1},\left\{\psi_{1}\right\}\right),\left(\wp_{2},\left\{\psi_{1}\right\}\right)\right\}$,
$\left(L_{2}, J_{2}\right)=\left\{\left(\wp_{1},\left\{\psi_{2}, \psi_{3}\right\}\right),\left(\wp_{2},\left\{\psi_{2}, \psi_{3}\right\}\right)\right\}$,
$\left(L_{3}, J_{3}\right)=\left\{\left(\wp_{3},\left\{\psi_{2}, \psi_{3}\right\}\right)\right\}$.
Remark 3.1. From the above illustration, it is clear that $\partial_{\eta}^{-1}(M)=\partial_{\eta}^{-1}\left(\partial_{\eta}(N)\right) \neq N$. This is because the given mappings $\partial: X \rightarrow Y$ and $\eta: P \rightarrow R$ are not bijective, so we get different result again. If we consider both of these mappings bijective, then $\partial_{\eta}^{-1}(M)=\partial_{\eta}^{-1}\left(\partial_{\eta}(N)\right)=N$.

Theorem 3.1. Let $\partial_{\eta}: N_{C} \rightarrow M_{C}$ be a mappings between two NCS-classes, where $ð: X \rightarrow Y$ and $\eta: P \rightarrow R$ are well-defined mappings. Then, for NCSSs $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$ and $M=\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle$ in the NCSS class $N_{C}$, we have

1. $\partial_{\eta}\left(\Phi_{N}\right)=\Phi_{N}$,
2. $\partial_{\eta}(N \cup M) \subseteq \partial_{\eta}(N) \cup \partial_{\eta}(M)$, (holds for union of Type-1 and Type-2). In general, $\partial_{\eta}\left(\cup_{\alpha}\left(N_{\alpha}\right)\right) \subseteq$ $\cup_{\alpha} \partial_{\eta}\left(N_{\alpha}\right)$,
3. $ð_{\eta}(N \cap M) \supseteq ð_{\eta}(N) \cap \partial_{\eta}(M)$, (holds for intersection of Type-1 and Type-2). In general, $\partial_{\eta}\left(\cap_{\alpha}\right.$ $\left.\left(N_{\alpha}\right)\right) \supseteq \cap_{\alpha} \partial_{\eta}\left(N_{\alpha}\right)$.
4. If $N \subseteq M$ then $\partial_{\eta}(N) \subseteq \partial_{\eta}(M)$.

## Proof.

1. Proof is obvious.
2. We have to show that $\partial_{\eta}(N \cup M) \subseteq \partial_{\eta}(N) \cup \partial_{\eta}(M)$. Since $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle, M=$ $\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle \in N_{C}$, where $\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right),\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right)$ and $\left(M_{3}, H_{3}\right)$ are soft sets on $X$ and $P$, then
Type-1: $N \cup M=\left\langle\left(N_{1}, G_{1}\right) \widetilde{\cup}\left(M_{1}, H_{1}\right),\left(N_{2}, G_{2}\right) \widetilde{\cup}\left(M_{2}, H_{2}\right),\left(N_{3}, G_{3}\right) \widetilde{\cap}\left(M_{3}, H_{3}\right)\right\rangle$.
$\partial_{\eta}(N \cup M)=\left\langle\partial_{\eta}\left[\left(N_{1}, G_{1}\right) \widetilde{\cup}\left(M_{1}, H_{1}\right)\right], \partial_{\eta}\left[\left(N_{2}, G_{2}\right) \widetilde{\cup}\left(M_{2}, H_{2}\right)\right], \partial_{\eta}\left[\left(N_{3}, G_{3}\right) \widetilde{\cap}\left(M_{3}, H_{3}\right)\right]\right\rangle$. From [22]
$\partial_{\eta}\left[\left(N_{1}, G_{1}\right) \widetilde{\cup}\left(M_{1}, H_{1}\right)\right]=ð_{\eta}\left[\left(N_{1}, G_{1}\right) \widetilde{\cup} \partial_{\eta}\left(M_{1}, H_{1}\right)\right.$,
$\partial_{\eta}\left[\left(N_{2}, G_{2}\right) \widetilde{\cup}\left(M_{2}, H_{2}\right)\right]=\partial_{\eta}\left(N_{2}, G_{2}\right) \widetilde{\cup} \mho_{\eta}\left(M_{2}, H_{2}\right)$, and
$\partial_{\eta}\left[\left(N_{3}, G_{3}\right) \widetilde{\cap}\left(M_{3}, H_{3}\right)\right] \supseteq \partial_{\eta}\left(N_{3}, G_{3}\right) \widetilde{\cap}_{\eta}\left(M_{3}, H_{3}\right)$.
Then, by the Definition 2.7, $\partial_{\eta}(N \cup M) \subseteq \partial_{\eta}(N) \cup \partial_{\eta}(M)$.
Type-2: $N \cup M=\left\langle\left(N_{1}, G_{1}\right) \widetilde{\cup}\left(M_{1}, H_{1}\right),\left(N_{2}, G_{2}\right) \widetilde{\cap}\left(M_{2}, H_{2}\right),\left(N_{3}, G_{3}\right) \widetilde{\cap}\left(M_{3}, H_{3}\right)\right\rangle$.
$\partial_{\eta}(N \cup M)=\left\langle\partial_{\eta}\left[\left(N_{1}, G_{1}\right) \widetilde{\cup}\left(M_{1}, H_{1}\right)\right], \partial_{\eta}\left[\left(N_{2}, G_{2}\right) \widetilde{\cap}\left(M_{2}, H_{2}\right)\right], \partial_{\eta}\left[\left(N_{3}, G_{3}\right) \widetilde{\cap}\left(M_{3}, H_{3}\right)\right]\right\rangle$. From [22]
$\partial_{\eta}\left[\left(N_{1}, G_{1}\right) \widetilde{\cup}\left(M_{1}, H_{1}\right)\right]=\check{\partial}_{\eta}\left[\left(N_{1}, G_{1}\right) \widetilde{\cup} \check{\partial}_{\eta}\left(M_{1}, H_{1}\right)\right.$,
$\partial_{\eta}\left[\left(N_{2}, G_{2}\right) \widetilde{\cap}\left(M_{2}, H_{2}\right)\right] \widetilde{\beth}_{\eta}\left(N_{2}, G_{2}\right) \widetilde{\cap} \partial_{\eta}\left(M_{2}, H_{2}\right)$, and
$\partial_{\eta}\left[\left(N_{3}, G_{3}\right) \widetilde{\cap}\left(M_{3}, H_{3}\right)\right] \widetilde{\supseteq} \partial_{\eta}\left(N_{3}, G_{3}\right) \widetilde{\cap} \partial_{\eta}\left(M_{3}, H_{3}\right)$.
Then, by the Definition 2.7, $\partial_{\eta}(N \cup M) \subseteq \partial_{\eta}(N) \cup \partial_{\eta}(M)$.
3. We have to show that $\partial_{\eta}(N \cap M) \supseteq \partial_{\eta}(N) \cap ð_{\eta}(M)$. Since $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle, M=$ $\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle \in N_{C}$, where $\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right),\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right)$ and ( $M_{3}, H_{3}$ ) are soft sets on $X$ and $P$, then

Type-1: $N \cap M=\left\langle\left(N_{1}, G_{1}\right) \widetilde{\cap}\left(M_{1}, H_{1}\right),\left(N_{2}, G_{2}\right) \widetilde{\cap}\left(M_{2}, H_{2}\right),\left(N_{3}, G_{3}\right) \widetilde{\cup}\left(M_{3}, H_{3}\right)\right\rangle$.
$\partial_{\eta}(N \cup M)=\left\langle\partial_{\eta}\left[\left(N_{1}, G_{1}\right) \tilde{\cap}\left(M_{1}, H_{1}\right)\right], \partial_{\eta}\left[\left(N_{2}, G_{2}\right) \widetilde{\cap}\left(M_{2}, H_{2}\right)\right], \partial_{\eta}\left[\left(N_{3}, G_{3}\right) \widetilde{\cup}\left(M_{3}, H_{3}\right)\right]\right\rangle$. From [22],
$\partial_{\eta}\left[\left(N_{1}, G_{1}\right) \widetilde{\cap}\left(M_{1}, H_{1}\right)\right] \widetilde{\Omega} \partial_{\eta}\left[\left(N_{1}, G_{1}\right) \widetilde{\cap} \partial_{\eta}\left(M_{1}, H_{1}\right)\right.$,
$\check{\partial}_{\eta}\left[\left(N_{2}, G_{2}\right) \widetilde{\cap}\left(M_{2}, H_{2}\right)\right] \tilde{\simeq} \partial_{\eta}\left(N_{2}, G_{2}\right) \widetilde{\cap} \check{ŋ}_{\eta}\left(M_{2}, H_{2}\right)$, and
$\partial_{\eta}\left[\left(N_{3}, G_{3}\right) \widetilde{\cup}\left(M_{3}, H_{3}\right)\right]=\partial_{\eta}\left(N_{3}, G_{3}\right) \widetilde{U}_{\eta}\left(M_{3}, H_{3}\right)$.
Then, by Definition 2.7, $\partial_{\eta}(N \cap M) \supseteq \partial_{\eta}(N) \cap \partial_{\eta}(M)$.
Type-2: $N \cap M=\left\langle\left(N_{1}, G_{1}\right) \widetilde{\cap}\left(M_{1}, H_{1}\right),\left(N_{2}, G_{2}\right) \widetilde{\cup}\left(M_{2}, H_{2}\right),\left(N_{3}, G_{3}\right) \widetilde{\cup}\left(M_{3}, H_{3}\right)\right\rangle$.
$\partial_{\eta}(N \cap M)=\left\langle\check{\partial}_{\eta}\left[\left(N_{1}, G_{1}\right) \widetilde{\cap}\left(M_{1}, H_{1}\right)\right], \partial_{\eta}\left[\left(N_{2}, G_{2}\right) \widetilde{\cup}\left(M_{2}, H_{2}\right)\right], \partial_{\eta}\left[\left(N_{3}, G_{3}\right) \widetilde{\cup}\left(M_{3}, H_{3}\right)\right]\right\rangle$. From [22]
$\partial_{\eta}\left[\left(N_{1}, G_{1}\right) \widetilde{\cap}\left(M_{1}, H_{1}\right)\right] \tilde{\underline{D}} \partial_{\eta}\left[\left(N_{1}, G_{1}\right) \tilde{\cap} \partial_{\eta}\left(M_{1}, H_{1}\right)\right.$,
$\partial_{\eta}\left[\left(N_{2}, G_{2}\right) \widetilde{\cup}\left(M_{2}, H_{2}\right)\right]=\check{\partial}_{\eta}\left(N_{2}, G_{2}\right) \widetilde{\cup} \check{\partial}_{\eta}\left(M_{2}, H_{2}\right)$, and
$\partial_{\eta}\left[\left(N_{3}, G_{3}\right) \widetilde{\cup}\left(M_{3}, H_{3}\right)\right]=\partial_{\eta}\left(N_{3}, G_{3}\right) \widetilde{\cup} \partial_{\eta}\left(M_{3}, H_{3}\right)$.
Then, by Definition 2.7, $\partial_{\eta}(N \cup M) \supseteq \partial_{\eta}(N) \cup \partial_{\eta}(M)$.
4. Type-1: Given is that $N \subseteq M=\left\langle\left(N_{1}, G_{1}\right) \widetilde{\subseteq}\left(M_{1}, H_{1}\right),\left(N_{2}, G_{2}\right) \widetilde{\subseteq}\left(M_{2}, H_{2}\right),\left(N_{3}, G_{3}\right) \widetilde{\cong}\left(M_{3}, H_{3}\right)\right\rangle$.
$\partial_{\eta}(N \subseteq M)=\left\langle\partial_{\eta}\left[\left(N_{1}, G_{1}\right) \widetilde{\subseteq}\left(M_{1}, H_{1}\right)\right], \partial_{\eta}\left[\left(N_{2}, G_{2}\right) \widetilde{\subseteq}\left(M_{2}, H_{2}\right)\right], \partial_{\eta}\left[\left(N_{3}, G_{3}\right) \widetilde{\varrho}\left(M_{3}, H_{3}\right)\right]\right\rangle$.
From [22],
$\widetilde{\partial}_{\eta}\left[\left(N_{1}, G_{1}\right) \widetilde{\subseteq}\left(M_{1}, H_{1}\right)\right]=\partial_{\eta}\left(N_{1}, G_{1}\right) \widetilde{\subseteq} \mathscr{\partial}_{\eta}\left(M_{1}, H_{1}\right)$,
$\partial_{\eta}\left[\left(N_{2}, G_{2}\right) \widetilde{\subseteq}\left(M_{2}, H_{2}\right)\right]=\partial_{\eta}\left(N_{2}, G_{2}\right) \widetilde{\subseteq} \partial_{\eta}\left(M_{2}, H_{2}\right)$,
$\check{\partial}_{\eta}\left[\left(N_{3}, G_{3}\right) \tilde{\cong}\left(M_{3}, H_{3}\right)\right]=\check{\partial}_{\eta}\left(N_{3}, G_{3}\right) \tilde{\simeq} \partial_{\eta}\left(M_{3}, H_{3}\right)$.
This shows that $\partial_{\eta}(N) \subseteq \partial_{\eta}(M)$.
Type-2: Let $N \subseteq M=\left\langle\left(N_{1}, G_{1}\right) \widetilde{\subseteq}\left(M_{1}, H_{1}\right),\left(N_{2}, G_{2}\right) \cong \simeq\left(M_{2}, H_{2}\right),\left(N_{3}, G_{3}\right) \widetilde{\cong}\left(M_{3}, H_{3}\right)\right\rangle$.
$\partial_{\eta}(N \subseteq M)=\left\langle\check{\partial}_{\eta}\left[\left(N_{1}, G_{1}\right) \widetilde{\subseteq}\left(M_{1}, H_{1}\right)\right], \partial_{\eta}\left[\left(N_{2}, G_{2}\right) \widetilde{\cong}\left(M_{2}, H_{2}\right)\right], \partial_{\eta}\left[\left(N_{3}, G_{3}\right) \widetilde{\cong}\left(M_{3}, H_{3}\right)\right]\right\rangle$.
From [22],
$\partial_{\eta}\left[\left(N_{1}, G_{1}\right) \widetilde{\subseteq}\left(M_{1}, H_{1}\right)\right]=\check{\partial}_{\eta}\left(N_{1}, G_{1}\right) \widetilde{\subseteq} \check{\partial}_{\eta}\left(M_{1}, H_{1}\right)$,
$\partial_{\eta}\left[\left(N_{2}, G_{2}\right) \check{\cong}\left(M_{2}, H_{2}\right)\right]=\partial_{\eta}\left(N_{2}, G_{2}\right) \tilde{\cong} \partial_{\eta}\left(M_{2}, H_{2}\right)$,
$\partial_{\eta}\left[\left(N_{3}, G_{3}\right) \tilde{\cong}\left(M_{3}, H_{3}\right)\right]=\check{\partial}_{\eta}\left(N_{3}, G_{3}\right) \tilde{\simeq} \partial_{\eta}\left(M_{3}, H_{3}\right)$.
This shows that $\partial_{\eta}(N) \subseteq \partial_{\eta}(M)$.
Remark 3.2. In soft set theory, if we take soft sets in the place of NCSSs, the part (2) of Theorem 3.1 holds for equality. But equality does not holds for NCSSs. This can be easily proved by using the same proof of part (2).

Theorem 3.2. Let $\partial_{\eta}: N_{C} \rightarrow M_{C}$ be a mapping between two NCS-classes, where $\partial: X \rightarrow Y$ and $\eta: P \rightarrow R$ are well-defined mappings. Then, for NCSSs $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$ and $M=\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle$ in the NCSS class $M_{C}$, we have

1. $\partial_{\eta}^{-1}\left(\Phi_{N}\right)=\Phi_{N}$,
2. $\partial_{\eta}^{-1}(N \cup M)=\partial_{\eta}^{-1}(N) \cup \partial_{\eta}^{-1}(M)$, (holds for union of Type-1 and Type-2). In general $\partial_{\eta}^{-1}\left(\cup_{\alpha}\right.$ $\left.\left(N_{\alpha}\right)\right)=\cup_{\alpha} \partial_{\eta}^{-1}\left(N_{\alpha}\right)$,
3. $\partial_{\eta}^{-1}(N \cap M)=\partial_{\eta}^{-1}(N) \cap \partial_{\eta}^{-1}(M)$, (holds for intersection of Type-1 and Type-2). In general $\partial_{\eta}^{-1}\left(\cap_{\alpha}\left(N_{\alpha}\right)\right)=\cap_{\alpha} \partial_{\eta}^{-1}\left(N_{\alpha}\right)$.
4. If $N \subseteq M$ then $\partial_{\eta}^{-1}(N) \subseteq \partial_{\eta}^{-1}(M)$.

Proof. The proof is obvious by using the same arguments used in the proof of Theorem 3.1.

## 4 Emerging Trends in SNS via NCS-Mapping with the Generation Gap

An online program utilized by societies to construct social links or social associations with other individuals who partake comparable personal or professional interests, events, experiences or real-life influences is called a social networking service (social networking site, or SNS or social media). In 2015, according to a study, 63 percent of the users of Twitter and Facebook in the USA consider that the major source of entertainment and update is SNS. A survey made in 2015 survey shows that 85 percent of people who are Millennials or of generation Y use SNS for their purchase decision-making.

Early SNS has begun in the form of online websites given as http://www.theglobe.com/ (1995), Geocities (1994) and Tripod.com (1995). Some web sites such as Classmates.com-took a diverse methodology by merely having people refer to one another via email addresses. PlanetAll was initiated in 1996.

In the recent 1990s, user profiles became a central characteristic of SNW. The i-generation uses SNW to boom with the development of SixDegrees.com in 1997, followed by Open Diary in 1998, mixi in 1999, Makeoutclub in 2000, Hub Culture and Friendster in 2002. In 2004, Facebook was launched, became the biggest SNW in the cosmos in early 2009. The term SNS were ushered in and soon became widespread.

Table 8 provides the list of the biggest SNS with the ranking of active users, as of January 2018, as issued by Statista.

Table 8: Largest social networking services
$\left.\begin{array}{llllll}\hline \text { SNS ranking } & 1 & 2 & 3 & 4 & 5 \\ \hline \text { Service } & \text { Facebook } & \text { You Tube } & \text { WhatsApp } & \text { Facebook } \\ \text { Messenger }\end{array}\right)$ WeChat

## Example 4.1.

We are presenting an application of social networking system (SNS), where we can analysis the emerging trends and issues of SNS for different generations by using the NCSS mapping. We construct a model of NCSS data containing some important purposes of SNS, we can increase the points of features used in SNS according to our requirement. In this example, we choose some specific features and discuss about some specific trends and issues for different generations. Our choice is random and gives us an idea about the trends and issues that how these factors changes with the generation gap.

## Algorithm:

## Input:

Step-1: Input the sample spaces $X$ and $Y$ with the sets of decision variables $P$ and $R$.
Step-2: Construct the NCS classes $N_{C}$ and $M_{C}$ on $X$ with $P$ and $Y$ with $R$, respectively.
Step-3: Define appropriate mappings $\partial: X \rightarrow Y$ and $\eta: P \rightarrow R$ by using the facts.
Step-4: Construct the NCS data for every generation separately. This implies that we can get different NCSSs for every individual generation.

## Output:

Step-5: Use Definition 3.1 of NCSS mapping and calculate the image NCSS for every generation.
Step-6: Resulting NCSSs show how the SNS is tending or creating issue with the change of time or with the gap of generation. Basically, this model is used for data collection at a large scale and can be used for checking the results and observing the changings in the collected information with the change of time.

The flow chart diagram of NCSS-algorithm is shown in Fig. 2.


Figure 2: Flow chart diagram of NCSS-algorithm

## Construction:

Let we have a sample space $X=\left\{\psi_{1}, \psi_{2}, \psi_{3}, \psi_{4}, \psi_{5}, \psi_{6}, \psi_{7}, \psi_{8}, \psi_{9}, \psi_{10}, \psi_{11}, \psi_{12}, \psi_{13}\right\}$ as the set of some purposes for which SNS is used. In the set X , let
$\psi_{1}=$ Science,
$\psi_{2}=$ Education,
$\psi_{3}=$ Curriculum use,
$\psi_{4}=$ Professional use (business models),
$\psi_{5}=$ Positive correlates,
$\psi_{6}=$ Social Interaction,
$\psi_{7}=$ Employment (trending networks),
$\psi_{8}=$ Spamming,
$\psi_{9}=$ Data mining,
$\psi_{10}=$ Trolling,
$\psi_{11}=$ Unauthorized access,
$\psi_{12}=$ Online bullying,
$\psi_{13}=$ Facilities laziness.
Let sample space $Y=\left\{\pi_{1}, \pi_{2}, \pi_{3}, \pi_{4}, \pi_{5}, \pi_{6}, \pi_{7}, \pi_{8}, \pi_{9}, \pi_{10}, \pi_{11}\right\}$ be the collection of some important benefits, bad impacts or drawbacks and unaffected or undecidable features of SNS according to the different generations. In set $Y$, suppose
$\pi_{1}=$ Privacy (risk of fraud and identity theft),
$\pi_{2}=$ negative effect of employability,
$\pi_{3}=$ Risk of child safety,
$\pi_{4}=$ Cyberbullying and crime against children,
$\pi_{5}=$ Interpersonal communication,
$\pi_{6}=$ Psychological effects of SNS (source of entertainment),
$\pi_{7}=$ Social overload (social anxiety),
$\pi_{8}=$ Worldwide connectivity (faster communication),
$\pi_{9}=$ Criminal investigation,
$\pi_{10}=$ Co-marketing opportunity,
$\pi_{11}=$ Time intensive (reduce family closeness).
The collection $P=\left\{\wp_{1}, \wp_{2}, \wp_{3}\right\}$ and $R=\left\{r_{1}, r_{2}, r_{3}\right\}$ are the set of decision variables where,
$\wp_{1}=$ Emerging trends,
$\wp_{2}=$ Neutral,
$\wp_{3}=$ Issues,
$r_{1}=$ Benefits,
$r_{2}=$ Unaffected or undecidable,
$r_{3}=$ Drawbacks or bad impacts.
In this application, we use the age factor for our discussion about SNS and its trends. We consider five generations given as:
(1): Traditional or silent generation: (Born in 1945 and before).
(2): Baby boomers: (Born in 1946 to 1964, roughly 50 to 70 years).
(3): Generation X: (Born in 1965 to 1976, 35-50 years).
(4): Millennials or generation Y: (Born in 1977 to 1995, 18-34 years).
(5): Centennials Generation Z or i-generation: (Born in 1996 and later).

Now, we construct five NCSSs which are $N, M, L, O$ and $Q$ for above five generations.
The tabular representation of $N=\left\langle\left(N_{1}, G_{1}\right),\left(N_{2}, G_{2}\right),\left(N_{3}, G_{3}\right)\right\rangle$ for the traditional or silent generation, $M=\left\langle\left(M_{1}, H_{1}\right),\left(M_{2}, H_{2}\right),\left(M_{3}, H_{3}\right)\right\rangle$ for baby boomers, $L=\left\langle\left(L_{1}, J_{1}\right),\left(L_{2}, J_{2}\right),\left(L_{3}, J_{3}\right)\right\rangle$ for generation $\mathrm{X}, O=\left\langle\left(O_{1}, K_{1}\right),\left(O_{2}, K_{2}\right),\left(O_{3}, K_{3}\right)\right\rangle$ for millennials or generation Y and $Q=$ $\left\langle\left(Q_{1}, I_{1}\right),\left(Q_{2}, I_{2}\right),\left(Q_{3}, I_{3}\right)\right\rangle$ for cantennials Generation Z or i-generation is given in Table 9.

Table 9: NCSS data with five generations

| NCSS data | First soft set |
| :--- | :--- |
| $N$ | $\left(N_{1}, G_{1}\right)=\left\{\left(\wp_{1},\left\{\psi_{1}, \psi_{2}, \psi_{3}\right\}\right)\right\}$ |
| $M$ | $\left(M_{1}, H_{1}\right)=\left\{\left(\wp_{1},\left\{\psi_{1}, \psi_{2}, \psi_{3}, \psi_{4}\right\}\right)\right\}$ |
| $L$ | $\left(L_{1}, J_{1}\right)=\left\{\left(\wp_{1},\left\{\psi_{1}, \psi_{2}, \psi_{3}, \psi_{4}, \psi_{5}\right\}\right)\right\}$ |
| $O$ | $\left(O_{1}, K_{1}\right)=\left\{\left(\wp_{1},\left\{\psi_{1}, \psi_{2}, \psi_{3}, \psi_{4}, \psi_{5}\right\}\right)\right\}$ |
| $Q$ | $\left(Q_{1}, I_{1}\right)=\left\{\left(\wp_{1},\left\{\psi_{1}, \psi_{2}, \psi_{3}, \psi_{4}, \psi_{5}, \psi_{6}, \psi_{7}\right\}\right)\right\}$ |
| NCSS data | Second soft set |
| $N$ | $\left(N_{2}, G_{2}\right)=\left\{\left(\wp_{2},\left\{\psi_{4}, \psi_{5}, \psi_{6}, \psi_{7}, \psi_{8}, \psi_{9}, \psi_{10}, \psi_{11}, \psi_{12}, \psi_{13}\right\}\right)\right\}$ |
| $M$ | $\left(M_{2}, H_{2}\right)=\left\{\left(\wp_{2},\left\{\psi_{5}, \psi_{6}, \psi_{7}, \psi_{8}, \psi_{9}, \psi_{10}, \psi_{11}, \psi_{12}, \psi_{13}\right\}\right)\right\}$ |
| $L$ | $\left.\left(L_{2}, J_{2}\right)=\left\{\wp_{2},\left\{\psi_{6}, \psi_{7}, \psi_{8}, \psi_{10}, \psi_{11}, \psi_{12}, \psi_{13}\right\}\right)\right\}$ |
| $O$ | $\left(O_{2}, K_{2}\right)=\left\{\left(\wp_{2},\left\{\psi_{6}, \psi_{7}, \psi_{8}, \psi_{11}, \psi_{12}, \psi_{13}\right\}\right)\right\}$ |
| $Q$ | $\left(Q_{2}, I_{2}\right)=\left\{\left(\wp_{2}, \phi\right)\right\}$, |
| NCSS data | Third soft set |
| $N$ | $\left(N_{3}, G_{3}\right)=\left\{\left(\wp_{3}, \phi\right)\right\}$ |
| $M$ | $\left(M_{3}, H_{3}\right)=\left\{\left(\wp_{3}, \phi\right)\right\}$ |
| $L$ | $\left(L_{3}, J_{3}\right)=\left\{\left(\wp_{3},\left\{\psi_{9}\right\}\right)\right\}$ |
| $O$ | $\left(O_{3}, K_{3}\right)=\left\{\left(\wp_{3},\left\{\psi_{9}, \psi_{10}\right\}\right)\right\}$ |
| $Q$ | $\left(Q_{3}, I_{3}\right)=\left\{\left(\wp_{3},\left\{\psi_{8}, \psi_{9}, \psi_{10}, \psi_{11}, \psi_{12}, \psi_{13}\right\}\right)\right\}$ |

Now, we define appropriate mappings $\delta: X \rightarrow Y$ and $\eta: P \rightarrow R$ by using the facts from the given information as

$$
\partial\left(\psi_{1}\right)=\pi_{8}, \partial\left(\psi_{2}\right)=\pi_{8}, \partial\left(\psi_{3}\right)=\pi_{8}, \partial\left(\psi_{4}\right)=\pi_{10},
$$

```
\(\partial\left(\psi_{5}\right)=\pi_{6}, \partial\left(\psi_{6}\right)=\pi_{5}, \partial\left(\psi_{7}\right)=\pi_{10}, \partial\left(\psi_{8}\right)=\pi_{7}\),
\(\partial\left(\psi_{9}\right)=\pi_{7}, \partial\left(\psi_{10}\right)=\pi_{1}, \partial\left(\psi_{11}\right)=\pi_{1}, \partial\left(\psi_{12}\right)=\pi_{4}, \partial\left(\psi_{13}\right)=\pi_{11}\).
```

and
$\eta\left(\wp_{1}\right)=r_{1}, \eta\left(\wp_{2}\right)=r_{2}, \eta\left(\wp_{3}\right)=r_{3}$.

## Calculations:

We use Definition 3.1 and calculate the images of N, M, L, O and Q respectively as
$\partial_{\eta}\left(N_{1}, G_{1}\right)\left(r_{1}\right)=\left\{\pi_{8}\right\}$,
$\partial_{\eta}\left(N_{2}, G_{2}\right)\left(r_{2}\right)=\left\{\pi_{1}, \pi_{4}, \pi_{5}, \pi_{6}, \pi_{7}, \pi_{10}, \pi_{11}\right\}$,
$\partial_{\eta}\left(M_{1}, H_{1}\right)\left(r_{1}\right)=\left\{\pi_{8}, \pi_{10}\right\}$,
$\partial_{\eta}\left(M_{2}, H_{2}\right)\left(r_{2}\right)=\left\{\pi_{1}, \pi_{4}, \pi_{5}, \pi_{6}, \pi_{7}, \pi_{10}, \pi_{11}\right\}$,
$\partial_{\eta}\left(L_{1}, J_{1}\right)\left(r_{1}\right)=\left\{\pi_{6}, \pi_{8}, \pi_{10}\right\}$,
$\partial_{\eta}\left(L_{2}, J_{2}\right)\left(r_{2}\right)=\left\{\pi_{1}, \pi_{4}, \pi_{5}, \pi_{7}, \pi_{10}, \pi_{11}\right\}$,
$\partial_{\eta}\left(L_{3}, J_{3}\right)\left(r_{3}\right)=\left\{\pi_{7}\right\}$,
$\partial_{\eta}\left(O_{1}, K_{1}\right)\left(r_{1}\right)=\left\{\pi_{6}, \pi_{8}, \pi_{10}\right\}$,
$\partial_{\eta}\left(O_{2}, K_{2}\right)\left(r_{2}\right)=\left\{\pi_{1}, \pi_{4}, \pi_{5}, \pi_{7}, \pi_{10}, \pi_{11}\right\}$,
$\partial_{\eta}\left(O_{3}, K_{3}\right)\left(r_{3}\right)=\left\{\pi_{1}, \pi_{7}\right\}$,
$\partial_{\eta}\left(Q_{1}, I_{1}\right)\left(r_{1}\right)=\left\{\pi_{5}, \pi_{6}, \pi_{8}, \pi_{10}\right\}$,
$\partial_{\eta}\left(Q_{3}, I_{3}\right)\left(r_{3}\right)=\left\{\pi_{1}, \pi_{4}, \pi_{7}, \pi_{11}\right\}$,
$\partial_{\eta}\left(N_{1}, G_{1}\right)\left(r_{2}\right)=\partial_{\eta}\left(N_{1}, G_{1}\right)\left(r_{3}\right)=\partial_{\eta}\left(N_{2}, G_{2}\right)\left(r_{1}\right)=\partial_{\eta}\left(N_{2}, G_{2}\right)\left(r_{3}\right)=\partial_{\eta}\left(N_{3}, G_{3}\right)\left(r_{1}\right)=\partial_{\eta}\left(N_{3}, G_{3}\right)\left(r_{2}\right)=$ $\partial_{\eta}\left(N_{3}, G_{3}\right)\left(r_{3}\right)=\phi$.
$\partial_{\eta}\left(M_{1}, H_{1}\right)\left(r_{2}\right)=\partial_{\eta}\left(M_{1}, H_{1}\right)\left(r_{3}\right)=\partial_{\eta}\left(M_{2}, H_{2}\right)\left(r_{1}\right)=\partial_{\eta}\left(M_{2}, H_{2}\right)\left(r_{3}\right)=\partial_{\eta}\left(M_{3}, H_{3}\right)\left(r_{1}\right)=$ $\partial_{\eta}\left(M_{3}, H_{3}\right)\left(r_{2}\right)=\partial_{\eta}\left(M_{3}, H_{3}\right)\left(r_{3}\right)=\phi$.
$\partial_{\eta}\left(L_{1}, J_{1}\right)\left(r_{2}\right)=\partial_{\eta}\left(L_{1}, J_{1}\right)\left(r_{3}\right)=\partial_{\eta}\left(L_{2}, J_{2}\right)\left(r_{1}\right)=\partial_{\eta}\left(L_{2}, J_{2}\right)\left(r_{3}\right)=\partial_{\eta}\left(L_{3}, J_{3}\right)\left(r_{1}\right)=\partial_{\eta}\left(L_{3}, J_{3}\right)$ $\left(r_{2}\right)=\phi$.
$\partial_{\eta}\left(O_{1}, K_{1}\right)\left(r_{2}\right)=\partial_{\eta}\left(O_{1}, K_{1}\right)\left(r_{3}\right)=ذ_{\eta}\left(O_{2}, K_{2}\right)\left(r_{1}\right)=\partial_{\eta}\left(O_{2}, K_{2}\right)\left(r_{3}\right)=\partial_{\eta}\left(O_{3}, K_{3}\right)\left(r_{1}\right)=$ $\partial_{\eta}\left(O_{3}, K_{3}\right)\left(r_{2}\right)=\phi$.
$\partial_{\eta}\left(Q_{1}, I_{1}\right)\left(r_{2}\right)=\partial_{\eta}\left(Q_{1}, I_{1}\right)\left(r_{3}\right)=\partial_{\eta}\left(Q_{2}, I_{2}\right)\left(r_{1}\right)=\partial_{\eta}\left(Q_{2}, I_{2}\right)\left(r_{2}\right)=\partial_{\eta}\left(Q_{2}, I_{2}\right)\left(r_{3}\right)=\partial_{\eta}\left(Q_{3}, I_{3}\right)\left(r_{1}\right)=$ $\partial_{\eta}\left(Q_{3}, I_{3}\right)\left(r_{2}\right)=\phi$.

The tabular representation of images of $\partial_{\eta}(N)$ for the traditional or silent generation, $\partial_{\eta}(M)$ for baby boomers, $\partial_{\eta}(L)$ for generation $\mathrm{X}, \partial_{\eta}(O)$ for millennials or generation Y and $\partial_{\eta}(Q)$ for cantennials Generation Z or i -generation is given in Table 10.

Table 10: Images of NCSSs for five generations

| NCSS data | Image of first soft set |
| :--- | :--- |
| $N$ | $\searrow_{\eta}\left(N_{1}, G_{1}\right)=\left\{\left(r_{1},\left\{\pi_{8}\right\}\right)\right\}$ |
| $M$ | $\partial_{\eta}\left(M_{1}, H_{1}\right)=\left\{\left(r_{1},\left\{\pi_{8}, \pi_{10}\right\}\right)\right\}$ |

## Table 10 (continued)

| NCSS data | Image of first soft set |
| :---: | :---: |
| $L$ | $\partial_{\eta}\left(L_{1}, J_{1}\right)=\left\{\left(r_{1},\left\{\pi_{6}, \pi_{8}, \pi_{10}\right\}\right)\right\}$ |
| $O$ | $\partial_{\eta}\left(O_{1}, K_{1}\right)=\left\{\left(r_{1},\left\{\pi_{6}, \pi_{8}, \pi_{10}\right\}\right)\right\}$ |
| Q | $\partial_{\eta}\left(Q_{1}, I_{1}\right)=\left\{\left(r_{1},\left\{\pi_{5}, \pi_{6}, \pi_{8}, \pi_{10}\right\}\right)\right\}$ |
| NCSS data | Image of second soft set |
| $N$ | $\partial_{\eta}\left(N_{2}, G_{2}\right)=\left\{\left(r_{2},\left\{\pi_{1}, \pi_{4}, \pi_{5}, \pi_{6}, \pi_{7}, \pi_{10}, \pi_{11}\right\}\right)\right\}$ |
| M | $\left.\partial_{\eta}\left(M_{2}, H_{2}\right)=\left\{\left(r_{2},\left\{\pi_{1}, \pi_{4}, \pi_{5}, \pi_{6}, \pi_{7}, \pi_{10}, \pi_{11}\right\}\right\}\right)\right\}$ |
| $L$ | $\partial_{\eta}\left(L_{2}, J_{2}\right)=\left\{\left(r_{2},\left\{\pi_{1}, \pi_{4}, \pi_{5}, \pi_{7}, \pi_{10}, \pi_{11}\right\}\right)\right\}$ |
| $O$ | $\partial_{\eta}\left(O_{2}, K_{2}\right)=\left\{\left(r_{2},\left\{\pi_{1}, \pi_{4}, \pi_{5}, \pi_{7}, \pi_{10}, \pi_{11}\right\}\right)\right\}$ |
| $\underline{Q}$ | $\partial_{\eta}\left(Q_{2}, I_{2}\right)=\left\{\left(r_{2}, \phi\right)\right\}$ |
| NCSS data | Image of third soft set |
| $N$ | $\partial_{\eta}\left(N_{3}, G_{3}\right)=\left\{\left(r_{3}, \phi\right)\right\}$ |
| M | $\partial_{\eta}\left(M_{3}, H_{3}\right)=\left\{\left(r_{3}, \phi\right)\right\}$ |
| $L$ | $\partial_{\eta}\left(L_{3}, J_{3}\right)=\left\{\left(r_{3},\left\{\pi_{7}\right\}\right)\right\}$ |
| O | $\partial_{\eta}\left(O_{3}, K_{3}\right)=\left\{\left(r_{3},\left\{\pi_{1}, \pi_{7}\right\}\right)\right\}$ |
| $Q$ | $\partial_{\eta}\left(Q_{3}, I_{3}\right)=\left\{\left(r_{3},\left\{\pi_{1}, \pi_{4}, \pi_{7}, \pi_{11}\right\}\right)\right\}$ |

The information of SNS via NCSS data for five generations (Input) and effects of SNS to different generations (Output) is expressed in Tables 9 and 10, respectively.

In the Table 10, five NCSSs show the benefits, drawbacks and unaffected factors of SNS for the five generations. With the change of generation, we can see that emerging trends and benefits of SNS are increasing day by day. Similarly, some drawbacks are also increasing with the very much use of SNS. In the Table 10, the image of first soft set column shows the increase in benefits of SNS with the change of time and image of second soft set column shows that indeterminacy decreases with the generation gap, because people are searching and exploring things and collect information about the facts and easily decide that these features are beneficial or not for themselves. In the same way, the image of third soft set column also increasing which means the drawbacks or bad impacts of SNS are increasing for different generations, because when the use of SNS increases, then its bad impacts also affects the personal and social life of people and create disturbance in many areas of life.

## 5 A Comparison Analysis and Discussion

In this section, to corroborate the practicability of proposed NCSS and NCS-mapping we compare it with other hybrid structures of soft and fuzzy sets. In this inquiry, we present a new concept of NCSS with different characters and operations and use illustrations to flesh out the concept. Essentially, we construct three parameterized families of subsets of the creation of discourse with the given set of decision variables and the triplet becomes an NCSS. The presented idea of parameters or decision variables in the NC set contributes to the new approach of NCSS and covers the NC set theory as easily.

Resulting NCSSs show how the SNS is tending or creating issue with the change of time or with the gap of generation. Basically, this model is used for data collection at a large scale and can be used
for checking the results and observing the changings in the collected information with the change of time.

The relations between neutrosophic hybrid sets and other hybrid structures are shown in Fig. 3.


Figure 3: The relations between neutrosophic hybrid sets and other hybrid structures
In previous parts, we establish NCS-mapping of NCS classes and establish some results of mathematical functions which hold in soft set theory but do not hold in NCSS theory. In the application regarding SNS, we can utilize our set and its mapping to develop a robust mathematical modeling, and it can be easily used for data collection at a large scale and for a long time period. This constructed model through NCSSs can be used for discussion about the emerging trends, disadvantages and unpretentious factors of the social networking system for the five generations at the same time. If we examine the hybrid structures of soft and fuzzy sets, then we can well conclude that there is a chain relationship between NCSS and other hybrid structures like fuzzy set, IFS, GIFS, GNS. This relationship can be easily determined through the given flow chart diagram Fig. 3.

## 6 Conclusion

We introduced some new concepts of the neutrosophic crisp soft set (NCSS) and established different types of NCSSs. We defined some operations on NCSSs and illustrated these concepts with the help of some examples of NCSSs. We introduced NCS-mapping and developed images and inverse images of NCS classes. We derived some significant results regarding NCS-mapping. We further discussed some consequences of mapping which hold in soft set theory but do not hold in NCSS theory. We used NCS-mapping to develop a robust mathematical modeling to analyze emerging trends in social networking systems and the generation gap. The benefits, disadvantages and unpretentious factors of social networking systems for five generations were investigated. This article is an innovative approach to data science and information fusion at a large scale. Proposed theories and models are more efficient in addressing vagueness and uncertainties in real-life problems. This theory can be
extended to various fields like geographical information systems, networking systems, robotics, pattern recognition, computational intelligence, medical diagnosis, etc.

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