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ARTICLE

Distributionally Robust Newsvendor Model for Fresh Products under Cap-and-Offset Regulation

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ABSTRACT

The cap-and-offset regulation is a practical scheme to lessen carbon emissions. The retailer selling fresh products can adopt sustainable technologies to lessen greenhouse gas emissions. We aim to analyze the optimal joint strategies on order quantity and sustainable technology investment when the retailer faces stochastic market demand and can only acquire the mean and variance of distribution information. We construct a distributionally robust optimization model and use the Karush-Kuhn-Tucker (KKT) conditions to solve the analytic formula of optimal solutions. By comparing the models with and without investing in sustainable technologies, we examine the effect of sustainable technologies on the operational management decisions of the retailer. Finally, some computational examples are applied to analyze the impact of critical factors on operational strategies, and some managerial insights are given based on the analysis results.

KEYWORDS

Distributionally robust optimization; KKT conditions; cap-and-offset regulation; fresh products

1 Introduction

With the intensification of climate warming and enhancement of sustainable awareness, carbon emissions reduction has become one of the critical issues in the world. Carbon dioxide is a major contributor to global warming. Many countries and regions have set short-term and long-term targets to lessen emissions. In September 2020, the Chinese government promised to lower carbon emissions, and successively released a series of supporting measures to achieve carbon peaking and carbon neutrality goals. Great efforts, such as implementing carbon regulations, have been made to achieve the goal of sustainable development [1,2]. The carbon tax, carbon trading and cap-and-offset regulations are most frequently adopted and researched [3–5]. In this paper, we assume that a fresh product retailer operates under the cap-and-offset regulation. Under cap-and-offset regulation, a certain threshold on carbon emissions is allocated to the retailer. The retailer can emit more than the threshold but will be penalized for the emissions exceeding the threshold.

In this context, the operational objective of the retailer has been changed to pursue economic benefits and protect environment simultaneously. Many enterprises take active measures to protect the environment. For example, WalMart, the largest department store in the United States, actively adopted new technologies to save energy and has built a low-carbon distribution center and a low-carbon supermarket. The development of cold chain market urges retailers to concentrate on the operational management of fresh products. The fresh products have perishable physical properties and are preserved in special temperature-controlled equipment that generates higher carbon emissions. Lekkerland, a famous retailer in Germany, implemented a "multi-temperature logistics" distribution strategy to sell perishable products. However, using such special multi-temperature equipment leads Lekkerland to produce much greenhouse gases compared with standard warehousing and logistics systems [6]. In this scenario, new challenges have been brought and raised our research interest in studying the influences of cap-and-offset regulation on retailer management strategies.

Affected by market fluctuations and uncertainties, it is more difficult for fresh product retailers to forecast the full distribution information of demand [7–9]. Partial distribution information of the demand is easier to specified accurately. In this scenario, when carbon emission reduction is considered in the operational management of fresh product retailers, the following issues arise. (i) How do fresh product retailers make robust strategies to pursue maximum economic benefits under cap-and-offset policy? (ii) How does the implementation of cap-and-offset affect the economic benefits and environmental performance of fresh product retailers? (iii) With incomplete distribution information of consumer demand, how can retailers achieve a win-win situation between the economy and the environment?

To address the major issues mentioned above, we consider the operational strategy of a fresh product retailer under cap-and-offset regulation. The retailer has to decide whether to invest in sustainable technology to lessen greenhouse gas emissions and find the optimal order quantity only according to the mean and variance of the stochastic demand. In order to give valuable suggestions to the retailer, we construct a distributionally robust optimization model and work out the analytic formula of joint order quantity and low-carbon technology investment. We further explore the situation without technology investment and compare two distributionally robust optimization models. Finally, some computational studies are conducted to validate the impact of principal factors on the robustness of operational decisions.

Our work has the following research contributions. First, we consider limited distribution information of stochastic demand and low-carbon technology investment in operational decisions of the fresh product retailer. We use a distributionally robust newsvendor method to work out the analytic formula of joint order quantity and sustainable technology investment. Second, we theoretically and numerically provide some conditions where investing in sustainable technology leads the fresh product retailer to gain higher expected profit and emit lower greenhouse gases under cap-and-offset regulation. Finally, we numerically investigate how carbon parameters affect the robustness of the optimal joint strategies on order quantity and sustainable technology investment.

The reminders of this paper are arranged as follows. Section 2 reviews the relevant literature and shows the research gap. Section 3 interprets the considered problem and relevant notations, and constructs two distributionally robust models. Section 4 contains some numerical studies to illustrate and complement the theoretical outcomes. Conclusions and implications are revealed in the last section.

2 Literature Review

Two research branches are connected with the considered topic. Operational decisions of the fresh product retailer and robust decisions of the retailer under carbon regulations.

The first branch of research concentrates on operational decisions of the fresh product retailer. This topic has received extensive attentions. In this context, Cai et al. [10] characterized a continuous variable to affect the fresh quality and survival quantity of fresh products. The fresh quality impacts the price-dependent stochastic demand with a complete probability distribution function. This system is coordinated with price discount scheme and compensation scheme. On this foundation, Cai et al. [11] developed the impact of transportation time on the perishability of quantity and quality of fresh products by simplifying the influence of random factors. In addition, a new mechanism is proposed to promote the cooperation between producer and distributor. Wu et al. [12] investigated how the channel power structure affects the decision-making behavior of managers in the game. The authors deem that logistics service level and pricing standard affect the sales demand of fresh products. Ma et al. [13] argued that asymmetric demand messages can cause the loss of profit in decentralized system. Combined with the particularity of agricultural products, the authors propose a new mechanism to promote cooperation and make up for system losses. Wu et al. [14] explored the impact of adopting blockchain technology on optimal strategies for e-sales of fresh products. The authors design an incentive mechanism to realize the overall optimization of the supply chain. Other literature on fresh products includes Chen et al. [15], Xu et al. [16], Duan et al. [17].

The articles mentioned above do not consider the impact of carbon emission reduction on developing models for different types of supply chains with fresh products. However, how to lessen carbon emissions has become a major issue in optimizing the fresh product supply chain. It is due to the fact that employing special packaging, cryogenic devices and other equipment may release more greenhouse gases. Bai et al. [18] integrated emission trading policy in a manufacturer-retailer supply chain for perishable items. The authors construct some optimization system models and coordinated mechanisms. Wang et al. [19] put forward pricing strategies of fresh foods and proclaim the relationship between carbon trading and cold logistics services. Wang et al. [20] proposed three replenishment scenarios to solve the optimal scheme between a supplier and several retailers under cap-and-trade policy. Ma et al. [21] considered the impact of freshness-keeping efforts on a three-tier cold chain under cap-and-trade policy. The authors propose a coordinated scheme to improve the profit of the whole system. Although these scholars introduce cap-and-trade regulation into the fresh product system, they assume that market demand is deterministic or stochastic with full distribution information. Many scholars assume that full information of the demand distribution is known and use a newsvendor mechanism to work out the optimal tactics under different types of carbon regulations [22–25]. Unlike them, in this paper, we integrate cap-and-offset regulation and limited distribution information of the demand into the decision-making background of a fresh product retailer. We utilize a distributionally robust optimization mechanism to work out the optimal joint strategies on the order quantity and sustainable technology investment.

The second branch of research concentrates on frims' robust decisions under carbon regulations. This topic is popular in the operational management field. Due to the difficulties of acquiring full information of demand distribution, distributionally robust optimization approach is proposed to work out the optimal tactics with limited distribution information [26–29]. Recently, Liu et al. [30] extend the newsvendor problem to an independent remanufacturing structure under three different kinds of carbon regulations. The authors assume that only the mean and variance of stochastic demand are specified. And they work out the optimal robust remanufacturing quantity. Bai et al. [31] exploited a distributionally robust newsvendor model to work out the optimal order tactic on dual sources when the firm is regulated by carbon tax and carbon trading schemes. Xu et al. [32] studied and compare the impacts of carbon cap and carbon trading schemes on the firms' robust order tactics with limited distribution information of the demand. Bai et al. [33] considered a remanufacturing structure under carbon trading scheme and develop a distributionally robust newsvendor model to find out the optimal decisions on collection and production quantities. Bai et al. [34] combined robust optimization method and Hurwicz-decision theory to solve the optimal decision of the sustainable manufacturer. The authors comparatively analyze the impact of carbon tax and mandatory carbon quota on decision-making. Although these scholars relax the information of demand distribution, they do not take the carbon policies into account in the operational decisions for fresh products. However, in this paper, we exploit a distributionally robust optimization model under cap-and-offset regulation for the fresh product retailer. We emphasize the impacts of both cap-and-offset regulation and limited distribution information on the optimal joint order quantity and low-carbon technology investment.

3 The Model

3.1 Problem Description and Hypothesis

The retailer purchases q units of a kind of fresh products at a unit order cost c_1, and sells them at the unit price p to fulfill the random consumption demand. Referencing to Bai et al. [18], the transportation time can be standardized to 1 and the unit transportation cost is c_2. Due to the perishable property of fresh products, referring to Cai et al. [10], we characterize surviving index τ and freshness index ϑ, where τ is defined over $(0, 1]$, $\tau = 1$ indicates that all products survive during transportation. The survival quantity reaching the market that is available for sale becomes $q\tau$ with $q\tau > 0$. ϑ is defined over $(0, 1]$, and $\vartheta = 1$ represents the products are completely fresh before the sales period. In addition, ε represents the other influence factors of demand that is random variable. Consequently, the demand function can be expressed as

$$D(\vartheta, p, \varepsilon) = y_0 \vartheta p^{-k_0} \varepsilon \tag{1}$$

In Eq. (1), y_0 is the potential sales scale, k_0 is the price elasticity of demand, and ε represents the random demand fluctuation.

If actual demand D during selling period does not exceed $q\tau$, then the revenue is pD. Due to the perishability of fresh products over time, its salvage value is not considered temporarily. Alternatively, if demand D exceeds $q\tau$, then the revenue is $pq\tau$, and the $D(\vartheta, p, \varepsilon) - q\tau$ shortages is assessed by the per-unit penalty cost s. The expected profit is expressed as

$$\Pi_0(q) = p\mathbb{E}_\varepsilon[min(D(\vartheta, p, \varepsilon), q\tau))] - (c_1 + c_2)q - s\mathbb{E}_\varepsilon[(D(\vartheta, p, \varepsilon) - q\tau)^+] \tag{2}$$

In Eq. (2), the first item indicates expected income, the second item indicates the order and transportation cost, and the third item indicates the expected shortage cost.

The transportation is the main link to emit greenhouse gases. The retailer can reduce carbon emissions by investing in sustainable technologies, equipments or machineries. The marginal reduction amount of carbon emissions deceases as the increment of the investment cost that identifies with the principle of "Increasing Marginal Cost" in economics. Referring to Huang et al. [35] and Toptal et al. [36], if the retailer invests in technologies, then the carbon emissions can be cut down by amount of $aR - bR^2$ as the return on investment of R currency per year $\left(0 \le R \le \dfrac{a}{2b}\right)$. Here, a

reflects the efficacy of sustainable technology in decreasing emissions, and b is the decreasing return coefficient. Let e be the carbon emissions per unit product before investment. The carbon emissions are eq if the retailer does not invest in technologies. Consequently, the total carbon emissions are expressed as

$$Y(q, R) = eq - aR + bR^2 \tag{3}$$

Considering the fact that the carbon emissions cannot be completely lessened to 0 even if investing in sustainable technologies, we assume that carbon emissions satisfies $Y(q, R) > 0$ in Eq. (3). The relevant notations involved in the paper are summarized in Table 1.

Table 1: Summary of relevant notations

Decision variables	Explanation
q	Order quantity
R	The investment cost of sustainable technology
Other parameters	**Explanation**
c_1	Unit order cost
c_2	Unit transportation cost
s	The shortage cost per unit product
p	Unit sales price
τ	Surviving index of fresh products
ϑ	Freshness index of fresh products
ε	Random factor of the market demand. Assuming that the retailer can only acquire the mean μ and variance σ^2
$D(\vartheta, p, \varepsilon)$	The stochastic market demand
y_0	The potential sales scale
k_0	The price elasticity of demand
e	Carbon emission per unit product before investment
$Y(q,R)$	Total carbon emissions
$\Pi_0(q)$	Expected profit function without carbon regulations

3.2 Robust Optimization Strategies under Cap-and-Offset Policy

This subsection utilizes the distributionally robust optimization approach to analyze the optimal joint strategies on order quantity and sustainable technology investment. The fresh product retailer faces the fact that the probability distribution of demand is difficult to acquire and operation management is constrained by cap-and-offset regulation.

Referring to Chen et al. [5], under the cap-and-offset regulation, the government allocates the fixed number of carbon emission permits to the retailer, that is denoted by carbon cap K. When the total carbon emissions $Y(q,R)$ exceed the carbon cap, the retailer needs to pay $c_3[Y(q, R) - K]$ for excess emissions where c_3 is carbon tax charged on unit redundant carbon emissions. Otherwise, the retailer does not need to pay carbon tax.

Based on historical data information, the retailer can only acquire the mean μ and variance σ^2 of the random factor but cannot obtain the complete distribution function G. Let Ψ represent the set of distribution functions whose mean and variance is μ and σ^2, respectively. That is, $G \in \Psi$. Using Eqs. (2) and (3), we establish the distributionally robust optimization model for the fresh product retailer under cap-and-offset regulation as follows:

$$(M_1) \quad \max_{q,R} \min_{G\in\Psi} \; \Pi_1(q,R) = \Pi_0(q) - R - c_3(Y(q,R) - K)^+ \tag{4}$$

$$s.t. \quad q \geq 0, R \geq 0$$

Here, $x^+ = \max\{x, 0\}$. According to characteristic of Eq. (4), we first solve the following two sub-models M_{11} and M_{12} before solving model M_1.

$$(M_{11}) \quad \max_{q,R} \min_{G\in\Psi} \Pi_{11}(q,R) = \Pi_0(q) - R - c_3(Y(q,R) - K) \tag{5}$$

$$s.t. \quad \begin{cases} Y(q,R) \geq K \\ q \geq 0, R \geq 0 \end{cases} \tag{6}$$

$$(M_{12}) \quad \max_{q,R} \min_{G\in\Psi} \Pi_{12}(q,R) = \Pi_0(q) - R \tag{7}$$

$$s.t. \quad \begin{cases} Y(q,R) \leq K \\ q \geq 0, R \geq 0 \end{cases} \tag{8}$$

We solve the distributionally robust optimization models M_{11} and M_{12} with a two-stage optimization approach. That is, first we focus on the expected profit in the worst demand scenario, namely $\min_{G\in\Psi} \Pi(q,R)$. Then we maximize the profit function $\min_{G\in\Psi} \Pi(q,R)$ to obtain the optimal joint strategies on order quantity and sustainable technology investment.

Referring to Gallego et al. [27], the following lemma is given to show the relevant properties of the distribution function of stochastic demand.

Lemma 3.1. If the distribution function G belongs to set Ψ on $[0, +\infty)$ satisfied $\mathbb{E}(\varepsilon) = \mu$ and $Var(\varepsilon) = \sigma^2$, then the following inequality holds.

$$\mathbb{E}(\varepsilon - z)^+ \leq \frac{\sqrt{\sigma^2 + (z-\mu)^2} - (z-\mu)}{2}. \tag{9}$$

And there exists a distribution function $G^* \in \Psi$ that makes Eq. (9) holds with equality.

For the convenience of solving the model, set $A_0 = \dfrac{y_0 \vartheta p^{-k_0}}{\tau}$ and let $z = \dfrac{q}{A_0}$ represent the stock factor [37]. Combined with Lemma 3.1 to solve model M_{11}, there exists a two-point cumulative distribution function $G^* \in \Psi$ to minimize $\Pi_{11}(q,R)$. The expected profit function is further simplified as

$$\begin{aligned} \min_{G^*\in\Psi} \Pi_{11}(z,R) &= A_0 \left\{ \frac{\tau(p+s)\left[(z+\mu) - \sqrt{\sigma^2 + (z-\mu)^2}\right]}{2} - s\mu\tau - (c_1 + c_2 + c_3 e) z \right\} \\ &\quad - R(1 - ac_3 + bc_3 R) + c_3 K \end{aligned} \tag{10}$$

To ensure the feasibility of the solution, we assume that $p > \left(c_1 + c_2 + c_3 e + \frac{e}{a}\right)\frac{1}{\tau}$. For simplicity, we define two parameters z_α and z_β.

$$z_\alpha = \mu + \frac{\sigma}{2}\left[\sqrt{\frac{(p+s)\tau - (c_1 + c_2 + c_3 e)}{c_1 + c_2 + c_3 e}} - \sqrt{\frac{c_1 + c_2 + c_3 e}{(p+s)\tau - (c_1 + c_2 + c_3 e)}}\right], \tag{11}$$

$$z_\beta = \mu + \frac{\sigma}{2}\left[\sqrt{\frac{a(p+s)\tau - (ac_1 + ac_2 + e)}{ac_1 + ac_2 + e}} - \sqrt{\frac{ac_1 + ac_2 + e}{a(p+s)\tau - (a_1 + ac_2 + e)}}\right]. \tag{12}$$

Using Eqs. (6), (10)–(12), the following theorem can be obtained by solving the model M_{11}.

Theorem 3.1. For model M_{11}, there exists the only optimal stock factor z_{11}^* and sustainable technology investment R_{11}^* to maximize expected profit. They can be expressed as

$$\left(z_{11}^*, R_{11}^*\right) =$$
$$\begin{cases} (z_\alpha, 0), & 0 < K \le ez_\alpha A_0 \text{ and } 0 < c_3 < \frac{1}{a}; \\ \left(\frac{K}{eA_0}, 0\right), & K > ez_\alpha A_0 \text{ and } 0 < c_3 < \frac{1}{a} \text{or} \\ & K > ez_\beta A_0 \text{ and } c_3 \ge \frac{1}{a}; \\ (z_{11}, R_{11}), & ez_\alpha A_0 - \frac{a^2}{4b} + \frac{1}{4bc_3^2} < K \le ez_\beta A_0 \text{ and } c_3 \ge \frac{1}{a}; \\ \arg\max_{z,R}\left\{\Pi_{11}^G\left(z_\alpha, \frac{ac_3 - 1}{2bc_3}\right), \Pi_{11}^G(z_{11}, R_{11})\right\}, & ez_\alpha A_0 - \frac{a^2}{4b} < K \le ez_\alpha A_0 - \frac{a^2}{4b} + \frac{1}{4bc_3^2} \\ & \text{and } c_3 \ge \frac{1}{a}; \\ \left(z_\alpha, \frac{ac_3 - 1}{2bc_3}\right), & 0 < K \le ez_\alpha A_0 - \frac{a^2}{4b} \text{ and } c_3 \ge \frac{1}{a}. \end{cases} \tag{13}$$

where

$$z_{11} = u + \frac{\sigma}{2}\left\{\sqrt{\frac{(a - 2bR_{11})(p+s)\tau - [(a - 2bR_{11})(c_1 + c_2) + e]}{(a - 2bR_{11})(c_1 + c_2) + e}} - \sqrt{\frac{(a - 2bR_{11})(c_1 + c_2) + e}{(a - 2bR_{11})(p+s)\tau - [(a - 2bR_{11})(c_1 + c_2) + e]}}\right\}$$

and

$$R_{11} = \frac{a - \sqrt{a^2 - 4b\left(ez_{11}A_0 - K\right)}}{2b}.$$

Please refer to Appendix A for the specific certification process. It can be seen that the optimal solution depends on the parameters of cap-and-offset regulation. Particularly, when $ez_\alpha A_0 - \frac{a^2}{4b} + \frac{1}{4bc_3^2} < K \le ez_\beta A_0$ and $c_3 \ge \frac{1}{a}$, the optimal joint strategies of model M_{11} is $(z_{11}^*, R_{11}^*) = (z_{11}, R_{11})$.

When $0 < K \le e z_\alpha A_0 - \frac{a^2}{4b}$ and $c_3 \ge \frac{1}{a}$, the optimal joint strategies of model M_{11} can be expressed as $\left(z_{11}^*, R_{11}^*\right) = \left(z_\alpha, \frac{ac_3 - 1}{2bc_3}\right)$. In the overlapping interval $e z_\alpha A_0 - \frac{a^2}{4b} < K \le e z_\alpha A_0 - \frac{a^2}{4b} + \frac{1}{4bc_3^2}$, we choose the solution that maximizes the objective function as the optimal solution.

From the proof of Theorem 3.1, we can also draw the following corollary that reflects the conditions of investment and relation between the total carbon emissions and carbon cap for model M_{11}.

Corollary 3.1. For the model M_{11}, the following conclusions hold.

(i) When $K > e z_\alpha A_0$ and $0 < c_3 < \frac{1}{a}$, or $K > e z_\beta A_0$ and $c_3 \ge \frac{1}{a}$, or $0 < K \le e z_\alpha A_0$ and $0 < c_3 < \frac{1}{a}$, the fresh product retailer does n't invest in low-carbon technology, i.e., $R_{11}^* = 0$.

(ii) When $K > e z_\alpha A_0$ and $0 < c_3 < \frac{1}{a}$, or $K > e z_\beta A_0$ and $c_3 \ge \frac{1}{a}$, or $e z_\alpha A_0 - \frac{a^2}{4b} + \frac{1}{4bc_3^2} < K \le e z_\beta A_0$ and $c_3 \ge \frac{1}{a}$, the carbon emissions generated by the retailer are equal to the carbon cap, i.e., $Y(q_{11}^*(z_{11}^*), R_{11}^*) = K$.

(iii) When $e z_\alpha A_0 - \frac{a^2}{4b} < K \le e z_\alpha A_0 - \frac{a^2}{4b} + \frac{1}{4bc_3^2}$ and $c_3 \ge \frac{1}{a}$, if $\Pi_{11}^G \left(z_\alpha, \frac{ac_3 - 1}{2bc_3}\right) < \Pi_{11}^G (z_{11}, R_{11})$ is true, then $Y(q_{11}^*(z_{11}^*), R_{11}^*) = K$; otherwise, $Y(q_{11}^*(z_{11}^*), R_{11}^*) > K$.

Next, we solve the distributionally robust optimization model M_{12}. Similar to the analysis of model M_{11}, using Lemma 3.1 and Eq. (7), the expected profit can be expressed as

$$\min_{G^* \in \Psi} \Pi_{12}(z, R) = A_0 \left\{ \frac{\tau (p+s) \left[(z+\mu) - \sqrt{\sigma^2 + (z-\mu)^2}\right]}{2} - s\mu\tau - (c_1 + c_2) z \right\} - R. \tag{14}$$

Defined z_0 as

$$z_0 = \mu + \frac{\sigma}{2} \left[\sqrt{\frac{(p+s)\tau - (c_1 + c_2)}{c_1 + c_2}} - \sqrt{\frac{c_1 + c_2}{(p+s)\tau - (c_1 + c_2)}} \right]. \tag{15}$$

Using Eqs. (8), (12), (14) and (15), we can draw the following theorem by solving model M_{12}.

Theorem 3.2. For model M_{12}, there exists the only optimal stock factor z_{12}^* and sustainable technology investment R_{12}^* to maximize expected profit under the condition of $K > e z_\beta A_0 - \frac{a^2}{4b}$. They can be expressed as

$$\left(z_{12}^*, R_{12}^*\right) = \begin{cases} (z_0, 0), & K \ge e z_0 A_0; \\ \left(\frac{K}{eA_0}, 0\right), & e z_\beta A_0 < K < e z_0 A_0; \\ (z_{11}, R_{11}), & e z_\beta A_0 - \frac{a^2}{4b} < K \le e z_\beta A_0. \end{cases} \tag{16}$$

Otherwise, there does not exist optimal solutions.

Above theorem indicates that when carbon emissions are restricted by carbon cap, the optimal joint strategy is easier to determine than in the case where carbon emissions exceed carbon cap. Additionally, the selection of optimal solution only depends on carbon cap K. However, in Theorem 3.1, the selection of optimal solution depends on both carbon cap K and carbon tax c_3.

According to Theorem 3.2, we can also draw the following corollary that reflects the conditions of investment and relation between the total carbon emissions and carbon cap for model M_{12}.

Corollary 3.2. For model M_{12}, the following conclusions hold.

(i) When $K \geq ez_\beta A_0$ holds, the fresh product retailer does not invest in low-carbon technology, i.e., $R^*_{12} = 0$.

(ii) When $ez_\beta A_0 - \dfrac{a^2}{4b} < K < ez_0 A_0$ holds, the carbon emissions generated by the retailer are equal to the carbon cap, i.e., $Y(q^*_{12}(z^*_{12}), R^*_{12}) = K$.

Combining with Theorem 3.1 and Theorem 3.2, we can get Theorem 3.3.

Theorem 3.3. For model M_1, there exists the only optimal stock factor z^*_1 and sustainable technology investment R^*_1 that maximizes expected profit function. They can be expressed as

$$\left(z^*_1, R^*_1\right) = \arg\max_{z,R} \left\{\Pi^G_{11}\left(z^*_{11}, R^*_{11}\right), \Pi^G_{12}\left(z^*_{12}, R^*_{12}\right)\right\} \tag{17}$$

Theorem 3.3 shows that the fresh product retailer can make the distributionally robust optimal decisions to maximize expected profit function when the random demand information is limited to know.

3.3 Performance Analysis

This subsection constructs another distributionally robust optimization model, denoted as M_2. M_2 represents the situation where the retailer doesn't invest in sustainable technology under cap-and-offset regulation. By comparing M_1 and M_2, we can further explore the impact of cap-and-offset regulation and emission reduction technology investment on operational decisions of the retailer.

Similar to model M_1, model M_2 can be expressed as Eq. (18).

$$\begin{aligned} (M_2) \qquad & \max_{q} \min_{G\in\Psi} \ \Pi_2(q) = \Pi_0(q) - c_3(Y(q,0) - K)^+ \\ s.t. \qquad & q \geq 0. \end{aligned} \tag{18}$$

Without investing in sustainable technology, M_{21} represents the robust optimization model when the emissions exceed carbon cap. M_{22} represents the robust optimization model when the emissions are no more than carbon cap. Similar to the previous proof, Theorem 3.4 can be obtained by solving model M_{21} and model M_{22} using Eq. (18).

Theorem 3.4. When the fresh product retailer does not invest in sustainable technology, the following conclusions hold.

(i) For Model M_{21}, there exists the only optimal stock factor $z^*_{21} = \max\left\{z_\alpha, \dfrac{K}{eA_0}\right\}$ to maximize the target profit $\Pi^G_{21}(z)$.

(ii) For Model M_{22}, there exists the only optimal stock factor $z_{22}^* = \min\left\{z_0, \frac{K}{eA_0}\right\}$ to maximize the target profit $\Pi_{22}^G(z)$.

(iii) For Model M_2, there exists the only optimal stock factor $z_2^* = \arg\max_z \left\{\Pi_{21}^G\left(z_{21}^*\right), \Pi_{22}^G\left(z_{22}^*\right)\right\}$ to maximize the target profit $\Pi_2^G(z)$.

Theorem 3.4 solves the analytic formula for the optimal solution of M_{21}, M_{22} and M_2. This shows that there exists the only optimal order strategy for the retailer without investing in sustainable technology under cap-and-offset regulation.

According to Theorem 3.3 and Theorem 3.4, we can obtain the following theorem.

Theorem 3.5. Comparing with M_1 and M_2, the following conclusions hold:

(i) $\Pi_1^G(z_1^*, R_1^*) \geq \Pi_2^G(z_2^*)$ and $Y(z_{12}^*, R_{12}^*) = Y(z_{22}^*, 0) \leq K$;

(ii) If $\left(z_{11}^*, R_{11}^*\right) = \left(\frac{K}{eA_0}, 0\right)$ or (z_{11}, R_{11}), then $K = Y(z_{11}^*, R_{11}^*) \leq Y(z_{21}^*, 0)$;

If $\left(z_{11}^*, R_{11}^*\right) = \left(z_\alpha, \frac{ac_3 - 1}{2bc_3}\right)$ and $z_{21}^* = z_\alpha$, then $K \leq Y(z_{11}^*, R_{11}^*) \leq Y(z_{21}^*, 0)$;

If $(z_{11}^*, R_{11}^*) = (z_\alpha, 0)$ and $z_{21}^* = z_\alpha$, then $K \leq Y(z_{11}^*, R_{11}^*) = Y(z_{21}^*, 0)$.

Theorem 3.5 compares the expected profits and carbon emissions between models M_1 and M_2. Compared with the case of not investing in sustainable technology, the retailer will earn more profits and generate lower carbon emissions. This may be due to the fact that investing in low-carbon technology promotes an increase of order quantity under cap-and-offset regulation. Thereby the fresh product retailer can achieve the target of reducing carbon emissions while increasing expected profits. It is conducive to achieve sustainable development for the fresh product retailer.

4 Computational Studies

In this subsection, the computational examples are reported. We explore the impact of some critical parameters, such as carbon cap K, unit carbon tax c_3, surviving index τ and freshness index ϑ, on the joint order quantity and sustainable technology investment strategies. The basic parameters are chosen as: $c_1 = 5$, $c_2 = 2$, $p = 15$, $y_0 = 600$, $k_0 = 2.32$, $s = 2$, $e = 6$, $a = 5$, $b = 0.01$, $\tau = 0.8639$, $\vartheta = 0.8930$, $c_3 = 0.34$, $K = 3800$, $\mu = 600$, $\sigma = 100$.

Based on the above situation, we solved the expected profits and carbon emissions of the retailer with only acquiring the mean and variance of stochastic demand. The calculation results are shown in Table 2.

Table 2: The optimal strategies under the cap-and-offset policy

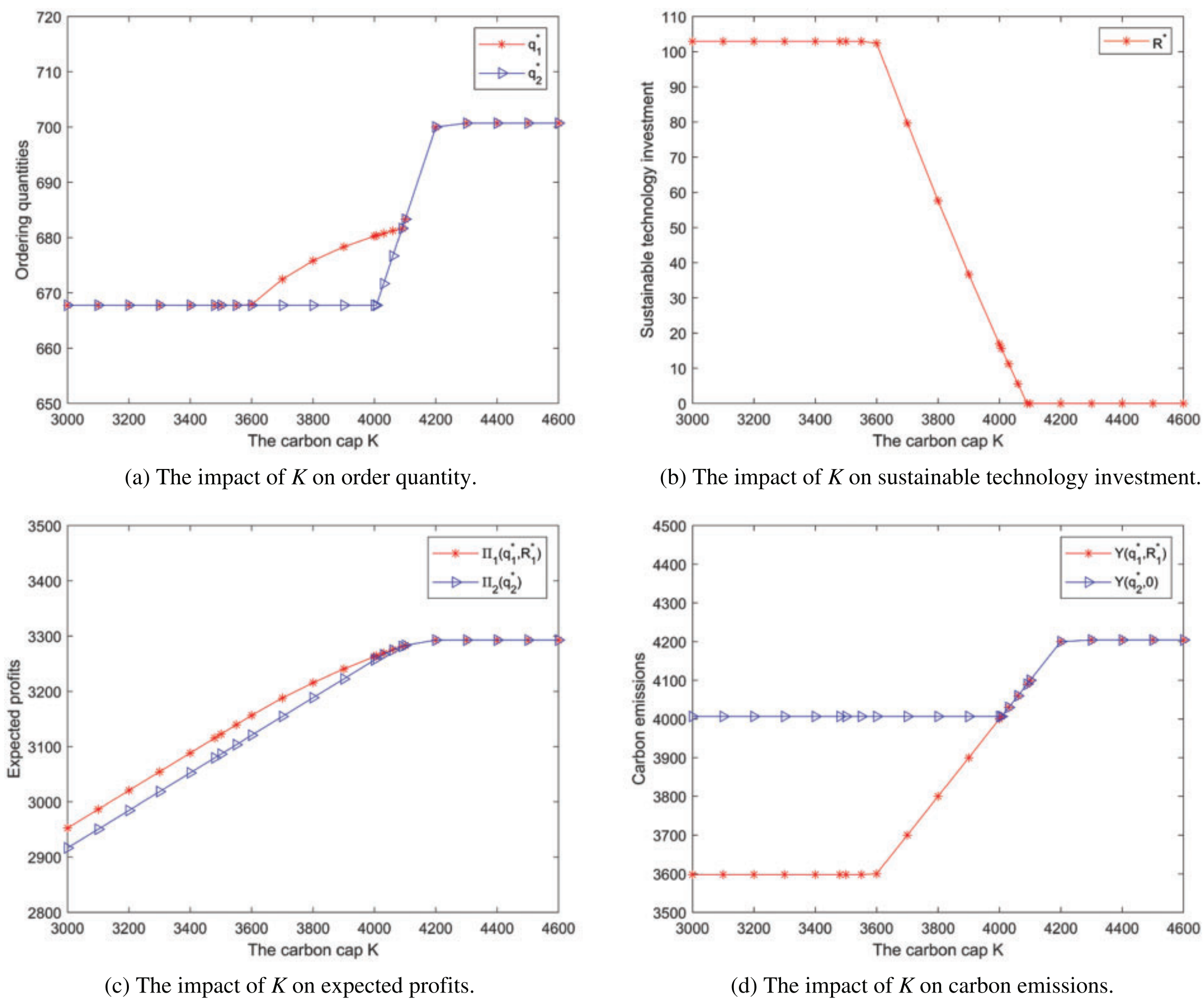
	Ordering quantities	Sustainable technology investments	Expected profits	Carbon emissions
M_1	675.82	57.62	3215.68	3800
M_2	667.76	0	3188.56	4006.54

From Table 2, the following observations can be made.

From the aspect of emissions, if the retailer decides to invest in sustainable technology under cap-and-offset regulation with limited distribution information, the optimal sustainable technology investments are 57.62, and the carbon emissions are 3800, which are equivalent to the carbon cap stipulated by authorities. When the retailer does not invest in sustainable technology, the carbon emissions are 4006.54 that exceed the carbon cap. It shows that investing in sustainable technologies can effectively lessen releasing greenhouse gases.

Moreover, the expected profits in the case of investing in sustainable technology increase by 0.8% and the carbon emissions decrease by 8.2% compared with the case of no investment. It demonstrates that under the cap-and-offset regulation, investing in sustainable technology is more conducive for the retailer to achieving higher expected profits and lower carbon emissions.

We further analyze the sensitivity effects of carbon cap K and carbon tax c_3 on the retailer's operational decisions. The corresponding changes are shown in Figs. 1 and 2.



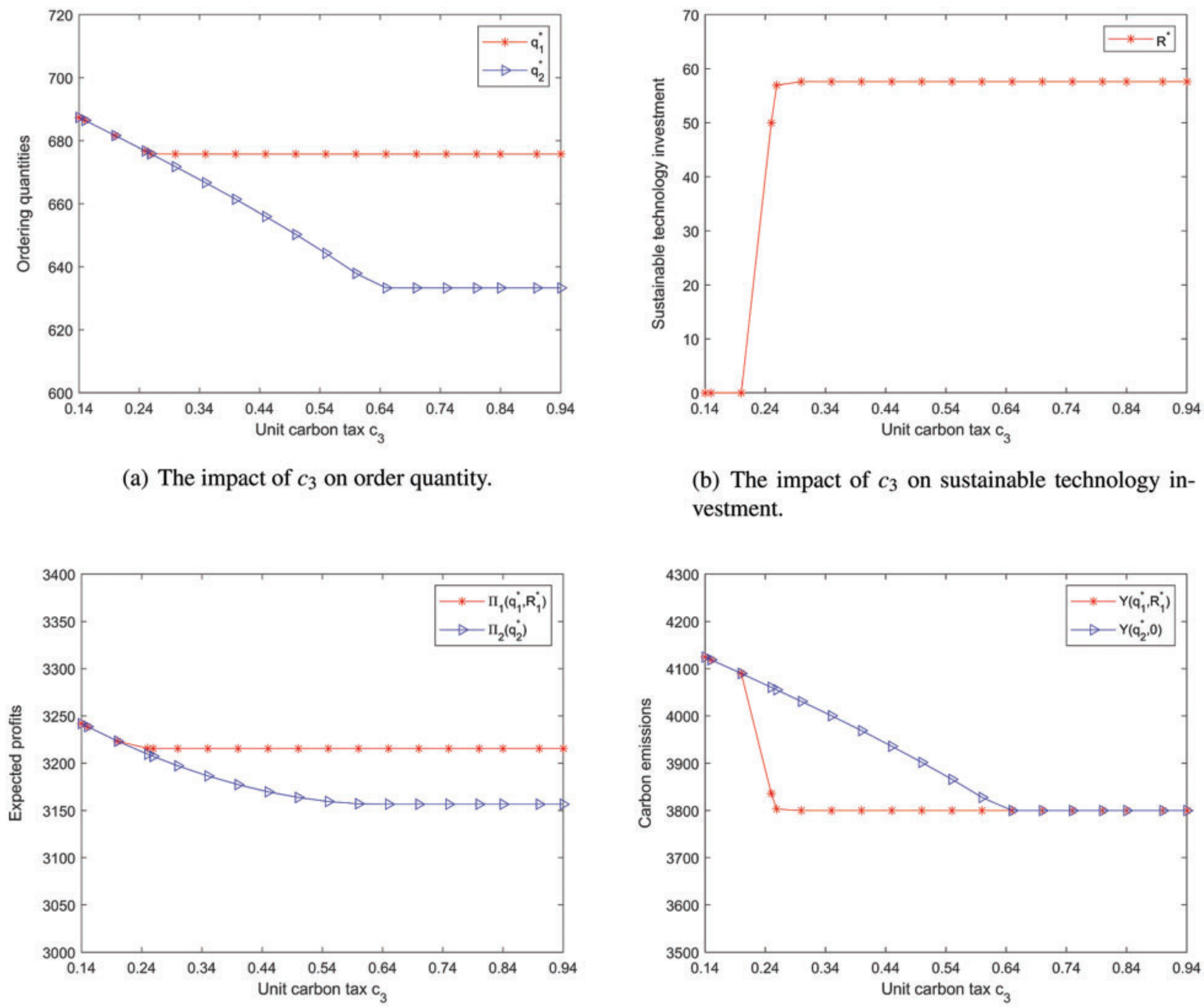
(a) The impact of K on order quantity.

(b) The impact of K on sustainable technology investment.

(c) The impact of K on expected profits.

(d) The impact of K on carbon emissions.

Figure 1: The impact of K on the retailer's operational decisions



(a) The impact of c_3 on order quantity.

(b) The impact of c_3 on sustainable technology investment.

(c) The impact of c_3 on expected profits.

(d) The impact of c_3 on carbon emissions.

Figure 2: The impact of c_3 on the retailer's operational decisions

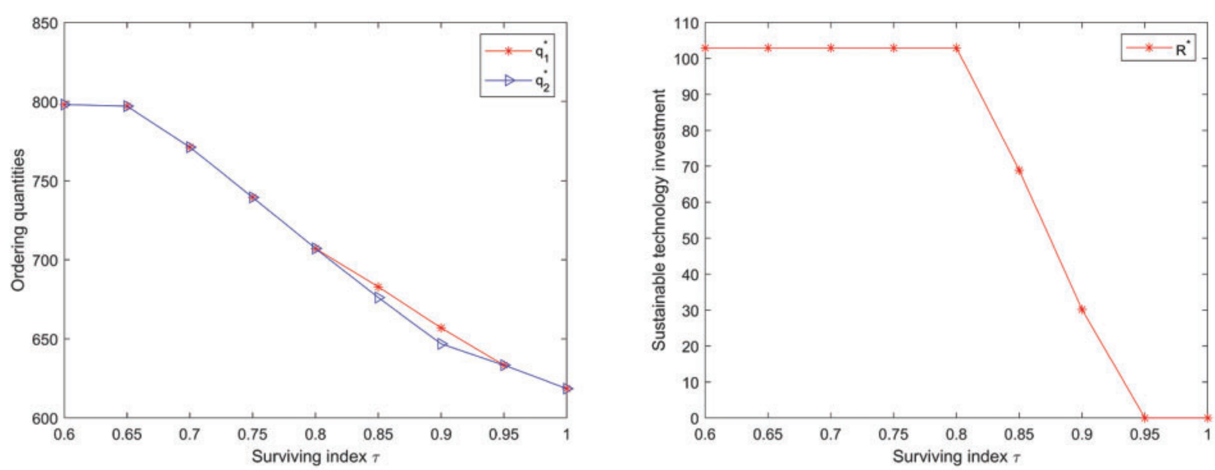
Fig. 1 depicts the impact of carbon cap K on the retailer's optimal strategies and operational effectiveness. It can be seen that, with the increase of K, the change trend of optimal order quantity and carbon emissions will firstly remain unchanged, then increase, and finally remain unchanged; the change trend of sustainable technology investments will firstly remain unchanged, then decrease, and finally remain unchanged; the change trend of expected profits will firstly increase and then remain unchanged. The change trend of the corresponding indicators in the cases of with and without investing in sustainable technology is similar. Moreover, when $K < 4000$, even if the retailer invests in sustainable technology, its expected profits still exceed that without investment, and its carbon emissions are lower than that without investment. However, when $K > 4000$, the retailer's expected profits and carbon emissions are equivalent in the both cases. This suggests that if the carbon cap is set too high, the restrictive efficacy of the cap-and-offset regulation on the retailer will be lost.

Fig. 2 depicts the impact of carbon tax c_3 on the retailer's optimal strategies and operational effectiveness. It can be seen that, with the increase of c_3, the change trend of optimal order quantity and carbon emissions will firstly decrease, and finally remain unchanged; the change trend of sustainable technology investment will firstly remain unchanged, then increase, and finally remain unchanged; the change trend of expected profits will firstly decrease and then remain unchanged. The change trend of the corresponding indicators in the cases of with and without investing in sustainable technology is similar. Moreover, when $c_3 < 0.65$, the carbon emissions after investment are significantly lower than that without investment. When $c_3 > 0.65$, the carbon emissions of investing and not investing in sustainable technology are equivalent and remain unchanged. This indicates that when c_3 is set too high, it is not only unfavorable to reduce carbon emissions, but also increases the heavy tax burden of retailers.

According to the above discussion, the increase of carbon cap could encourage retailer to order and increase operating profit. But the increase of carbon tax could restrict the retailer to ordering less products and lessen operating profit. Under the cap-and-offset regulation, the optimal order quantity and expected profits after investment are always no less than that without investment. As well as the carbon emissions are always no more than that without investment. This suggests that the cap-and-offset regulation can effectively promote the retailer to invest in sustainable technology. It is beneficial for the retailer to achieve the target of higher expected profits and lower carbon emissions.

Next, we analyze the impact of surviving index τ and freshness index ϑ on optimal strategies and operational effectiveness. The results are shown in Figs. 3 and 4.

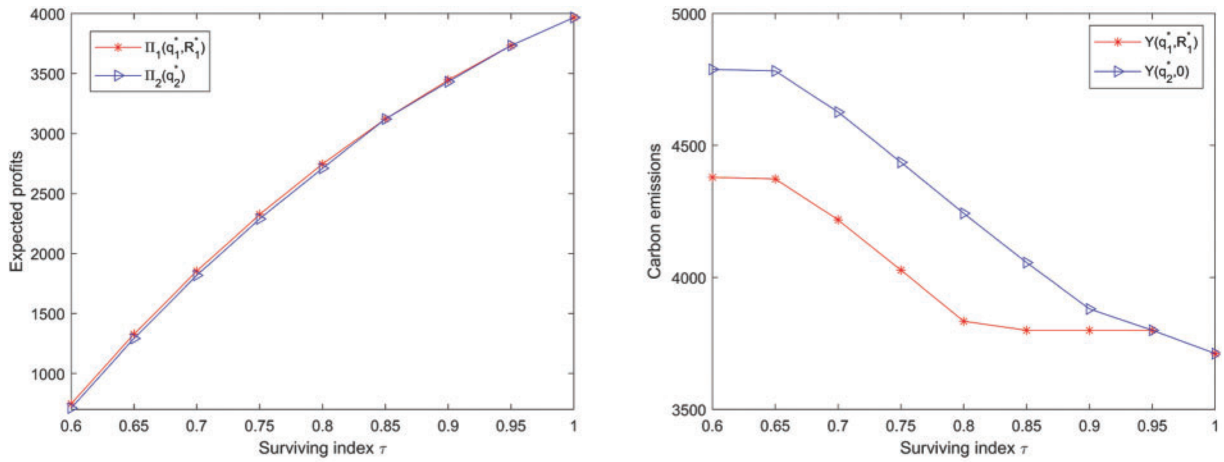
Fig. 3 depicts the impact of surviving index τ on optimal strategies. When the product has a high survivability and is not easy to decay, the fresh product retailer does not need to maintain a high inventory. So it will appropriately reduce the order quantity and will not invest too much capital in sustainable technology. Thus, the order cost, inventory cost and investment cost of the retailer are reduced. The retailer can obtain higher expected profits and release fewer emissions. If the survival rate of the product is too low, for example, when $\tau < 0.6$, the retailer cannot gain positive profit because the output cost is higher than the income.



(a) The impact of τ on order quantity.

(b) The impact of τ on sustainable technology investment.

Figure 3: (Continued)



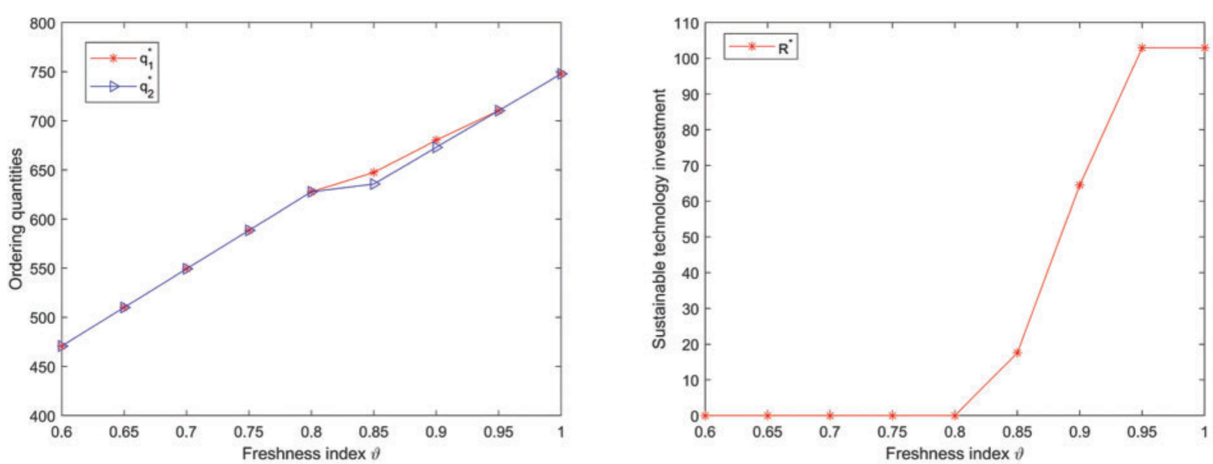
(c) The impact of τ on expected profits.

(d) The impact of τ on carbon emissions.

Figure 3: The impact of τ on the retailer's operational decisions

Fig. 4 depicts the impact of freshness index ϑ on optimal strategies that have a great difference from the impact of τ. This is because the freshness index affects the demand of consumers. When the product has a high freshness level, it stimulates the increase in consumer demand which further promotes the retailer to expand order quantity. At the same time, the retailer will increase its investment in sustainable technologies and achieve higher expected profits.

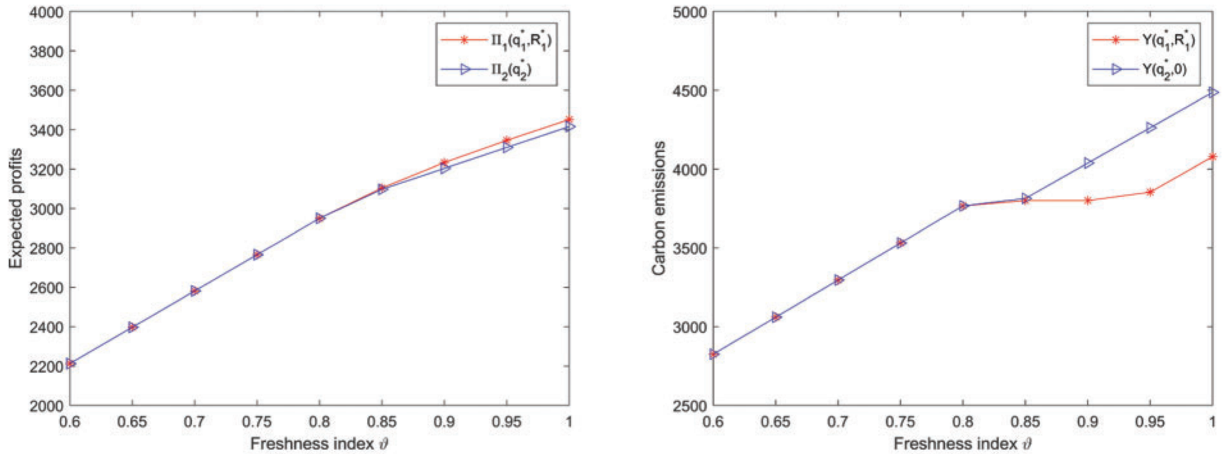
In addition, we can also find that no matter how the survival and freshness of products affect business operations, investing in sustainable technologies is more profitable and emits fewer emissions than that without investment under the cap-and-offset policy.



(a) The impact of ϑ on order quantity.

(b) The impact of ϑ on sustainable technology investment.

Figure 4: (Continued)



(c) The impact of ϑ on expected profits. (d) The impact of ϑ on carbon emissions.

Figure 4: The impact of ϑ on the retailer's operational decisions

5 Conclusions

With the enhancement of environmental awareness, the sustainable management concept brings new opportunities and challenges to the operation of fresh product retailers. Based on this background, we combine cap-and-offset regulation to research the optimal decisions of the fresh product retailer. Firstly, we construct distributionally robust newsvendor models where the information of stochastic factors in the market demand is limited to the mean and variance. We propose the analytic formula of joint decisions on order quantity and low-carbon technology investment by KKT conditions. A further comparison between the problems with investment and without investment is revealed. Finally, numerical studies are carried out to verify the impact of the critical carbon factors on the robustness of operational tactics. The results show that the cap-and-offset regulation can effectively encourage the retailer to invest in sustainable technologies and lead to higher expected profits and lower carbon emissions. This is consistent with the retailer's long-term goals and is conducive to achieving sustainable development. This paper mainly studies the optimal strategies from the angle of a single fresh product retailer. Further research can be conducted from the angle of the whole supply chain with fresh products in the future.

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Appendix A.

Proof of Theorem 3.1.

Let $\Pi_{11}^{G}(z,R) = \min_{G^*\in\Psi} \Pi_{11}(z,R)$. Taking the second partial derivative of $\Pi_{11}^{G}(z,R)$ with respect to z and R, we have $\frac{\partial^2 \Pi_{11}^{G}(z,R)}{\partial R^2} = -2bc_3 < 0$, $\frac{\partial^2 \Pi_{11}^{G}(z,R)}{\partial z^2} = \frac{-(p+s)\tau A_0\sigma^2}{2\left[\sigma^2+(z-\mu)^2\right]^{\frac{3}{2}}} < 0$, and $\frac{\partial^2 \Pi_{11}^{G}(z,R)}{\partial z \partial R} = 0$. As a result, it can be seen that Hessian matrix is a negative definite matrix and $\Pi_{11}^{G}(z,R)$ is strictly concave with respect to z and R.

Using Eqs. (3), (6) and (10), the Lagrange function of the model M_{11} is constructed as

$$\begin{aligned} L_1(z,R,\lambda,u_1,u_2) &= A_0\left\{\frac{\tau(p+s)\left[(z+\mu)-\sqrt{\sigma^2+(z-\mu)^2}\right]}{2} - s\mu\tau - (c_1+c_2+c_3e)z\right\} \\ &\quad + c_3K - R(1-ac_3+bc_3R) + \lambda\left(ezA_0 - aR + bR^2 - K\right) + u_1 z + u_2 R. \end{aligned} \tag{19}$$

KKT conditions are as follows:

$$A_0\left\{\frac{(p+s)\tau}{2}\left[1-\frac{z-\mu}{\sqrt{\sigma^2+(z-\mu)^2}}\right] - (c_1+c_2+c_3e)\right\} + \lambda e A_0 + u_1 = 0 \tag{20}$$

$$-1 + ac_3 - 2bc_3R - a\lambda + 2b\lambda R + u_2 = 0 \tag{21}$$

$$\lambda(ezA_0 - aR + bR^2 - K) = 0 \tag{22}$$

$$u_1 z = 0 \tag{23}$$

$$u_2 R = 0 \tag{24}$$

$$ezA_0 - aR + bR^2 \ge K \tag{25}$$

$$z \ge 0, R \ge 0, \lambda \ge 0, u_1 \ge 0, u_2 \ge 0 \tag{26}$$

In actual operation, the retailer pursues profit maximization, and the stock factor satisfies the condition $z > 0$. According to Eq. (23), it is easy to get $u_1 = 0$. Therefore, we solve the model M_{11} by analyzing the following cases.

Case 1. $\lambda = 0, u_1 = 0, u_2 > 0$. According to Eq. (24), it is easy to get $R = 0$. Further putting $\lambda = 0$ and $R = 0$ into Eq. (21) obtains $u_2 = 1 - ac_3 > 0$, i.e., $0 < c_3 < \frac{1}{a}$. Using $\lambda = 0$ and $u_1 = 0$ to simplify Eq. (20), we can get $z = z_\alpha$. Combined with $R = 0$, Eq. (25) can be simplified to $K \le ez_\alpha A_0$. Thus, when $0 < K \le ez_\alpha A_0$ and $0 < c_3 < \frac{1}{a}$, the optimal joint strategy of model M_{11} is $(z_{11}^*, R_{11}^*) = (z_\alpha, 0)$.

Case 2. $\lambda > 0, u_1 = 0, u_2 > 0$. In this case, $R = 0$ still holds. Further using $\lambda > 0$ and Eq. (22), we can get $z = \frac{K}{eA_0}$. Using $R = 0$ to simplify Eq. (21), we obtain $u_2 = 1 + a\lambda - ac_3$. Because of $u_2 > 0$, we have $\lambda > c_3 - \frac{1}{a}$. Using $u_1 = 0$ to simplify Eq. (20), we can get the following formula:

$$\lambda = \frac{c_1 + c_2 + c_3 e}{e} - \frac{(p+s)\tau}{2e}\left[1 - \frac{z-u}{\sqrt{\sigma^2 + (z-u)^2}}\right]. \tag{27}$$

Since $\frac{\mathrm{d}\lambda}{\mathrm{d}z} = \frac{(p+s)\,\tau\sigma^2\left[\sigma^2 + (z-u)^2\right]^{-\frac{3}{2}}}{2e} > 0$, λ is an increasing function of z.

If $c_3 < \frac{1}{a}$ is true, then λ satisfies the condition $\lambda > 0$. Combined with Eq. (27), we can get $z > z_\alpha$, i.e., $\frac{K}{eA_0} > z_\alpha$. On the other hand, if $c_3 \geq \frac{1}{a}$ is true, then λ satisfies the condition $\lambda > c_3 - \frac{1}{a} \geq 0$. Combined with Eq. (27), we have $z > z_\beta$, i.e., $\frac{K}{eA_0} > z_\beta$. Thus, when $K > eA_0 z_\alpha$ and $0 < c_3 < \frac{1}{a}$, or $K > eA_0 z_\beta$ and $c_3 \geq \frac{1}{a}$, the optimal joint strategy of model M_{11} is $\left(z_{11}^*, R_{11}^*\right) = \left(\frac{K}{eA_0}, 0\right)$.

Case 3. $\lambda > 0, u_1 = 0, u_2 = 0$. Substituting $u_2 = 0$ into Eq. (21) deduces the analytical formula of λ.

$$\lambda = c_3 - \frac{1}{a - 2bR} \tag{28}$$

Substituting Eqs. (28) into (20), the optimal stock factor of model M_{11} can be expressed as $z_{11}^* = z_{11}$. Combined with $0 \leq R \leq \frac{a}{2b}$, we have $a - 2bR > 0$ and $0 < \lambda \leq c_3 - \frac{1}{a}$, i.e., $c_3 \geq \frac{1}{a}$. Using Eq. (27), we know that λ is an increasing function of z. Thus, when $0 < \lambda \leq c_3 - \frac{1}{a}$, $z_\alpha < z_{11} \leq z_\beta$ holds. Otherwise, when $\lambda > 0$, using z_{11} to simplify Eq. (22), we obtain $ez_{11}A_0 - aR + bR^2 = K$. When $K < ez_\alpha A_0 - \frac{a^2}{4b}$, there does not exist any feasible solution of sustainable technology investment. When $K > ez_\alpha A_0 - \frac{a^2}{4b}$, combined with $0 \leq R < \frac{a}{2b}$, the optimal sustainable technology investment of model M_{11} is $R_{11}^* = R_{11} = \frac{a - \sqrt{a^2 - 4b\,(ez_{11}A_0 - K)}}{2b}$. In this case, $K \leq ez_\beta A_0$ holds. If not, the above equation yields $e(z_{11} - z_\beta)A_0 > aR_{11} - bR_{11}^2 \geq 0$. Thereby, $z_{11} > z_\beta$ holds, which contradicts $z_\alpha < z_{11} \leq z_\beta$. Thus, when $ez_\alpha A_0 - \frac{a^2}{4b} < K \leq ez_\beta A_0$ and $c_3 \geq \frac{1}{a}$, the optimal joint strategy of model M_{11} is $(z_{11}^*, R_{11}^*) = (z_{11}, R_{11})$.

Case 4. $\lambda = 0, u_1 = 0, u_2 = 0$. Substituting $\lambda = 0$ and $u_1 = 0$ into Eq. (20) obtains $z = z_\alpha$. Substitute $\lambda = 0$ and $u_2 = 0$ into Eq. (21), we have $R = \frac{ac_3 - 1}{2bc_3} < \frac{a}{2b}$. According to the non-negativity of sustainable technology investment, $c_3 \geq \frac{1}{a}$ holds. Using $R = \frac{ac_3 - 1}{2bc_3}$ and $z = z_\alpha$ to simplify Eq. (25), we get $K \le ez_\alpha A_0 - \frac{a^2}{4b} + \frac{1}{4bc_3^2}$. Thus, when $0 < K \le ez_\alpha A_0 - \frac{a^2}{4b} + \frac{1}{4bc_3^2}$ and $c_3 \ge \frac{1}{a}$, the optimal joint strategy of model M_{11} is $\left(z_{11}^*, R_{11}^*\right) = \left(z_\alpha, \frac{ac_3 - 1}{2bc_3}\right)$.

Case 5. Combining with Case 3 and Case 4 in the overlapping interval $ez_\alpha A_0 - \frac{a^2}{4b} < K \le ez_\alpha A_0 - \frac{a^2}{4b} + \frac{1}{4bc_3^2}$, we choose the solution that maximizes the objective function as the optimal solution.

Proof of Theorem 3.2.

Let $\Pi_{12}^G(z, R) = \min_{G^* \in \Psi} \Pi_{12}(z, R)$. Taking the second order partial derivative on $\Pi_{12}^G(z, R)$ with respect to z and R, we have $\frac{\partial^2 \Pi_{12}^G(z, R)}{\partial z^2} = \frac{-(p+s)\,\tau A_0 \sigma^2}{2\left[\sigma^2 + (z-\mu)^2\right]^{\frac{3}{2}}} < 0$, $\frac{\partial^2 \Pi_{12}^G(z, R)}{\partial R^2} = 0$, and $\frac{\partial^2 \Pi_{12}^G(z, R)}{\partial z \partial R} = 0$. As a result, it can be seen that $\Pi_{12}^G(z, R)$ is a concave function of z and R.

Using Eqs. (3), (8) and (14), the Lagrange function of the model M_{12} is constructed as

$$\begin{aligned} L_2(z, R, \lambda, u_1, u_2) &= A_0 \left\{ \frac{\tau\,(p+s)\left[(z+\mu) - \sqrt{\sigma^2 + (z-\mu)^2}\right]}{2} - s\mu\tau - (c_1 + c_2)\, z \right\} \\ &\quad - R + \lambda\left(K - ezA_0 + aR - bR^2\right) + u_1 z + u_2 R \end{aligned} \tag{29}$$

KKT conditions are as follows:

$$A_0 \left\{ \frac{(p+s)\tau}{2}\left[1 - \frac{z-\mu}{\sqrt{\sigma^2 + (z-\mu)^2}}\right] - (c_1 + c_2) \right\} - \lambda e A_0 + u_1 = 0 \tag{30}$$

$$-1 + a\lambda - 2b\lambda R + u_2 = 0 \tag{31}$$

$$\lambda(K - ezA_0 + aR - bR^2) = 0 \tag{32}$$

$$u_1 z = 0 \tag{33}$$

$$u_2 R = 0 \tag{34}$$

$$ezA_0 - aR + bR^2 \le K \tag{35}$$

$$z \ge 0, R \ge 0, \lambda \ge 0, u_1 \ge 0, u_2 \ge 0. \tag{36}$$

Similar to the proof of Theorem 3.1, we assume the stock factor satisfies $z > 0$. According to Eq. (33), it is easy to get $u_1 = 0$. Therefore, we solve the model M_{12} by analyzing the following four cases.

Case 1. $\lambda = 0, u_1 = 0, u_2 > 0$. According to Eqs. (34) and (31), it is easy to get $R = 0$ and $u_2 = 1$. Further putting $\lambda = 0$ and $u_1 = 0$ into Eq. (30), we can obtain $z = z_0$. Combined with Eq. (35), when $K \ge ez_0 A_0$, the optimal joint strategy of model M_{12} can be expressed as $(z_{12}^*, R_{12}^*) = (z_0, 0)$.

Case 2. $\lambda > 0, u_1 = 0, u_2 > 0$. According to Eq. (34), it is easy to get $R = 0$. Simplify Eq. (31), we can obtain $u_2 = 1 - a\lambda$ and $\lambda < \frac{1}{a}$. Further using $\lambda > 0$ and Eq. (32), we have $z = \frac{K}{eA_0}$. Substitute $u_1 = 0$ into Eq. (30), we obtain

$$\lambda = \frac{(p+s)\tau}{2e}\left[1 - \frac{z-u}{\sqrt{\sigma^2 + (z-u)^2}}\right] - \frac{(c_1+c_2)}{e}. \tag{37}$$

Taking the first derivative on λ with respect to z, we get $\frac{d\lambda}{dz} = -\frac{(p+s)\,\tau\sigma^2\left[\sigma^2 + (z-u)^2\right]^{-\frac{3}{2}}}{2e} < 0$, and λ is a decreasing function of z. Combined with $0 < \lambda < \frac{1}{a}$, the value range of z is $z_\beta < z < z_0$. Thus, when $ez_\beta A_0 < K < ez_0 A_0$, the optimal joint strategy of model M_{12} is $\left(z_{12}^*, R_{12}^*\right) = \left(\frac{K}{eA_0}, 0\right)$.

Case 3. $\lambda > 0, u_1 = 0, u_2 = 0$. Using $u_2 = 0$ and Eq. (31), we can get $\lambda = \frac{1}{a - 2bR}$. Combined with $0 \leq R < \frac{a}{2b}$, it is easy to know $\lambda \geq \frac{1}{a}$. Substitute it into Eq. (30), we have $z_{12}^* = z_{11}$. According to Eq. (37), we know that λ is a decreasing function of z. Because of $\lambda \geq \frac{1}{a}$, the formula of $z \leq z_\beta$ holds.

From Eq. (32), we can deduce $K - ezA_0 + aR - bR^2 = 0$. It is easy to verify when $K < ez_\beta A_0 - \frac{a^2}{4b}$, there doesn't exist any feasible solution to optimize the objective function. When $K > ez_\beta A_0 - \frac{a^2}{4b}$, according to $0 \leq R < \frac{a}{2b}$, the optimal sustainable technology investment of model M_{12} is $R_{12}^* = R_{11}$. In this case, $K \leq ez_\beta A_0$ holds. If not, the above equation yields $ezA_0 - aR + bR^2 > ez_\beta A_0$, i.e., $eA_0(z - z_\beta) > aR - bR^2 \geq 0$. It is in contradiction with $z \leq z_\beta$. Thus, when $ez_\beta A_0 - \frac{a^2}{4b} < K \leq ez_\beta A_0$, the optimal joint strategy of model M_{12} is $(z_{12}^*, R_{12}^*) = (z_{11}, R_{11})$.

Case 4. $\lambda = 0, u_1 = 0, u_2 = 0$. Substituting $\lambda = 0$ and $u_2 = 0$ into Eq. (31) shows that there doesn't exist any feasible solution to optimize the objective function.

Proof of Theorem 3.5.

(i) Using Eqs. (5) and (18), we have $\Pi_{11}(q, R) = \Pi_{21}(q) - R + c_3(aR - bR^2)$ or $\Pi_{11}(z, R) = \Pi_{21}(z) - R + c_3(aR - bR^2)$. Since the optimal solution $(z_{21}^*, 0)$ of model M_{21} is the feasible solution of model M_{11}, we obtain $\Pi_{11}^G(z_{11}^*, R_{11}^*) \geq \Pi_{11}^G(z_{21}^*, 0) = \Pi_{21}^G(z_{21}^*)$. Similarly, $\Pi_{12}(q, R) = \Pi_{22}(q) - R$ or $\Pi_{12}(z, R) = \Pi_{22}(z) - R$ can be deduced from Eqs. (7) and (18). Since the optimal solution $(z_{22}^*, 0)$ of model M_{22} is the feasible solution of model M_{12}, we have $\Pi_{12}^G(z_{12}^*, R_{12}^*) \geq \Pi_{12}^G(z_{22}^*, 0) = \Pi_{22}^G(z_{22}^*)$. Due to the fact that $\Pi_1^G(z_1^*, R_1^*) = \max\{\Pi_{11}^G(z_{11}^*, R_{11}^*), \Pi_{12}^G(z_{12}^*, R_{12}^*)\}$ and $\Pi_2^G(z_2^*) = \max\{\Pi_{21}^G(z_{21}^*), \Pi_{22}^G(z_{22}^*)\}$, the relationship between the expected profits of M_1 and M_2 is $\Pi_1^G(z_1^*, R_1^*) \geq \Pi_2^G(z_2^*)$.

According to the aforementioned proof, we know that if $K \geq ez_0 A_0$, then $(z_{12}^*, R_{12}^*) = (z_0, 0)$ and $z_{22}^* = z_0$. Substitute them into Eq. (3), we get $Y(z_{12}^*, R_{12}^*) = Y(z_{22}^*, 0) \leq K$. If $K < ez_0 A_0$, then $\left(z_{12}^*, R_{12}^*\right) = \left(\frac{K}{eA_0}, 0\right)$ or $(z_{12}^*, R_{12}^*) = (z_{11}, R_{11})$ and $z_{22}^* = \frac{K}{eA_0}$. Thus, $Y(z_{12}^*, R_{12}^*) = Y(z_{22}^*, 0) = K$ is obtained. The relationship between carbon emissions of M_1 and M_2 is $Y(z_{12}^*, R_{12}^*) = Y(z_{22}^*, 0) \leq K$.

(ii) According to Theorem 3.1 and Theorem 3.4, it is easy to verify the conclusion by taking (z_{11}^*, R_{11}^*) and z_{21}^* into the carbon emission function.