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ARTICLE

Fractal Fractional Order Operators in Computational Techniques for Mathematical Models in Epidemiology

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ABSTRACT

New fractional operators, the COVID-19 model has been studied in this paper. By using different numerical techniques and the time fractional parameters, the mechanical characteristics of the fractional order model are identified. The uniqueness and existence have been established. The model's Ulam-Hyers stability analysis has been found. In order to justify the theoretical results, numerical simulations are carried out for the presented method in the range of fractional order to show the implications of fractional and fractal orders. We applied very effective numerical techniques to obtain the solutions of the model and simulations. Also, we present conditions of existence for a solution to the proposed epidemic model and to calculate the reproduction number in certain state conditions of the analyzed dynamic system. COVID-19 fractional order model for the case of Wuhan, China, is offered for analysis with simulations in order to determine the possible efficacy of Coronavirus disease transmission in the Community. For this reason, we employed the COVID-19 fractal fractional derivative model in the example of Wuhan, China, with the given beginning conditions. In conclusion, again the mathematical models with fractional operators can facilitate the improvement of decision-making for measures to be taken in the management of an epidemic situation.

KEYWORDS

COVID-19 model; fractal-fractional operator; Ulam-Hyers stability; existence and uniqueness; numerical simulation

1 Introduction

Coronavirus (COVID-19) is a new phenomenon in recent days, which affected the entire world while it was emerging. According to reference [1], it is reported that a mysterious outbreak of atypical pneumonia was traced to a seafood market in Wuhan, China. In December 2019 the first case of novel coronavirus was reported. The symptoms of coronavirus are dry cough, fever, fatigue and in severe cases, acute respiratory syndrome that appears in 2–10 days which further causes pneumonia, kidney failure and even death [2]. In between March and April, coronavirus became a global phenomenon with the whole world facing an emergency situation. Initial cases were reported in the wet seafood market of Wuhan, China [3]. That is why some researchers thought that it is transmitted by animals to humans. This virus is transmitted from one person to another through physical contact, droplets during sneezing and coughing [4]. Researchers in the field of epidemiology and other fields of biology are trying hard to develop the cure based on ongoing clinical trials, but different researching companies from different countries have developed the vaccine for COVID-19 in [5]. Developed countries like the USA, UK, Italy, Spain and many others are affected very badly, most of the global deaths are being reported from these countries [6]. Mathematical modeling is used to understand the dynamics and behavior of disease and then develop the procedures for the treatment of disease. For this purpose, many researchers developed the COVID-19 models (see [7–12]). Reproductive number has a notable role in the analysis of mathematical models. Reproductive number explains the behavior of the simulation of COVID-19. The fractional order mathematical models of a few more infectious diseases have recently been studied in [13].

In order to address problems in the real world, fractional calculus (FC) is essential. It is widely utilized in a variety of scientific, engineering, and financial sectors. The key characteristics of FC are fractional integrals and derivatives of fractional order. Researchers' interest in fractional calculus and the numerous aspects of that study under inquiry has grown in recent years. This is due to the fact that genetic mutations are a crucial tool for characterizing the dynamic operation of diverse biological systems. These component operators' non-local properties, which are absent from the integer separator operator, give them their power [14–16]. For the development of an artificial pancreas, Farman et al. [17] employed an Atangana Baleanu derivative to manage glucose levels in insulin treatments. Differential equations with various generalized fractional derivatives have been solved using a variety of numerical techniques [18–20]. In [21], a generalization of the squared remainder minimization method for resolving multi-term fractional differential equations was developed. The Caputo time-fractional derivative and redefined extended B-spline functions have been used for the time and spatial discretization, respectively in [22–25] and some details are also given in [26–28]. COVID-19 outbreaks have been well modeled in [29–31] for a variety of geographic locations. Additionally, some publications [32–34] explored the impact of quarantine and social isolation on the viral load in the environment. For the COVID-19 epidemic, some researchers suggested the best control approaches including cost-effectiveness assessments [35,36].

2 Basic Concepts of Fractional Operators

In this section, we consider some definition related to fractal fractional operator given in [31,34,37,38].

Definition 2.1: Let $0 \leq \sigma, \sigma_1 \leq 1$, then with power law type kernel the fractal-fractional derivative is described by:

$$ {}^{FFP}J_{0,t}^{\sigma,\sigma 1}\left(g\left(t\right)\right) = \frac{1}{\Gamma\left(m-\sigma\right)} \frac{d}{dt^{\sigma_1}} \int_0^t \left(t-s\right)^{m-\sigma-1} g\left(s\right) ds, \tag{1} $$

$$\frac{d}{ds^{\sigma_1}} g(s) = \lim_{t \to s} \frac{g(t) - g(s)}{t^{\sigma_1} - s^{\sigma_1}}.$$

Definition 2.2: Let $0 \le \sigma, \sigma_1 \le 1$, then with the exponential-decay type kernel the fractal fractional derivative is described by:

$$^{FFP}J_{0,t}^{\sigma,\sigma 1}(g(t)) = \frac{\mathrm{M}(\sigma)}{\Gamma(m-\sigma)} \frac{d}{dt^{\sigma_1}} \int_0^t exp\left[-\frac{\sigma}{1-\sigma}(t-s)^{\nu-\sigma-1}\right] g(s)\, ds, \tag{2}$$

where $\sigma > 0,\ \sigma_1 \le \nu \in N$, and $M(0) = M(1) = 1$.

Definition 2.3: Let $0 \le \sigma, \sigma_1 \le 1$, then with the generalized Mittag-Leffler type kernel the fractal fractional derivative is described by:

$$^{FFM}J_{0,t}^{\sigma,\sigma 1}(g(t)) = \frac{\mathrm{AB}(\sigma)}{1-\sigma} \frac{d}{dt^{\sigma_1}} \int_0^t E_\sigma\left[-\frac{\sigma}{1-\sigma}(t-s)^{\sigma}\right] g(s)\, ds, \tag{3}$$

where $\sigma > 0,\ \sigma_1 \le 1$, and $AB(\sigma) = 1 - \sigma + \dfrac{\sigma}{\Gamma(\sigma)}$.

Definition 2.4: The function $g(t)$ of order (σ, σ_1), for fractal-fractional integral with power law type kernel is described by:

$$^{FFP}I_{0,t}^{\sigma,\sigma 1}(g(t)) = \frac{1}{\Gamma(\sigma_1)} \int_0^t s^{1-\sigma_1}(t-s)^{\sigma-1} g(s)\, ds. \tag{4}$$

Definition 2.5: The function $g(t)$ of order (σ, σ_1), for fractal-fractional integral with exponential-decay type kernel is described by:

$$^{FFP}I_{0,t}^{\sigma,\sigma 1}(g(t)) = \frac{\sigma_1(1-\sigma)t^{\sigma_1-1}g(t)}{\mathrm{M}(\sigma)} + \frac{\sigma\sigma_1}{\mathrm{M}(\sigma)} \int_0^t s^{\sigma-1} g(s)\, ds. \tag{5}$$

Definition 2.6: The function $g(t)$ of order (σ, σ_1), for fractal-fractional integral with Mittag-Leffler type kernel is described by:

$$^{FFM}J_{0,t}^{\sigma,\sigma 1}(g(t)) = \frac{\sigma_1(1-\sigma)t^{\sigma_1-1}g(t)}{\mathrm{AB}(\sigma)} + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)} \int_0^t s^{\sigma-1}(t-s)\, g(s)\, ds. \tag{6}$$

3 Fractal Fractional Order Model

We suppose the COVID-19 model formulated by Ahmad et al. [39]. In this model, $S(t)$ represents susceptible individuals, $H(t)$ represents resistant or healthy individuals, infected individuals are represented by $I(t)$ and $Q(t)$ represents quarantined individuals. We suppose that the used parameters in the model are non-negative. Hence, $N(t) = S(t) + H(t) + I(t) + Q(t)$. In this model, recruitment rate of susceptible individuals is represented by λ, γ denotes disease transmission rate, Recruitment rate of healthy people is α, Healthy people transmission rate is denoted by β, Cure rate of the infected individuals in the quarantined compartment is θ, δ which represent the rate at which quarantined people get infections, μ represents death rate of suspected or infected individuals due to disease and d denotes natural death rate. We present the COVID-19 classical model [39] using fractal-fractional Atangana–Baleanu derivative. We have the following model:

$$\begin{cases} ^{FF}D_{0,t}^{\sigma,\sigma 1}S(t) = \lambda - \gamma S(t)I(t) - (d+\mu)S(t) \\ ^{FF}D_{0,t}^{\sigma,\sigma 1}H(t) = \alpha - \beta H(t)I(t) + \theta I(t) - (d+\mu)H(t) \\ ^{FF}D_{0,t}^{\sigma,\sigma 1}I(t) = \gamma S(t)I(t) + \beta H(t)I(t) + \delta Q(t) - (d+\mu+\eta+\theta)I(t) \\ ^{FF}D_{0,t}^{\sigma,\sigma 1}Q(t) = \eta I(t) - (d+\mu+\delta)Q(t) \end{cases}, \tag{7}$$

with initial conditions are

$$S(0) \geq 0,\ H(0) \geq 0,\ I(0) \geq 0 \ and \ Q(0) \geq 0. \tag{8}$$

3.1 Equilibrium Points

In this section, we will discuss the equilibrium points of the given COVID-19 model (7). Equilibrium points have two types namely as disease free equilibrium and endemic equilibrium. We obtained these points by putting the number zero on the right side of the system (7). We suppose that E' represents disease free equilibrium and endemic equilibrium is represented by E*. If we take both of our equilibriums, we have

$$E' = (S', H', I', Q') = \left(\frac{\lambda}{d+\mu}, \frac{\alpha}{d+\mu}, 0, 0\right)$$

$$S^* = \frac{\lambda}{\gamma I^* + d + \mu},\ H^* = \frac{\lambda + \theta I^*}{\beta I^* + d + \mu},\ I^* = \frac{\delta Q^*}{(d+\mu+\eta+\theta) - \gamma S^* - \beta H^*},\ Q^* = \frac{\eta I^*}{(d+\mu+\delta)}.$$

We obtain the basic reproductive number R_0 by [37], we have

$$R_0 = \frac{(\gamma\lambda + \beta\alpha)(d+\mu+\delta)}{(d+\mu)[(d+\mu+\eta+\theta)(d+\mu+\delta) - \delta\eta]}.$$

4 Existence and Stability Theory

4.1 Existence

We consider [40]

$$\begin{cases} {}_0^{ABR}D_0^{\sigma} S(t) = \sigma_1 t^{\sigma_1 - 1} C(t, S, H, I, Q) \\ {}_0^{ABR}D_0^{\sigma} H(t) = \sigma_1 t^{\sigma_1 - 1} D(t, S, H, I, Q) \\ {}_0^{ABR}D_0^{\sigma} I(t) = \sigma_1 t^{\sigma_1 - 1} E(t, S, H, I, Q) \\ {}_0^{ABR}D_0^{\sigma} Q(t) = \sigma_1 t^{\sigma_1 - 1} F(t, S, H, I, Q) \end{cases}, \tag{9}$$

where

$$C(t, S, H, I, Q) = \lambda - \gamma S(t) I(t) - (d+\mu) S(t),$$

$$D(t, S, H, I, Q) = \alpha - \beta H(t) I(t) + \theta I(t) - (d+\mu) H(t),$$

$$E(t, S, H, I, Q) = \gamma S(t) I(t) + \beta H(t) I(t) + \delta Q(t) - (d+\mu+\eta+\theta) I(t),$$

$$F(t, S, H, I, Q) = \eta I(t) - (d+\mu+\delta) Q(t).$$

We can write system (9) as:

$$\begin{cases} {}_0^{ABR}D_t^{\sigma} \Pi(t) = \sigma_1 t^{\sigma_1 - 1} \Lambda(t, \Pi(t)) \\ \Pi(0) = \Pi_0 \end{cases}, \tag{9'}$$

By replacing ${}_0^{ABR}D_0^{\sigma,\sigma 1}$ by ${}_0^{ABC}D_0^{\sigma,\sigma 1}$ and applying fractional integral, we get

$$\Pi(t) = \Pi(0) + \frac{\sigma_1 t^{\sigma_1 - 1}(1-\sigma)}{AB(\sigma)} \Lambda(t, \Pi(t)) + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)} \int_0^t \tau^{\sigma_1 - 1}(t-\tau)^{\sigma_1 - 1} \Lambda(\tau, \Pi(\tau))\, d\tau,$$

where

$$\Pi(t) = \begin{cases} S(t) \\ H(t) \\ I(t) \\ Q(t) \end{cases}, \quad \Pi(0) = \begin{cases} S(0) \\ H(0) \\ I(0) \\ Q(0) \end{cases}, \quad \Lambda(t, \Pi(t)) = \begin{cases} C(t, S, H, I, Q) \\ D(t, S, H, I, Q) \\ E(t, S, H, I, Q) \\ F(t, S, H, I, Q) \end{cases}$$

We describe a Banach space $B = C \times C \times C \times C$, where $C = [0, T]$ under the norm

$$\|\Pi\| = \max_{t\in[0,T]} |S(t) + H(t) + I(t) + Q(t)|$$

Define as operator $\aleph : B \to B$ as:

$$\aleph(\Pi)(t) = \Pi(0) + \frac{\sigma_1 t^{\sigma_1-1}(1-\sigma)}{AB(\sigma)}\Lambda(t, \Pi(t)) + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}\int_0^t \tau^{\sigma_1-1}(t-\tau)^{\sigma_1-1}\Lambda(\tau, \Pi(\tau))\, d\tau. \quad (10)$$

We suppose that

- For each $\Pi \in \mathrm{B}, \exists$ constants $C_\Lambda > 0$ and M_Λ such that

$$|\Lambda(t, \Pi(t))| \le C_\Lambda |\Pi(t)| + M_\Lambda. \quad (11)$$

- Considering $\Pi, \overline{\Pi} \in \mathrm{B}, \exists$ a constant $L_\Lambda > 0$ such that

$$\left|\Lambda(t, \Pi(t)) - \Lambda\left(t, \overline{\Pi}(t)\right)\right| \le L_\Lambda \left|\Pi(t) - \overline{\Pi}(t)\right|. \quad (12)$$

Theorem 4.1: Suppose that the state (11) exists. Let $\Lambda : [0, T] \times B \to R$ be a continuous function. The system having at least one solution, the condition given in [41,42].

Proof: First of all, considering the Eq. (10) is completely continuous which is described by operator $\aleph$. Since Λ and $\aleph$ are continuous operators,

Suppose that $H = \backslash\{\Pi \in \mathrm{B} : \|\Pi\| \le R,\ R > 0\backslash\}$. For some $\Pi \in \mathrm{B}$, we have

$$|\aleph(\Pi)| = \max_{t\in[0,T]}\left|\Pi(0) + \frac{\sigma_1 t^{\sigma_1-1}(1-\sigma)}{AB(\sigma)}\Lambda(t, \Pi(t)) + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}\int_0^t \tau^{\sigma_1-1}(t-\tau)^{\sigma_1-1}\Lambda(\tau, \Pi(\tau))\, d\tau\right|$$

$$\le \Pi(0) + \frac{\sigma_1 T^{\sigma_1-1}(1-\sigma)}{AB(\sigma)}(C_\Lambda\|\Pi\| + M_\Lambda) + \max_{t\in[0,T]}\frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}\int_0^t \tau^{\sigma_1-1}(t-\tau)^{\sigma_1-1}\Lambda(\tau, \Pi(\tau))\, d\tau$$

$$\le \Pi(0) + \frac{\sigma_1 T^{\sigma_1-1}(1-\sigma)}{AB(\sigma)}(C_\Lambda\|\Pi\| + M_\Lambda) + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}(C_\Lambda\|\Pi\| + M_\Lambda)T^{\sigma+\sigma_1-1}H(\sigma, \sigma_1)$$

$$\le R.$$

Therefore, we get

$$|\aleph(\Pi)(t_2) - \aleph(\Pi)(t_1)|$$

$$= \left|\frac{\sigma_1 t_2^{\sigma_1-1}(1-\sigma)}{AB(\sigma)}\Lambda(t_2, \Pi(t_2)) + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}\int_0^{t_2} \tau^{\sigma_1-1}(t_2-\tau)^{\sigma_1-1}\Lambda(\tau, \Pi(\tau))\, d\tau\right.$$

$$\left.- \frac{\sigma_1 t_1^{\sigma_1-1}(1-\sigma)}{AB(\sigma)}\Lambda(t_1, \Pi(t_1)) + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}\int_0^{t_1} \tau^{\sigma_1-1}(t_1-\tau)^{\sigma_1-1}\Lambda(\tau, \Pi(\tau))\, d\tau\right|$$

$$\leq \left| \frac{\sigma_1 t_2^{\sigma_1-1}(1-\sigma)}{AB(\sigma)}(C_\Lambda |\Pi(t)| + M_\Lambda) + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}(C_\Lambda |\Pi(t)| + M_\Lambda)\, t_2^{\sigma+\sigma_1-1} H(\sigma,\sigma_1) \right.$$

$$\left. - \frac{\sigma_1 t_1^{\sigma_1-1}(1-\sigma)}{AB(\sigma)}(C_\Lambda |\Pi(t)| + M_\Lambda) + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}(C_\Lambda |\Pi(t)| + M_\Lambda)\, t_1^{\sigma+\sigma_1-1} H(\sigma,\sigma_1) \right|$$

When $t_1 \to t_2$ then $|\aleph(\Pi)(t_2) - \aleph(\Pi)(t_1)| \to 0|$.

$\|\aleph(\Pi)(t_2) - \aleph(\Pi)(t_1)\| \to 0,\ as\ t_1 \to t_2.$

Thus, $\aleph$ is equicontinuous. Then, by Schauder's fixed point result, the condition is held.

Theorem 4.2: [38] Suppose that the condition (12) holds. If $\rho < 1$, where

$$\rho = \left(\frac{\sigma_1 T^{\sigma_1-1}(1-\sigma)}{AB(\sigma)} + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}(C_\Lambda \|\Pi\| + M_\Lambda)\, T^{\sigma+\sigma_1-1} H(\sigma,\sigma_1) \right) L_\Lambda.$$

Then the solution of the system is unique.

Proof: For all $\Pi, \overline{\Pi} \in \mathrm{B}$, acquire the following:

$$\left|\aleph(\Pi) - \aleph\left(\overline{\Pi}\right)\right| = \max_{t\in[0,T]} \left| \frac{\sigma_1 t^{\sigma_1-1}(1-\sigma)}{AB(\sigma)} \left(\Lambda(t,\Pi(t)) - \Lambda\left(t,\overline{\Pi}(t)\right)\right) \right.$$

$$\left. + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)} \int_0^t \tau^{\sigma_1-1}(t-\tau)^{\sigma_1-1} d\tau \left(\Lambda(\tau,\Pi(\tau)) - \Lambda\left(\tau,\overline{\Pi}(\tau)\right)\right) \right|$$

$$\leq \left[\frac{\sigma_1 T^{\sigma_1-1}(1-\sigma)}{AB(\sigma)} + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}(C_\Lambda \|\Pi\| + M_\Lambda)\, T^{\sigma+\sigma_1-1} H(\sigma,\sigma_1) \right] \|\Pi - \overline{\Pi}\|$$

$$\leq \rho \|\Pi - \overline{\Pi}\|$$

Therefore, $\aleph$ is a contraction. Thus, the solution of the system is unique according to Banach contraction principle [43].

We denote by:

$$\begin{cases} T_1(S(t), H(t), I(t), Q(t)) = S(t) - {}_0^{ABR}D_0^{\sigma} S(t) + \sigma_1 t^{\sigma_1-1} C(t,S,H,I,Q) \\ T_2(S(t), H(t), I(t), Q(t)) = H(t) - {}_0^{ABR}D_0^{\sigma} H(t) + \sigma_1 t^{\sigma_1-1} D(t,S,H,I,Q) \\ T_3(S(t), H(t), I(t), Q(t)) = I(t) - {}_0^{ABR}D_0^{\sigma} I(t) + \sigma_1 t^{\sigma_1-1} E(t,S,H,I,Q) \\ T_4(S(t), H(t), I(t), Q(t)) = Q(t) - {}_0^{ABR}D_0^{\sigma} Q(t) + \sigma_1 t^{\sigma_1-1} F(t,S,H,I,Q) \end{cases} . \tag{13}$$

Let $Pcl(R)$ be the set of all nonempty closed subsets of R.

Theorem 4.3: [44] Let $A \in M_{m,m}(R_+)$. The following are equivalents:

(i) A is a matrix which converges to zero;

(ii) $A^n \to 0$ as $n \to \infty$;

(iii) The modulus for every eigen-values of A is lower than *1*;

(iv) The matrix I—A is non-singular, with $(I-A)^{-1} = I + A + \cdots + A^n + \cdots$.

We give the following theorem, for the hypothesis that $T_i : R \to Pcl(R)$ for $i \in \{1,\ldots,4\}$ are contractions. $Pcl(R)$ is the set of all nonempty closed subsets of R, where R represents the set of real numbers. Examples of conditions for them to be contractions are, for instance, the cases in which the absolute values of the derivatives are lower than 1. This situation is possible when the variations of functions $S(t)$, $H(t)$, $I(t)$, $Q(t)$ are low.

Theorem 4.4: Let $T_i : R \to Pcl(R)$ for $i \in \{1, \ldots, 4\}$ be contractions and $0 \le a_{kk} \le 1, k \in \{1, \ldots, 4\}$. Let $(S(t_1), H(t_1), I(t_1), Q(t_1)), (S(t_2), H(t_2), I(t_2), Q(t_2)) \in R^4$ where t_1 and t_2 are moments from a time interval J. If for each $y_k = T_k(S(t_1), H(t_1), I(t_1), Q(t_1))$, $k \in \{1, \ldots, 4\}$ there exists $z_k = T_k(S(t_1), H(t_1), I(t_1), Q(t_1))$ such that for all $k \in \{1, \ldots, 4\}$ in [44]:

$$|y_k - z_k| \le a_{11} |S(t_2) - S(t_1)|,$$

$$|y_k - z_k| \le a_{22} |H(t_2) - H(t_1)|,$$

$$|y_k - z_k| \le a_{33} |I(t_2) - I(t_1)|,$$

$$|y_k - z_k| \le a_{44} |Q(t_2) - Q(t_1)|,$$

then, the semi linear inclusion system:

$$\begin{cases} S(t) \in T_1(S(t), H(t), I(t), Q(t)) \\ H(t) \in T_2(S(t), H(t), I(t), Q(t)) \\ I(t) \in T_3(S(t), H(t), I(t), Q(t)) \\ Q(t) \in T_4(S(t), H(t), I(t), Q(t)) \end{cases} \tag{14}$$

has at least one solution in R^4.

Proof: The theorem is a particular case of Theorem 3.11 from [44]. It is demonstrated the same as Theorem 3.11 from [44], with $(u_1, u_2, u_3, u_4) = (S(t_1), H(t_1), I(t_1), Q(t_1))$ and $(v_1, v_2, v_3, v_4) = (S(t_2), H(t_2), I(t_2), Q(t_2))$ and working with $\|u - v\| = \begin{pmatrix} |u_1 - v_1| \\ |u_2 - v_2| \\ |u_3 - v_3| \\ |u_4 - v_4| \end{pmatrix}$. In this case, the diagonal matrix $\mathrm{A} = (\mathrm{a_{ij}})$ converges to 0.

The demonstration of the theorem uses elements from the fixed-point theory and results from the fact that $T = (T_1, \ldots, T_4) : R^4 \to Pcl(R^4)$ is a multivalued operator A-contraction to the left, thus T is an MWP operator. The concept of multivalued weakly Picard operator (briefly MWP operator) was introduced by Rus et al. in [45]. The authors created this concept in connection to the successive approximation technique for the fixed-point set of multivalued operators defined on a complete metric space. As R^4 is a Banach space, T has at least one fixed point [44], therefore, the conclusion to this theorem is verified.

4.2 Ulam-Hyers Stability

Definition 4.1: The system is Ulam-Hyers stable if $\exists\ \aleph_{\sigma,\sigma_1} \ge 0$ such that for any $\varepsilon > 0$ and for every $\Pi \in (C[0,T], R)$ is satisfied the following:

$$\left|{}_0^{FFM}D_t^{\sigma,\sigma 1}\Pi(t) - \Lambda(t, \Pi(t))\right| \le \varepsilon,\ t \in [0,T],$$

And there exists unique solution $\Omega \in (C[0,T], R)$ such that

$$|\Pi(t) - \Omega(t)| \le \aleph_{\sigma,\sigma_1}\varepsilon,\ t \in [0,T],$$

- $\Phi(t) \le \varepsilon$ *for* $\varepsilon > 0$.
- ${}_{0}^{FFM}D_{t}^{\sigma,\sigma 1}\Pi(t) = \Lambda(t, \Pi(t)) + \Phi(t)$.

Lemma 4.1: Perturbed solution for the system according the given result in [37].

${}_{0}^{FFM}D_{t}^{\sigma,\sigma 1}\Pi(t) = \Lambda(t, \Pi(t)) + \phi(t)$

$\Pi(0) = \Pi_0$

satisfies the following relation:

$$\left|\aleph(t) - \left(\Pi(0) + \frac{\sigma_1 t^{\sigma_1-1}(1-\sigma)}{AB(\sigma)}\Lambda(t,\Pi(t)) + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}\int_0^t \tau^{\sigma_1-1}(t-\tau)^{\sigma_1-1}\Lambda(\tau,\Pi(\tau))\,d\tau\right)\right|$$
$$\le \left(\frac{\sigma_1 T^{\sigma_1-1}(1-\sigma)}{AB(\sigma)} + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}T^{\sigma+\sigma_1-1}H(\sigma,\sigma_1)\right)\varepsilon, \tag{15}$$

We note:

$$\sigma^*_{\sigma,\sigma_1} = \frac{\sigma_1 T^{\sigma_1-1}(1-\sigma)}{AB(\sigma)} + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}T^{\sigma+\sigma_1-1}H(\sigma,\sigma_1). \tag{16}$$

Lemma 4.2: The solution of the system is Ulam-Hyers stable if $\rho < 1$, in condition (12) along with Lemma (4.1).

Proof: Suppose that $\Omega \in \mathrm{B}$ and $\Pi \in \mathrm{B}$ is unique and any solution, respectively, we have

$$|\Pi(t) - \Omega(t)| =$$
$$\left|\Pi(t) - \left(\Omega(0) + \frac{\sigma_1 t^{\sigma_1-1}(1-\sigma)}{AB(\sigma)}\Lambda(t,\Omega(t)) + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}\int_0^t \tau^{\sigma_1-1}(t-\tau)^{\sigma_1-1}\Lambda(\tau,\Omega(\tau))\,d\tau\right)\right|$$
$$\le \left|\Pi(t) - \left(\Pi(0) + \frac{\sigma_1 t^{\sigma_1-1}(1-\sigma)}{AB(\sigma)}\Lambda(t,\Pi(t)) + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}\int_0^t \tau^{\sigma_1-1}(t-\tau)^{\sigma_1-1}\Lambda(\tau,\Pi(\tau))\,d\tau\right)\right|$$
$$+ \left|\Pi(0) + \frac{\sigma_1 t^{\sigma_1-1}(1-\sigma)}{AB(\sigma)}\Lambda(t,\Pi(t)) + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}\int_0^t \tau^{\sigma_1-1}(t-\tau)^{\sigma_1-1}\Lambda(\tau,\Pi(\tau))\,d\tau\right|$$
$$- \left|\Omega(0) + \frac{\sigma_1 t^{\sigma_1-1}(1-\sigma)}{AB(\sigma)}\Lambda(t,\Omega(t)) + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}\int_0^t \tau^{\sigma_1-1}(t-\tau)^{\sigma_1-1}\Lambda(\tau,\Omega(\tau))\,d\tau\right|$$
$$\le \sigma^*_{\sigma,\sigma_1}\varepsilon + \left(\frac{\sigma_1 T^{\sigma_1-1}(1-\sigma)}{AB(\sigma)} + \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)}T^{\sigma+\sigma_1-1}H(\sigma,\sigma_1)\right)L_\Lambda|\Pi(t) - \Omega(t)|$$
$$\le \sigma^*_{\sigma,\sigma_1}\varepsilon + \rho|\Pi(t) - \Omega(t)|.$$

Consequently, one can write

$$||\Pi(t) - \Omega(t)|| \le \sigma^*_{\sigma,\sigma_1}\varepsilon + \rho||\Pi(t) - \Omega(t)||.$$

We can write the above relation is

$$||\Pi(t) - \Omega(t)|| \le \aleph_{\sigma,\sigma_1}\varepsilon,$$

where $\aleph_{\sigma,\sigma_1} = \dfrac{\sigma^*_{\sigma,\sigma_1}}{1-\rho}$. Therefore, system is stable.

4.3 Fractal-Fractional Integral with Mittag-Leffler Kernel

Consider:

$$\begin{cases} S(t) = S(0) + \dfrac{\sigma_1 t^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_1(t, S, H, I, Q) + \dfrac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)} \int_0^t \tau^{\sigma_1 - 1}(t-\tau)^{\sigma - 1} P_1(\tau, S, H, I, Q)\, d\tau \\ H(t) = H(0) + \dfrac{\sigma_1 t^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_2(t, S, H, I, Q) + \dfrac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)} \int_0^t \tau^{\sigma_1 - 1}(t-\tau)^{\sigma - 1} P_2(\tau, S, H, I, Q)\, d\tau \\ I(t) = I(0) + \dfrac{\sigma_1 t^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_3(t, S, H, I, Q) + \dfrac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)} \int_0^t \tau^{\sigma_1 - 1}(t-\tau)^{\sigma - 1} P_3(\tau, S, H, I, Q)\, d\tau \\ Q(t) = Q(0) + \dfrac{\sigma_1 t^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_4(t, S, H, I, Q) + \dfrac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)} \int_0^t \tau^{\sigma_1 - 1}(t-\tau)^{\sigma - 1} P_4(\tau, S, H, I, Q)\, d\tau \end{cases}. \tag{17}$$

We construct the numerical scheme at $t = t_{n+1}$:

$$\begin{cases} S^{n+1} = S^0 + \dfrac{\sigma_1 t_n^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_1(t_n, S^n, H^n, I^n, Q^n) + \dfrac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)} \int_0^t \tau^{\sigma_1 - 1}(t_{n+1}-\tau)^{\sigma - 1} P_1(\tau, S, H, I, Q)\, d\tau \\ H^{n+1} = H^0 + \dfrac{\sigma_1 t_n^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_2(t_n, S^n, H^n, I^n, Q^n) + \dfrac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)} \int_0^t \tau^{\sigma_1 - 1}(t_{n+1}-\tau)^{\sigma - 1} P_2(\tau, S, H, I, Q)\, d\tau \\ I^{n+1} = I^0 + \dfrac{\sigma_1 t_n^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_3(t_n, S^n, H^n, I^n, Q^n) + \dfrac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)} \int_0^t \tau^{\sigma_1 - 1}(t_{n+1}-\tau)^{\sigma - 1} P_3(\tau, S, H, I, Q)\, d\tau \\ Q^{n+1} = Q^0 + \dfrac{\sigma_1 t_n^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_4(t_n, S^n, H^n, I^n, Q^n) + \dfrac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)} \int_0^t \tau^{\sigma_1 - 1}(t_{n+1}-\tau)^{\sigma - 1} P_4(\tau, S, H, I, Q)\, d\tau \end{cases} \tag{18}$$

Applying the approximation of the integrals on the right hand side of system (18) yields:

$$\begin{cases} S^{n+1} = S^0 + \dfrac{\sigma_1 t_n^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_1(t_n, S^n, H^n, I^n, Q^n) + \\ \dfrac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)} \sum_{j=0}^n \int_{t_j}^{t_{j+1}} \tau^{\sigma_1 - 1}(t_{n+1}-\tau)^{\sigma - 1} P_1(\tau, S, H, I, Q)\, d\tau \\ H^{n+1} = H^0 + \dfrac{\sigma_1 t_n^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_2(t_n, S^n, H^n, I^n, Q^n) \\ + \dfrac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)} \sum_{j=0}^n \int_{t_j}^{t_{j+1}} \tau^{\sigma_1 - 1}(t_{n+1}-\tau)^{\sigma - 1} P_2(\tau, S, H, I, Q)\, d\tau \\ I^{n+1} = I^0 + \dfrac{\sigma_1 t_n^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_3(t_n, S^n, H^n, I^n, Q^n) \\ + \dfrac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)} \sum_{j=0}^n \int_{t_j}^{t_{j+1}} \tau^{\sigma_1 - 1}(t_{n+1}-\tau)^{\sigma - 1} P_3(\tau, S, H, I, Q)\, d\tau \\ Q^{n+1} = Q^0 + \dfrac{\sigma_1 t_n^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_4(t_n, S^n, H^n, I^n, Q^n) \\ + \dfrac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma)} \sum_{j=0}^n \int_{t_j}^{t_{j+1}} \tau^{\sigma_1 - 1}(t_{n+1}-\tau)^{\sigma - 1} P_4(\tau, S, H, I, Q)\, d\tau \end{cases}. \tag{19}$$

We consider

$$L_j(\tau) = \frac{\tau - t_{j-1}}{t_j - t_{j-1}} t_j^{\sigma_1 - 1} P_1\left(t^j, S^j, H^j, I^j, Q^j\right) - \frac{\tau - t_j}{t_j - t_{j-1}} t_{j-1}^{\sigma_1 - 1} P_1\left(t^{j-1}, S^{j-1}, H^{j-1}, I^{j-1}, Q^{j-1}\right),$$

$$M_j(\tau) = \frac{\tau - t_{j-1}}{t_j - t_{j-1}} t_j^{\sigma_1 - 1} P_2\left(t^j, S^j, H^j, I^j, Q^j\right) - \frac{\tau - t_j}{t_j - t_{j-1}} t_{j-1}^{\sigma_1 - 1} P_2\left(t^{j-1}, S^{j-1}, H^{j-1}, I^{j-1}, Q^{j-1}\right),$$

$$N_j(\tau) = \frac{\tau - t_{j-1}}{t_j - t_{j-1}} t_j^{\sigma_1 - 1} P_3\left(t^j, S^j, H^j, I^j, Q^j\right) - \frac{\tau - t_j}{t_j - t_{j-1}} t_{j-1}^{\sigma_1 - 1} P_3\left(t^{j-1}, S^{j-1}, H^{j-1}, I^{j-1}, Q^{j-1}\right),$$

$$O_j(\tau) = \frac{\tau - t_{j-1}}{t_j - t_{j-1}} t_j^{\sigma_1 - 1} P_4\left(t^j, S^j, H^j, I^j, Q^j\right) - \frac{\tau - t_j}{t_j - t_{j-1}} t_{j-1}^{\sigma_1 - 1} P_4\left(t^{j-1}, S^{j-1}, H^{j-1}, I^{j-1}, Q^{j-1}\right).$$

Then, we have

$$\begin{aligned} S^{n+1} = S^0 + \frac{\sigma_1 t_n^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_1(t_n, S^n, H^n, I^n, Q^n) + \frac{\sigma_1 (\Delta t)^\sigma}{\mathrm{AB}(\sigma)\Gamma(\sigma+2)} \sum_{j=0}^{n} \Big[t_j^{\sigma_1 - 1} P_1\left(t^j, S^j, H^j, I^j, Q^j\right) \\ \times \left((n+1-j)^\sigma (n-j+2+\sigma) - (n-j)^\sigma (2+2\sigma+n-j)\right) \\ - t_{j-1}^{\sigma_1 - 1} P_1\left(t^{j-1}, S^{j-1}, H^{j-1}, I^{j-1}, Q^{j-1}\right) \times \left((1+n-j)^{\sigma+1} - (n-j)^\sigma (1+\sigma+n-j)\right)\Big], \end{aligned} \tag{20}$$

$$\begin{aligned} H^{n+1} = H^0 + \frac{\sigma_1 t_n^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_2(t_n, S^n, H^n, I^n, Q^n) + \frac{\sigma_1 (\Delta t)^\sigma}{\mathrm{AB}(\sigma)\Gamma(\sigma+2)} \sum_{j=0}^{n} \Big[t_j^{\sigma_1 - 1} P_2\left(t^j, S^j, H^j, I^j, Q^j\right) \\ \times \left((n+1-j)^\sigma (n-j+2+\sigma) - (n-j)^\sigma (2+2\sigma+n-j)\right) \\ - t_{j-1}^{\sigma_1 - 1} P_2\left(t^{j-1}, S^{j-1}, H^{j-1}, I^{j-1}, Q^{j-1}\right) \times \left((1+n-j)^{\sigma+1} - (n-j)^\sigma (1+\sigma+n-j)\right)\Big], \end{aligned} \tag{20'}$$

$$\begin{aligned} I^{n+1} = I^0 + \frac{\sigma_1 t_n^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_3(t_n, S^n, H^n, I^n, Q^n) + \frac{\sigma_1 (\Delta t)^\sigma}{\mathrm{AB}(\sigma)\Gamma(\sigma+2)} \sum_{j=0}^{n} \Big[t_j^{\sigma_1 - 1} P_3\left(t^j, S^j, H^j, I^j, Q^j\right) \\ \times \left((n+1-j)^\sigma (n-j+2+\sigma) - (n-j)^\sigma (2+2\sigma+n-j)\right) \\ - t_{j-1}^{\sigma_1 - 1} P_3\left(t^{j-1}, S^{j-1}, H^{j-1}, I^{j-1}, Q^{j-1}\right) \times \left((1+n-j)^{\sigma+1} - (n-j)^\sigma (1+\sigma+n-j)\right)\Big], \end{aligned} \tag{20''}$$

$$\begin{aligned} Q^{n+1} = Q^0 + \frac{\sigma_1 t_n^{\sigma_1 - 1}(1-\sigma)}{\mathrm{AB}(\sigma)} P_4(t_n, S^n, H^n, I^n, Q^n) + \frac{\sigma_1 (\Delta t)^\sigma}{\mathrm{AB}(\sigma)\Gamma(\sigma+2)} \sum_{j=0}^{n} \Big[t_j^{\sigma_1 - 1} P_4\left(t^j, S^j, H^j, I^j, Q^j\right) \\ \times \left((n+1-j)^\sigma (n-j+2+\sigma) - (n-j)^\sigma (2+2\sigma+n-j)\right) \\ - t_{j-1}^{\sigma_1 - 1} P_4\left(t^{j-1}, S^{j-1}, H^{j-1}, I^{j-1}, Q^{j-1}\right) \times \left((1+n-j)^{\sigma+1} - (n-j)^\sigma (1+\sigma+n-j)\right)\Big]. \end{aligned} \tag{20'''}$$

5 Computational Result and Discussion

COVID-19 fractional order model for the case of Wuhan, China, is offered for analysis with simulations in order to determine the possible efficacy of Coronavirus disease transmission in the Community. For this reason, we employed the COVID-19 fractal fractional derivative model in the example of Wuhan, China, with the given beginning conditions. The parameters of actual data are described in detail in [46]. By using different numerical techniques and the time fractional parameters, the mechanical characteristics of the fractional order model are identified. The findings of fractional value calculations were used to detect the outcomes of the nonlinear system memory. It provides a better way than wanting to control the disease without defining other parameters.

In Figs. 1–4, simulations were obtained by fractal fractional method. It is noted that physical procedures are far better explained using the fractional order derivatives which are the most notable and sustainable component compared to the classical-order case with order at 1. The behaviors of the dynamics found in the various fractional orders are shown in the form of numerical results that have been reported.

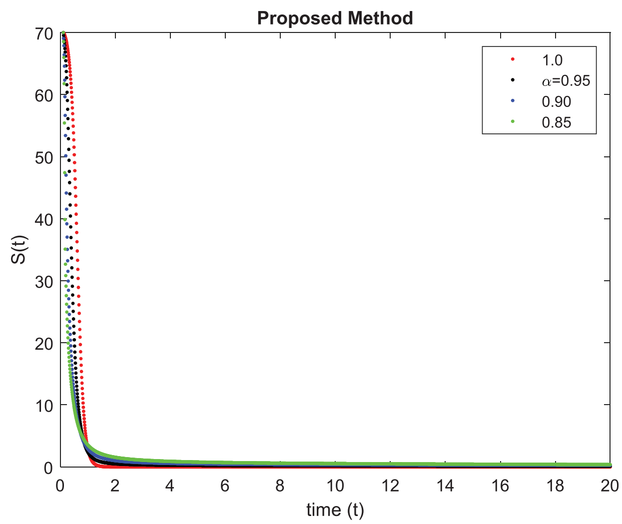

Figure 1: Simulation of $S\left(t\right)$ at different fractal orders and fractional order is 1.0

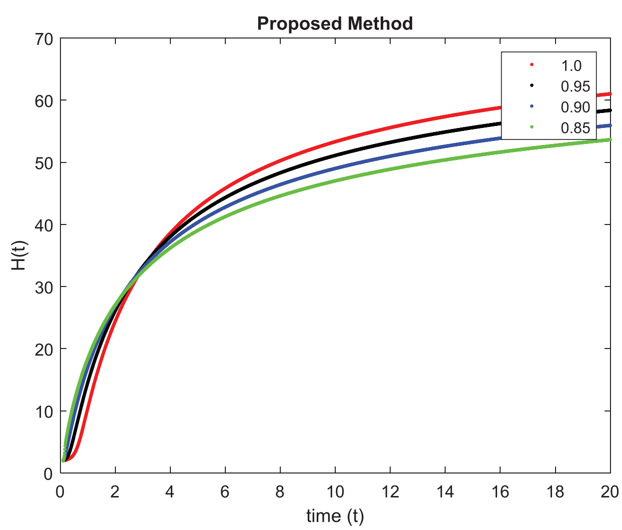

Figure 2: Simulation of $H\left(t\right)$ at different fractal orders and fractional order is 1.0

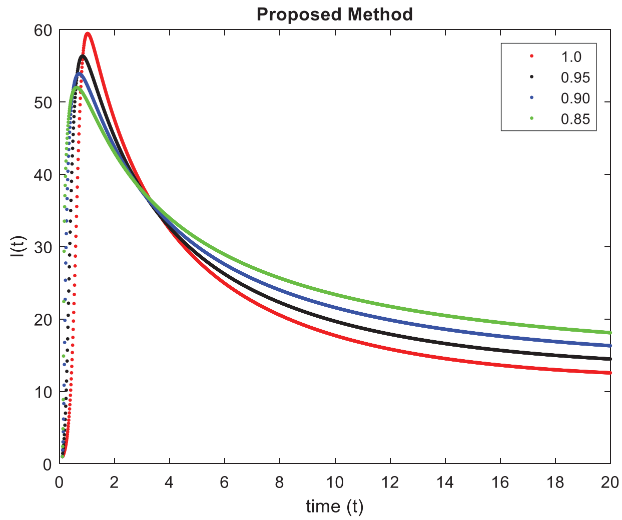

Figure 3: Results of $I(t)$ for different fractal orders and fractional order is 1.0

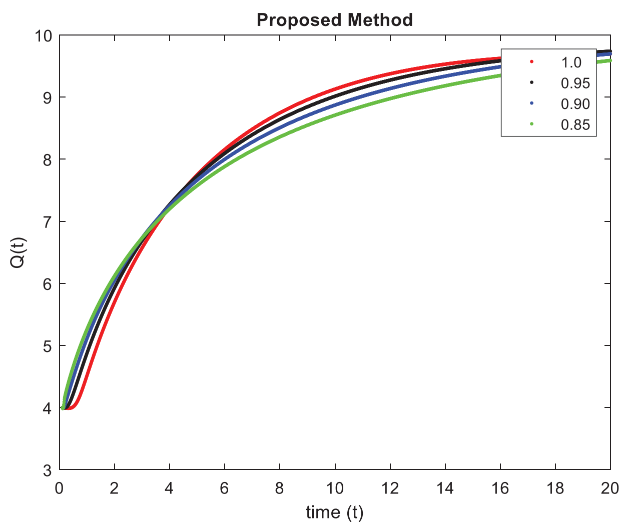

Figure 4: Results of $Q(t)$ at different fractal orders and fractional order is 1.0

In Figs. 5–8, simulations were obtained by fractal fractional method. It is noted that physical procedures are far better explained using the fractional order derivatives which are the most notable and sustainable component compared to the classical-order case with order at 0.9. The behaviors of the dynamics found in the various fractional orders are shown in the form of numerical results that have been reported.

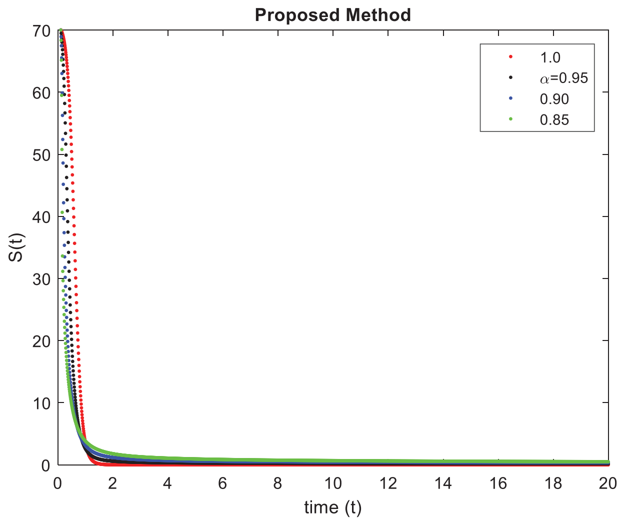

Figure 5: Results of $S(t)$ at different fractional value with dimension 0.9

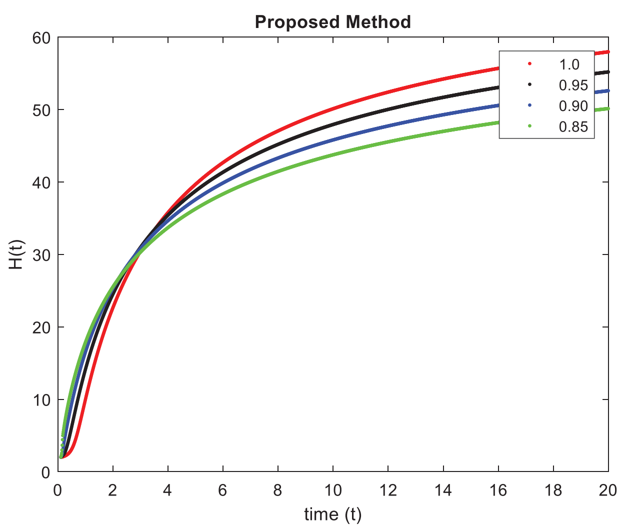

Figure 6: Results of $H(t)$ at different fractional value with dimension 0.9

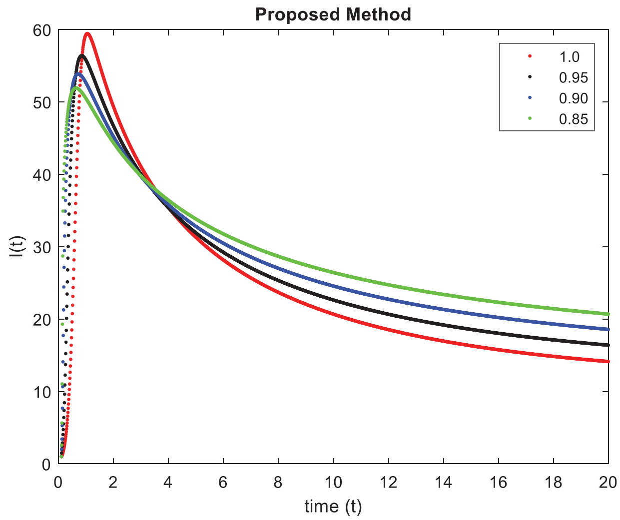

Figure 7: Results of $I(t)$ at different fractional value with dimension 0.9

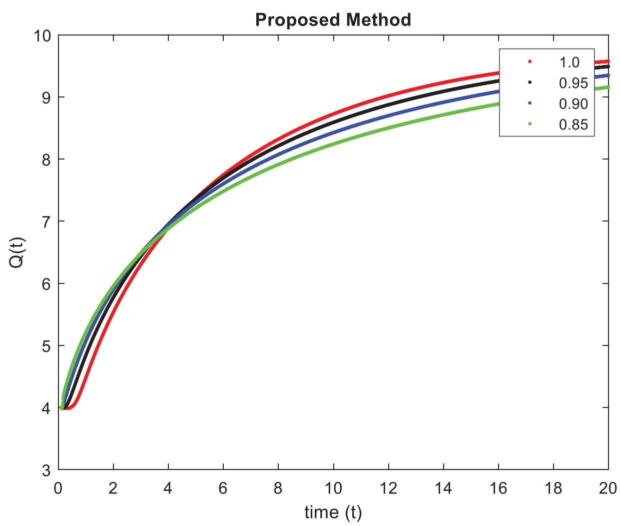

Figure 8: Simulation of $Q(t)$ at different fractional value with dimension 0.9

In Figs. 9–12, simulations were obtained by fractal fractional method. It is noted that physical procedures are far better explained using the fractional order derivatives which are the most notable and sustainable component compared to the classical-order case with order at 0.8. The behaviors of the dynamics found in the various fractional orders are shown in the form of numerical results that have been reported.

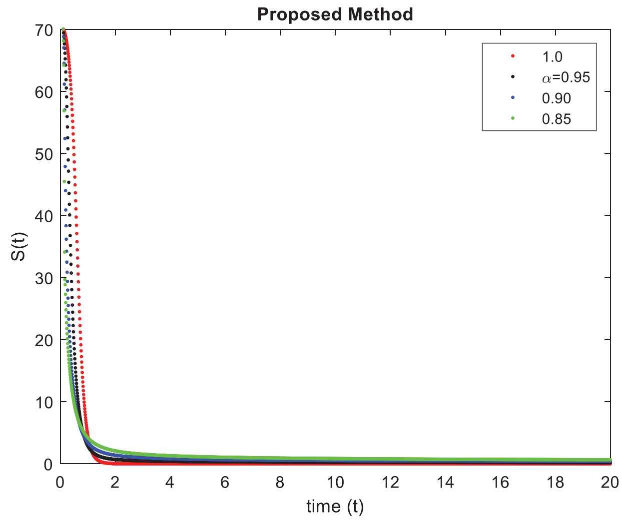

Figure 9: Results of $S(t)$ at different fractional value with dimension 0.8

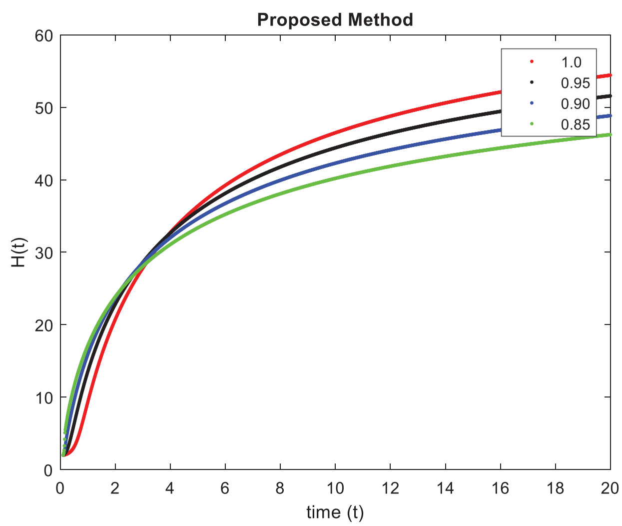

Figure 10: Results of $H(t)$ at different fractional value with dimension 0.8

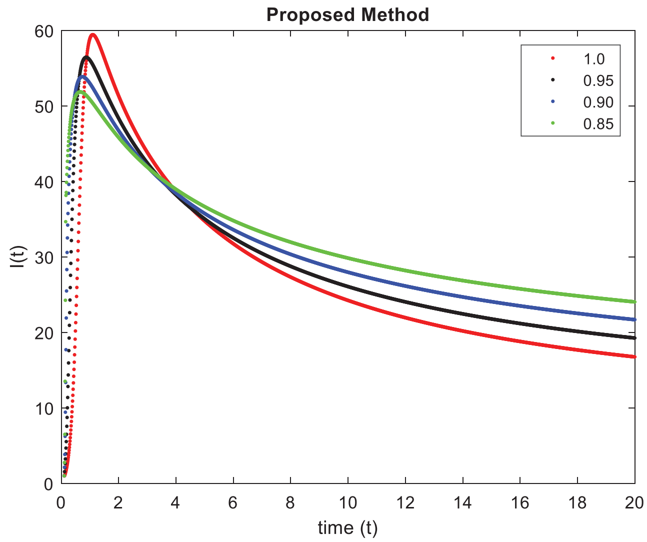

Figure 11: Results of $I(t)$ at different fractional value with dimension 0.8

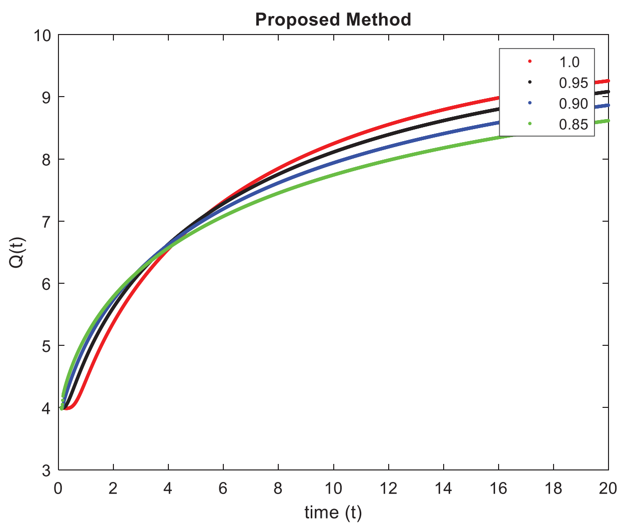

Figure 12: Results of $Q(t)$ at different fractional value with dimension 0.8

6 Conclusion

In this paper, the fractal-fractional differential equation model for COVID-19 disease has been investigated with fractal fractional operator. The steady state and fundamental characteristics of the model equilibria are investigated. Fixed point theory is used to demonstrate in detail the existence and uniqueness of solutions for the model with FFM derivative. The Ulam-Hyers technique is used to conduct the stability analysis of the system which fulfills all properties. The two-step fractional Lagrange polynomial approach with FFM derivative is used to generate the model's numerical solution. The numerical simulations are obtained and briefly described by choosing various values of the fractional order and dimension. We applied very effective numerical techniques to obtain the solutions of the model. We analyzed our obtained results and concluded that they are effective for the proposed model. Some theoretical results were also discussed for the model. This model turns out to be quite trustworthy when precise estimations of transmission structures are provided in real-time. The new suggested improvement will shed some modeling-related light on problems with and without singularity at the origin.

The analysis of the following problems forms further directions of research and developments: creating and exploring families of other epidemiological models based on fractal-fractional differential equations for diseases; the exploration of distinctions, taking into account the types of distinctions between fractional models; creating the output information for subsequent methodological recommendations, for diseases expansions analysis and for anti-diseases interventions plans.

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