Computer Modeling in
Engineering & Sciences

Tech Science Press

DOI: 10.32604/cmes.2023.030030

ARTICLE

The Spherical q-Linear Diophantine Fuzzy Multiple-Criteria Group Decision-Making Based on Differential Measure

Huzaira Razzaque[1], Shahzaib Ashraf[1,*], Muhammad Naeem[2] and Yu-Ming Chu[3,*]

[1]Institute of Mathematics, Khwaja Fareed University of Engineering & Information Technology, Rahim Yar Khan, 64200, Pakistan

[2]Department of Mathematics, Deanship of Applied Sciences, Umm Al-Qura University, Mecca, Saudi Arabia

[3]Department of Mathematics, Huzhou University, Huzhou, 313000, China

*Corresponding Authors: Shahzaib Ashraf. Email: shahzaib.ashraf@kfueit.edu.pk; Yu-Ming Chu. Email: chuyuming@zjhu.edu.cn

Received: 19 March 2023 Accepted: 19 June 2023 Published: 17 November 2023

ABSTRACT

Spherical q-linear Diophantine fuzzy sets (Sq-LDFSs) proved more effective for handling uncertainty and vagueness in multi-criteria decision-making (MADM). It does not only cover the data in two variable parameters but is also beneficial for three parametric data. By Pythagorean fuzzy sets, the difference is calculated only between two parameters (membership and non-membership). According to human thoughts, fuzzy data can be found in three parameters (membership uncertainty, and non-membership). So, to make a compromise decision, comparing Sq-LDFSs is essential. Existing measures of different fuzzy sets do, however, can have several flaws that can lead to counterintuitive results. For instance, they treat any increase or decrease in the membership degree as the same as the non-membership degree because the uncertainty does not change, even though each parameter has a different implication. In the Sq-LDFSs comparison, this research develops the differential measure (DFM). The main goal of the DFM is to cover the unfair arguments that come from treating different types of FSs opposing criteria equally. Due to their relative positions in the attribute space and the similarity of their membership and non-membership degrees, two Sq-LDFSs form this preference connection when the uncertainty remains same in both sets. According to the degree of superiority or inferiority, two Sq-LDFSs are shown as identical, equivalent, superior, or inferior over one another. The suggested DFM's fundamental characteristics are provided. Based on the newly developed DFM, a unique approach to multiple criterion group decision-making is offered. Our suggested method verifies the novel way of calculating the expert weights for Sq-LDFSS as in PFSs. Our proposed technique in three parameters is applied to evaluate solid-state drives and choose the optimum photovoltaic cell in two applications by taking uncertainty parameter zero. The method's applicability and validity shown by the findings are contrasted with those obtained using various other existing approaches. To assess its stability and usefulness, a sensitivity analysis is done.

KEYWORDS

Multi-criteria group decision-making; spherical q-linear Diophantine fuzzy sets; difference measures; photovoltaic cells; medical diagnosis

1 Literature Review

Multiple criteria decision-making (GCDM) is a subfield of operations research and a sophisticated mathematical method that explicitly evaluates viable options to competing with many criteria in order to get the best answer [1]. Due to the growing complexity of decisions in these contexts, group decision-making (GDM) becomes important since it is typically difficult for a single person to fully analyze all the aspects of a situation [2]. Group decision-making (GDM) is a procedure in which several people interact at once to analyze issues, evaluate various alternatives characterized by several conflicting criteria, and select a suitable alternative [1]. Science, engineering, and medicine all depend on logic and crisp set theory. A crisp set indicates a digital element's belongingness by indicating whether or not it is part of a set. However, in many real-life circumstances, partial belongingness becomes essential for describing ambiguity. We have to cope with circumstances where the crisp-set theory fails on mostly real-world issues. The choice of such a person is ambiguous; for instance, if a decision maker (GDM) has to choose the best instructor, a bright student, a gorgeous female, a young researcher, etc., what constitutes being the best, young, and youthful varies from person to person. A fuzzy set consists of partial grade gain values in the unit close interval [0, 1] to show the element's partial belongingness to a set under some specific characteristics. The fuzzy set concept created by Zadeh [3] is a powerful set-theoretic concept for expressing ambiguous information. The fuzzy set caught the researchers' attention due to its effectiveness and usefulness. It is used all around the globe in a variety of domains, including computational intelligence, information fusion, pattern recognition, engineering, medical diagnostics, artificial intelligence, machine learning, neural networks, and solving issues with (GCDM). The intuitionistic FS (IFS) concept by Atanassova et al. [4] is one of the well-known extended structures. Since the IFS contains two grades: truth-membership grade (TM-grade) and falsity-membership grade (FM-grade), which fulfil the requirement that the total of their grade values gained from $[0, 1]$ cannot be surpassed by one also $0 \leq T + F \leq 1$, the selection is limited to the satisfaction and dissatisfaction classes. In the event that $00.7 + 0.5 > 1$, it fails. Yager [5] was able to overcome this problem by creating the PyFS principle, which imposes the condition that the square sum of the TM and FM grades cannot be more than $0 \leq T^2 + F^2 \leq 1$. Pythagorean fuzzy information measures and their applications in pattern recognition, cluster analysis, and medical diagnostics were proposed by Peng et al. [6]. The Pythagorean fuzzy idea proposed by Peng et al. [6], used in information measures, has applications in pattern recognition, cluster analysis, and medical diagnostics. Peng et al. [7] highlighted some noteworthy PFS features. In addition, Yager [8] created a powerful conceptual thought called the qROFS, which is an extension of both the PFS and the IFS. IFS and PFS are not enough for modeling when $0.71^2 + 0.72^2 = 1.0225 > 0$. Wang et al. [9] put out a concept for q-ortho-pair based on the conventional cosine and cotangent similarity metrics. He defined ten similarity measures and ten weighted similarity measures between q-ROFS for (GADM) issues such as pattern identification and scheme selection. Also for Q-rung ortho-pair fuzzy sets with and without hesitation degree, Donyatalab et al. [10] provided a novel similarity measure that is expanded based on square root cosine similarity. For q-rung ortho-pair fuzzy sets, four distinct types of square root cosine similarity metrics (q-ROFSs) created. Jana et al. [11] utilized this article, in which they employed Dombi operations to produce a number of Pythagorean fuzzy Dombi aggregation operators to address the problems associated with multiple-attribute decision-making. Jana et al. [12] built a couple Q-rung orthopair fuzzy Dombi aggregation operators using Dombi operations to address the problems associated with multiple-attribute decision-making in this setting. Qiyas et al. [13] provided a collection of distances and similarity measures utilizing q-rung linear diophantine fuzzy (q-ROLDF). He proposed a Jaccard similarity measure, an exponential similarity measure, and cosine and cotangent function-based similarity measures for q-LDFSs. More recently, Kutlu Gündogdu et al. [14] established the concept of the spherical fuzzy set (SFS). With the eye-catching characteristic that, if one model does not work, the other will handle the uncertain circumstances, these models have the potential to deal with uncertainties in actual issues. With the constraint that the sum of the squares of the three indexes, membership grade, hesitation, and non-membership grade, is less than or equal to 1, an SFS is very significant since it may communicate hazy and ambiguous information. By utilising SFSs, Ahmmad et al. [15] created novel average aggregation operators. Kutlu Gündoğdu et al. [16] introduced numerous properties and arithmetic operational laws of SFSs and their scores and accuracy functions. Shanshan was impressed by the similarity measure in fuzzy sets, which is one of the most powerful and essential methods for assessing how similar two things are in comparison. Under a spherical fuzzy environment, Shishavan et al. [17] developed the Jaccard, exponential, and square root cosine similarity metrics. It has several uses in decision-making, data mining, medical diagnosis, and pattern detection. Rafiq et al. [18] discussed the family of unique cosine-based similarity measures for spherical fuzzy sets. He also evaluated them in light of all prior reflections. Khan et al. [19] investigated the distances and cosines with cotangent similarity measures in the selection of megaprojects in underdeveloped countries. Rayappan et al. [20] used the spherical fuzzy weighted cross entropy between the ideal alternative and an alternative to rank the alternatives according to the cross entropy values and choosed the most desired one(s). Mao et al. [21] used a simple designed aggregation operator to propose a neutrosophic data envelopment analysis model with undesired results in his article. Kouatli et al. [22] used fuzzy logic in this paper to identify technical analyses (TAs). For fuzzyification of these TA(s), he used the modular approach of fuzzy logic and "fuzzimetric sets" to achieve the "fuzzy spectrum" of forecasted price tolerances when making buying and selling decisions. As an innovation, Iqbal et al. [23] used the geometric core of a trapezoidal linear diophantine fuzzy number while ranking trapLDFNs using a centroid index. Hasan et al. [24] set up a series of picture fuzzy mean operators in the decision-making process to aggregate the picture fuzzy set, namely picture fuzzy harmonic mean, picture fuzzy weighted harmonic mean, picture fuzzy arithmetic mean, picture fuzzy weighted arithmetic mean (PFWAM), picture fuzzy geometric mean, and picture fuzzy weighted geometric mean. Wang et al. [25] presented some innovative dice similarity measures of SFSs and its generalization, GDSM. He indicates that the dice similarity measures and asymmetric measures (projection measures) are the specific instances of the GDSM in some parameter values. Wei et al. [26] applied the cosine function over SFS to find similarity measures in pattern recognition. Mahmood et al. [27] introduced SFS-based cosine similarity and data measures for pattern recognition and medical diagnosis. For linear Diophantine fuzzy sets, Mohammad et al. [28] proposed some novel distance and similarity measures, including the normalized distance measure, generalized hamming distance, euclidian distance measure, generalized distance measure, jaccard similarity measure, exponential similarity measure, and cosine and cotangent function similarity measures. In a spherical fuzzy environment, Donyatalab et al. [29] developed several innovative distances based on Minkowski and Minkowski-Hausdorff distances. He discussed innovative SF-similarity measures based on Minkowski and Minkowski-Hausdorff distances with some properties.

In order to evaluate logistics AVs, Bonab et al. [30] created an enhanced MCDM framework based on the Choquet integral (CI) and a spherical fuzzy set (SFS). By employing spherical fuzzy sets and the spherical fuzzy weight correlation coefficient, Ma et al. [31] initiated a novel failure mode and effect analysis (FMEA) that enhanced the performance of FMEA. Ali et al. [32] introduced a novel approach for making decisions called MCDM-total area based on orthogonal vectors with a few innovative distance measurements made possible by matrix norms. The limitations of the current distance measurements are removed by this measure, which also meets all axiomatic requirements.

Das et al. [33] provided a way for resolving group decision-making issues using intuitionistic fuzzy parameterized intuitionistic multi fuzzy N-soft sets of dimension q by providing its induced IFP-hesitant N-soft set as an addition to the multi-fuzzy N-soft set-based method. Khan et al. [34] provided multi-attribute decision-making in a T-spherical fuzzy environment using the Archimedean aggregation operator for three-dimensional data with n powers.

Measures of similarity and distance are two sides of the same coin. The majority of similarity tests rely on distance measurements. A lot depends on both metrics when comparing two PFSs. Even though several studies suggested various distance measurements, the ones that are currently in use still have certain shortcomings [35]. They can first result in counterintuitive consequences [36]. Second, it is possible that they will not be able to calculate the maximum distance measurement value [35]. This will skew the ranking values of the alternatives and result in inaccurate findings. As a result, Shraf has solved the open problem of determining the distance between two PFSs in the Pythagorean fuzzy environment [36]. But there is still a problem with the idea, which is that it might not function if the raw data is in triplet form. To remove this flaw, we created a new fuzzy set to address this problem.

An integer-coefficiented linear Diophantine equation is included in the fuzzy membership function of Q-linear Diophantine fuzzy sets, a particular kind of fuzzy set. A solution to a q-linear Diophantine equation is the value that is specifically described as the membership function of a q-linear Diophantine fuzzy set. Applications that utilise such fuzzy sets in fuzzy set theory include decision-making, control, pattern recognition, and image processing. Overall, q-linear Diophantine fuzzy sets are a practical tool in fuzzy set theory for representing ambiguous and imprecise information and have many applications in a variety of domains. The human mind extends much beyond TM and FM. As a result, a new fuzzy set was required to take human thinking into account, and Ashraf et al. [37] proposed the concept of a spherical fuzzy set that provides a more adaptable and expressive framework for the ideas of classical (crisp) sets. In spherical fuzzy sets, an element of a set is given a range of membership values rather than a single membership value to represent the degree of uncertainty or ambiguity in that element's categorization. This study's objective is to identify the differential measure among three parameters using the Sq-LDFS specified fuzzy set to cover the three degrees of membership (membership, non-membership and indeterminacy). Razzaque et al. [38] proposed a collection of innovative Einstein aggregation operations by using sq-LDF data. Additionally, a better VIKOR method is presented to address the uncertainty in categorising viral hepatitis.

This article introduces the idea of differential measure (DFM), derived from [39] as a novel method for contrasting spherical q-linear Diophantine fuzzy sets (Sq-LDFSs) [40]. When the uncertainty grade has equally in both sets, there is a preference connection between two Sq-LDFS because of their relative positions in the attribute space and the proximity of their membership and non-membership degrees. Due to the equal consideration of the membership and non-membership degrees, and uncertainty grade, although each direction has a distinct connotation, the existing Distance measures and similarity measures across various fuzzy sets, namely PFSs with a zero uncertainty grade have several shortcomings. To take into To account the effects of each element of an Sq-LDFSs evaluation, the DFM uses signed distance. According to the degree of superiority or inferiority, two Sq-LDFSs are shown as the same, equal, superior, or inferior to one another. Based on the newly established DFM, a novel MCGDM approach is certified by our proposed idea. Since Sq-LDFS are more adaptable than IFSs, PFSs, q-LDFSs and SFSs to deal with absurdity and ambiguity, as well as to deal with human evaluation information, it is vital to pay greater attention to collective decision-making in this context. Therefore,

1. The proposal of DFM is proposed as a revolutionary technique for contrasting Sq-LDFSs. It is an association between two Sq-LDFSs based on the position in the attribute space. To cover the limitations of the current distance in discriminating, a DFM maintains the identity of the Sq-LDFSs' parameters.
2. A novel (GCGDM) technique is presented based on the proposed DFM.
3. The experts' weights in (GCGDM) issues are computed using a novel method that is put forth. Furthermore, the (GCGDM) problems may have a solution using this method.

The rest of the part is structured as follows: The most recent information regarding Sq-LDFSs is provided in the following sections: Literature review in Section 1, preliminary definitions in Section 2, and an explanation of the differential measure in Section 3. We provided the differential rules suggested for Sq-LDFSs with short numerical examples in Section 4. In Section 5, an enhanced framework for MCGDM is proposed to explain all phases of the technique and offer some counter-examples for previously proposed differential measures by our technique. In Subsection 5.1, a numerical example of diagnosing cancer patients using a variety of tests is provided to bolster the differential measure. In Sections 6 and 7, a discussion and a conclusion are offered.

2 Preliminaries

Definition 2.1. [40] A spherical q-linear Diophantine fuzzy set $(Sq - LDFS)\ \wp$ over a fixed set $\mathfrak{K}$ is defined as

$$\wp = \left\{\ell, \left\langle T_{\wp}(\ell), I_{\wp}(\ell), F_{\wp}(\ell)\right\rangle, \left\langle \mathfrak{i}_{\wp}(\ell), \mathfrak{j}_{\wp}(\ell), \mathfrak{k}_{\wp}(\ell)\right\rangle : \forall \ell \in \mathfrak{K}\right\} \tag{1}$$

where each membership grade $T(\ell), I(\ell), F(\ell) : \wp \to [0,1]$, and each control parameter $\mathfrak{i}(\ell), \mathfrak{j}(\ell), \mathfrak{k}(\ell) : \wp \to [0,1]$, for an element $\ell \in \mathfrak{K}$ with the limitations.

1. $0 \le \mathfrak{i}^q T_{\wp}(\ell) + \mathfrak{j}^q I_{\wp}(\ell) + \mathfrak{k}^q F_{\wp}(\ell) \le 1$
2. $0 \le \mathfrak{i}^q_{\wp}(\ell) + \mathfrak{j}^q_{\wp}(\ell) + \mathfrak{k}^q_{\wp}(\ell) \le 1$.

Definition 2.2. A compliment of $Sq - LDFS\wp$ over a fixed set $\mathfrak{K}$ is defined as

$$\wp^c = \left[\ell, \left\{F_{\wp}(\ell), I_{\wp}(\ell), T_{\wp}(\ell)\right\}, \left\{\mathfrak{k}_{\wp}(\ell), \mathfrak{j}_{\wp}(\ell), \mathfrak{i}_{\wp}(\ell)\right\} : \forall \ell \in \mathfrak{K}\right]$$

Definition 2.3. For a Sq-LDFS a score function $(SF)\ \Xi$ is defined by the transformation $\Xi : \acute{S}q - LDFS\,(e) \to [-1, 1]$, and given by:

$$\Xi\,(\mathfrak{A}) = \frac{1}{2N}\left\{(1 + T - I - F) + \frac{\left(1 + \mathfrak{i}^q - \mathfrak{j}^q - \mathfrak{k}^q\right)}{2}\right\}; q \ge 1 \tag{2}$$

where $\acute{S}q - LDFS(\ell)$ is a collection on $\acute{S}q - LDFNs$ over $\ell, \forall \ell \in \mathfrak{K}$.

Definition 2.4. The hamming distance using Sq-LDFSs [38] can be defined as:

$$HD = \frac{1}{6}\left\{\left|T_1^3 - T_2^3\right| + \left|I_1^3 - I_2^3\right| + \left|F_1^3 - F_2^3\right| + \left|\mathfrak{i}_1^3 - \mathfrak{i}_2^3\right| + \left|\mathfrak{j}_1^3 - \mathfrak{j}_2^3\right| + \left|\mathfrak{k}_1^3 - \mathfrak{k}_2^3\right|\right\} \tag{3}$$

Definition 2.5. The euclidean function using Sq-LDFSs can be defined as:

$$ED = \sqrt{\frac{1}{n}\left[\sum_{i=1}^{n}\left|T_1^3 - T_2^3\right| + \left|I_1^3 - I_2^3\right| + \left|F_1^3 - F_2^3\right| + \left|\mathfrak{i}_1^3 - \mathfrak{i}_2^3\right| + \left|\mathfrak{j}_1^3 - \mathfrak{j}_2^3\right| + \left|\mathfrak{k}_1^3 - \mathfrak{k}_2^3\right|\right]} \tag{4}$$

3 The Differential Measure

Song et al. [41] presented the analysis of the distinction between psychological distance measure and classical distance measurement. The classical distance measurements' surprising potential to ignore alternative background information and their competing interactions resolved by psychological distance. To illustrate, we use the SF to evaluate the three alternatives $\mathfrak{A}_1 = \{0.8300, 0.2100, 0.4300, 0.7200, 0.1300, 0.1400\}$, $\mathfrak{A}_2 = \{0.8300, 0.2100, 0.4300, 0.5100, 0.1800, 0.1700\}$ and $\mathfrak{A}_3 = \{0.9300, 0.2100, 0.4300, 0.7200, 0.2100, 0.1300\}$ using Sq-LDF evaluation. We observe that $\mathfrak{A}_3$ is the best alternative with large value of membership grade and control variable. Also $\mathfrak{A}_1$ is better alternative than $\mathfrak{A}_2$ with same degree of support and indeterminacy degree but less degree of opposition. But by using the hamming distances according to Sq-LDFS rules we found that $D_{Hm}(\mathfrak{A}_2, \mathfrak{A}_3) = 0.08 > D_{Hm}(\mathfrak{A}_1, \mathfrak{A}_3) = 0.04$, this shows that $\mathfrak{A}_1$ is closed to $\mathfrak{A}_3$, i.e., $\mathfrak{A}_3$ is preferable substitute than $\mathfrak{A}_2$. Therefore, we can not rely on traditional distance measurements to accurately depict the preferences of different choices. The reader is recommended to take an eye view at Xiao et al. [36] and Huang et al. [35] for counter-examples that demonstrate the shortcomings of the existing distance metrics on pythagorean fuzzy sets. $\mathfrak{A}_3$ is superior to $\mathfrak{A}_1$ since it has more support and less opposition. Since $\mathfrak{A}_3$ is superior to $\mathfrak{A}_1$, we can also claim that $\mathfrak{A}_1$ is inferior to $\mathfrak{A}_3$. Comparing options $\mathfrak{A}_3$ and $\mathfrak{A}_2$, it is noticeable that $\mathfrak{A}_3$ is preferable to $\mathfrak{A}_2$ due to its equal support and lesser opposition and indeterminacy. While comparing options $\mathfrak{A}_3$ and $\mathfrak{A}_1$, $\mathfrak{A}_3$ is superior to $\mathfrak{A}_2$ regarding support, but $\mathfrak{A}_1$ has superiority to $\mathfrak{A}_3$ regarding the opposition with CFs. Even if each parameter has a different effect, an increase in the membership degree is treated in the traditional distance measures in the same way as an increase in the non-membership degree. Therefore, support, indeterminacy, and opposition should be measured using signed distance. A step toward membership is a positive step, whereas a step away from membership shows as a negative step. For the three parameters, the differential measures show by using an Sq-LDFSs. Support T-grade increases as the difference between membership degrees rises, but opposition F-grade increases as the difference between non-membership degrees gets bigger and vice versa. Consider about the alternatives $\mathfrak{A}_2$ and $\mathfrak{A}_3$ that have the Sq-LD's fuzzy evaluations $\mathfrak{A}_2 = \{\langle 0.83, 0.21, 0.43\rangle, \langle 0.51, 0.18, 0.17\rangle\}$, and $\mathfrak{A}_3 = \{\langle 0.93, 0.21, 0.41\rangle, \langle 0.72, 0.21, 0.13\rangle\}$. Utilizing the accuracy function (11), as well as score function (10) $Sc(\mathfrak{A}_2) = 0.1284$, $Sc(\mathfrak{A}_3) = 0.1473$ Utilizing the accuracy function (11), as well as score function (10) $\mathfrak{A}_3 = \{\langle 0.93, 0.21, 0.41\rangle, \langle 0.72, 0.21, 0.13\rangle\}$ $\mathfrak{A}_2 = \{\langle 0.83, 0.21, 0.43\rangle, \langle 0.51, 0.18, 0.17\rangle\}$ and $Sc(\mathfrak{A}_2) = 0.1284$, $Sc(\mathfrak{A}_3) = 0.1473$. Human thoughts are not limited to satisfaction and dissatisfaction. There are chances to refuse or remain neutral except to accept or not accept. The elements of an Sq-LDFS $\{\langle T, I, F\rangle, \langle \mathfrak{i}, \mathfrak{j}, \mathfrak{k}\rangle\}$ have a physical interpretation like "who votes in favor" (satisfaction degree), "who votes against" (dissatisfaction degree) "who refuses to vote" (refusal grade) "who abstain" (uncertainty or abstinence grade). Hence for alternative $\mathfrak{A}_1$, the vote for the resolution in favor, remains neutral, and against is equal amount. In evaluation, a tie occurs and Sq-LDF evaluations $\mathfrak{A}_1$, $\mathfrak{A}_2$, and $\mathfrak{A}_3$ with their control factors have the same impact. The accuracy function, which takes the amount of information under consideration, is biased in favor of $\mathfrak{A}_1$ in this situation. Naturally, $\mathfrak{A}_1$ communicates more information than $\mathfrak{A}_2$ and $\mathfrak{A}_3$. However, in this situation, the Sq-LDFS's information is significant since it makes one or two solutions appear superior. Either $\mathfrak{A}_1$ is superior to $\mathfrak{A}_2$ and $\mathfrak{A}_3$ or tie between three and vice versa. That leads us to the fundamental idea behind a scoring function: better Sq-LDFSs have high values for T and low values for I and F with their control variables. This is simply the difference between support, neutral, and opposition is all that matters. In research of sharaf, he found the difference between support and opposition. This research [39] encourages us to find differences among three variables.

4 Rules for Differentiation

The differential measure between two Sq-linear Diophantine fuzzy sets $\mathfrak{A} = \{\langle T_{\mathfrak{A}}(\ell), I_{\mathfrak{A}}(\ell), F_{\mathfrak{A}}(\ell)\rangle, \langle \mathfrak{i}_{\mathfrak{A}}(\ell), \mathfrak{j}_{\mathfrak{A}}(\ell), \mathfrak{k}_{\mathfrak{A}}(\ell)\rangle\}$ and $\mathfrak{B} = \{\langle T_{\mathfrak{B}}(\ell), I_{\mathfrak{B}}(\ell), F_{\mathfrak{B}}(\ell)\rangle, \langle \mathfrak{i}_{\mathfrak{B}}(\ell), \mathfrak{j}_{\mathfrak{B}}(\ell), \mathfrak{k}_{\mathfrak{B}}(\ell)\rangle\}, \forall \sim \ell \in \mathfrak{K}$ is determine as follows.

For triplet grades, membership determinacy and non-membership degrees, the datum is one. In case where A's membership degree exceeds B's, the difference between the grades is added to one, showing a positive step; in case where A's membership degree is less than B's, the difference between the grades is deducted from one, showing a negative step.

Definition 4.1. A differential measurement between two spherical q-linear Diophantine fuzzy sets (Sq-LDFSs) $\mathfrak{A} = \{\langle T_{\mathfrak{A}}(\ell), I_{\mathfrak{A}}(\ell), F_{\mathfrak{A}}(\ell)\rangle, \langle \mathfrak{i}_{\mathfrak{A}}(\ell), \mathfrak{j}_{\mathfrak{A}}(\ell), \mathfrak{k}_{\mathfrak{A}}(\ell)\rangle\}$ and $\mathfrak{B} = \{\langle T_{\mathfrak{B}}(\ell), I_{\mathfrak{B}}(\ell), F_{\mathfrak{B}}(\ell)\rangle, \langle \mathfrak{i}_{\mathfrak{B}}(\ell), \mathfrak{j}_{\mathfrak{B}}(\ell), \mathfrak{k}_{\mathfrak{B}}(\ell)\rangle\}, \forall \ell \in \mathfrak{K}$ is a preference relationship based on the closeness of two Sq-LDFSs' membership, indeterminacy, and non-membership degrees about their positions in the attribute space with cotrol parameters. It can be defined as

$$diff(\mathfrak{A},\mathfrak{B}) = \left[\begin{array}{l} \left\langle \dfrac{1+(T_{\mathfrak{A}}-T_{\mathfrak{B}})}{3+(T_{\mathfrak{A}}+I_{\mathfrak{A}}+F_{\mathfrak{A}})-(T_{\mathfrak{B}}+I_{\mathfrak{B}}+F_{\mathfrak{B}})}, \dfrac{1+(I_{\mathfrak{A}}-I_{\mathfrak{B}})}{3+(T_{\mathfrak{A}}+I_{\mathfrak{A}}+F_{\mathfrak{A}})-(T_{\mathfrak{B}}+I_{\mathfrak{B}}+F_{\mathfrak{B}})}, \dfrac{1+(F_{\mathfrak{A}}-F_{\mathfrak{B}})}{3+(T_{\mathfrak{A}}+I_{\mathfrak{A}}+F_{\mathfrak{A}})-(T_{\mathfrak{B}}+I_{\mathfrak{B}}+F_{\mathfrak{B}})} \right\rangle \\ \left\langle \dfrac{1+(\mathfrak{i}_{\mathfrak{A}}-\mathfrak{i}_{\mathfrak{B}})}{3+(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})-(\mathfrak{i}_{\mathfrak{B}}+\mathfrak{j}_{\mathfrak{B}}+\mathfrak{k}_{\mathfrak{B}})}, \dfrac{1+(\mathfrak{j}_{\mathfrak{A}}-\mathfrak{j}_{\mathfrak{B}})}{3+(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})-(\mathfrak{i}_{\mathfrak{B}}+\mathfrak{j}_{\mathfrak{B}}+\mathfrak{k}_{\mathfrak{B}})}, \dfrac{1+(\mathfrak{k}_{\mathfrak{A}}-\mathfrak{k}_{\mathfrak{B}})}{3+(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})-(\mathfrak{i}_{\mathfrak{B}}+\mathfrak{j}_{\mathfrak{B}}+\mathfrak{k}_{\mathfrak{B}})} \right\rangle \end{array}\right] \tag{5}$$

Definition 4.2. Two Sq-LDFSs $\mathfrak{A} = \{\langle T_{\mathfrak{A}}(\ell), I_{\mathfrak{A}}(\ell), F_{\mathfrak{A}}(\ell)\rangle, \langle \mathfrak{i}_{\mathfrak{A}}(\ell), \mathfrak{j}_{\mathfrak{A}}(\ell), \mathfrak{k}_{\mathfrak{A}}(\ell)\rangle\}$ and $\mathfrak{B} = \{\langle T_{\mathfrak{B}}(\ell), I_{\mathfrak{B}}(\ell), F_{\mathfrak{B}}(\ell)\rangle, \langle \mathfrak{i}_{\mathfrak{B}}(\ell), \mathfrak{j}_{\mathfrak{B}}(\ell), \mathfrak{k}_{\mathfrak{B}}(\ell)\rangle\}, \forall \ell \in \mathfrak{K}$ can be categories using differential measurement (4.1) as below:

1. If $diff(\mathfrak{A},\mathfrak{B}) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$, then $\mathfrak{A}$ and $\mathfrak{B}$ are equivalent $\mathfrak{A} \cong \mathfrak{B}$, with a degee of superiority $DoS(\mathfrak{A},\mathfrak{B}) = DoS(\mathfrak{B},\mathfrak{A}) = 0$, and a degree of inferiority $DoN(\mathfrak{A},\mathfrak{B}) = DoN(\mathfrak{B},\mathfrak{A}) = 0$. If $T_{\mathfrak{A}} = T_{\mathfrak{B}}, I_{\mathfrak{A}} = I_{\mathfrak{B}}, F_{\mathfrak{A}} = F_{\mathfrak{B}}, \mathfrak{i}_{\mathfrak{A}} = \mathfrak{i}_{\mathfrak{B}}, \mathfrak{j}_{\mathfrak{A}} = \mathfrak{j}_{\mathfrak{B}}, \mathfrak{k}_{\mathfrak{A}} = \mathfrak{k}_{\mathfrak{B}}$, then "$\mathfrak{A}$" and "$\mathfrak{B}$" are the identical "$\mathfrak{A} \equiv \mathfrak{B}$." **2.** If $diff(\mathfrak{A},\mathfrak{B}) = \{\langle T_{\mathfrak{D}}, I_{\mathfrak{D}}, F_{\mathfrak{D}}\rangle, \langle \mathfrak{i}_{\mathfrak{D}}, \mathfrak{j}_{\mathfrak{D}}, \mathfrak{k}_{\mathfrak{D}}\rangle\}$ with $\mathfrak{i}_{\mathfrak{D}}^q T_{\mathfrak{D}} > 0.5$, then $\mathfrak{A}$ is superior to $\mathfrak{B}$. $\mathfrak{A} \succ \mathfrak{B}$ with the superior degree $\mathrm{DoS}(\mathfrak{A},\mathfrak{B}) = \frac{1}{2n}[(1+T-I-F)+\left(1+\mathfrak{i}^q-\mathfrak{j}^q-\mathfrak{k}^q\right)/2] > 0$. **3.** If $diff(\mathfrak{A},\mathfrak{B}) = \{\langle T_{\mathfrak{D}}, I_{\mathfrak{D}}, F_{\mathfrak{D}}\rangle, \langle \mathfrak{i}_{\mathfrak{D}}, \mathfrak{j}_{\mathfrak{D}}, \mathfrak{k}_{\mathfrak{D}}\rangle\}$ with $\mathfrak{k}_{\mathfrak{D}}^q F_{\mathfrak{D}} > 0.5$, then $\mathfrak{A}$ is inferior to $\mathfrak{B}$. $\mathfrak{A} \prec \mathfrak{B}$ with the superior degree $DoN(\mathfrak{A},\mathfrak{B}) = \frac{1}{2n}[(1+T-I-F)+\left(1+\mathfrak{i}^q-\mathfrak{j}^q-\mathfrak{k}^q\right)/2] < 0$.

Proposition 4.1. For Sq-LDFSs $\mathfrak{A} = \{\langle T_{\mathfrak{A}}, I_{\mathfrak{A}}, F_{\mathfrak{A}}\rangle, \langle \mathfrak{i}_{\mathfrak{A}}, \mathfrak{j}_{\mathfrak{A}}, \mathfrak{k}_{\mathfrak{A}}\rangle\}, \mathfrak{B} = \{\langle T_{\mathfrak{B}}, I_{\mathfrak{B}}, F_{\mathfrak{B}}\rangle, \langle \mathfrak{i}_{\mathfrak{B}}, \mathfrak{j}_{\mathfrak{B}}, \mathfrak{k}_{\mathfrak{B}}\rangle\}$ and $\mathfrak{C} = \{\langle T_{\mathfrak{C}}, I_{\mathfrak{C}}, F_{\mathfrak{C}}\rangle, \langle \mathfrak{i}_{\mathfrak{C}}, \mathfrak{j}_{\mathfrak{C}}, \mathfrak{k}_{\mathfrak{C}}\rangle\}$

1. If $\mathfrak{A} \succ \mathfrak{B}, \mathfrak{B} \succ \mathfrak{C} \Longrightarrow \mathfrak{A} \succ \mathfrak{C}$.
2. If $\mathfrak{A} \prec \mathfrak{B}, \mathfrak{B} \prec \mathfrak{C} \Longrightarrow \mathfrak{A} \prec \mathfrak{C}$.

Proposition 4.2. For the Sq-LDFSs $\mathfrak{A} = \{\langle T_{\mathfrak{A}}, I_{\mathfrak{A}}, F_{\mathfrak{A}}\rangle, \langle \mathfrak{i}_{\mathfrak{A}}, \mathfrak{j}_{\mathfrak{A}}, \mathfrak{k}_{\mathfrak{A}}\rangle\}$ and $\mathfrak{A}^c = \{\langle F_{\mathfrak{A}}, I_{\mathfrak{A}}, T_{\mathfrak{A}}\rangle, \langle \mathfrak{k}_{\mathfrak{A}}, \mathfrak{j}_{\mathfrak{A}}, \mathfrak{i}_{\mathfrak{A}}\rangle\}$ $diff\left(\mathfrak{A},\mathfrak{A}^c\right) = diff\left(\mathfrak{A}^c,\mathfrak{A}\right)$, And if $\mathfrak{A}$ is superior to $\mathfrak{A}^c$, then $DoN\left(\mathfrak{A}^c,\mathfrak{A}\right) = -DoN\left(\mathfrak{A},\mathfrak{A}^c\right)$, and vice versa. Proof. If we apply the definition of difference between the set $\mathfrak{A}$ and its compliment $\mathfrak{A}^c$, we get

$$diff\left(\mathfrak{A},\mathfrak{A}^c\right)=\left\{\begin{array}{c}\left\langle\begin{array}{c}\frac{1+(T_{\mathfrak{A}}-F_{\mathfrak{A}})}{3+(T_{\mathfrak{A}}+I_{\mathfrak{A}}+F_{\mathfrak{A}})-(F_{\mathfrak{A}}+I_{\mathfrak{A}}+T_{\mathfrak{A}})},\frac{1+(I_{\mathfrak{A}}-I_{\mathfrak{A}})}{3+(T+I_{\mathfrak{A}}+F_{\mathfrak{A}})-(F_{\mathfrak{A}}+I_{\mathfrak{A}}+T_{\mathfrak{A}})},\\ \frac{1+(F_{\mathfrak{A}}-T_{\mathfrak{A}})}{3+(T_{\mathfrak{A}}+I_{\mathfrak{A}}+F_{\mathfrak{A}})-(F_{\mathfrak{A}}+I_{\mathfrak{A}}+T_{\mathfrak{A}})}\end{array}\right\rangle\\ \left\langle\begin{array}{c}\frac{1+(\mathfrak{i}_{\mathfrak{A}}-\mathfrak{k}_{\mathfrak{A}})}{3+(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})-(\mathfrak{k}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{i}_{\mathfrak{A}})},\frac{1+(\mathfrak{j}_{\mathfrak{A}}-\mathfrak{j}_{\mathfrak{A}})}{3+(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})-(\mathfrak{k}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{i}_{\mathfrak{A}})},\\ \frac{1+(\mathfrak{k}_{\mathfrak{A}}-\mathfrak{i}_{\mathfrak{A}})}{3+(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})-(\mathfrak{k}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{i}_{\mathfrak{A}})}\end{array}\right\rangle\end{array}\right\}$$

$$=\left\{\begin{array}{c}\left\langle\begin{array}{c}\frac{1+(T_{\mathfrak{A}}-F_{\mathfrak{A}})}{3+(T_{\mathfrak{A}}+I_{\mathfrak{A}}+F_{\mathfrak{A}})-(F_{\mathfrak{A}}+I_{\mathfrak{A}}+T_{\mathfrak{A}})},\frac{1}{3+(T_{\mathfrak{A}}+I_{\mathfrak{A}}+F_{\mathfrak{A}})-(F_{\mathfrak{A}}+I_{\mathfrak{A}}+T_{\mathfrak{A}})},\\ \frac{1+(F_{\mathfrak{A}}-T_{\mathfrak{A}})}{3+(T_{\mathfrak{A}}+I_{\mathfrak{A}}+F_{\mathfrak{A}})-(F_{\mathfrak{A}}+I_{\mathfrak{A}}+T_{\mathfrak{A}})}\end{array}\right\rangle\\ \left\langle\begin{array}{c}\frac{1+(\mathfrak{i}_{\mathfrak{A}}-\mathfrak{k}_{\mathfrak{A}})}{3+(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})-(\mathfrak{k}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{i}_{\mathfrak{A}})},\frac{1}{3+(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})-(\mathfrak{k}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{i}_{\mathfrak{A}})},\\ \frac{1+(\mathfrak{k}_{\mathfrak{A}}-\mathfrak{i}_{\mathfrak{A}})}{3+(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})-(\mathfrak{k}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{i}_{\mathfrak{A}})}\end{array}\right\rangle\end{array}\right\}$$

$$diff\left(\mathfrak{A}^c,\mathfrak{A}\right)=\left\{\begin{array}{c}\left\langle\begin{array}{c}\frac{1+(F_{\mathfrak{A}}-T_{\mathfrak{A}})}{3+(F_{\mathfrak{A}}+I_{\mathfrak{A}}+T_{\mathfrak{A}})-(T_{\mathfrak{A}}+I_{\mathfrak{A}}+F_{\mathfrak{A}})},\frac{1+(I_{\mathfrak{A}}-I_{\mathfrak{A}})}{3+(F_{\mathfrak{A}}+I_{\mathfrak{A}}+T_{\mathfrak{A}})-(T_{\mathfrak{A}}+I_{\mathfrak{A}}+F_{\mathfrak{A}})},\\ \frac{1+(T_{\mathfrak{A}}-F_{\mathfrak{B}})}{3+(F_{\mathfrak{A}}+I_{\mathfrak{A}}+T_{\mathfrak{A}})-(T_{\mathfrak{A}}+I_{\mathfrak{A}}+F_{\mathfrak{A}})}\end{array}\right\rangle\\ \left\langle\begin{array}{c}\frac{1+(\mathfrak{k}_{\mathfrak{A}}-\mathfrak{i}_{\mathfrak{A}})}{3+(\mathfrak{k}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{i}_{\mathfrak{A}})-(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})},\frac{1+(\mathfrak{j}_{\mathfrak{A}}-\mathfrak{j}_{\mathfrak{A}})}{3+(\mathfrak{k}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{i}_{\mathfrak{A}})-(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})},\\ \frac{1+(\mathfrak{i}_{\mathfrak{A}}-\mathfrak{k}_{\mathfrak{A}})}{3+(\mathfrak{k}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{i}_{\mathfrak{A}})-(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})}\end{array}\right\rangle\end{array}\right\}$$

$$=\left\{\begin{array}{c}\left\langle\begin{array}{c}\frac{1+(F_{\mathfrak{A}}-T_{\mathfrak{A}})}{3+(F_{\mathfrak{A}}+I_{\mathfrak{A}}+T_{\mathfrak{A}})-(T_{\mathfrak{A}}+I_{\mathfrak{A}}+F_{\mathfrak{A}})},\frac{1}{3+(F_{\mathfrak{A}}+I_{\mathfrak{A}}+T_{\mathfrak{A}})-(T_{\mathfrak{A}}+I_{\mathfrak{A}}+F_{\mathfrak{A}})},\\ \frac{1+(T_{\mathfrak{A}}-F_{\mathfrak{B}})}{3+(F_{\mathfrak{A}}+I_{\mathfrak{A}}+T_{\mathfrak{A}})-(T_{\mathfrak{A}}+I_{\mathfrak{A}}+F_{\mathfrak{A}})}\end{array}\right\rangle\\ \left\langle\begin{array}{c}\frac{1+(\mathfrak{k}_{\mathfrak{A}}-\mathfrak{i}_{\mathfrak{A}})}{3+(\mathfrak{k}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{i}_{\mathfrak{A}})-(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})},\frac{1}{3+(\mathfrak{k}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{i}_{\mathfrak{A}})-(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})},\\ \frac{1+(\mathfrak{i}_{\mathfrak{A}}-\mathfrak{k}_{\mathfrak{A}})}{3+(\mathfrak{k}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{i}_{\mathfrak{A}})-(\mathfrak{i}_{\mathfrak{A}}+\mathfrak{j}_{\mathfrak{A}}+\mathfrak{k}_{\mathfrak{A}})}\end{array}\right\rangle\end{array}\right\}$$

We noticed that $diff\left(\mathfrak{A},\mathfrak{A}^c\right)=diff^c\left(\mathfrak{A}^c,\mathfrak{A}\right)$. And if $\mathfrak{A}$ is superior than $\mathfrak{A}^c$, then $DON\left(\mathfrak{A}^c,\mathfrak{A}\right)=-DOS\left(\mathfrak{A},\mathfrak{A}^c\right)$ and vice versa.

Numerical Examples In this subsection a few examples are provided to demonstrate the use of differential measures.

Example 1. Let $\mathfrak{A}_1 = \{\langle 1.0, 0.0, 0.0\rangle, \langle 1., 0.0, 0.0\rangle\}, \mathfrak{A}_2 = \{\langle 1.0, 0.0, 0.0\rangle, \langle 1., 0.0, 0.0\rangle\}$, then

$$diff\ (\mathfrak{A}_1, \mathfrak{A}_2) = \left\{ \begin{array}{l} \left\langle \dfrac{1+(1.0-1.0)}{3+(1.0+0.0+0.0)-(1.0+0.0+0.0)}, \dfrac{1+(0.0-0.0)}{3+(0.0+0.0+0.0)-(1.0+0.0+0.0)}, \dfrac{1+(0.0-0.0)}{3+(1.0+0.0+0.0)-(1.0+0.0+0.0)} \right\rangle, \\ \left\langle \dfrac{1+(1.0-1.0)}{3+(1.0+0.0+0.0)-(1.0+0.0+0.0)}, \dfrac{1+(0.0-0.0)}{3+(0.0+0.0+0.0)-(1.0+0.0+0.0)}, \dfrac{1+(0.0-0.0)}{3+(1.0+0.0+0.0)-(1.0+0.0+0.0)} \right\rangle \end{array} \right\}$$

$$= \{\langle 0.33, 0.33, 0.33\rangle\ \langle 0.33, 0.33, 0.33\rangle\}$$

$$diff\ (\mathfrak{A}_2, \mathfrak{A}_1) = \left\{ \begin{array}{l} \left\langle \dfrac{1+(1.0-1.0)}{3+(1.0+0.0+0.0)-(1.0+0.0+0.0)}, \dfrac{1+(0.0-0.0)}{3+(0.0+0.0+0.0)-(1.0+0.0+0.0)}, \dfrac{1+(0.0-0.0)}{3+(1.0+0.0+0.0)-(1.0+0.0+0.0)} \right\rangle, \\ \left\langle \dfrac{1+(1.0-1.0)}{3+(1.0+0.0+0.0)-(1.0+0.0+0.0)}, \dfrac{1+(0.0-0.0)}{3+(0.0+0.0+0.0)-(1.0+0.0+0.0)}, \dfrac{1+(0.0-0.0)}{3+(1.0+0.0+0.0)-(1.0+0.0+0.0)} \right\rangle \end{array} \right\}$$

$$= \{\langle 0.33, 0.33, 0.33\rangle\ \langle 0.33, 0.33, 0.33\rangle\}$$

Example 2. Let $\mathfrak{A}_1 = \{\langle 0.95, 0.24, 0.38\rangle, \langle 41., 0.21, 0.11\rangle\}, \mathfrak{A}_2 = \{\langle 0.85, 0.24, 0.45\rangle, \langle 0.25, 0.34, 0.18\rangle\}$

$$diff\ (\mathfrak{A}_1, \mathfrak{A}_2) = \left\{ \begin{array}{l} \left\langle \dfrac{1+(0.95-0.85)}{3+(0.95+0.24+0.38)-(0.85+0.24+0.45)}, \dfrac{1+(0.24-0.24)}{3+(0.95+0.24+0.38)-(0.85+0.24+0.45)}, \dfrac{1+(0.38-0.45)}{3+(0.95+0.24+0.38)-(0.85+0.24+0.45)} \right\rangle, \\ \left\langle \dfrac{1+(0.41-0.25)}{3+(0.41+0.21+0.11)-(0.25+0.34+0.18)}, \dfrac{1+(0.24-0.24)}{3+(0.41+0.21+0.11)-(0.25+0.34+0.18)}, \dfrac{1+(0.38-0.45)}{3+(0.41+0.21+0.11)-(0.25+0.34+0.18)} \right\rangle \end{array} \right\}$$

$$= \{\langle 0.45, 0.50, 0.55\rangle\ \langle 0.65, 0.50, 0.35\rangle\}$$

$$diff\ (\mathfrak{A}_2, \mathfrak{A}_1) = \left\{ \begin{array}{l} \left\langle \dfrac{1+(0.85-0.95)}{3+(0.85+0.24+0.45)-(0.95+0.24+0.38)}, \dfrac{1+(0.24-0.24)}{3+(0.85+0.24+0.45)-(0.95+0.24+0.38)}, \dfrac{1+(0.45-0.38)}{3+(0.85+0.24+0.45)-(0.95+0.24+0.38)} \right\rangle, \\ \left\langle \dfrac{1+(0.41-0.25)}{3+(0.41+0.21+0.11)-(0.25+0.34+0.18)}, \dfrac{1+(0.24-0.24)}{3+(0.41+0.21+0.11)-(0.25+0.34+0.18)}, \dfrac{1+(0.38-0.45)}{3+(0.41+0.21+0.11)-(0.25+0.34+0.18)} \right\rangle \end{array} \right\}$$

$$= \{\langle 0.55, 0.50, 0.45\rangle, \langle 0.35, 0.50, 0.65\rangle\}$$

Following that, either $\mathfrak{A}_2$ is suppior than $\mathfrak{A}_1$ with a degree of superiority DoS $(\mathfrak{A}_2, \mathfrak{A}_1) = 0.2$, or $\mathfrak{A}_1$ is inferior than $\mathfrak{A}_2$ with a degree of inferiority DoN $(\mathfrak{A}_1, \mathfrak{A}_2) = -0.2$.

5 An Improved Proposed Framework for MCGDM

(1) Problem identification. Evaluation of the criteria $\{\mathfrak{C}_1, \mathfrak{C}_2, ..., \mathfrak{C}_q\}$ that experts attribute to alternatives $\{\mathfrak{A}_1, \mathfrak{A}_2, ..., \mathfrak{A}_p\}$. Experts' $\{E_1, E_2, ..., E_k\}$ formation of DM about each alternative in columns and each criterion in rows.

(2) The creation of decision matrix and weight vector assigned by experts as Sq-LDFSs.

$$A = [\mathfrak{a}_{ij}]_{t\times s} = \begin{pmatrix} \mathfrak{a}_{11}^{\partial} & \mathfrak{a}_{12}^{\partial} & \cdots & \mathfrak{a}_{1t}^{\partial} \\ \mathfrak{a}_{21}^{\partial} & \mathfrak{a}_{22}^{\partial} & \cdots & \mathfrak{a}_{1t}^{\partial} \\ \mathfrak{a}_{31}^{\partial} & \mathfrak{a}_{32}^{\partial} & \cdots & \mathfrak{a}_{1t}^{\partial} \\ \vdots & \vdots & \ddots & \vdots \\ \mathfrak{a}_{s1}^{\partial} & \mathfrak{a}_{s2}^{\partial} & \cdots & \mathfrak{a}_{st}^{\partial} \end{pmatrix}$$

and $W = \{w_1^{\partial}, w_2^{\partial}, ..., w_t^{\partial}\}$,

(3) Normalize the information in DM to get the best results.

$$A_{st} = 1 - \frac{a_{st} - \min a_{st}}{\max a_{st} - \min a_{st}} \tag{6}$$

(4) Create the aggregated decision matrix using the formula (7), and then determine the best rating for each criterion.

$$M_{st} = [a_{st}], a_{st} = \left\{\left(\sum_{\partial=1}^{z} w_{\partial} T_{st}^{\partial}, \sum_{\partial=1}^{z} w_{\partial} I_{st}^{\partial}, \sum_{\partial=1}^{z} w_{\partial} F_{st}^{\partial}\right), \left(\sum_{\partial=1}^{z} w_{\partial} \mathfrak{i}_{st}^{\partial}, \sum_{\partial=1}^{z} w_{\partial} \mathfrak{j}_{st}^{\partial}, \sum_{\partial=1}^{z} w_{\partial} \mathfrak{k}_{st}^{\partial}\right)\right\} \tag{7}$$

(5) Determine the DFM between the ideal rating and the rating of each alternative for a criterion to create the differential matrix.

$$IR_t = \left\{\left(\max_s T_{st}^{\partial}, \min_s I_{st}^{\partial}, \min_s F_{st}^{\partial}\right), \max_s \mathfrak{i}_{st}^{\partial}, \min_s \mathfrak{j}_{st}^{\partial}, \min_s \mathfrak{k}_{st}^{\partial}\right\}, F_{st} = [f_{st}], f_{st} = Diff\,(IR_s, \mathfrak{a}_{st}) \tag{8}$$

(6) Calculate the collective differential measure (CDFM) for each option.

$$CDFM\,(f_{st}) = \left\{\left(\sum_{s=1}^{z} w_s T_{f_{st}}, \sum_{s=1}^{z} w_s I_{f_{st}}, \sum_{s=1}^{z} w_s F_{f_{st}}\right), \left(\sum_{s=1}^{z} w_s \mathfrak{i}_{f_{st}}, \sum_{s=1}^{z} w_s \mathfrak{j}_{f_{st}}, \sum_{s=1}^{z} w_s \mathfrak{k}_{f_{st}}\right)\right\} \tag{9}$$

(7) Calculate the CDFM's score function and rank. The crisp value of the CDFM of each A is estimated using the score formula. The total degree to which the ideal ratings for the evaluation criteria outperform the ratings of an alternative is the CDFM. The superiority of the alternative rating decreases with decreasing ideal rating superiority. The option with the lowest value is the best, and the alternatives are arranged in ascending order.

The expert weights evaluation technique: With the socioeconomic environment becoming more complicated, switching from a single expert to a group of experts became necessary since it is challenging for a single expert to manage all the relevant components of a complex problem [42]. A homogenous group of specialists with comparable attitudes, knowledge, and experience is virtually impossible when group decision-making. Consideration should be given to the credibility of expert opinions and how they could influence the choice made. Results may be inaccurate if the relative importance of the experts is ignored. Studies on calculating the weights of the criteria are abundant, but studies on calculating the weights of the experts are few [43]. Usually, the weights of the experts are chosen based on personal preference. They are decided upon by a manager or through peer reviews. Credibility is low for this method of allocating the experts' weights. To give the experts' opinions greater weight from a more objective standpoint, objective procedures that make use of quantitative methodologies are used [44]. Based on the similarity between the opinions of the individual experts and the group, Zhang [45] suggested a method to calculate the experts' unknown weights. Experts are more significant and should be given more weight when their opinions are more in line with the group's viewpoint. This study suggests a precise approach for calculating the weights of the unknown experts based on their agreement with the ranking of the alternatives. The weighted sum approach is employed to rank each expert (WSM). Spearman's correlation coefficient is then used to determine the correlation between each pair of experts' ranks. The correlation between an expert and other experts is added to determine the expert's overall correlation. The final step is to divide the sum of the total correlations of each expert by that of the experts as a whole. First, each expert's assessments of a potential replacement for the evaluation criteria are combined using the Pythagorean fuzzy weighted averaging operator (7).

Step1: Create each expert's judgment vector. The pth expert's evaluations for the ith alternative are combined utilizing (7). $B_s^{\partial} = \begin{bmatrix} \sum_{j=1}^{s} \wp_j^p \mathfrak{A}_{1j}^p \\ \sum_{j=1}^{s} \wp_j^p \mathfrak{A}_{2j}^p \\ \vdots \\ \sum_{j=1}^{s} \wp_j^p \mathfrak{A}_{tj}^p \end{bmatrix}$.

Step2: Calculate the total rating score for each option and place it in order. $B_s^{\partial} = \begin{bmatrix} Sc\left(B_1^{\partial}\right) \\ Sc\left(B_2^{\partial}\right) \\ \vdots \\ Sc\left(B_t^{\partial}\right) \end{bmatrix}$. The choice with the highest score is considered to be the best one, with the alternatives sorted in descending order. Next, the pth expert is ranked.

Step3: Use Spearman's correlation coefficient to calculate the correlation between one expert's ranking and the other. Using Spearman's correlation coefficient, the ranking of the pth expert and each of the other experts is correlated. $C\left(E_{\partial}, E_k\right) = 1 - \frac{6\sum d_{\partial k}^2}{n\left(n^2 - 1\right)}$, for $1 \leq \partial, k \leq R, \partial \neq k$ where $d_{\partial k}$ is the difference in the Sth alternative's rank as ordered by the ∂th and kth experts, and $\left(E_{\partial}, E_k\right)$, $C\left(E_{\partial}, E_k\right)$ is the correlation between the rankings of the E_{∂} and E_k experts.

Step4: Calculate each expert's overall correlation. The expert's total correlation is the result of adding up all of the expert's correlations. Assign a weight to the experts. $C\left(E_{\partial}\right) = \sum_{k=1 k\neq\partial}^{R} C\left(E_{\partial}, E_k\right)$.

Step5: Each expert's weight is determined by dividing his or her overall correlation by the sum of the total correlations of the other experts. $\mathrm{w}\left(E_{\partial}\right) = \frac{C\left(E_{\partial}\right)}{\sum_{\partial=1}^{R} C\left(E_{\partial}\right)}$.

Fig. 1 depicts the prescribed MCGDM framework.

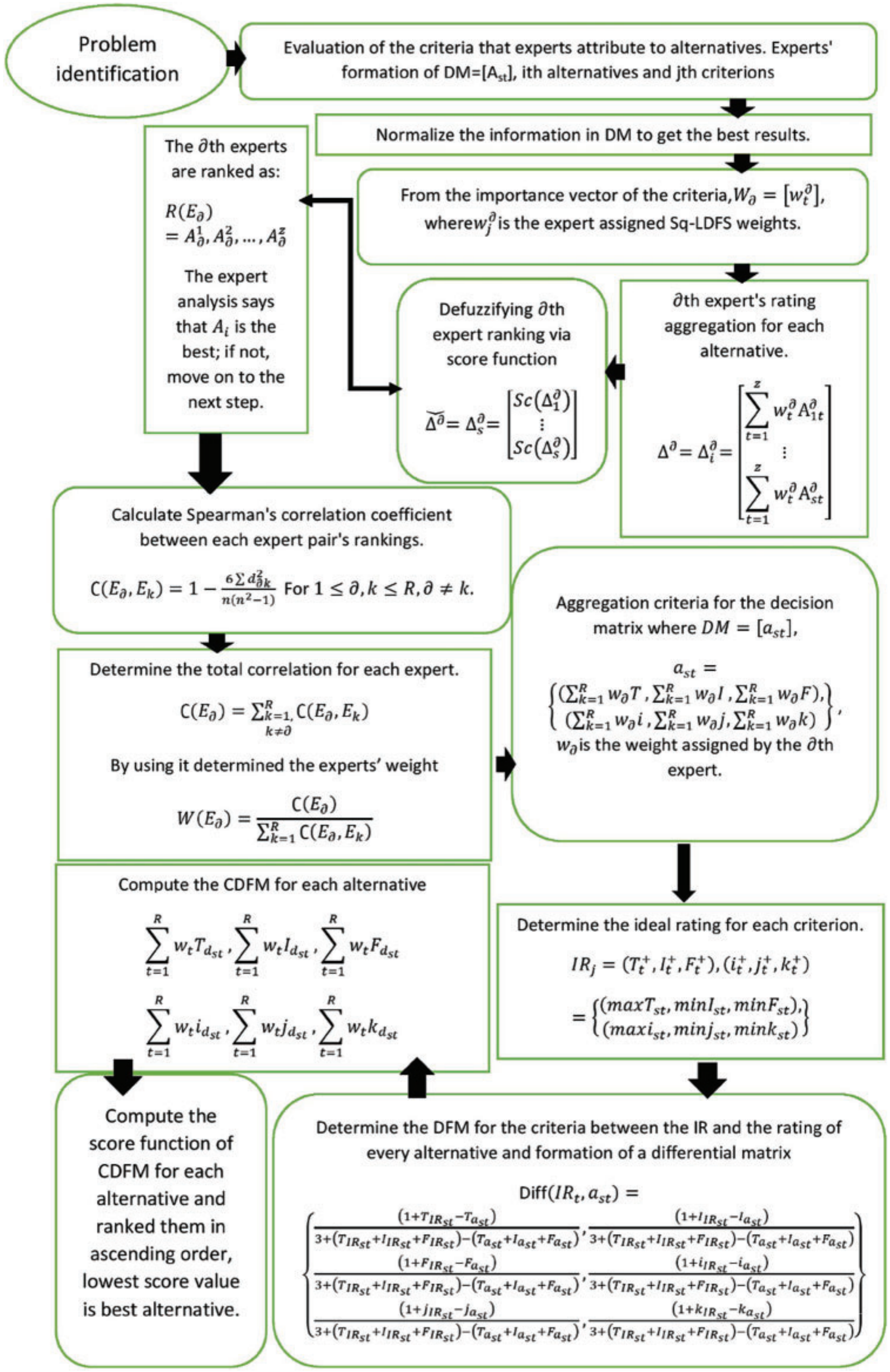

Figure 1: Improved technique for MCGDM

Practical examples: In decision making problem, one of the efficient way to identifying an eligible preferred alternative from a group of alternatives is its similarity degree to the ideal rating. The best alternative is chosen by the degree of similarity. The least differential measurement indicates the ideal option when evaluating the differential measure. In this part, two real-world applications utilizing Pythagorean fuzzy information are discussed in two practically applications. The first application is the evaluation of solid-state drives (SSDs) derived from Huang et al. [35]. The second application is the identification of the finest photovoltaic cell taken from Zhang [45].

Evaluation of SSDs: Flash memory is the primary storage technology used in computers and mobile devices today. Modern computing systems frequently substitute solid-state drives (SSDs) for magnetic hard disc drives as secondary memory (HDDs). HDD performance remained sluggish due to limits on the search time of actuator arms and the rotational speed of magnetic platters. SSDs, on the other side, do not contain complex mechanical parts. Compared to HDDs, this has a reduced failure rate and latency. In addition, SSDs have several advantages over HDDs, including more bandwidth, reduced power use, better random I/O performance, stronger shock tolerance, and increased system dependability [46]. An enterprise wishes to select one type of SSD from the five accessible types, symbolized by the letters "$\mathfrak{A}_1$, $\mathfrak{A}_2$, $\mathfrak{A}_3$, $\mathfrak{A}_4$, $\mathfrak{A}_5$". Performance ($\beth_1$), dependability ($\beth_2$), capacity ($\beth_3$), form factor and connections ($\beth_4$), battery life ($\beth_5$), speed ($\beth_6$), durability ($\beth_7$), and pricing ($\beth_8$) all weigh into the evaluation. Every criterion is a benefit criterion, except the eighth, which is the price, which is a cost criterion. The criteria have a weights vector {0.19, 0.09, 0.11, 0.12, 0.12, 0.13, 0.07, 0.17}. A business expert uses PFSs to assess the SSDs according to the specified criteria. The best rating for the Pythagorean fuzzy decision matrix is in Table 1.

Table 1: The decision matrix for the evaluation of SSDs based on PythFNs

	$\beth_1$	$\beth_2$	$\beth_3$	$\beth_4$	$\beth_5$	$\beth_6$	$\beth_7$	$\beth_8$
$\mathfrak{A}_1$	{.7, .1}	{.5, .2}	{.6, .1}	{.6, .1}	{.6, .1}	{.7, .1}	{.6, .4}	{.6, .2}
$\mathfrak{A}_2$	{.6, .1}	{.6, .2}	{.6, .3}	{.5, .1}	{.5, .3}	{.6, .1}	{.6, .4}	{.5, .3}
$\mathfrak{A}_3$	{.8, .1}	{.6, .1}	{.7, .1}	{.6, .4}	{.6, .3}	{.7, .1}	{.7, .3}	{.5, .2}
$\mathfrak{A}_4$	{.3, .6}	{.3, .1}	{.5, .6}	{.5, .1}	{.4, .2}	{.6, .4}	{.2, .5}	{.7, .1}
$\mathfrak{A}_5$	{.4, .5}	{.3, .2}	{.5, .3}	{.4, .2}	{.3, .3}	{.4, .5}	{.3, .4}	{.6, .1}
IR	{.8, .1}	{.6, .1}	{.7, .1}	{.6, .1}	{.6, .1}	{.7, .1}	{.7, .3}	{.7, .1}

Utilizing DM, the differential measurement between the ideal rating (IR) and the rating for each criterion is computed with the help of the differential measure formula, as shown in Table 2. To obtain its integrated DiFM, the differential measurements of each alternative are aggregated. After that, the CDiFMs are defuzzified using the score function (2.3), allowing for the evaluation of the total degree of the best ratings' superiority over the ratings of the alternatives available for the evaluating criterion.

Table 2: The matrix based on difference measure in pythFSs for SSDs evaluation

	$\beth_1$	$\beth_2$	$\beth_3$	$\beth_4$	$\beth_5$	$\beth_6$	$\beth_7$	$\beth_8$
$\mathfrak{A}_1$	{.4, .3}	{.4, .3}	{.4, .3}	{.3, .3}	{.3, .3}	{.3, .3}	{.4, .3}	{.4, .3}
$\mathfrak{A}_2$	{.4, .3}	{.3, .3}	{.4, .3}	{.4, .3}	{.4, .3}	{.4, .3}	{.4, .3}	{.4, .3}
$\mathfrak{A}_3$	{.3, .3}	{.3, .3}	{.3, .3}	{.4, .3}	{.3, .3}	{.3, .3}	{.3, .3}	{.4, .3}

(Continued)

Table 2 (continued)

	$\beth_1$	$\beth_2$	$\beth_3$	$\beth_4$	$\beth_5$	$\beth_6$	$\beth_7$	$\beth_8$
$\mathfrak{A}_4$	{.5, .2}	{.4, .3}	{.4, .2}	{.4, .3}	{.4, .3}	{.4, .3}	{.4, .3}	{.3, .3}
$\mathfrak{A}_5$	{.5, .2}	{.4, .3}	{.4, .3}	{.4, .3}	{.4, .3}	{.4, .3}	{.4, .3}	{.4, .3}

The options are ranked as shown in Table 3 in ascending order. According to Table 3, the ranking order is $A_3 > A_1 > A_2 > A_4 > A_5$. The outcomes of other decision-making methodologies, including the TOPSIS method proposed by Zhang et al. [47], the TODIM approach developed by Ren et al. [48], the distance and similarity measure introduced by Zeng et al. [49], the fuzzy weighted and ordered weighted aggregation operators presented by Garg [50], and the differential measure for pythagorean fuzzy sets, are compared with the outcomes of the proposed methodology. Table 4 displays the rankings that these techniques produced [39]. Table 4 demonstrates that the suggested technique produced the same optimal alternative as the ones previously employed. The finest choice is the third substitute. The ranking results of our proposed method are the same as the Zeng et al. [49], Ren et al. [48], Garg [50], and Huang et al. [35] techniques.

Table 3: Collective differential measures

	Collective difference measure	DoS (IR, A)	Rank
$\mathfrak{A}_1$	{0.3514, 0.3164}	0.0959	2
$\mathfrak{A}_2$	{0.3671, 0.3045}	0.0977	3
$\mathfrak{A}_3$	{0.3396, 0.3199}	0.0950	1
$\mathfrak{A}_4$	{0.4039, 0.2695}	0.1021	5
$\mathfrak{A}_5$	{0.4042, 0.2765}	0.1017	4

Table 4: Ranking comparison of SSDs for different methods

Methods	Rankings
Zhang et al.'s method [47]	$\mathfrak{A}_3 > \mathfrak{A}_1 > \mathfrak{A}_2 > \mathfrak{A}_5 > \mathfrak{A}_4$
Ren et al.'s method [48]	$\mathfrak{A}_3 > \mathfrak{A}_1 > \mathfrak{A}_2 > \mathfrak{A}_5 > \mathfrak{A}_4$
Zeng et al.'s method [49]	$\mathfrak{A}_3 > \mathfrak{A}_1 > \mathfrak{A}_2 > \mathfrak{A}_4 > \mathfrak{A}_5$
Garg's method [50]	$\mathfrak{A}_3 > \mathfrak{A}_2 > \mathfrak{A}_1 > \mathfrak{A}_5 > \mathfrak{A}_4$
Huang et al.'s method [35]	$\mathfrak{A}_3 > \mathfrak{A}_2 > \mathfrak{A}_1 > \mathfrak{A}_5 > \mathfrak{A}_4$
Sharaf's method [39]	$\mathfrak{A}_3 > \mathfrak{A}_1 > \mathfrak{A}_2 > \mathfrak{A}_4 > \mathfrak{A}_5$
Proposed method	$\mathfrak{A}_3 > \mathfrak{A}_1 > \mathfrak{A}_2 > \mathfrak{A}_5 > \mathfrak{A}_4$

Photovoltaic cells: Photovoltaic (PV) thermoelectric technology has gained a great deal of attention as a result of the present global crisis caused by the shortage of natural fuels in the earth's crust [51]. The most effective alternative to conventional sources of energy power is photovoltaic solar energy, which offers both sustainable and environmentally friendly energy sources [52]. Photovoltaic systems produce zero pollution as they directly harness solar energy to produce electricity [53]. They also provide many benefits, including noiseless operation, low maintenance requirements, and excellent dependability [54]. The advancement of new photovoltaic technologies has recently been the topic of extensive research focused on boosting the effectiveness and sustainability of these devices, integrating with using cheap materials and techniques [52]. These significant efforts combined with outstanding efforts to identify the best alternative and choice based on the requirements, using multi-criteria decision approaches, which have been kept parallel to this ongoing search [55]. A photovoltaic cell is an apparatus that uses the photovoltaic effect, a well-known physical and chemical phenomenon, to transform light energy into electricity. A generational division of several PV cell types in use. The first generation, which is the most widely used and conventional kind consisting of monocrystalline or polycrystalline silicon, controls more than 90% of the market for solar energy today. Due to the expensive manufacture of crystalline silicon, the experts made more efforts to create alternative materials with low-cost fabrication methods. The second generation was created of thin-film cells using less expensive techniques. Amorphous silicon (a-Si), nanocrystalline silicon (nc-Si), cadmium telluride (CdTe), and copper indium gallium selenide are employed the most frequently (CIGS). However, they do not perform much better than first-generation cells. The third generation was created to be highly efficient and inexpensive. They include a range of thin film technologies, some of which employ organic materials to produce energy while others rely on inorganic materials, such as dye-sensitized cells (DSSCs), quantum dot-sensitized cells (QDSSCs), organic solar cells, and hybrid perovskite cells [51]. Table 5 represents the aggregated decision matrix consists of photovoltaic cell data.

Table 5: Aggregated decision matrix

	$\beth_1$	$\beth_2$	$\beth_3$	$\beth_4$	$\beth_5$
$\mathfrak{A}_1$	$\{0.43, 0.82\}$	$\{0.72, 0.59\}$	$\{0.52, 0.75\}$	$\{0.36, 0.62\}$	$\{0.29, 0.59\}$
$\mathfrak{A}_2$	$\{0.66, 0.46\}$	$\{0.89, 0.20\}$	$\{0.79, 0.23\}$	$\{0.29, 0.52\}$	$\{0.39, 0.46\}$
$\mathfrak{A}_3$	$\{0.33, 0.56\}$	$\{0.56, 0.62\}$	$\{0.66, 0.36\}$	$\{0.39, 0.56\}$	$\{0.46, 0.43\}$
$\mathfrak{A}_4$	$\{0.39, 0.75\}$	$\{0.62, 0.52\}$	$\{0.62, 0.20\}$	$\{0.29, 0.62\}$	$\{0.36, 0.69\}$
$\mathfrak{A}_5$	$\{0.26, 0.66\}$	$\{0.72, 0.26\}$	$\{0.85, 0.29\}$	$\{0.59, 0.59\}$	$\{0.36, 0.62\}$
IR	$\{0.66, 0.46\}$	$\{0.88, 0.20\}$	$\{0.85, 0.20\}$	$\{0.59, 0.52\}$	$\{0.46, 0.43\}$

After selecting the ideal site for a photovoltaic solar plant installation, it is necessary to choose the kind of cell among the many PV cell options that best optimize the construction, for example, enhances production or efficiency, reduces costs, and provides the highest maturation and reliability [55].

The selection of photovoltaic cells: The selection of optimal photovoltaic is required from the listed alternatives: CdTe and CIGS ($\mathfrak{A}_1$), amorphous silicon ($\mathfrak{A}_2$), thin-film III-V with tracking systems ($\mathfrak{A}_3$), organic and hybrid cells ($\mathfrak{A}_4$), and crystalline silicon ($\mathfrak{A}_5$). These parameters are used to evaluate these cells: manufacturing cost ($\Im_1$), energy conversion efficiency ($\Im_2$), market share ($\Im_3$), emission of greenhouse gas from production ($\Im_4$), and energy payback period ($\Im_5$). The evaluation involved in this procedure used weights given by three experts (0.3191, 0.3533, 0.3276). The criteria's weighting vector is (0.2, 0.4, 0.1, 0.1, 0.2). PFSs present how the experts rated the potential solutions to the evaluation criteria. The decision matrices are created and normalized to consider the cost criterion. Table 6 presents the aggregated decision matrix using the PFWAY (7) and the ideal rating (IR) utilizing (8). Zhang [45] provided detailed information on Pythagorean fuzzy decision matrices. Then the diff. measure between IR and the alternative's rating calculated for evaluation criterion by using

(4.1) and creating a differential matrix as shown in Table 6. The total differential estimation can be obtained, and the differential measurements of each alternative are combined. The options are then rated accordingly to those shown in Table 7 using the scoring function (2.3). The ultimate ranking is $\mathfrak{A}_2 > \mathfrak{A}_5 > \mathfrak{A}_3 > \mathfrak{A}_4 > \mathfrak{A}_1$, as shown in Table 7. As a result, $\mathfrak{A}_2$ is the ideal solar cell (amorphous silicon). A technique for multi-criteria group decision-making (MCGDM) based on similarity measures and newly created aggregation operators was described by Zhang [45]. An approach for MCGDM using similarity measures based on point operators was also put out by Biswas et al. [56]. They created the Pythagorean fuzzy-dependent averaging operator and the Pythagorean fuzzy-dependent geometric operator using these similarity measurements as a starting point. These operators are used to combine the opinions of the experts. The ranking achieved using the methods of Zhang [45] and Biswas et al. [56] are compared to the results of the suggested technique. Table 8 provides a comparison. Thus according to Table 8, the ranking produced by the suggested strategy matches the ranking of Biswas et al. [56].

Table 6: Differential measure matrix

	$\beth_1$	$\beth_2$	$\beth_3$	$\beth_4$	$\beth_5$
$\mathfrak{A}_1$	$\{0.43, 0.22\}$	$\{0.42, 0.22\}$	$\{0.48, 0.16\}$	$\{0.39, 0.29\}$	$\{0.30, 0.26\}$
$\mathfrak{A}_2$	$\{0.33, 0.33\}$	$\{0.33, 0.33\}$	$\{0.35, 0.32\}$	$\{0.39, 0.30\}$	$\{0.27, 0.30\}$
$\mathfrak{A}_3$	$\{0.41, 0.28\}$	$\{0.46, 0.20\}$	$\{0.40, 0.28\}$	$\{0.38, 0.31\}$	$\{0.26, 0.30\}$
$\mathfrak{A}_4$	$\{0.43, 0.24\}$	$\{0.43, 0.23\}$	$\{0.38, 0.31\}$	$\{0.41, 0.28\}$	$\{0.28, 0.24\}$
$\mathfrak{A}_5$	$\{0.44, 0.25\}$	$\{0.38, 0.30\}$	$\{0.34, 0.31\}$	$\{0.34, 0.32\}$	$\{0.28, 0.25\}$

Table 7: Collective differential measures

	Collective difference measure	DoS (IR, A)	Rank
$\mathfrak{A}_1$	$\{0.4008, 0.2288\}$	0.1672	5
$\mathfrak{A}_2$	$\{0.3293, 0.3204\}$	0.1509	1
$\mathfrak{A}_3$	$\{0.3940, 0.2529\}$	0.1641	3
$\mathfrak{A}_4$	$\{0.3922, 0.2454\}$	0.1647	4
$\mathfrak{A}_5$	$\{0.3626, 0.2840\}$	0.1578	2

Table 8: Rank of PV cells by different methods

Techniques	Rankings
Zhang's technique [45]	$\mathfrak{A}_2 > \mathfrak{A}_5 > \mathfrak{A}_4 > \mathfrak{A}_3 > \mathfrak{A}_1$
Biswas et al.'s technique [56]	$\mathfrak{A}_2 > \mathfrak{A}_5 > \mathfrak{A}_3 > \mathfrak{A}_4 > \mathfrak{A}_1$
Sharaf's technique [39]	$\mathfrak{A}_2 > \mathfrak{A}_5 > \mathfrak{A}_3 > \mathfrak{A}_4 > \mathfrak{A}_1$
Proposed technique	$\mathfrak{A}_2 > \mathfrak{A}_5 > \mathfrak{A}_3 > \mathfrak{A}_4 > \mathfrak{A}_1$

It is also obvious that the ranking of the suggested technique is virtually identical to Zhang's [45] results. The two best technologies and the poorest technologies are unchanged, however, the moderately performing technologies have altered somewhat.

Propose weighting method application: The proposed technique may be used to calculate the weights of the professionals for the previous PV cell selection task. The illustrations of the stages are listed below.

Step 1: From the given weight vector by each decision professional, by using (2.3) $\mathrm{WV}_{P_1} = \begin{bmatrix} (0.59, 0.67) \\ (0.73, 0.39) \\ (0.39, 0.54) \\ (0.57, 0.54) \\ (0.51, 0.54) \end{bmatrix}$, $\mathrm{WV}_{P_2} = \begin{bmatrix} (0.48, 0.68) \\ (0.67, 0.32) \\ (0.53, 0.56) \\ (0.54, 0.59) \\ (0.62, 0.44) \end{bmatrix}$, $\mathrm{WV}_{P_3} = \begin{bmatrix} (0.52, 0.65) \\ (0.65, 0.33) \\ (0.56, 0.54) \\ (0.39, 0.64) \\ (0.517, 0.39) \end{bmatrix}$

Step 2: Evaluate each alternative's score function (2.3) using Table 10 to rank them. $\widehat{WV}_{P_1} = \begin{bmatrix} -0.1008 \\ 0.3808 \\ -0.1395 \\ 0.1649 \\ -0.0315 \end{bmatrix}$, $\widehat{WV}_{P_2} = \begin{bmatrix} -0.2320 \\ 0.3465 \\ -0.0327 \\ -0.0565 \\ 0.1908 \end{bmatrix}$, $\widehat{WV}_{P_3} = \begin{bmatrix} -0.1521 \\ 0.3201 \\ 0.0220 \\ -0.2575 \\ 0.1728 \end{bmatrix}$. Using the score function's values, each expert's rank is $\mathrm{R(E_1)} = \begin{bmatrix} \mathfrak{A}_2 \\ \mathfrak{A}_4 \\ \mathfrak{A}_5 \\ \mathfrak{A}_1 \\ \mathfrak{A}_3 \end{bmatrix}$, $R(E_2) = \begin{bmatrix} \mathfrak{A}_2 \\ \mathfrak{A}_5 \\ \mathfrak{A}_3 \\ \mathfrak{A}_4 \\ \mathfrak{A}_1 \end{bmatrix}$, $R(E_3) = \begin{bmatrix} \mathfrak{A}_2 \\ \mathfrak{A}_5 \\ \mathfrak{A}_3 \\ \mathfrak{A}_1 \\ \mathfrak{A}_4 \end{bmatrix}$

Step 3: Utilize Spearman's correlation to identify the relationship between the expert ranks. Table 9 summarizes the results.

Table 9: The proposed weighting method used for the PV cell collective differential measure and ranking

Alternatives	R (σ_1)	R (σ_2)	δ_{12}^2	R (σ_1)	R (σ_3)	δ_{13}^2	R (σ_2)	R (σ_3)	δ_{23}^2
$\mathfrak{A}_1$	4	5	1	4	4	0	5	4	1
$\mathfrak{A}_2$	1	1	0	1	1	0	1	1	0
$\mathfrak{A}_3$	5	3	4	5	3	4	3	3	0
$\mathfrak{A}_4$	2	4	4	2	5	9	4	5	1
$\mathfrak{A}_5$	3	2	1	3	2	1	2	2	0
	C (σ_1, σ_2) = 0.5			C (σ_1, σ_3) = 0.3			C (σ_2, σ_3) = 0.9		

Step 4: Computation of each professional's total correlation $C(\mathrm{E}_1) = C(\mathrm{E}_1, \mathrm{E}_2) + C(\mathrm{E}_1, \mathrm{E}_3) = 0.8$, $C(\mathrm{E}_2) = C(\mathrm{E}_1, \mathrm{E}_2) + C(\mathrm{E}_2, \mathrm{E}_3) = 1.4$, $C(\mathrm{E}_3) = C(\mathrm{E}_1, \mathrm{E}_3) + C(\mathrm{E}_2, \mathrm{E}_3) = 1.2$

Step 5: Quantify the weights of the experts $w\ (\mathrm{E}_1) = \dfrac{0.8}{3.4} = 0.24$, $w\ (\mathrm{E}_1) = \dfrac{01.4}{3.4} = 0.41$, $w\ (\mathrm{E}_1) = \dfrac{1.2}{3.4} = 0.35$.

After the resolution of PV cell selection by qualities, the weights are calculated by the proposed technique, and the obtained data are summarized in Table 10. The obtained ranking is consistent with previous results. It is worth noting that this methodology can detect the optimum alternative for selecting expert weights. After determining how each expert ranked the choices, if one alternative comes in first among all experts, it is the best option, and thereafter step is not necessary unless a complete ranking list is required. The three professionals select the PV cell at number second $\mathfrak{A}_2$ first in the preceding case. In that case, it is the best choice.

5.1 Case Study

Cancer [57] is not only a disease but rather a collection of diseases that collectively cause the body's cells to alter and proliferate uncontrollably. Cancers are classed either based on the type of fluid or tissue from which they arise or based on where in the body they first manifested themselves. Some cancers are of mixed kinds. The origins of cancer in the tissues and blood may be subdivided into the five major categories below:

Carcinoma: A carcinoma cancer develops in epithelial tissue, which covers or lines the surfaces of organs, glands, and other body organs. A carcinoma, for instance, is a cancer of the stomach lining. Numerous carcinomas attack organs or glands that secrete substances, such as milk-producing breasts. 80%–90% of all cancer cases are carcinomas.

Table 10: The modified SSD problem's over all differential measure and ranking

Alternatives	CDFM	DoS $(IR, \mathfrak{A}_i)$	Ranking
$\mathfrak{A}_1$	$\{0.4315, 0.2271\}$	0.2131	5
$\mathfrak{A}_2$	$\{0.3455, 0.3259\}$	0.1899	1
$\mathfrak{A}_3$	$\{0.4162, 0.2621\}$	0.2068	3
$\mathfrak{A}_4$	$\{0.4274, 0.2430\}$	0.2105	4
$\mathfrak{A}_5$	$\{0.3846, 0.2893\}$	0.1994	2

Sarcoma: Sarcomas are cancerous tumors that grow from connective tissues such as cartilage, fat, muscle, tendons, and bones. The most prevalent sarcoma, a bone tumor, usually affects young individuals. Chondrosarcoma (cartilage) and osteosarcoma are two examples of sarcoma.

Lymphoma: Lymphoma-type cancer develops in the lymph systems nodes or glands, whose job is to produce white blood cells and clean body fluids, as in organs like the brain and breast. The two types of lymphomas are Hodgkin's lymphoma and Hodgkin's non-lymphoma.

Leukemia: Leukemia, widely known as blood cancer, is a bone marrow cancer that resists the marrow from creating healthy red, white, and platelet blood cells. To fight infection, white blood cells are necessary. To avoid anemic, red blood cells are necessary. The presence of platelets helps to prevent rapid bleeding and bruising.

Myeloma: Bone marrow plasma cells are where myeloma develops. Myeloma cells can sometimes gather in one bone to create a single tumor known as plasmacytoma. However, in some instances, the myeloma cells assemble in several bones, producing numerous bone tumors. Several medical tests [58] are used to diagnose these cancer types namely CT scan, MRI, Nuclear scan, Bone scan, PET scan, Ultrasound, X-rays, and Biopsy. Suppose a medical expert have five cancer patients namely $\mathfrak{A}_1$ = Jown, $\mathfrak{A}_2$ = Jerry, $\mathfrak{A}_3$ = Wilson, $\mathfrak{A}_4$ = Hang, and $\mathfrak{A}_5$ = Wang, who is suffering from cancer of different types at the last stages. After some medical tests report namely CT scan, MRI, Nuclear scan, Bone scan, PET scan, ultrasound, X-rays, and Biopsy reports, the medical expert's target is to find the cancer patient who is in the most critical condition due to one of the listed cancer types. Let us say a medical expert examines five cancer patients, each of whom has a different type of cancer. According to their tests reports respective values of $\mathfrak{f} = \{\mathfrak{f}_1$ (CT scan), $\mathfrak{f}_2$ (MRI), $\mathfrak{f}_3$ (Nuclearscan), $\mathfrak{f}_4$ (PETscan), and $\mathfrak{f}_5$ (Biopsy)$\}$ of five patients are given in SLDFNs. Different values for medical reports are associated with the cancer types $\mathfrak{R} = \{\mathfrak{r}_1$(Leukemia), $\mathfrak{r}_2$ (Carcinoma), $\mathfrak{r}_3$ (Sarcoma), $\mathfrak{r}_4$ (Lymphoma), and $\mathfrak{r}_5$ (Myeloma)$\}$. The criteria's weights are $\{0.45, 0.21, 0.20, 0.10, 0.04\}$ specialists diagnose the patients for the specified cancer types based on Sq-LDFSs. The table provides the spherical q-linear Diophantine fuzzy decision matrix with the optimal rating.

Step1: Creation of a decision matrix based on Sq-LDFSs w.r.t the reports of five cancer patients in Table 11.

Table 11: Sq-LDF decision matrix $\{\langle T_{\mathfrak{A}}, I_{\mathfrak{A}}, F_{\mathfrak{A}}\rangle, \langle \mathrm{i}_{\mathfrak{A}}, \mathrm{j}_{\mathfrak{A}}, \mathfrak{k}_{\mathfrak{A}}\rangle\}$

Alternatives	Criterias
$\mathfrak{A}_1$	$\mathfrak{f}_1 = \{\langle .85, .24, .45\rangle, \langle .25, .34, .18\rangle\}$, $\mathfrak{f}_2 = \{\langle .73, .31, .48\rangle, \langle .34, .11, .23\rangle\}$ $\mathfrak{f}_3 = \{\langle .63, .45, .38\rangle, \langle .41, .28, .11\rangle\}$, $\mathfrak{f}_4 = \{\langle .81, .41, .32\rangle, \langle .31, .23, .31\rangle\}$ $\mathfrak{f}_5 = \{\langle .78, .17, .45\rangle, \langle .33, .12, .27\rangle\}$
$\mathfrak{A}_2$	$\mathfrak{f}_1 = \{\langle .77, .41, .52\rangle, \langle .34, .21, .22\rangle\}$, $\mathfrak{f}_2 = \{\langle .82, .51, .43\rangle, \langle .13, .25, .21\rangle\}$ $\mathfrak{f}_3 = \{\langle .58, .43, .41\rangle, \langle .31, .23, .15\rangle\}$, $\mathfrak{f}_4 = \{\langle .78, .45, .31\rangle, \langle .51, .11, .18\rangle\}$ $\mathfrak{f}_5 = \{\langle .83, .21, .43\rangle, \langle .72, .13, .14\rangle\}$
$\mathfrak{A}_3$	$\mathfrak{f}_1 = \{\langle .95, .41, .38\rangle, \langle .41, .25, .18\rangle\}$, $\mathfrak{f}_2 = \{\langle .77, .62, .43\rangle, \langle .31, .25, .21\rangle\}$ $\mathfrak{f}_3 = \{\langle .86, .41, .38\rangle, \langle .41, .23, .17\rangle\}$, $\mathfrak{f}_4 = \{\langle .89, .38, .46\rangle, \langle .46, .32, .11\rangle\}$ $\mathfrak{f}_5 = \{\langle .83, .21, .38\rangle, \langle .51, .18, .17\rangle\}$
$\mathfrak{A}_4$	$\mathfrak{f}_1 = \{\langle .82, .41, .38\rangle, \langle .41, .21, .11\rangle\}$, $\mathfrak{f}_2 = \{\langle .91, .61, .53\rangle, \langle .38, .21, .22\rangle\}$ $\mathfrak{f}_3 = \{\langle .73, .61, .48\rangle, \langle .25, .31, .18\rangle\}$, $\mathfrak{f}_4 = \{\langle .83, .63, .47\rangle, \langle .38, .21, .17\rangle\}$ $\mathfrak{f}_5 = \{\langle .76, .58, .43\rangle, \langle .31, .23, .33\rangle\}$
$\mathfrak{A}_5$	$\mathfrak{f}_1 = \{\langle .73, .61, .53\rangle, \langle .41, .21, .18\rangle\}$, $\mathfrak{f}_2 = \{\langle .83, .51, .68\rangle, \langle .31, .21, .15\rangle\}$ $\mathfrak{f}_3 = \{\langle .73, .61, .58\rangle, \langle .41, .23, .16\rangle\}$, $\mathfrak{f}_4 = \{\langle .81, .32, .58\rangle, \langle .38, .31, .14\rangle\}$ $\mathfrak{f}_5 = \{\langle .93, .21, .41\rangle, \langle .41, .21, .13\rangle\}$

Step2: Normalized fuzzy data given in Table 12.

Table 12: Normalized Sq-LDF decision matrix

Alternatives	Criterias
$\mathfrak{A}_1$	$\mathfrak{f}_1 = \{\langle .45, 1.0, .53\rangle, \langle 1.00, .00, .36\rangle\}$, $\mathfrak{f}_2 = \{\langle .82, .54, .07\rangle, \langle .44, 1.0, .00\rangle\}$ $\mathfrak{f}_3 = \{\langle .00, .54, 1.0\rangle, \langle .00, .79, .36\rangle\}$, $\mathfrak{f}_4 = \{\langle .59, .54, 1.0\rangle, \langle .00, 1.0, 1.0\rangle\}$ $\mathfrak{f}_5 = \{\langle 1.0, .00, .00\rangle, \langle .00, 1.0, .36\rangle\}$
$\mathfrak{A}_2$	$\mathfrak{f}_1 = \{\langle 1.0, 1.0, .80\rangle, \langle .16, 1.0, .00\rangle\}$, $\mathfrak{f}_2 = \{\langle .50, .35, 1.0\rangle, \langle 1.0, .00, .25\rangle\}$ $\mathfrak{f}_3 = \{\langle .78, .00, 1.0\rangle, \langle .28, .00, .63\rangle\}$, $\mathfrak{f}_4 = \{\langle .00, .03, .60\rangle, \langle .00, .29, .13\rangle\}$ $\mathfrak{f}_5 = \{\langle .44, .35, .00\rangle, \langle .28, .29, 1.0\rangle\}$

(Continued)

Table 12 (continued)

Alternatives	Criterias
$\mathfrak{A}_3$	$\mathfrak{f}_1 = \{\langle .82, .80, 1.00\rangle, \langle .00, .38, 1.0\rangle\}$, $\mathfrak{f}_2 = \{\langle 1.0, .90, .85\rangle, \langle .63, 1.0, .43\rangle\}$ $\mathfrak{f}_3 = \{\langle .00, 1.0, 1.0\rangle, \langle .00, 1.0, .14\rangle\}$, $\mathfrak{f}_4 = \{\langle .46, .00, .50\rangle, \langle 1.0, .00, .00\rangle\}$ $\mathfrak{f}_5 = \{\langle .46, .00, .00\rangle, \langle .00, 1.0, .29\rangle\}$
$\mathfrak{A}_4$	$\mathfrak{f}_1 = \{\langle .73, .71, .96\rangle, \langle 1.0, .43, .00\rangle\}$, $\mathfrak{f}_2 = \{\langle 1.0, .58, 1.0\rangle, \langle .00, 1.0, .65\rangle\}$ $\mathfrak{f}_3 = \{\langle .00, .81, .44\rangle, \langle .25, .00, 1.0\rangle\}$, $\mathfrak{f}_4 = \{\langle .55, .00, .41\rangle, \langle .65, .52, .70\rangle\}$ $\mathfrak{f}_5 = \{\langle .73, 1.0, .00\rangle, \langle .65, .05, .85\rangle\}$
$\mathfrak{A}_5$	$\mathfrak{f}_1 = \{\langle .88, 1.0, .00\rangle, \langle .95, 1.0, .30\rangle\}$, $\mathfrak{f}_2 = \{\langle .59, .90, .29\rangle, \langle .00, .91, .95\rangle\}$ $\mathfrak{f}_3 = \{\langle .59, .90, 1.0\rangle, \langle .51, .45, .80\rangle\}$, $\mathfrak{f}_4 = \{\langle 1.0, .00, .29\rangle, \langle 1.0, .00, .00\rangle\}$ $\mathfrak{f}_5 = \{\langle .00, .90, .57\rangle, \langle .76, .18, 1.0\rangle\}$

Step3: The differential measure between the ideal ratings and the ratings of each alternative for the criterion is derived using the decision matrix. Following that, the differential matrix shows the results in Table 13.

Table 13: Weighted measurement among parameters

Alternatives	Criterias
$\mathfrak{A}_1$	$\mathfrak{f}_1 = \{\langle 1.5, .00, .46\rangle, \langle .61, .61, .39\rangle\}$, $\mathfrak{f}_2 = \{\langle .75, .29, .59\rangle, \langle 1.0, .00, .64\rangle\}$ $\mathfrak{f}_3 = \{\langle 1.4, .31, .00\rangle, \langle 1.0, .16, .33\rangle\}$, $\mathfrak{f}_4 = \{\langle 1.6, .53, .00\rangle, \langle 2.0, .00, .00\rangle\}$ $\mathfrak{f}_5 = \{\langle .50, .50, .50\rangle, \langle 1.2, .00, .39\rangle\}$
$\mathfrak{A}_2$	$\mathfrak{f}_1 = \{\langle 5.0, .00, 1.0\rangle, \langle 1.0, .00, .54\rangle\}$, $\mathfrak{f}_2 = \{\langle 1.3, .56, .00\rangle, \langle .57, .57, .43\rangle\}$ $\mathfrak{f}_3 = \{\langle 1.0, .82, .00\rangle, \langle .82, .48, .18\rangle\}$, $\mathfrak{f}_4 = \{\langle .84, .41, .17\rangle, \langle .77, .28, .34\rangle\}$ $\mathfrak{f}_5 = \{\langle .71, .29, .45\rangle, \langle 1.2, .50, .00\rangle\}$
$\mathfrak{A}_3$	$\mathfrak{f}_1 = \{\langle 3.1, .53, .00\rangle, \langle 1.2, .38, .00\rangle\}$, $\mathfrak{f}_2 = \{\langle 4.0, .40, .60\rangle, \langle 1.5, .00, .60\rangle\}$ $\mathfrak{f}_3 = \{\langle 2.0, .00, .00\rangle, \langle 1.1, .00, .46\rangle\}$, $\mathfrak{f}_4 = \{\langle .75, .49, .25\rangle, \langle .50, .50, .50\rangle\}$ $\mathfrak{f}_5 = \{\langle .61, .39, .39\rangle, \langle 1.2, .00, .42\rangle\}$
$\mathfrak{A}_4$	$\mathfrak{f}_1 = \{\langle 2.1, .48, .06\rangle, \langle .64, .36, .64\rangle\}$, $\mathfrak{f}_2 = \{\langle 2.4, 1.0, .00\rangle, \langle 1.5, .00, .26\rangle\}$ $\mathfrak{f}_3 = \{\langle 1.1, .11, .32\rangle, \langle 1.0, .57, .00\rangle\}$, $\mathfrak{f}_4 = \{\langle .71, .49, .29\rangle, \langle 1.2, .42, .27\rangle\}$ $\mathfrak{f}_5 = \{\langle 1.0, .00, .79\rangle, \langle .93, .66, .10\rangle\}$
$\mathfrak{A}_5$	$\mathfrak{f}_1 = \{\langle 1.0, .00, .89\rangle, \langle 1.4, .00, .93\rangle\}$, $\mathfrak{f}_2 = \{\langle 1.2, .08, .58\rangle, \langle 1.8, .08, .04\rangle\}$ $\mathfrak{f}_3 = \{\langle 2.8, .19, .00\rangle, \langle 1.2, .44, .16\rangle\}$, $\mathfrak{f}_4 = \{\langle .58, .58, .42\rangle, \langle .50, .50, .50\rangle\}$ $\mathfrak{f}_5 = \{\langle 1.3, .06, .28\rangle, \langle 1.2, .77, .00\rangle\}$

The collective differential measure and ranks for cancer patients can be seen in Table 14. The ranking of patients according to their condition is presented in Table 15.

Table 14: By proposed weighting method

	CDFM	DoS $(IR, \mathfrak{A}_i)$	Ranks
$\mathfrak{A}_1$	$\{\langle 0.6869, 0.9310, 0.7043\rangle, \langle 0.6216, 0.3679, 0.3756\rangle\}$	0.0620	2
$\mathfrak{A}_2$	$\{\langle 0.7967, 0.5919, 0.5214\rangle, \langle 0.5319, 0.7864, 0.2412\rangle\}$	0.1008	4
$\mathfrak{A}_3$	$\{\langle 0.1869, 0.5600, 0.9444\rangle, \langle 0.1043, 0.5297, 0.4555\rangle\}$	0.0061	1
$\mathfrak{A}_4$	$\{\langle 0.4533, 0.2500, 0.7282\rangle, \langle 0.3050, 0.5624, 0.5463\rangle\}$	0.0819	3
$\mathfrak{A}_5$	$\{\langle 0.7089, 0.2106, 0.0229\rangle, \langle 0.1540, 0.7220, 0.5558\rangle\}$	0.1703	5

Table 15: Ranking of patients according to their condition

Method	Ranking
Riaz's method	$\mathfrak{A}_3 > \mathfrak{A}_1 > \mathfrak{A}_4 > \mathfrak{A}_2 > \mathfrak{A}_5$
Proposed method	$\mathfrak{A}_3 > \mathfrak{A}_1 > \mathfrak{A}_4 > \mathfrak{A}_2 > \mathfrak{A}_5$

6 Disscusion

The purpose of this paper is to discriminate the Sq-LDFSs by the proposed notion of DFM. It is a preference connection between two Sq-LDFSs because of where they are in the attribute space and how closely their degrees of membership, indeterminacy and non-membership with the control parameter are correlated. Overcoming potential flaws in the current measures between fuzzy sets is the primary goal of the DFM. The shortcomings of the current distance and similarity metrics come from treating membership, indeterminacy, and non-membership degree identically, although each direction has a different connotation. Differently, a DFM handles variations in a spherical q-linear diophantine fuzzy assessment's parameters. A step taken in the support direction is viewed as a +ve point, while an opposite step is shown as −ve with the same indeterminacy values. The degree of superiority or inferiority between two Sq-LDFSs is determined by whether they are classed as identical, equivalent, superior, or inferior to one another. Based on the newly introduced DFM, a new method for calculating the expert weights is created, and an MCGDM method is also defined. To demonstrate the approach, two real-world issues were resolved. The suggested method's results are contrasted with those of several other existing MCDM approaches. The comparison's primary goal is to confirm and show the method's applicability. There is no worst or best methodology; all of the MCDM methods can be compared to one another. Even when processing the same data and information, different methodologies can yield different conclusions. We can determine which approach is most appropriate in a specific circumstance [59]. When the MCDM methods based on distance or similarity measurements are impacted by the aforementioned flaws, resulting to inaccurate findings, the proposed method would be preferable. The assessment of solid-state drives is the first illustration. The proposed method's outcome is contrasted with that of other decision-making techniques, including the TOPSIS method put forth by Zhang et al. [47], the TODIM approach created by Ren et al. [48], the distance and similarity measures introduced by Zeng et al. [49], the fuzzy weighted and ordered weighted aggregation operators put forth by Garg [50], and the PF-MULTIMOORA method put forth by Huang et al. [35]. The best choice was found using the same techniques. With a few minor differences from previous ranking lists, the ranking list produced by the suggested method is consistent with the outcomes of Zeng et al. The choice of the best photovoltaic cell serves as the second illustration. The outcome of the suggested strategy is contrasted with that of Zhang [45] and Biswas et al. [56]. The best alternative was determined using all three procedures. The ranking list of the suggested technique is almost identical to the ranking list of Zhang [45] and correlates with the ranking list of Biswas et al. [56]. Finally, using Spearman's correlation coefficient as a base, an improved method for calculating the expert weights is created and used for the photovoltaic cell problem. The weights obtained from the experts differ slightly from those reported by Zhang [45] in some respects. Nevertheless, the experts' relative weights are equal, and the same answer is reached when the problem is solved using the suggested weights. Practical applications for the proposed DFM include image processing, pattern recognition, machine learning, information retrieval, medical diagnosis, and decision-making. The suggested framework might reduce preferences that might develop in the decision-making process as a result of giving decision-makers subjective weights, which frequently results in outcomes that are not accurate [60]. The framework offers decision-makers objective weights to counteract the unfavorable impact of subjective weights. The suggested approach can therefore help managers make better judgments. It is predicted that the created method will work successfully in any GDM setting, including corporate management and industrial engineering. Fig. 2 represents the differential measurement in both PyFS and Sq-LDFS graphically.

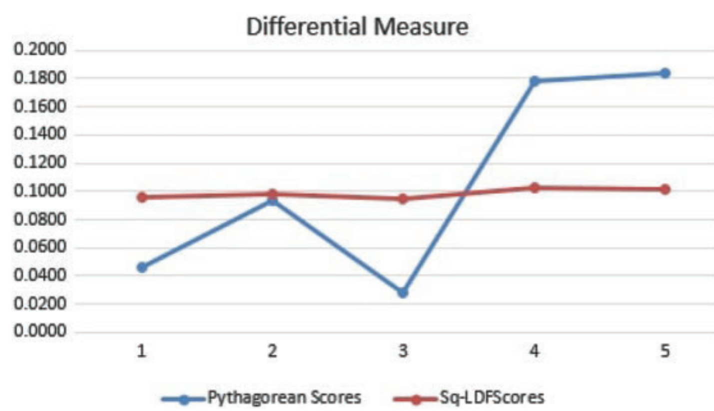

Figure 2: Differential measure comparison

7 Conclusion

The construction of an appropriate evaluation method for the fuzzy data to find differential measures is essential. There are a lot of techniques to find a difference in fuzzy sets, i.e., similarity measures, distance measures, and methods such as VIKOR, TOPSIS, EDAS, etc. In this article, we proposed differential measures among three grades membership, indeterminacy, and non-membership related to their control variables. The results of our technique show efficient performance. we use the previous examples from Pythagorean fuzzy sets data but got the same results as our proposed technique of differential measure in Sq-LDFSs. The MCGDM method proposed by finding the weights by Spearman's correlation coefficient got identical results in three grades. "Differential Measure for Pythagorean Fuzzy Sets" only covers two membership grades; however, there is a need for a fuzzy set to fill in this gap and cover the data for the three grades when the Pythagorean fuzzy set constraints fall short in specific situations. Therefore, using data from three grades, we determine the

Sq-LDFS and find the differential measure. The outcome validates the effectiveness of the suggested method. We infer that our approach is useful for three parametric data in addition to two degree data based on our ranking results and comparison.

The proposed Sq-LDFS concept can be used in other existing methods, such as MAIRCA, ELECTREE, AHP, or PROMETHEE, to enhance its performance and accuracy. The proposed technique can be applicable to several real-life problems that involve decision-making under uncertainty and imprecision. For instance, in green supplier selection, the proposed method can be used to evaluate and rank potential suppliers based on multiple criteria, such as quality, price, and delivery time etc. In medical field, the proposed method can be used to analyze patient data and provide a diagnosis based on multiple symptoms and test results.

Acknowledgement: Muhammad Naeem would like to thank the Deanship of Scientific Research at Umm Al-Qura University for supporting this work.

Funding Statement: This work is supported by the Deanship of Scientific Research at Umm Al-Qura University (Grant Code: 22UQU4310396DSR65).

Author Contributions: The authors confirm contribution to the paper as follows: study conception and design: H. Razzaque, S. Ashraf; data collection: H. Razzaque; analysis and interpretation of results: H. Razzaque, S. Ashraf, M. Naeem, Y.M. Chu; draft manuscript preparation: H. Razzaque, S. Ashraf, M. Naeem, Y.M. Chu. All authors reviewed the results and approved the final version of the manuscript.

Availability of Data and Materials: The data used in this paper are hypothetical and can be used by anyone by just citing this article.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

References

1. Akram, M., Dudek, W. A., Ilyas, F. (2019). Group decision-making based on pythagorean fuzzy TOPSIS method. *International Journal of Intelligent Systems, 34(7),* 1455–1475.
2. Wu, Y., Gao, Y., Zhang, B., Pedrycz, W. (2023). Minimum information-loss transformations to support heterogeneous group decision making in a distributed linguistic context. *Information Fusion, 89,* 437–451.
3. Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8(3),* 338–353.
4. Atanassova, K., Stoeva, S. (1983). Intuitionistic fuzzy sets. *Polish Symposium on Interval and Fuzzy Mathematics*, pp. 23–26. Poznan, Poland.
5. Yager, R. R. (2013). Pythagorean fuzzy subsets. *2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS)*, pp. 57–61. Edmonton, AB, Canada.
6. Peng, X., Yuan, H., Yang, Y. (2017). Pythagorean fuzzy information measures and their applications. *International Journal of Intelligent Systems, 32(10),* 991–1029.
7. Peng, X., Selvachandran, G. (2019). Pythagorean fuzzy set: State of the art and future directions. *Artificial Intelligence Review, 52,* 1873–1927.
8. Yager, R. R. (2016). Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems, 25(5),* 1222–1230.
9. Wang, P., Wang, J., Wei, G., Wei, C. (2019). Similarity measures of q-rung orthopair fuzzy sets based on cosine function and their applications. *Mathematics, 7(4),* 340.

10. Donyatalab, Y., Farrokhizadeh, E., Seyfi Shishavan, S. A. (2021). Similarity measures of q-rung orthopair fuzzy sets based on square root cosine similarity function. *Intelligent and Fuzzy Techniques: Smart and Innovative Solutions: Proceedings of the INFUS 2020 Conference*, pp. 475–483. Istanbul, Turkey.
11. Jana, C., Senapati, T., Pal, M. (2019). Pythagorean fuzzy Dombi aggregation operators and its applications in multiple attribute decision-making. *International Journal of Intelligent Systems, 34(9),* 2019–2038.
12. Jana, C., Muhiuddin, G., Pal, M. (2019). Some Dombi aggregation of Q-rung orthopair fuzzy numbers in multiple-attribute decision making. *International Journal of Intelligent Systems, 34(12),* 3220–3240.
13. Qiyas, M., Naeem, M., Abdullah, S., Khan, N., Ali, A. (2022). Similarity measures based on q-rung linear diophantine fuzzy sets and their application in logistics and supply chain management. *Journal of Mathematics, 2022,* 1–19.
14. Kutlu Gündoğdu, F., Kahraman, C. (2019). Spherical fuzzy sets and spherical fuzzy TOPSIS method. *Journal of Intelligent & Fuzzy Systems, 36(1),* 337–352.
15. Ahmmad, J., Mahmood, T., Chinram, R., Iampan, A. (2021). Some average aggregation operators based on spherical fuzzy soft sets and their applications in multi-criteria decision making. *AIMS Mathematics, 6(7),* 7798–7833.
16. Kutlu Gündoğdu, F., Kahraman, C. (2021). Properties and arithmetic operations of spherical fuzzy sets. In: *Decision making with spherical fuzzy sets: Theory and applications*, pp. 3–25. Springer.
17. Shishavan, S. A. S., Gündoğdu, F. K., Farrokhizadeh, E., Donyatalab, Y., Kahraman, C. (2020). Novel similarity measures in spherical fuzzy environment and their applications. *Engineering Applications of Artificial Intelligence, 94,* 103837.
18. Rafiq, M., Ashraf, S., Abdullah, S., Mahmood, T., Muhammad, S. (2019). The cosine similarity measures of spherical fuzzy sets and their applications in decision making. *Journal of Intelligent & Fuzzy Systems, 36(6),* 6059–6073.
19. Khan, M. J., Kumam, P., Deebani, W., Kumam, W., Shah, Z. (2020). Distance and similarity measures for spherical fuzzy sets and their applications in selecting mega projects. *Mathematics, 8(4),* 519.
20. Rayappan, P., Mohana, K. (2021). Spherical fuzzy cross entropy for multiple attribute decision making problems. *Journal of Fuzzy Extension and Applications, 2(4),* 355–363.
21. Mao, X., Zhao, G., Fallah, M., Edalatpanah, S. A. (2020). A neutrosophic-based approach in data envelopment analysis with undesirable outputs. *Mathematical Problems in Engineering, 2020,* 1–8.
22. Kouatli, I. (2022). Modeling fuzzimetric cognition of technical analysis decisions: Reducing emotional trading. *Journal of Fuzzy Extension and Applications, 3(1),* 45–63.
23. Iqbal, S., Yaqoob, N. (2023). Ranking of linear Diophantine fuzzy numbers using circumcenter of centroids. *AIMS Mathematics, 8(4),* 9840–9861.
24. Hasan, M. K., Ali, M., Sultana, A., Mitra, N. K. (2022). Some picture fuzzy mean operators and their applications in decision-making. *Journal of Fuzzy Extension and Applications, 3(4),* 349–361.
25. Wang, P., Wang, J., Wei, G. (2021). The generalized dice similarity measures for spherical fuzzy sets and their applications. In: *Decision making with spherical fuzzy sets: Theory and applications*, pp. 85–110. Springer.
26. Wei, G., Wang, J., Lu, M., Wu, J., Wei, C. (2019). Similarity measures of spherical fuzzy sets based on cosine function and their applications. *IEEE Access, 7,* 159069–159080.
27. Mahmood, T., Ilyas, M., Ali, Z., Gumaei, A. (2021). Spherical fuzzy sets-based cosine similarity and information measures for pattern recognition and medical diagnosis. *IEEE Access, 9,* 25835–25842.
28. Mohammad, M. M. S., Abdullah, S., Al-Shomrani, M. M. (2022). Some linear Diophantine fuzzy similarity measures and their application in decision making problem. *IEEE Access, 10,* 29859–29877. https://doi.org/10.1109/ACCESS.2022.3151684
29. Donyatalab, Y., Gündoğdu, F. K., Farid, F., Seyfi-Shishavan, S. A., Farrokhizadeh, E. et al. (2022). Novel spherical fuzzy distance and similarity measures and their applications to medical diagnosis. *Expert Systems with Applications, 191,* 116330.

30. Bonab, S. R., Ghoushchi, S. J., Deveci, M., Haseli, G. (2023). Logistic autonomous vehicles assessment using decision support model under spherical fuzzy set integrated Choquet Integral approach. *Expert Systems with Applications, 214,* 119205.
31. Ma, Q. X., Zhu, X. M., Bai, K. Y., Zhang, R. T., Liu, D. W. (2023). A novel failure mode and effect analysis method with spherical fuzzy entropy and spherical fuzzy weight correlation coefficient. *Engineering Applications of Artificial Intelligence, 122,* 106163.
32. Ali, J., Garg, H. (2023). On spherical fuzzy distance measure and TAOV method for decision-making problems with incomplete weight information. *Engineering Applications of Artificial Intelligence, 119,* 105726.
33. Das, A. K., Granados, C. (2023). IFP-intuitionistic multi fuzzy N-soft set and its induced IFP-hesitant N-soft set in decision-making. *Journal of Ambient Intelligence and Humanized Computing, 14,* 10143–10152. https://doi.org/10.1007/s12652-021-03677-w
34. Khan, M. R., Ullah, K., Khan, Q. (2023). Multi-attribute decision-making using Archimedean aggregation operator in T-spherical fuzzy environment. *Reports in Mechanical Engineering, 4(1),* 18–38.
35. Huang, C., Lin, M., Xu, Z. (2020). Pythagorean fuzzy MULTIMOORA method based on distance measure and score function: Its application in multicriteria decision making process. *Knowledge and Information Systems, 62,* 4373–4406.
36. Xiao, F., Ding, W. (2019). Divergence measure of Pythagorean fuzzy sets and its application in medical diagnosis. *Applied Soft Computing, 79,* 254–267.
37. Ashraf, S., Abdullah, S. (2019). Spherical aggregation operators and their application in multiattribute group decision-making. *International Journal of Intelligent Systems, 34(3),* 493–523.
38. Razzaque, H., Ashraf, S., Kallel, W., Naeem, M., Sohail, M. (2023). A strategy for hepatitis diagnosis by using spherical q-linear Diophantine fuzzy Dombi aggregation information and the VIKOR method. *Aims Mathematics, 8(6),* 14362–14398.
39. Sharaf, I. M. (2023). The differential measure for Pythagorean fuzzy multiple criteria group decision-making. *Complex & Intelligent Systems, 9,* 3333–3354. https://doi.org/10.1007/s40747-022-00913-4
40. Ashraf, S., Razzaque, H., Naeem, M., Botmart, T. (2023). Spherical q-linear Diophantine fuzzy aggregation information: Application in decision support systems. *AIMS Mathematics, 8(3),* 6651–6681.
41. Song, C., Xu, Z., Hou, J. (2021). An improved TODIM method based on the hesitant fuzzy psychological distance measure. *International Journal of Machine Learning and Cybernetics, 12(4),* 973–985.
42. Yang, Q., Du, P. A. (2015). A straightforward approach for determining the weights of decision makers based on angle cosine and projection method. *International Journal of Industrial and Manufacturing Engineering, 9(10),* 3335–3341.
43. Koksalmis, E., Kabak, Ö. (2019). Deriving decision makers' weights in group decision making: An overview of objective methods. *Information Fusion, 49,* 146–160.
44. Qu, S., Xu, Y., Wu, Z., Xu, Z., Ji, Y. et al. (2021). An interval-valued best-worst method with normal distribution for multi-criteria decision-making. *Arabian Journal for Science and Engineering, 46,* 1771–1785.
45. Zhang, X. (2016). A novel approach based on similarity measure for Pythagorean fuzzy multiple criteria group decision making. *International Journal of Intelligent Systems, 31(6),* 593–611.
46. Jin, Y., Lee, B. (2019). A comprehensive survey of issues in solid state drives. *Advances in Computers, 114,* 1–69.
47. Zhang, X., Xu, Z. (2014). Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets. *International Journal of Intelligent Systems, 29(12),* 1061–1078.
48. Ren, P., Xu, Z., Gou, X. (2016). Pythagorean fuzzy TODIM approach to multi-criteria decision making. *Applied Soft Computing, 42,* 246–259.

49. Zeng, W., Li, D., Yin, Q. (2018). Distance and similarity measures of Pythagorean fuzzy sets and their applications to multiple criteria group decision making. *International Journal of Intelligent Systems, 33(11),* 2236–2254.
50. Garg, H. (2017). Confidence levels based Pythagorean fuzzy aggregation operators and its application to decision-making process. *Computational and Mathematical Organization Theory, 23(4),* 546–571.
51. Ananthakumar, S., Kumar, J. R., Babu, S. M. (2019). Third-generation solar cells: Concept, materials and performance-an overview. *Emerging Nanostructured Materials for Energy and Environmental Science, 23,* 305–339.
52. Giannouli, M. (2021). Current status of emerging PV technologies: A comparative study of dye-sensitized, organic, and perovskite solar cells. *International Journal of Photoenergy, 2021,* 1–19.
53. Liu, Z., Jin, Z., Li, G., Zhao, X., Badiei, A. (2022). Study on the performance of a novel photovoltaic/thermal system combining photocatalytic and organic photovoltaic cells. *Energy Conversion and Management, 251,* 114967.
54. Zhang, F., Wu, M., Hou, X., Han, C., Wang, X. et al. (2021). The analysis of parameter uncertainty on performance and reliability of photovoltaic cells. *Journal of Power Sources, 507,* 230265.
55. Socorro García-Cascales, M., Teresa Lamata, M., Miguel Sánchez-Lozano, J. (2012). Evaluation of photovoltaic cells in a multi-criteria decision making process. *Annals of Operations Research, 199,* 373–391.
56. Biswas, A., Sarkar, B. (2018). Pythagorean fuzzy multicriteria group decision making through similarity measure based on point operators. *International Journal of Intelligent Systems, 33(8),* 1731–1744.
57. U.S. National Cancer Institute's Surveillance, Epidemiology and End Results (SEER) Program. https://training.seer.cancer.gov/disease/categories/classification.html (Accessed on March 2023).
58. National Cancer Institute at the National Institutes of Health, USA. https://www.cancer.gov/about-cancer/diagnosis-staging/diagnosis (Accessed on March 2023).
59. Akram, M., Luqman, A., Kahraman, C. (2021). Hesitant pythagorean fuzzy ELECTRE-II method for multi-criteria decision-making problems. *Applied Soft Computing, 108,* 107479.
60. Akram, M., Ilyas, F., Al-Kenani, A. N. (2021). Two-phase group decision-aiding system using ELECTRE III method in Pythagorean fuzzy environment. *Arabian Journal for Science and Engineering, 46,* 3549–3566.