Computer Modeling in Engineering & Sciences

Tech Science Press

DOI: 10.32604/cmes.2023.029138

ARTICLE

New Antenna Array Beamforming Techniques Based on Hybrid Convolution/Genetic Algorithm for 5G and Beyond Communications

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Received: 03 February 2023 Accepted: 25 July 2023 Published: 15 December 2023

ABSTRACT

Side lobe level reduction (SLL) of antenna arrays significantly enhances the signal-to-interference ratio and improves the quality of service (QOS) in recent and future wireless communication systems starting from 5G up to 7G. Furthermore, it improves the array gain and directivity, increasing the detection range and angular resolution of radar systems. This study proposes two highly efficient SLL reduction techniques. These techniques are based on the hybridization between either the single convolution or the double convolution algorithms and the genetic algorithm (GA) to develop the Conv/GA and DConv/GA, respectively. The convolution process determines the element's excitations while the GA optimizes the element spacing. For M elements linear antenna array (LAA), the convolution of the excitation coefficients vector by itself provides a new vector of excitations of length $N = (2M - 1)$. This new vector is divided into three different sets of excitations including the odd excitations, even excitations, and middle excitations of lengths M, $M - 1$, and M, respectively. When the same element spacing as the original LAA is used, it is noticed that the odd and even excitations provide a much lower SLL than that of the LAA but with a much wider half-power beamwidth (HPBW). While the middle excitations give the same HPBW as the original LAA with a relatively higher SLL. To mitigate the increased HPBW of the odd and even excitations, the element spacing is optimized using the GA. Thereby, the synthesized arrays have the same HPBW as the original LAA with a two-fold reduction in the SLL. Furthermore, for extreme SLL reduction, the DConv/GA is introduced. In this technique, the same procedure of the aforementioned Conv/GA technique is performed on the resultant even and odd excitation vectors. It provides a relatively wider HPBW than the original LAA with about quad-fold reduction in the SLL.

KEYWORDS

Array synthesis; convolution process; genetic algorithm (GA); half power beamwidth (HPBW); linear antenna array (LAA); side lobe level (SLL); quality of service (QOS)

1 Introduction

The synthesis of antenna arrays has become an important study now, as it has many applications in the fields of communications, radar, etc. Therefore, the requirement for an array with a directive beam and reduced SLL prompted the search for promising antenna array synthesis techniques. SLL reduction or cancellation can be achieved by adjusting the amplitude and phase of excitations while keeping the spacing between the elements as in conventional arrays. They can also be accomplished by adjusting the spacing between antenna elements while keeping the excitation's amplitude and phase. This is beneficial for avoiding interfering and undesired signals. There are two ways to improve the synthesis of antenna array patterns, the deterministic method, and the stochastic method [1]. The deterministic method is divided into analytical and semi-analytical methods. The deterministic method is characterized by the fact that the total calculation time for the synthesis process is very low, which makes it very suitable for real-time applications that distinguish it from the stochastic method. On the other hand, the stochastic method is known for its expensive calculations. However, it is more efficient and flexible than the deterministic method. In recent years, many algorithms have been developed to solve the synthesis problems of antenna arrays. A new mathematical formula of the array factor of linear antenna array was introduced in [2] to improve its performance in terms of directivity and SLL. It is based on the principle of mixing or hybridizing two different linear arrays and the resultant array is treated as a new linear array to fulfill the required purposes. The simulations revealed that the suggested formula outperformed the Chebyshev array that has the same number of antenna elements in terms of SLL, excitation coefficients, and directivity. The radiation pattern of non-uniform LAA is improved iteratively to obtain the optimum radiation pattern using a reduced number of elements and achieve SLL lower than −20 dB through Genetic algorithm (GA) optimization of the elements spacing and number of elements [3]. Lower numbers of elements and iterations were used to obtain SLL lower than −20 dB than other algorithms. In [4], the ant lion optimization technique (ALO) was used to synthesize the LAAs for SLL reduction, pattern nulling at specific directions, and precise beam steering by optimizing the element spacing and excitation currents. The ALO's parameters to be defined for each problem are fewer than the other optimization algorithms. The ALO results showed its excellence in SLL reduction over other optimization techniques such as particle swarm optimization (PSO), ant colony optimization (ACO), cat swarm optimization (CSO), and biogeography-based optimization (BBO) at the expense of widening the HPBW of the synthesized array. In [5], the grey wolf optimization (GWO) algorithm was introduced for SLL reduction and minimization of the first side lobe level (FSLL) by the optimization of interelement spacing and excitations. The GWO algorithm introduced a significant reduction in the SLL of LAA compared to other state-of-the-art optimization techniques such as PSO, BBO and CSO. However, it also suffers from the increased HPBW. In [6], a new scatter-search based technique was introduced for SLL reduction and pattern nulling of LAAs. It employed a weighted cost function in finding the optimal positions of the array elements, which improved its performance compared to the non-weighted cost function-based techniques. Another SLL reduction technique based on pulse compression (PC) using a WOO filter was introduced in [7]. This method showed a significant improvement in integrated SLL (ISL) and peak SLL compared with other PC techniques. In [8], the convolution method and time scaling property are used to obtain the element's excitations to reduce the SLL of any uniform spacing LAA. The results showed that the SLL is reduced by two times or more, but the HPBW is increased. To control the SLL and address the issues with the other window functions that were unable to improve the SLL due to the high value of the dynamic range ratio (DRR) of the excitation currents, a fast iterative method based on the Fourier relationship between the array factor and the excitation currents was introduced in [9]. It applied the raised cosine window function to improve the SLL with acceptable DRR. It improved the SLL compared to the Kaiser window function but with a wider HPBW. In [10,11], two optimization-based synthesis techniques for SLL reduction of linear and circular antenna arrays were introduced. They are based on the inverse weed optimization (IWO) technique and the whale optimization algorithm (WOA). These techniques depend on the optimal selection of the excitation currents to obtain the desired radiation patterns. However, they did not provide significant reductions in the SLL. In [12], a two-stage piecewise linear frequency modulation (PWLFM) signal model was used to instead of the linear frequency modulation (LFM) signal to reduce the first side lobe level in radar systems. That is because the autocorrelation of the PWLFM signal has a low peak SLL ratio compared to the LFM signal. In addition, various types of convolution window functions were applied for more SLL reduction such as (Cauchy, Power of cosine, Papoulis, and Parzen). However, the power of the cosine window function proved its superiority in reducing the SLL. In [13], a new iterative method was developed. This algorithm relies on the iterative addition by rotation or translation of antenna array elements, then the synthesized pattern proves its approximation to the desired pattern in the sense of the Hilbert space specified standard. This iterative algorithm can also be used to synthesize several patterns, but with different excitations, for the same array shape.

Newly proposed hybrid beamforming techniques for minimizing the size of uniform planar antenna arrays (UPAA) and reducing their SLL were presented in [14]. They are based on the hybridization between the two-dimensional (2D) convolution and GA. The SLL reduction is performed by controlling both the elements' excitations and separation. The differential evolutionary (DE) algorithm was used in [15] to introduce a new approach for reducing SLL and rejecting interference signals in concentric hexagonal antenna arrays (CHAA). In the proposed approach, the CHAA was chosen because it would offer a lower SLL compared to circular and linear antenna arrays, which is considered a drawback of the approach.

In [16], the Eisenstein fractile array was presented to use the special geometrical characteristics of fractiles that permit multiband and wideband operation and prevent the appearance of grating lobes. In order to reduce the high SLL experienced at large-scale arrays, the proposed antenna array was thinned using the GA optimization technique. This was done by identifying the optimal set of "on" and "off" antenna elements that correspond to the minimum SLL without sacrificing the directivity of the array radiation pattern.

In [17], new hybrid array beamforming techniques based on the integration between the virtual antenna array (VAA) concept, PSO technique, and hyper-beamforming were introduced for SLL reduction and thinning of elliptical cylindrical antenna arrays (ECAA) in radar systems. The VAA decomposes the ECAA into created LAA and EAA. The number of antenna elements, element spacing, and excitations of the created LAA and EAA are optimized using PSO to produce efficient patterns with low SLL. Additional SLL reductions were achieved by using hyper-beamforming. In [18], new low SLL wideband planar antenna array designs based on the space-filling curves technique were presented. These arrays' special geometrical properties are used to provide wideband operation and prevent the emergence of grating lobes. Wideband operation, SLL reduction, and grating lobe removal are only a few of the advantages of the newly proposed array designs over their traditional periodic planar array equivalents in terms of the radiation pattern. In [19], a new structure for a 1×8 slotted S-band waveguide array with non-uniform offset slot locations was proposed for SLL reduction. The slots on the upper metallic wall of the waveguide are distributed following a binomial distribution.

The simulations revealed that the proposed array provided significantly reduced SLL compared to the slot array with a uniform offset location. In [20], a hybrid beamforming technique based on the combination of the genetic algorithm and the Gauss elimination algorithm denoted as GA/GE has been introduced for a specific side lobe cancellation of LAAs that reduces the received interference in the appropriate directions, which improves the receiver sensitivity. However, the GA/GE did not result in a general decrease in SLL. The performance of upcoming 5G cellular networks that utilize the mm-wave spectrum could be greatly improved by employing SLL reduction, which is considered one of the most crucial array beamforming techniques. The use of effective beamforming techniques such as SLL reduction and beam steering for antenna arrays at mobile base stations has several advantages, including longer battery life, a lower probability of an outage, much higher bit rates across a larger coverage area, lower infrastructure costs, and a higher capacity for numerous simultaneous users in both licensed and unlicensed spectrums. Due to its substantial unlicensed capacity, mm-wave communication is viewed as a key player for 5G and subsequent networks. There are significant difficulties, though, such as penetration and propagation losses. Antenna array beamforming is used to solve these problems as introduced in [21,22]. In [23], a highly efficient approach to improve the performance of multi-antenna elements-based spectrum sensing (SS) techniques in cognitive radio (CR) systems using SLL reduction has been introduced. It significantly improved the probability of detection of the CR system at much lower signal-to-noise ratio and signal-to-interference ratio scenarios. Furthermore, it improved the SS capability of the CR system further than SS based on array beamforming for maximum gain realization introduced in [24] and SS without array beamforming introduced in [25].

In this paper, two highly efficient SLL reduction techniques based on the hybrid combination between the convolution algorithms (single convolution algorithm (Conv) and the double convolution algorithm (DConv)) and the GA denoted as Conv/GA and DConv/GA, respectively, are proposed. In the Conv/GA technique, the convolution between the array excitation vector of length (M) and itself is used to adjust the excitations of the array elements. However, the convolution process generates a longer excitation vector with a length of ($2M - 1$), from which the appropriate set of (M) excitations can be selected for SLL reduction. Generally, the convolution process reduces the SLL but at the expense of widening the array HPBW. To alleviate this issue, the element spacing is optimized using the GA to optimize the array size that achieves the same HPBW as the original array. The proposed Conv/GA technique provides almost the same HPBW as the original LAA with a two-fold reduction in the SLL. While the proposed DConv/GA technique provides a relatively wider HPBW and an almost four-fold reduction in the SLL compared to the original LAA. Several simulations are performed to verify the effectiveness of the proposed techniques.

The rest of the paper is organized as follows: Section 2 presents the related work. In Section 3, the proposed Conv/GA and DConv/GA beamforming techniques are introduced. Section 4 introduces the simulation results and discussions. Finally, the conclusion is presented in Section 5.

The following is a summary of the contributions provided by this paper:

1. The convolution approach has been employed in a unique way to synthesize LAAs using the proposed Conv/GA technique, achieving the same HPBW as the original LAA pattern while reducing the SLL by two-fold.
2. When applying the proposed DC/GA technique to LAAs, the SLL can be reduced by a quad-fold with minimal changes in the HPBW. This is beneficial for avoiding interfering, jamming, and unwanted signals.

3. When applying the proposed Conv/GA technique, the uniform linear antenna array (ULAA) and Chebyshev array synthesis using middle excitations provided a slightly higher SLL than the original arrays but with a much narrower HPBW. This main beam thinning effectively minimizes multipath signals and makes it suitable for positioning systems to increase location accuracy in 5G and beyond communications. Additionally, the outer antenna elements of the synthesized arrays can be turned off in order to execute array thinning. However, this array thinning causes the synthesized arrays' SLL and HPBW to increase. Nevertheless, by adjusting the element spacing, the HPBW can be readjusted.

2 Linear Antenna Array Geometry and Genetic Algorithm

This section explains the geometrical construction of the linear antenna array (LAA) and the GA optimization technique while highlighting their key variables and parameters. Firstly, the GA is one of the most commonly used population-based metaheuristics optimization techniques that employ multiple candidate solutions during the search process. These metaheuristics preserve diversity in the population and avoid stuck solutions in the local optima. The GA is an optimization algorithm inspired by natural selection, which uses the concept of survival of the fit test [26]. New populations are produced through the frequent use of genetic operators over the existing individuals in the population. The major components of GA are the chromosome representation, selection, crossover, mutation, and computation of fitness functions. The GA procedure started by random initialization of a population (Y) of n chromosomes. Each chromosome's fitness in Y is determined. According to the fitness value, two chromosomes denoted as C_1 and C_2 are selected from the population Y. The crossover likelihood (Cp) single-point crossover operator is applied to C_1 and C_2 to generate an offspring denoted as O. A uniform mutation operator with mutation probability (Mp) is then applied to the generated offspring O to generate O'. In the new population, the new offspring O' is put. The selection, crossover, and mutation operations in the current population will be replicated before the completion of the new population. Through the probabilities of crossover and mutation, GA dynamically alters the search process and reaches the optimal solution. The encoded genes can be changed by GA. GA can analyze multiple individuals and create multiple optimal solutions. GA, therefore, has greater potential for global search. It is likely that the offspring formed from the crossover of parent chromosomes would abolish the admirable genetic schemes, and the crossover formula R is given by [26]:

$$R = (G + 2\sqrt{g})/3G \tag{1}$$

where G is the cumulative number of population-specified evolutionary generations and g is the number of generations. Eq. (1) shows that R is dynamically updated and increases with an increase in the number of evolutionary generations. The resemblance between individuals is very poor in the initial stage of GA. To ensure that the new population does not ruin the excellent genetic schema of individuals, the value of R should be small. The resemblance between individuals is very high at the end of evolution and the value of R should be high.

Secondly, the geometrical structure of the LAA is described considering an LAA consisting of M identical antenna elements arranged on a Z-axis with a uniform element spacing $d = \lambda/2$ as shown in Fig. 1. The array factor $\mathcal{AF}(\theta)$ of the LAA is given by:

$$\mathcal{AF}(\theta) = \sum_{m=1}^{M} a_m e^{j\beta(m-1)d\cos(\theta)} \tag{2}$$

where a_m is the complex excitation coefficient of the m^{th} antenna element, $\beta = 2\pi/\lambda$ is the wave number, and λ is the wavelength. Let A be the vector of the M-element excitations that can be written as:

$$A = [a_1, a_2, a_3, \ldots, a_M] \quad (3)$$

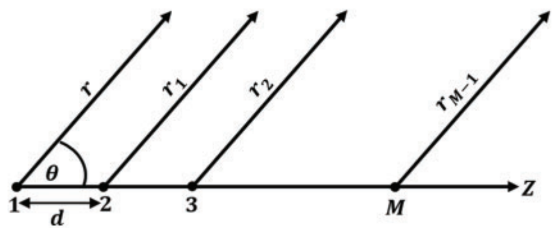

Figure 1: Structure of linear antenna array

3 Proposed Conv/GA and DConv/GA Beamforming Techniques

In this section, two proposed highly efficient SLL reduction techniques based on the hybrid combination between the convolution algorithms (single convolution algorithm (Conv) and the double convolution algorithm (DConv)) and the genetic algorithm (GA) denoted as Conv/GA and DConv/GA, respectively, are introduced. The convolution process determines the element's excitations while the GA optimizes the element spacing.

In general, it is widely understood from Mathematical theory that the convolution of two sequences of lengths K and M yields a new sequence of length $N = (K + M - 1)$ [8]. Given two vectors $A = [a_1, a_2, a_3, \ldots, a_M]$ and $B = [b_1, b_2, b_3, \ldots, b_K]$, then the convolution of A and B ($A * B$) is a vector $C = [c_1, c_2, c_3, \ldots, c_{K+M-1}]$, $x = 1, 2, 3, \ldots, K + M - 1$ where c_x is given by:

$$c_x = \sum_{u=u_{min}}^{u_{max}} a_u b_{x-u+1} \quad (4)$$

where u ranges over all legal subscripts for a_u and b_{x-u+1}, and the upper and lower limits of u are given by:

$$u = \begin{cases} max(1, x - K + 1) \\ min(x, M) \end{cases} \quad (5)$$

Accordingly, $u_{min} = min(u)$ and $u_{max} = max(u)$.

3.1 Proposed Conv/GA Technique

In this section, the proposed Conv/GA beamforming technique is introduced. It is based on calculating the convolution of the vector of excitations A by itself, which provides a new vector A_c of length $N = (2M - 1)$ that is given by:

$$A_c = A * A \quad (6)$$

$$A_c = [c_1, c_2, c_3, \ldots, c_{2M-1}] \quad (7)$$

where c_x can be calculated by (4) as follows:

$$c_1 = a_1 a_1$$

$$c_2 = a_1 a_2 + a_2 a_1$$

$$c_3 = a_1 a_3 + a_2 a_2 + a_3 a_1$$

$$\ldots$$

$$c_M = a_1 a_M + a_2 a_{M-1} + \ldots + a_M a_1$$

$$\ldots$$

$$c_{2M-1} = a_M a_M \tag{8}$$

The resultant vector A_c is divided into three different sets of excitations including the odd excitations A_{odd}, even excitations A_{even}, and middle excitations A_{mid} of lengths M, $M-1$, and M, respectively. When the same element spacing of the original LAA is used, it is noticed that the odd and even excitations provide much lower SLL than that of the LAA but with much wider HPBW. While the middle excitations give the same HPBW as the original LAA with higher SLL. So, the middle excitation solution is avoided. To mitigate the problem of odd and even excitations, the GA optimization is used for beam thinning of the synthesized arrays by determining the optimum element spacing d_s that minimizes the designed cost function C_f given below:

$$C_f = \left\{ \|HPBW_s - HPBW\| + \frac{1}{\|\|SLL_s\| - \|SLL\|\|} \right\} \Bigg|_{\|SLL_s\| > \|SLL\|} \tag{9}$$

where $HPBW_s$ and $HPBW$ are the half power beamwidths of the synthesized and original arrays, respectively. SLL_s and SLL are the side lobe levels of the synthesized and original arrays in dB, respectively. In order to minimize the designed cost function, the first term should be as minimum as possible, while the denominator of the second term $\|\|SLL_s\| - \|SLL\|\|$ should be as maximum as possible under the constraint that the synthesized SLL is larger than that of the original one ($\|SLL_s\| > \|SLL\|$). The GA optimizes the element spacing within the range $0.5\lambda < d_s < 0.9\lambda$ to avoid the appearance of grating lobes in the synthesized patterns, which are calculated by:

$$\mathcal{AF}_s(\theta)|_{odd} = \sum_{x=1}^{M} A_{odd}(x) e^{j\beta(x-1)d_s\cos(\theta)} \tag{10}$$

$$\mathcal{AF}_s(\theta)|_{even} = \sum_{x=1}^{M-1} A_{even}(x)\, e^{j\beta(x-1)d_s\cos(\theta)} \tag{11}$$

where $\mathcal{AF}_s(\theta)|_{odd}$ and $\mathcal{AF}_s(\theta)|_{even}$ are the synthesized patterns using odd excitations C_{xo} and even excitations C_{xe}, respectively that are calculated from c_x given in (4) such that:

$$A_{odd} = [c_1, c_3, c_5 \ldots, c_{2M-1}] \tag{12}$$

$$A_{even} = [c_2, c_4, c_6, \ldots, c_{2M-2}] \tag{13}$$

3.2 Proposed DConv/GA Technique

In this section, the proposed DConv/GA beamforming technique is introduced. It is based on calculating the convolution of the two aforementioned excitation vectors A_{odd} and A_{even} by themselves such that:

$$B_O = A_{odd} * A_{odd} \tag{14}$$

$$B_E = A_{even} * A_{even} \tag{15}$$

where

$$B_O = [b_{1o}, b_{2o}, b_{3o}, \ldots, b_{(2M-1)o}] \tag{16}$$

$$B_E = [b_{1e}, b_{2e}, b_{3e}, \ldots, b_{(2M-3)e}] \tag{17}$$

The vector B_O is divided again into two sets of excitations including the odd excitations B_{O-odd} and even excitations B_{O-even} of lengths M and $M-1$, respectively as follows:

$$B_{O-odd} = [b_{1o}, b_{3o}, \ldots, b_{(2M-1)o}] \tag{18}$$

$$B_{O-even} = [b_{2o}, b_{4o}, \ldots, b_{(2M-2)o}] \tag{19}$$

In this case, the two synthesized patterns are calculated by:

$$\mathcal{AF}_{DO}(\theta)|_{odd} = \sum\nolimits_{x=1}^{M} B_{O-odd}(x)\, e^{j\beta(x-1)d_s\cos(\theta)} \tag{20}$$

$$\mathcal{AF}_{DO}(\theta)|_{even} = \sum\nolimits_{x=1}^{M-1} B_{O-even}(x)\, e^{j\beta(x-1)d_s\cos(\theta)} \tag{21}$$

While the vector B_E is divided again into two sets of excitations including the odd excitations B_{E-odd} and even excitations B_{E-even} of lengths $M-1$ and $M-2$, respectively as follows:

$$B_{E-odd} = [b_{1e}, b_{3e}, \ldots, b_{(2M-3)e}] \tag{22}$$

$$B_{E-even} = [b_{2e}, b_{4e}, \ldots, b_{(2M-4)e}] \tag{23}$$

In this case, the two synthesized patterns are calculated by:

$$\mathcal{AF}_{DE}(\theta)|_{odd} = \sum\nolimits_{x=1}^{M-1} B_{E-odd}(x)\, e^{j\beta(x-1)d_s\cos(\theta)} \tag{24}$$

$$\mathcal{AF}_{DE}(\theta)|_{even} = \sum\nolimits_{x=1}^{M-2} B_{E-even}(x)\, e^{j\beta(x-1)d_s\cos(\theta)} \tag{25}$$

Also, to mitigate the HPBW problem, GA optimization is used for beam thinning of the synthesized arrays by determining the optimum element spacing d_s that minimizes the same designed cost function C_f given by (9). In general, after elaboration, a simplified map was made to explain the proposed algorithms, as shown in Fig. 2.

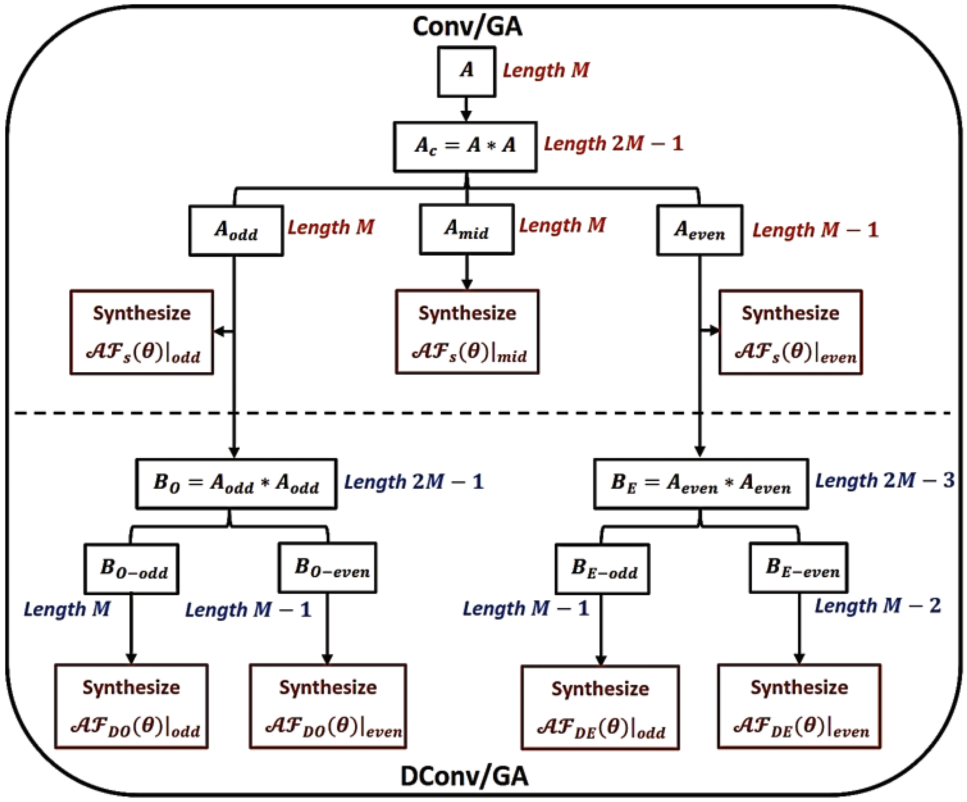

Figure 2: The flow chart that explains the proposed Conv/GA and DConv/GA algorithms

4 Simulation Results

In this section, the simulation results of the proposed Conv/GA and DConv/GA beamforming techniques are introduced for SLL reduction of non-uniform Chebyshev arrays and uniform linear antenna arrays.

Test Case 1: SLL Reduction of Odd Chebyshev Array Using Convolution Method

Consider a broadside Chebyshev array with odd number of elements $M = 15$ elements, $d = 0.5\,\lambda$, and $SLL = -23$ dB as in [8]. By taking the convolution of the excitation coefficient vector A by itself, the resultant excitation coefficient vector A_c of length $N = 29$ is listed in Table 1. This vector is divided into three groups including the odd excitation coefficient vector A_{odd} of length $N_{odd} = 15$, even excitation coefficient vector A_{even} of length $N_{even} = 14$, and the middle excitation coefficient vector A_{mid} of length $N_{mid} = 15$. These excitation vectors are applied to the antenna elements with the same element spacing $d_s = d = 0.5\,\lambda$ as the original array to generate three different array factors as shown in Fig. 3. By analyzing the results listed in Table 2, it is clear that both the odd and even excitations provide $SLLs$ of -41.6 dB and -46 dB, respectively, which are much lower than the $SLL = -23$ dB of the original Chebyshev pattern. However, their $HPBW_s = 10.9^{\circ}$ are 3.18° wider than that of the original pattern $HPBW = 7.72^{\circ}$. While the middle excitations provide the same HPBW as the original pattern with an increase of 4.4 dB in the SLL.

Table 1: The excitation coefficients vector A of the original Chebyshev array and the resultant excitation coefficient vector A_c for $M = 15$ elements

Excitation coefficients (A)				Synthesized excitation coefficients (A_C)							
m	a_m	m	a_m	n	c_n	n	c_n	n	c_n	n	c_n
1	1.0000	9	1.6765	1	1.0000	9	15.2047	17	23.5422	25	5.7171
2	0.7675	10	1.5770	2	1.5350	10	17.7275	18	22.0279	26	3.9789
3	0.9989	11	1.4212	3	2.5868	11	20.0482	19	20.0482	27	2.5868
4	1.2228	12	1.2228	4	3.9789	12	22.0279	20	17.7275	28	1.5350
5	1.4212	13	0.9989	5	5.7171	13	23.5422	21	15.2047	29	1.0000
6	1.5770	14	0.7675	6	7.7783	14	24.4930	22	12.6210	—	—
7	1.6765	15	1.0000	7	10.1081	15	25.7254	23	10.1081	—	—
8	1.7107	—	—	8	12.6210	16	24.4930	24	7.7783	—	—

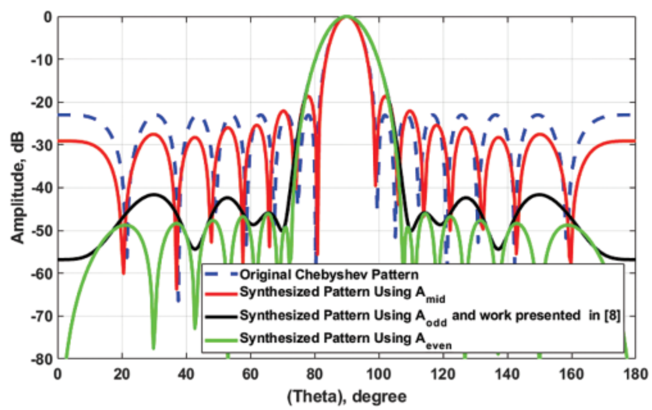

Figure 3: Synthesized array patterns using convolution method compared with the original Chebyshev pattern for $M = 15$ elements and $d = 0.5\,\lambda$

Table 2: The HPBWs and SLLs of the synthesized arrays using the convolution method compared with the original Chebyshev array for $M = 15$ elements

Parameter	Traditional Chebyshev	Synthesized pattern using A_{odd} [8]	Synthesized pattern using A_{even}	Synthesized pattern using A_{mid}
HPBW	7.72°	10.9°	10.9°	7.72°
SLL	−23 dB	−41.6 dB	−46 dB	−18.6 dB
M	15	15	14	15
Element spacing	$d = 0.5\,\lambda$	$d_s = 0.5\,\lambda$	$d_s = 0.5\,\lambda$	$d_s = 0.5\,\lambda$

Test Case 2: SLL Reduction of Odd Chebyshev Array Using the Proposed Conv/GA Technique

In this case, the HPBW problem that occurred in the test case (1) is mitigated by using the proposed Conv/GA technique where the element spacing is optimized to minimize the designed cost function in

Eq. (9). Fig. 4 shows the synthesized patterns using the odd, even, and middle excitation coefficients with optimized element spacing $d_s = 0.79\,\lambda$. By analyzing the results listed in Table 3, it is clear that the synthesized patterns using odd and even excitations provide the same $HPBW_s = 7.72^\circ$ as the original pattern with SLL_s of -41.6 dB and -46 dB, respectively, which are much lower than that of the original pattern $SLL = -23$ dB as listed in Table 3. While the synthesized pattern using middle excitations still has high $SLL_s = -18.6$ dB but with narrower $HPBW_s = 4.67^\circ$. From the simulation results, we can conclude that the synthesized array using the even excitation coefficients provides the lowest SLL_s, which is exactly equal to twice the original SLL, provides the same $HPBW_s$ as the original array $HPBW$, and saves the number of elements by one element.

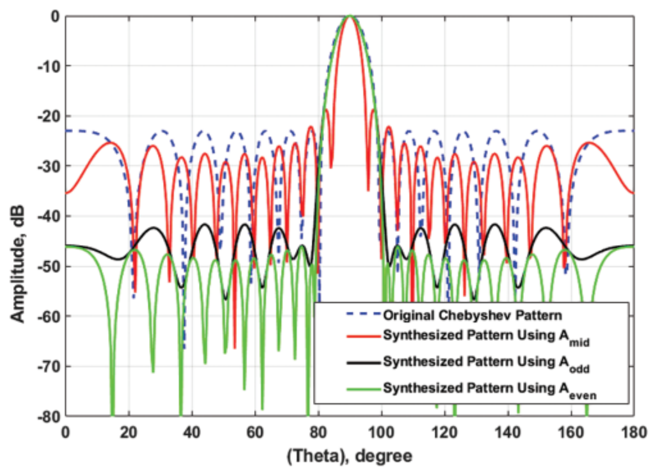

Figure 4: Synthesized array patterns using the proposed Conv/GA compared with the original Chebyshev pattern for $M = 15$ elements and $d_s = 0.79\,\lambda$

Table 3: The HPBWs and SLLs of the synthesized arrays using the proposed Conv/GA technique compared with the original Chebyshev array for $M \leq 15$ elements

Parameter	Traditional Chebyshev	Synthesized pattern using A_{odd}	Synthesized pattern using A_{even}	Synthesized pattern using A_{mid}
HPBW	7.72°	7.72°	7.72°	4.67°
SLL	−23 dB	−41.6 dB	−46 dB	−18.6 dB
M	15	15	14	15
Element spacing	$d = 0.5\,\lambda$	$d_s = 0.79\,\lambda$	$d_s = 0.79\,\lambda$	$d_s = 0.79\,\lambda$

Test Case 3: SLL Reduction of Even Chebyshev Array Using Convolution Method

Consider a broadside Chebyshev array with even number of elements $M = 20$ elements, $d = 0.5\,\lambda$, and $SLL = -23$ dB. By taking the convolution of the excitation coefficient vector A by itself, the resultant excitation coefficient vector A_c of length $N = 39$ is listed in Table 4. This vector is divided into three groups including the odd excitation coefficients A_{odd} of length $N_{odd} = 20$, even excitation coefficients A_{even} of length $N_{even} = 19$, and the middle excitation coefficients A_{mid} of length $N_{mid} = 20$. These new excitation vectors are used with the same element spacing $d_s = d = 0.5\,\lambda$ as the original array in order to produce three different array factors as seen in Fig. 5. By analyzing the results listed in Table 5, it is clear that both the odd and even excitations provide SLL_s of -47.64 dB and -40 dB, respectively, which are much lower than the $SLL = -23$ dB of the original Chebyshev pattern. However, their HPBWs are 2.21° wider than that of the original pattern. While the middle excitations provide the same HPBW as the original pattern with an increase of 5.1 dB in the SLL.

Table 4: The excitation coefficients vector A of the original Chebyshev array and the resultant excitation coefficient vector A_c for $M = 20$ elements

Excitation coefficients (A)				Synthesized excitation coefficients (A_C)							
m	a_m	m	a_m	n	c_n	n	c_n	n	c_n	n	c_n
1	1.000	11	1.3989	1	1.0000	11	12.2638	21	22.3861	31	8.8757
2	0.5752	12	1.3682	2	1.1504	12	14.0016	22	21.9070	32	7.3098
3	0.7145	13	1.3085	3	1.7599	13	15.7039	23	21.1281	33	5.8737
4	0.8549	14	1.2226	4	2.5317	14	17.3183	24	20.0775	34	4.5905
5	0.9903	15	1.1148	5	3.4746	15	18.7927	25	18.7927	35	3.4746
6	1.1148	16	0.9903	6	4.5905	16	20.0775	26	17.3183	36	2.5317
7	1.2226	17	0.8549	7	5.8737	17	21.1281	27	15.7039	37	1.7599
8	1.3085	18	0.7145	8	7.3098	18	21.9070	28	14.0016	38	1.1504
9	1.3682	19	0.5752	9	8.8757	19	22.3861	29	12.2638	39	1.0000
10	1.3989	20	1.0000	10	10.5400	20	23.6630	30	10.5400	—	—

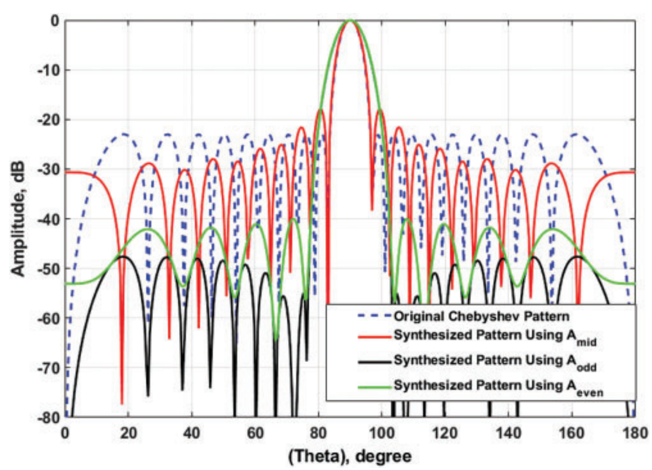

Figure 5: Synthesized array patterns using convolution method compared with the original Chebyshev pattern for $M = 20$ elements and $d = 0.5\,\lambda$

Test Case 4: SLL Reduction of Even Chebyshev Array Using the Proposed Conv/GA Technique

By applying the Conv/GA, the optimized element spacing is found to be $d_s = 0.79\,\lambda$. The synthesized array patterns using the odd, even, and middle excitation coefficients are shown in Fig. 6. It is clear that the synthesized patterns using odd and even excitations provide the same $HPBW_s = 5.69°$ as the original pattern and provide SLL_s of -47.76 dB and -40 dB, respectively that are much lower that of the original pattern $SLL = -23$ dB as listed in Table 6. While the synthesized pattern using middle excitations still has the high $SLL_s = -17.8$ dB but with narrower $HPBW_s = 3.86°$. From the simulation results, we can conclude that the array synthesis using the odd excitation coefficients provides the lowest $SLL_s = -47.76$ dB which is 2.076 times lower than that of the original array SLL. In addition, it provides the same HPBW as the original array.

Table 5: The HPBWs and SLLs of the synthesized arrays using the convolution method compared with the original Chebyshev array for $M = 20$ elements

Parameter	Traditional Chebyshev	Synthesized pattern using A_{odd}	Synthesized pattern using A_{even}	Synthesized pattern using A_{mid}
HPBW	5.69°	7.9°	7.9°	5.69°
SLL	−23 dB	−47.76 dB	−40 dB	−17.9 dB
M	20	20	19	20
Element spacing	$d = 0.5\,\lambda$	$d_s = 0.5\,\lambda$	$d_s = 0.5\,\lambda$	$d_s = 0.5\,\lambda$

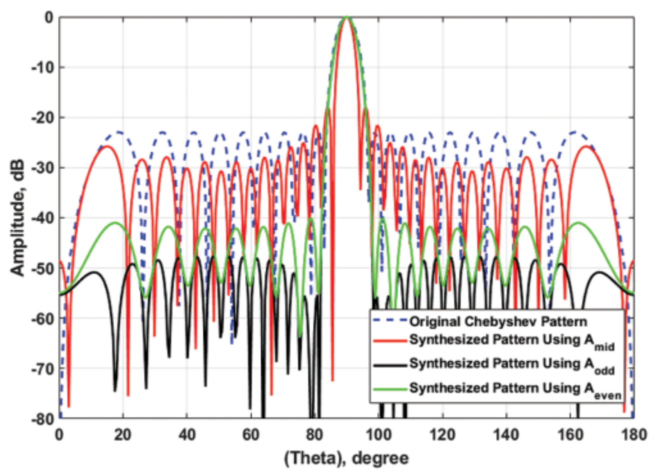

Figure 6: Synthesized array patterns using the proposed Conv/GA compared with the traditional Chebyshev pattern for $M = 20$ elements and $d_s = 0.79\,\lambda$

Table 6: The HPBWs and SLLs of the synthesized arrays using the proposed Conv/GA technique compared with the traditional Chebyshev array for $M = 20$ elements

Parameter	Traditional Chebyshev	Synthesized pattern using A_{odd}	Synthesized pattern using A_{even}	Synthesized pattern using A_{mid}
HPBW	5.69°	5.69°	5.69°	3.86°
SLL	−23 dB	−47.76 dB	−40 dB	−17.9 dB
M	20	20	19	20
Element spacing	$d = 0.5\,\lambda$	$d_s = 0.79\,\lambda$	$d_s = 0.79\,\lambda$	$d_s = 0.79\,\lambda$

Test Case 5: SLL Reduction of Chebyshev Array Using the Proposed DConv/GA Technique and B_O Excitations

In this section, the proposed DConv/GA technique is applied to the odd excitation vector A_{odd} of length $M = 15$. The resultant excitation vector B_O of length $2M - 1 = 29$ is listed in Table 7. By applying the excitations B_{O-odd} and B_{O-even} of lengths $M = 15$ and $M - 1 = 14$, respectively the optimized element spacings are found to be $d_s = 0.69\,\lambda$ and $d_s = 0.69\,\lambda$, respectively. The synthesized arrays patterns compared to the original Chebyshev array pattern are shown in Fig. 7. It is clear that the synthesized patterns using odd and even excitations have the same $HPBW_s = 11.36°$ that is wider than that of the original pattern by 3.64°. While B_{O-odd} and B_{O-even} excitations provide SLL_s of −81.1 dB and −87.2 dB, respectively which are much lower than the original pattern $SLL = -23$ dB as listed in Table 8. From the simulation results, we can conclude that the array synthesis using B_{O-even} excitation coefficients provides the lowest $SLL_s = -87.2$ dB that is 3.791 times lower than that of the original array SLL. In addition, it saves the number of array elements by one antenna element.

Table 7: The synthesized excitation coefficients vector $\boldsymbol{A_{odd}}$ and the resultant excitation coefficient vector $\boldsymbol{B_O}$ for $\boldsymbol{M = 15}$ elements Chebyshev array

Odd excitation coefficients (A_{odd})				Synthesized excitation coefficients ($\boldsymbol{B_O}$)							
m	a_m	m	a_m	n	b_{no}	n	b_{no}	n	b_{no}	n	b_{no}
1	1.0000	9	23.5422	1	0.0010	9	1.0858	17	2.9435	25	0.1154
2	2.5868	10	20.0282	2	0.0052	10	1.5416	18	2.5294	26	0.0498
3	5.7171	11	15.2047	3	0.0181	11	2.0411	19	2.0404	27	0.0181
4	10.1081	12	10.1081	4	0.0498	12	2.5301	20	1.5410	28	0.0052
5	15.2047	13	5.7171	5	0.1154	13	2.9441	21	1.0854	29	0.0010
6	20.0482	14	2.5868	6	0.2343	14	3.2220	22	0.7096	—	—
7	23.5422	15	1.0000	7	0.4268	15	3.3208	23	0.4267	—	—
8	25.7254	—	—	8	0.7099	16	3.2216	24	0.2343	—	—

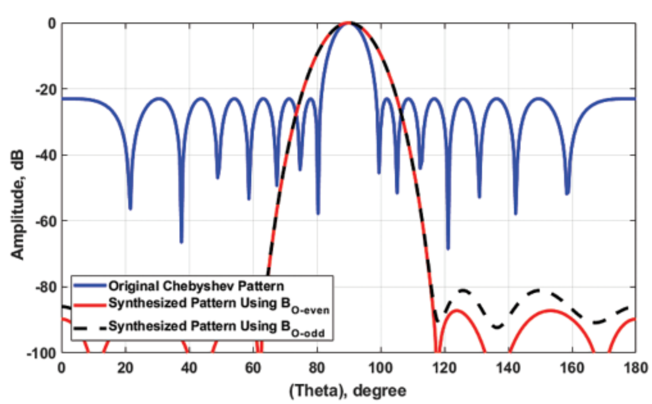

Figure 7: The synthesized arrays patterns using B_{O-odd} and B_{O-even} compared to the original Chebyshev array for $M = 15$ elements

Table 8: The HPBWs and SLLs of the synthesized arrays using the proposed DConv/GA technique compared with the traditional Chebyshev array for $M = 15$ elements and B_O excitations

Parameter	Traditional Chebyshev	Synthesized pattern using $\boldsymbol{B_{O-odd}}$	Synthesized pattern using $\boldsymbol{B_{O-even}}$
HPBW	7.72°	11.36°	11.36°
SLL	−23 dB	−81.1 dB	−87.2 dB
M	15	15	14

Test Case 6: SLL Reduction of Chebyshev Array Using the Proposed DConv/GA Technique and B_E Excitations

In this section, the proposed DConv/GA technique is applied to the even excitation vector A_{even} of length $M = 14$. The resultant excitation vector B_E of length $2M - 1 = 27$ is listed in Table 9. By applying the excitations B_{E-odd} and B_{E-even} of lengths $M = 14$ and $M - 1 = 13$, respectively the optimized element spacings are found to be $d_s = 0.69\,\lambda$ and $d_s = 0.69\,\lambda$, respectively. The synthesized array patterns compared to the original Chebyshev array pattern are shown in Fig. 8. It is clear that the synthesized patterns using odd and even excitations have the same $HPBW_s = 11.36°$ that is wider than that of the original pattern by 3.64°. While B_{E-odd} and B_{E-even} excitations provide SLL_s of −94 dB and −89.2 dB, respectively, which are much lower than the original pattern $SLL = -23$ dB as listed in Table 10. From the simulation results, we can conclude that the array synthesis using B_{E-odd} excitation coefficients provides the lowest $SLL_s = -94$ dB that is 4.08 times lower than that of the original array SLL. In addition, it saves the number of array elements by one antenna element.

Table 9: The synthesized excitation coefficients vector A_{even} and the resultant excitation coefficient vector B_E for $M = 15$ elements Chebyshev array

Even excitation coefficients ($\boldsymbol{A_{even}}$)				Synthesized excitation coefficients ($\boldsymbol{B_E}$)							
m	$\boldsymbol{a_m}$	***m***	$\boldsymbol{a_m}$	***n***	$\boldsymbol{b_{ne}}$	***n***	$\boldsymbol{b_{ne}}$	***n***	$\boldsymbol{b_{ne}}$	***n***	$\boldsymbol{b_{ne}}$
1	1.5350	9	22.0279	1	0.0024	9	1.5136	17	2.4944	25	0.0397
2	3.9789	10	17.7275	2	0.0122	10	2.0098	18	2.0100	26	0.0122
3	7.7739	11	12.6210	3	0.0397	11	2.4942	19	1.5139	27	0.0024
4	12.6210	12	7.7783	4	0.1006	12	2.9034	20	1.0603	–	–
5	17.7275	13	3.9789	5	0.2153	13	3.1778	21	0.6856	–	—
6	22.0279	14	1.5350	6	0.4049	14	3.2747	22	0.4050	—	—
7	24.4930	—	—	7	0.6854	15	3.1779	23	0.2154	—	—
8	24.4930	—	—	8	1.0601	16	2.9036	24	0.1006	—	—

Test Case 7: SLL Reduction of Uniform Linear Antenna Array Using the Proposed Conv/GA Technique

In this section, the proposed Conv/GA technique is used to synthesis a uniform linear antenna array (ULAA) consisting of $\boldsymbol{M = 10}$ elements with uniform element spacing $\boldsymbol{d = 0.5\,\lambda}$.

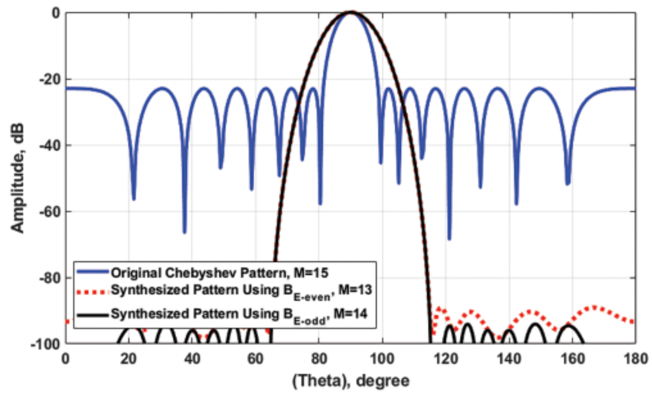

Figure 8: The synthesized arrays patterns using B_{E-odd} and B_{E-even} compared to the original Chebyshev array for $M = 15$ elements

Table 10: The HPBWs and SLLs of the synthesized arrays using the proposed DConv/GA technique compared with the traditional Chebyshev array for $M = 15$ elements and B_E excitations

Parameter	Traditional Chebyshev	Synthesized pattern using $\boldsymbol{B_{E-odd}}$	Synthesized pattern using $\boldsymbol{B_{E-even}}$
HPBW	7.72°	11.36°	11.36°
SLL	−23 dB	−94 dB	−89.2 dB
M	15	14	13
Element spacing	$d = 0.5\,\lambda$	$d_s = 0.69\,\lambda$	$d_s = 0.69\,\lambda$

By taking the convolution of the excitation coefficient vector A by itself, the resultant excitation coefficient vector A_c of length $N = 19$ is listed in Table 11. This vector is divided into three groups including the odd excitation coefficient vector A_{odd} of length $N_{odd} = 10$, even excitation coefficient vector A_{even} of length $N_{even} = 9$, and the middle excitation coefficient vector A_{mid} of length $N_{mid} = 9$. By applying the Conv/GA, the optimized element spacing is found to be $d_s = 0.67\lambda$. The synthesized array patterns using the odd, even, and middle excitation coefficients are shown in Fig. 9. It is clear that the synthesized patterns using odd and even excitations provide the same $HPBW_s = 10.44^\circ$ as the original uniform array pattern and provide SLL_s of −28.2 dB and −24 dB, respectively, that are much lower than that of the original pattern $SLL = -12.97$ dB as listed in Table 12. Also, the synthesized pattern using middle excitations provides lower SLL than the uniform array that equals $SLL_s = -17.8$ dB with narrower $HPBW_s = 9.4^\circ$. It is evident that the proposed Conv/GA approach significantly reduces SLL compared to the original uniform antenna array employing the synthesized middle, even, and odd excitations. In light of the simulation results, we can conclude that the array synthesis using the odd excitation coefficients provides the lowest $SLL_s = -28.2$ dB which is 2.174 times lower than that of the original uniform array. For more significant SLL reductions, the proposed DConv/GA technique can be utilized as in previous test cases.

Table 11: The excitation coefficients vector A of the original uniform array and the resultant excitation coefficient vector A_c for $M = 10$ elements

Excitation coefficients (A)				Synthesized excitation coefficients (A_C)							
m	a_m	m	a_m	n	c_n	n	c_n	n	c_n	n	c_n
1	1	6	1	1	1	6	6	11	9	16	4
2	1	7	1	2	2	7	7	12	8	17	3
3	1	8	1	3	3	8	8	13	7	18	2
4	1	9	1	4	4	9	9	14	6	19	1
5	1	10	1	5	5	10	10	15	5	–	——

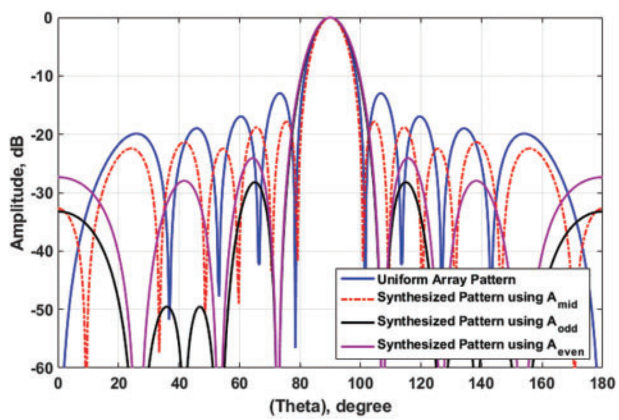

Figure 9: Synthesized array patterns using the proposed Conv/GA technique compared with the original uniform array pattern for $M = 10$ elements and $d = 0.5\,\lambda$

Table 12: The HPBWs and SLLs of the synthesized arrays using the proposed Conv/GA technique compared with the original uniform antenna array for $M = 10$ elements

Parameter	Uniform antenna array	Synthesized pattern using A_{odd}	Synthesized pattern using A_{even}	Synthesized pattern using A_{mid}
HPBW	10.44°	10.44°	10.44°	9.4°
SLL	−12.97 dB	−28.2 dB	−24 dB	−17.8 dB
M	10	10	9	9
Element spacing	$d = 0.5\,\lambda$	$d_s = 0.67\,\lambda$	$d_s = 0.67\,\lambda$	$d_s = 0.67\,\lambda$

5 Conclusion

In this paper, simple and highly effective SLL reduction techniques for LAAs denoted as Conv/GA and DConv/GA are introduced. The convolution method is used to determine the element's excitations that minimize the array SLL. GA is used to determine the optimum spacing between the elements, which guarantees the same HPBW as the original array in the case of the Conv/GA technique and guarantees the least change in HPBW in the case of the DConv/GA technique. For Chebyshev array consisting of an odd number of elements, the even excitation vector provides the lowest SLL that is two times lower than that of the original array and saves the number of elements by one element. While for LAA consisting of an even number of elements, the odd excitation vector provides the lowest SLL which is 2.076 times lower than that of the original array using the same number of elements. On the other hand, the DConv/GA provided about 4 times reduction in the SLL compared to the original array SLL with minimal changes in the HPBW of the main beam and saves one or two antenna elements according to the selected coefficients. In the case of the uniform antenna array, the proposed Conv/GA technique provides much lower SLL than the original array pattern using the synthesized middle, even, and odd excitations. It is evident that the proposed algorithms reduced the SLL by two fixed and distinct amounts for any type of linear antenna array, whether ULAA or Chebyshev array, which are two-fold, or quad-fold reduction compared to the original SLL.

Acknowledgement: The authors would like to thank the King Saud University, Saudi Arabia for supporting this work.

Funding Statement: This research is funded by Research Supporting Project Number (RSPD2023R 585), King Saud University, Riyadh, Saudi Arabia.

Author Contributions: Shimaa M. Amer, Ashraf A. M. Khalaf, Amr H. Hussein, Salman A. Alqahtani, Mostafa H. Dahshan, and Hossam M. Kassem are contributed equally in study conception and design, data collection, analysis and interpretation of results, reviewed the results and approved the final version of the manuscript.

Availability of Data and Materials: All the readers can contact Dr. Amr H. Hussein for any future inquiries.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding.

References

1. Buttazzoni, G., Babich, F., Vatta, F., Comisso, M. (2019). Comparison between deterministic and stochastic methods for the synthesis of aperiodic arrays. *2019 IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting*, pp. 51–52. Atlanta, GA, USA, IEEE.
2. Gassab, O., Azrar, A. (2016). Novel mathematical formulation of the antenna array factor for side lobe level reduction. *The Applied Computational Electromagnetics Society Journal (ACES), 31(12),* 1452–1462.
3. Todnatee, S., Phongcharoenpanich, C. (2016). Iterative GA optimization scheme for synthesis of radiation pattern of linear array antenna. *International Journal of Antennas and Propagation, 2016,* 1–8.
4. Saxena, P., Kothari, A. (2016). Ant lion optimization algorithm to control side lobe level and null depths in linear antenna arrays. *AEU-International Journal of Electronics and Communications, 70(9),* 1339–1349.
5. Prerna, S., Kothari, A. (2016). Optimal pattern synthesis of linear antenna array using grey wolf optimization algorithm. *International Journal of Antennas and Propagation, 2016,* 1–11.
6. Žarko, R., Mihić, O. (2016). Linear antenna array synthesis with minimum side lobe level and null controling. *Revue Roumaine des Sciences Techniques, Série Électrotechnique et Énergétique, 61(4),* 414– 417.
7. Ankur, T., Talluri Raju, S. (2017). A novel pulse compression technique for side-lobe reduction using woo filter concepts. *International Conference on Communication and Signal Processing (IEEE)*, pp. 1086–1090. Chennai, India.
8. Alijani, M. G. H., Neshati, M., Boozari, M. (2017). Side lobe level reduction of any type of linear equally spaced array using the method of convolution. *Progress in Electromagnetics Research Letters, 66,* 79–84.

9. Santos, F. E. S., Azevedo, J. A. R. (2017). Adapted raised cosine window function for array factor control with dynamic range ratio limitation. *11th European Conference on Antennas and Propagation (EUCAP)*, pp. 2020–2024. Paris, France, IEEE.
10. Geng, S., Liu, Y., Han, L., Liang, S., Wang, A. et al. (2018). An antenna array sidelobe level reduction approach through invasive weed optimization. *Hindawi International Journal of Antennas and Propagation, 2018,* 1–16.
11. Zhang C., Fu X., Ligthart L. P., Peng S., Xie M. (2018). Synthesis of broadside linear aperiodic arrays with sidelobe suppression and null steering using whale optimization algorithm. *IEEE Antennas and Wireless Propagation Letters, 17(2),* 347–350.
12. AdithyaValli, N., Elizabath Rani, D., Kavitha, C. (2019). Convolution windows for side lobe reduction. *International Journal of Innovative Technology and Exploring Engineering (IJITEE), 8(10),* 3526–3529.
13. Károly, M., Kracek, J., Bilicz, S. (2020). Antenna array pattern synthesis using an iterative method. *IEEE Transactions on Magnetics, 56(2),* 1–4.
14. Elkhawaga, A. M., Mahmoud, N. M., Abd Elnaby, M. M., Hussein, A. H. (2022). New hybrid approaches for SLL reduction and size miniaturization of planar antenna arrays based on 2D convolution and genetic algorithm. *Alexandria Engineering Journal, 61(9),* 7233–7264.
15. Akhitha, K., Ram, G. (2022). Side lobe reduction and interference rejection in concentric hexagonal antenna array using differential evolutionary algorithm. *2022 IEEE Microwaves, Antennas, and Propagation Conference (MAPCON)*, pp. 681–685. Bangalore, India.
16. El-Khamy, S. E., El-Sayed, H. F., Eltrass, A. S. (2022). Synthesis of wideband thinned eisenstein fractile antenna arrays with adaptive beamforming capability and reduced side-lobes. *IEEE Access, 10,* 122486–122500.
17. Dawood, H., El-Khobby, H., Abd Elnaby, M. M., Hussein, A. (2021). Optimized VAA based synthesis of elliptical cylindrical antenna array for SLL reduction and beam thinning using minimum number of elements. *IEEE Access, 9,* 50949–50960.
18. El-Khamy, S., El-Sayed, H. F., Eltrass, A. (2023). New wideband antenna arrays with low sidelobe based on space filling curves. *2023 International Microwave and Antenna Symposium (IMAS)*, pp. 56–61. Cairo, Egypt.
19. Dipankar, S., Saha, C., Siddiqui, J. Y. (2022). Binomially distributed slotted array antenna with highly reduced sidelobe level. *2022 IEEE 19th India Council International Conference (INDICON)*, pp. 1–3. Kochi, India, 2022
20. Abdelrahman, S. I., Hussein, A. H., Shaalan, A. E. A. (2021). A hybrid GA/GE beamforming technique for side lobe cancellation of linear antenna arrays. *Advanced Electromagnetics, 10(2),* 67–77.
21. Theodore, S. R., Sun, S., Mayzus, R., Zhao, H., Azar, Y. et al. (2013). Millimeter wave mobile communications for 5G cellular: It will work!. *IEEE Access, 1,* 335–349.
22. Nor, A. M., Halunga, S., Fratu, O. (2022). Survey on positioning information assisted mmWave beamforming training. *Ad Hoc Networks, 135(9),* 102947. https://doi.org/10.1016/j.adhoc.2022.102947
23. Hager, S. F., Nasr, M. E., Hussein, A. H. (2022). A highly efficient approach for performance enhancement of multiple antenna elements based spectrum sensing techniques using side lobe level reduction. *Alexandria Engineering Journal AEJ, 61(8),* 5983–5999.
24. Hussein, A., Shawky Fouda, H., Hussein Abdullah, H., Khalaf, A. A. M. (2020). A highly efficient spectrum sensing approach based on antenna arrays beamforming. *IEEE Access, 8,* 25184–25197.
25. Ahmed, A. K., Hussein, A. H., Khalaf, A. A. M., Hamed, H. F. A. (2019). A utilization of multiple antenna elements for matched filter based spectrum sensing performance enhancement in cognitive radio system. *AEU-International Journal of Electronics and Communications, 107,* 98–109.
26. Katoch, S., Chauhan, S. S., Kumar, V. (2021). A review on genetic algorithm: Past, present, and future. *Multimedia Tools and Applications, 80(5),* 8091–8126.