Computer Modeling in Engineering & Sciences

Tech Science Press

DOI: 10.32604/cmes.2024.031439

ARTICLE

Novelty of Different Distance Approach for Multi-Criteria Decision-Making Challenges Using *q*-Rung Vague Sets

Murugan Palanikumar[1], Nasreen Kausar[2,*], Dragan Pamucar[3,4], Seifedine Kadry[5,6,7,*], Chomyong Kim[8] and Yunyoung Nam[9]

[1]Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences, Chennai, 602105, India

[2]Department of Mathematics, Faculty of Arts and Science, Yildiz Technical University, Esenler, Istanbul, 34220, Turkey

[3]Department of Operations Research and Statistics, Faculty of Organizational Sciences, University of Belgrade, Belgrade, 11000, Serbia

[4]College of Engineering, Yuan Ze University, Taoyuan City, 320315, Taiwan

[5]Department of Applied Data Science, Noroff University College, Kristiansand, 4612, Norway

[6]Artificial Intelligence Research Center (AIRC), Ajman University, Ajman, 346, United Arab Emirates

[7]Department of Electrical and Computer Engineering, Lebanese American University, Byblos, 1102-2801, Lebanon

[8]ICT Convergence Research Center, Soonchunhyang University, Asan, 31538, South Korea

[9]Department of Computer Science and Engineering, Soonchunhyang University, Asan, 31538, South Korea

*Corresponding Authors: Nasreen Kausar. Email: kausar.nasreen57@gmail.com; Seifedine Kadry. Email: seifedine.kadry@noroff.no

Received: 16 June 2023 Accepted: 06 September 2023 Published: 11 March 2024

ABSTRACT

In this article, multiple attribute decision-making problems are solved using the vague normal set (VNS). It is possible to generalize the vague set (VS) and q-rung fuzzy set (FS) into the q-rung vague set (VS). A log q-rung normal vague weighted averaging (log q-rung NVWA), a log q-rung normal vague weighted geometric (log q-rung NVWG), a log generalized q-rung normal vague weighted averaging (log Gq-rung NVWA), and a log generalized q-rung normal vague weighted geometric (log Gq-rung NVWG) operator are discussed in this article. A description is provided of the scoring function, accuracy function and operational laws of the log q-rung VS. The algorithms underlying these functions are also described. A numerical example is provided to extend the Euclidean distance and the Humming distance. Additionally, idempotency, boundedness, commutativity, and monotonicity of the log q-rung VS are examined as they facilitate recognizing the optimal alternative more quickly and help clarify conceptualization. We chose five anemia patients with four types of symptoms including seizures, emotional shock or hysteria, brain cause, and high fever, who had either retrograde amnesia, anterograde amnesia, transient global amnesia, post-traumatic amnesia, or infantile amnesia. Natural numbers q are used to express the results of the models. To demonstrate the effectiveness and accuracy of the models we are investigating, we compare several existing models with those that have been developed.

KEYWORDS

Vague set; aggregating operators; euclidean distance; hamming distance; decision making

Abbreviations

DM	Decision-making
MADM	Multiple-attribute decision-making
MCDM	Multi-criteria decision-making
TOPSIS	Technique for order of preference by similarity to ideal solution
FS	Fuzzy set
IFS	Intuitionistic fuzzy set
PyFS	Pythagorean fuzzy set
PyIVFS	Pythagorean interval-valued fuzzy set
NSS	Neutrosophic set
SFS	Spherical fuzzy set
VS	Vague set
TMG	Truth membership grade
IMG	Indeterminacy membership grade
FMG	False membership grade
ED	Euclidean distance
HD	Hamming distance
AO	Aggregating operator
q-ROFS	q-rung orthogonal pair fuzzy set
q-ROFWABM	q-rung orthopair fuzzy weighted Archimedean Bonferroni mean
q-ROFABM	q-rung orthopair fuzzy Archimedean Bonferroni mean
q-ROFPA	q-rung orthopair fuzzy power averaging
q-ROFPWA	q-rung orthopair fuzzy power weighted average
q-ROFPWMSM	q-rung orthopair fuzzy power weighted Maclaurin symmetric mean
q-ROFWA	q-rung orthopair fuzzy weighted average
q-ROFWG	q-rung orthopair fuzzy weighted geometric
log q-rung VNN	log q-rung vague normal number
log q-rung NVWA	log q-rung normal vague weighted average
log q-rung NVWG	log q-rung normal vague weighted geometric
log G q-rung NVWA	log generalized q-rung normal vague weighted average
log G q-rung NVWG	log generalized q-rung normal vague weighted geometric

1 Introduction

Decision-makers find it increasingly difficult to identify the optimal solution as real-world systems become increasingly complex. Selecting the best option is possible difficulty of deciding between the alternatives. Opportunities, objectives, and viewpoint constraints are challenging to create for many firms. In line with this, when decisions making (DM), individuals or groups should consider multiple objectives at the same time. A wide variety of MADM-related issues are dealt with every day. Our DM abilities need to be improved as a result. This field of study has been studied by a variety of researchers using a variety of methods. There are several uncertain theories proposed by them to deal with the uncertainties, including fuzzy set (FS) [1], intuitionistic fuzzy set (IFS) [2], interval valued FS (IVFS) [3], vague set [4], Pythagorean fuzzy set (PFS) [5], IVPFS [6], spherical FS (SFS) [7]. A membership grade (MG) indicates how well an FS fits into the specified set with ranging from 0 to one. An IFS was defined by Atanassov [2] as having a total of membership grade (MG) and non-membership grade (NMG) less than one. The sum of the MG and NMG is sometimes greater than one when a DM method is applied. Yager [5] developed PFS, which is characterized by a square sum of its MG and NMG not exceeding one. In order to generalize IFS, Yager used PFS to build a model. A new concept has been proposed by Yager [8] in light of society's continuous complexity and theory development. The MG and NMG in the q-rung orthogonal pair FS (q-ROFS) have power q, but the sum can never exceed one. The IFSs and PFSs can all be considered special cases of q-ROFSs, therefore they are general. There are more orthopairs that meet the bounding constraint as rung q increases, and as rung q increases, the space of acceptable orthopairs increases. The use of q-ROFSs can thus express fuzzy information in a broader range. Because the parameter q can be adjusted, q-ROFSs are flexible and better suited to uncertain environments. An increase in q can be made as ambiguity in decision information increases. It is possible that some experts are influenced by both their own desires and their surroundings. Therefore, they may have an MDG of 0.95 and an NMG of 0.55 when evaluating certain decision-making things. The fuzzy information cannot be described by IFNs and PFNs, but q-ROFNs can be described if parameter q is increased. Due to this, the q-ROFS is more flexible and suitable for describing uncertain data.

A discussion of the q-Rung Orthopair fuzzy weighted Archimedean Bonferroni mean (q-ROFWABM) and q-Rung Orthopair fuzzy Archimedean Bonferroni mean (q-ROFABM) operators was given in Liu et al. [9]. Liu et al. [10] proposed a concept of q-rung orthopair fuzzy power averaging (q-ROFPA), q-rung orthopair fuzzy power weighted average (q-ROFPWA), q-ROFPMSM and q-rung orthopair fuzzy power weighted MSM (q-ROFPWMSM) operators for q-ROFNs, describing their properties. Liu et al. [11] discussed the q-rung orthopair fuzzy weighted average (q-ROFWA) and q-rung orthopair fuzzy weighted geometric (q-ROFWG) operators are introduced and their basic properties are discussed. The concept of an incomplete probabilistic linguistic preference relation (InPLPR) was introduced by Wang et al. [12]. In 2013, Liu et al. [13] presented the concept of unit cost consensus adjustment based on a group consensus decision model based on InPLPR that takes into account social trust networks, consistency, and social trust networks. In a recent study, Zhang et al. [14] discussed three types of multi-granularity q-rung orthopair fuzzy preference relations (PRS) as well as their interesting properties. With the MAGDM algorithm based on q-ROF multi-attribute rules, the MG-3WD approach can also be applied to q-ROF complex information systems. Zhang et al. [15] analyzed a UCI dataset using MGq-ROF PRSs, the MULTIMOORA method, and the TPOP method using the MAGDM method. Zhang et al. [16] discussed neutosophic fusion of RST based on basic models and soft sets models. Based on fuzzy granularity spaces with properties that correspond to fuzzy knowledge distances, Lian et al. [17] discussed fuzzy relative knowledge distances. Furthermore, it has been demonstrated that fuzzy knowledge distances contain different structure information than precise knowledge distances. The hybridization of archimedean copulas and generalized MSM operators, based on q-rung probabilistic dual hesitant fuzzy sets, was discussed by Anusha et al. [18]. Multi-attribute decision-making (MADM) [19,20] offers an efficient means of evaluating multiple alternatives based on their evaluation values. Usually, MADM problems can be solved in one of two ways. Traditional approaches, such as TOPSIS, VIKOR, ELECTRE, are examples. Information integration problems are more effectively solved by AOs than by traditional approaches. In contrast to traditional approaches, AOs provide comprehensive values of all alternatives, rather than ranking results only. In his article, Bairagi [21] used extended TOPSIS to select homogeneous groups of robotic systems.

This is insufficient for demonstrating neutrality (neither favor nor disfavor). It was developed by Cuong et al. [22] with a total grade no higher than one for three pointers such as positive, neutral, and negative. As a result, it would be appropriate for the DM method to use this set over IFS or PFS for selective applications. Liu et al. first presented the concept of an aggregation operator (AO) in generalized PFS [23]. A PIVFS algorithm for the problem of identifying truth membership grades

(TMGs), indeterminacy membership grades (IMGs) and false membership grades (FMGs) with AOs [6,24–26] have the feature that the sum of the three grades (TMG, IMG, and FMG) is greater than one. It has been suggested by Ashraf et al. [7] that the SFS should contain the following graph: this diagram shows that the sum of the squares of the TMG, IMG and FMG should be not exceeds one. To analyze the idea of SFS, Fatmaa et al. [27] used the TOPSIS technique as part of their study. There have been several different concepts of q-Rung picture FS with AO for DM that have been demonstrated by Liu et al. [28]. In addition to Biswas [4], there is a concept of VSs VS is called to the two functions TMG T_v and FMG F_v as well as a set of transformations. Suppose that $T_v(x)$ is the total likelihood estimate of x, derived from the evidence for x and $F_v(x)$ is the total likelihood estimate for x derived from the evidence against x. It can be noted that these functions fall into the interval [0,1], where their sum is less than one. Various extensions have been made to the VS such as the IVFS and the FS [29–31]. Zhang et al. [32] was first introduced that suggested PFS can be extended to multi-criteria decision making (MCDM) using TOPSIS. The application of the bipolar fuzzy soft set (BFSS) was explored by Jana et al. [33] for the purpose of discovering how to broaden the set of bipolar fuzzy terms. Ullah et al. [34] described how pattern recognition applications can be used to estimate PFS distances using complex separation algorithms. It has been discussed that MCDM can be utilized using the neutrosophic set as well as the Dombi power AOs [35]. A number of algebraic structures and their applications have been investigated by Palanikumar et al. [36,37]. The notion of fuzzy c-number clustering procedures for fuzzy data was discussed by Yang et al. [38,39].

As an alternative to algebraic operations, a log q-rung arithmetic operation can provide a smooth estimate quality that is similar to that of a continuous algebraic operation when compared with its smoothness. Compared to the log q-rung arithmetic operations on the IFS and PFS only a limited research has been done on log q-rung arithmetic operations. Our method of VS is based on log q-rung arithmetic AOs within VSs rather than using log q-rung arithmetic operations. The use of spherical fuzzy q-rung arithmetic AOs based on entropy in DM, as well as their real-life application to the problem, were introduced by Jin et al. [40]. Ashraf et al. [41] proposed by that linear-logarithmic hybrid AOs be used for single-valued neutrosophic sets. Palanikumar et al. have examined the new type Pythagorean fuzzy set with AO [42]. Yager [5] has also presented an average and geometric AO using PFS weighted and weighted power cases. A number of basic PFS features were discussed by Peng et al. [43]. A generalized PFS under AO has been developed by Liu et al. [23]. Adak et al. dicussed the concept of spherical distance measurement method for solving MCDM problems under PFS [44]. Some picture fuzzy mean operators and their applications in DM is discussed by Hasan et al. [45]. Mishra et al. [46] discussed the new concept of Pythagorean and Fermatean fuzzy sub-group redefined in context of T-norm and S-conorm. DM analysis of minimizing the death rate due to COVID-19 by using q-rung orthopair fuzzy soft bonferroni mean operator discussed by Abbas et al. [47]. Yaman et al. [48] discussed the new approach for warehouse location decisions changed in medical sector after pandemic study. Recently, FS and its extension including q-rung orthopair fuzzy set, T-spherical fuzzy set based on decision making approach [49–55]. The log q-rung information about the VNS was obtained utilizing OAs. Section 2 explains the given information about the FS and VS components. Section 3 explains the definition of q-rung vague sets as well as the different operations involved with them. There is a discussion on ED and HD in Section 4 using the log q-rung vague normal number (log q-rung VNN). A MADM connection is established through the Section 5 using log q-rung VNNs. Section 6 contains a numerical example and a description of log q-rung VS as well as the insert algorithm and log q-rung VS application. We provide a conclusion in Section 7. An overview of the key things that were taken into account during the research process is given below:

1. As a result of log-rung VNSs, ED and HD were introduced.

2. The log q-rung VNVWA, log q-rung VNVWG, log Gq-rung VNVWA, and log Gq-rung VNVWG operators were suggestions.
3. A log-rung VNS is used in order to explore the MADM technique.
4. We evaluate log q-rung VNVWA, log q-rung VNVWG, log Gq-rung VNVWA and log Gq-rung VNVWG in order to establish optimal value parameters.
5. An analysis of the proposed and early investigations is presented along with a comparative analysis.
6. DM outcomes for natural numbers with a value of q.

2 Basic Concepts

We will introduce some basic literature on Pythagorean fuzzy set, Pythagorean interval-valued fuzzy set, vague set and spherical fuzzy set in the section, which will be useful in later section. Additionally, the basic operating rules and zero vague and unit vague set of these concepts are discussed.

Definition 2.1. [5] Let U be the universal, PFS M in U is $M = \left\{\rho, \langle \eta_M^T(\rho), \eta_M^F(\rho)\rangle \middle| \rho \in U\right\}$, $\eta_M^T, \eta_M^F : U \to [0,1]$ denote MG and NMG of $\rho \in U$, respectively and $0 \preceq (\eta_M^T(\rho))^2 + (\eta_M^F(\rho))^2 \preceq 1$. For $M = \langle \eta_M^T, \eta_M^F \rangle$ is represents a Pythagorean fuzzy number (PFN).

Definition 2.2. [6] The PIVFS M in U is $M = \left\{\rho, \left\langle \widetilde{\eta_M^T}(\rho), \widetilde{\eta_M^F}(\rho)\right\rangle \middle| \rho \in U\right\}$, where $\widetilde{\eta_M^T}, \widetilde{\eta_M^F} : U \to Int([0,1])$ denote MG and NMG of $\rho \in U$, respectively, and $0 \preceq (\eta_M^{TU}(\rho))^2 + (\eta_M^{FU}(\rho))^2 \preceq 1$.

For $M = \left\langle \left[\eta_M^{TL}, \eta_M^{TU}\right], \left[\eta_M^{FL}, \eta_M^{FU}\right] \right\rangle$ is represent a PIVFN.

Definition 2.3. [4] (i) A VS M in U is a pair (M^T, M^F), $M^T, M^F : U \to [0,1]$ are mappings such that $M^T(\rho) + M^F(\rho) \preceq 1, \forall \rho \in U$, M^T and M^F are called the TMG and FMG, respectively.

(ii) $M(\rho) = [M^T(\rho), 1 - M^F(\rho)]$ is represent the vague value of ϱ in M.

Definition 2.4. [4] (i) A VS M is contained in VS M_1, $M \subseteq M_1$ if and only if $M(\rho) \preceq M_1(\rho)$. That is, $M^T(\rho) \preceq T_{M_1}(\rho)$ and $1 - M^F(\rho) \preceq 1 - F_{M_1}(\rho), \forall \rho \in U$.

(ii) Union of M and M_1, as $X = M \cup M_1$, $T_X = max\{M^T, T_{M_1}\}$ and $1 - F_X = max\{1 - M^F, 1 - F_{M_1}\} = 1 - min\{M^F, F_{M_1}\}$.

(iii) Intersection of M and M_1 as $X = M \cap M_1$, $T_X = min\{M^T, T_{M_1}\}$ and $1 - F_X = min\{1 - M^F, 1 - F_{M_1}\} = 1 - max\{M^F, F_{M_1}\}$.

Definition 2.5. [4] A VS M of $U, \forall \rho \in U$. Then

(i) $M^T(\rho) = 0$ and $M^F(\rho) = 1$ is represent a zero VS of U.

(ii) $M^T(\rho) = 1$ and $M^F(\rho) = 0$ is represent a unit VS of U.

Definition 2.6. [38] The fuzzy number $M(x) = exp^{-\frac{(x-\chi)^2}{\psi^2}}, (\psi > 0)$ and $M = (\chi, \psi)$ is represent a normal fuzzy number (NFN), here R is a real numbers.

Definition 2.7. [39] Let $L_1 = (\chi_1, \psi_1) \in N$ and $L_2 = (\chi_2, \psi_2) \in N$, $(\psi_1, \psi_2 > 0)$, then their distance is $\Upsilon(L_1, L_2) = \sqrt{(\chi_1 - \chi_2)^2 + \frac{1}{2}(\psi_1 - \psi_2)^2}$.

3 log q-rung NVN and Its Basic Operations

In this section, we develop some novel logarithmic operational laws for normal vague numbers and discuss their properties. The main purpose of this section is to propose novel logarithmic aggregation operators based on normal vague information. The logarithmic function offers versatility in supporting the decision experts choices during object appraisal due to its periodic and symmetric character. An important fundamental operation of the log q-rung normal number is defined.

Definition 3.1. The log VS M in U is $M = \left\{\rho, \left\langle \left[\log M^{T}(\rho), \log(M^{1-F}(\rho))\right],\right.\right.$ $\left.\left.\left[\log M^{F}(\rho), \log(M^{1-T}(\rho))\right]\right\rangle \middle| \rho \in U\right\}$, $\widetilde{\eta_M^T} : U \to Int([0,1])$ and $\widetilde{\eta_M^F} : U \to Int([0,1])$ denote the TMG, IMG and FMG of $\rho \in U$ to M, respectively and $0 \preceq (\log_{\Omega_i} M^{1-F}(\rho))^q + (\log_{\Omega_i} M^{1-T}(\rho))^q \preceq 1$, where $\Omega_i = \prod[M^T, M^{1-F}], [M^F, M^{1-T}]$. For, $M = \left\langle \left[\log_{\Omega_i} M^T, \log_{\Omega_i}(M^{1-F})\right], \left[\log_{\Omega_i} M^F, \log_{\Omega_i}(M^{1-T})\right]\right\rangle$ is called a log q-rung vague number.

Definition 3.2. Let $(\chi, \psi) \in N$, $M = \left\langle (\chi, \psi); [\log M^T, \log(M^{1-F})], [\log M^F, \log(M^{1-T})]\right\rangle$ be a log q-rung normal vague number (NVN). The TMG, IMG and FMG are defined as $\left[\log_{\Omega_i} M^T, \log_{\Omega_i}(M^{1-F})\right] = \left[\log_{\Omega_i} M^T \cdot exp^{-\frac{(x-\chi)^2}{\psi^2}}, \log_{\Omega_i}(M^{1-F}) \cdot exp^{-\frac{(x-\chi)^2}{\psi^2}}\right]$, $\left[\log_{\Omega_i} M^F, \log_{\Omega_i}(M^{1-T})\right] = \left[1-\left(1-\log_{\Omega_i} M^F\right) \cdot exp^{-\frac{(x-\chi)^2}{\psi^2}}, 1-\left(1-\log_{\Omega_i}(M^{1-T})\right) \cdot exp^{-\frac{(x-\chi)^2}{\psi^2}}\right]$ respectively, $\left[\log_{\Omega} M^T, \log_{\Omega}(M^{1-F})\right]$ and $0 \preceq \left(\log_{\Omega}(M^{1-F})(\rho)\right)^q + \left(\log_{\Omega}(M^{1-T})(\rho)\right)^q \preceq 1$, where $\Omega = \prod[M^T, M^{1-F}], [M^F, M^{1-T}]$, where $x \in X$ is a non-empty set.

Definition 3.3. Let $M = \left\langle (\chi, \psi); \left[\log M^T, \log(M^{1-F})\right], \left[\log M^F, \log(M^{1-T})\right]\right\rangle$ be the log q-rung NVN.

The score function of M is $\mathbb{S}(M) = \dfrac{\frac{\chi}{2}\left(\frac{\mathbb{X}_1}{2} + 1 - \frac{\mathbb{Z}_1}{2}\right) + \frac{\psi}{2}\left(\frac{\mathbb{X}_2}{2} + 1 - \frac{\mathbb{Z}_2}{2}\right)}{2}$, where $-1 \preceq \mathbb{S}(M) \preceq 1$.

The accuracy function of M is $\mathbb{A}(M) = \dfrac{\frac{\chi}{2}\left(\frac{\mathbb{X}_1}{2} + 1 + \frac{\mathbb{Z}_1}{2}\right) + \frac{\psi}{2}\left(\frac{\mathbb{X}_2}{2} + 1 + \frac{\mathbb{Z}_2}{2}\right)}{2}$, where $0 \preceq \mathbb{A}(M) \preceq 1$.

where $\mathbb{X}_1 = (\log M^T)^q, \mathbb{Z}_1 = (\log M^F)^q$ and $\mathbb{X}_2 = (\log(M^{1-F}))^q, \mathbb{Z}_2 = (\log(M^{1-T}))^q$.

Definition 3.4. Let $M = \left\langle (\chi, \psi); [\log M^T, \log(M^{1-F})], [\log M^F, \log(M^{1-T})]\right\rangle$,

$M_1 = \left\langle (\chi_1, \psi_1); [\log M_1^T, \log(M_1^{1-F})], [\log M_1^F, \log({M_1}^{1-T})]\right\rangle$ and

$M_2 = \left\langle (\chi_2, \psi_2); [\log M_2^T, \log(M_2^{1-F})], [\log M_2^F, \log({M_2}^{1-T})]\right\rangle$ be the three log q-rung NVNs, q is a natural number and $\Omega = \prod[T_{M_i}, M_i^{1-F}], [F_{M_i}, M_i^{1-T}]$, then

1. $$M_1 \bigoplus M_2 = \left[\begin{array}{c} (\chi_1+\chi_2, \psi_1+\psi_2); \\ \left[\begin{array}{c} \sqrt[q]{(\log_{\Omega_i} M_1^T)^q + (\log_{\Omega_i} M_2^T)^q - (\log_{\Omega_i} M_1^T)^q \cdot (\log_{\Omega_i} M_2^T)^q}, \\ \sqrt[q]{(\log_{\Omega_i}(M_1^{1-F}))^q + (\log_{\Omega_i}(M_2^{1-F}))^q - (\log_{\Omega_i}(M_1^{1-F}))^q \cdot (\log_{\Omega_i}(M_2^{1-F}))^q} \end{array}\right], \\ \left[\log_{\Omega_i} M_1^F \cdot \log_{\Omega_i} M_2^F, \log_{\Omega_i}({M_1}^{1-T}) \cdot \log_{\Omega_i}({M_2}^{1-T})\right] \end{array}\right],$$

2. $$M_1 \bigotimes M_2 = \left[\begin{array}{c} (\chi_1 \cdot \chi_2, \psi_1 \cdot \psi_2); \\ \left[\log_{\Omega_i} M_1^T \cdot \log_{\Omega_i} M_2^T, \log_{\Omega_i}(M_1^{1-F}) \cdot \log_{\Omega_i}(M_2^{1-F})\right], \\ \left[\begin{array}{c} \sqrt[q]{(\log_{\Omega_i} M_1^F)^q + (\log_{\Omega_i} M_2^F)^q - (\log_{\Omega_i} M_1^F)^q \cdot (\log_{\Omega_i} M_2^F)^q}, \\ \sqrt[q]{(\log_{\Omega_i}({M_1}^{1-T}))^q + (\log_{\Omega_i}({M_2}^{1-T}))^q - (\log_{\Omega_i}({M_1}^{1-T}))^q \cdot (\log_{\Omega_i}({M_2}^{1-T}))^q} \end{array}\right] \end{array}\right],$$

3. $$\Theta \cdot M = \left[\begin{array}{c} (\Theta \cdot \chi, \Theta \cdot \psi); \\ \left[\sqrt[q]{1-\left(1-(\log_{\Omega_i} M^T)^q\right)^q}, \sqrt[q]{1-\left(1-(\log_{\Omega_i}(M^{1-F}))^q\right)^q}\right], \\ \left[(\log_{\Omega_i} M^F)^q, (\log_{\Omega_i}(M^{1-T}))^q\right] \end{array}\right],$$

4. $$M^\Theta = \left[\begin{array}{c} (\chi^\Theta, \psi^\Theta); \left[(\log_{\Omega_i} M^T)^q, (\log_{\Omega_i}(M^{1-F}))^q\right], \\ \left[\sqrt[q]{1-\left(1-(\log_{\Omega_i} M^F)^q\right)^q}, \sqrt[q]{1-\left(1-(\log_{\Omega_i}(M^{1-T}))^q\right)^q}\right] \end{array}\right].$$

4 Distance between log q-rung Normal Vague Numbers

The Euclidean distance and Hamming distance are useful technique for calculating the distance between two elements, two sets, etc. In order to define the Euclidean distance and Hamming distance, first, we will define a distance measure. Basically, a distance measure has to accomplish the following properties. This study examined the mathematical properties of log q-rung NVNs and measured the expectation of ED and HD.

Definition 4.1. Let $M_1 = \left\langle(\chi_1, \psi_1); [\log M_1^T, \log(M_1^{1-F})], [\log M_1^F, \log({M_1}^{1-T})]\right\rangle$ and $M_2 = \left\langle(\chi_2, \psi_2); [\log M_2^T, \log(M_2^{1-F})], [\log M_2^F, \log({M_2}^{1-T})]\right\rangle$ be any two log q-rung NVNs. Then ED between M_1 and M_2 is

$$\Upsilon_E\left(M_1, M_2\right) = \frac{1}{2}\sqrt{\begin{array}{l} \left[\begin{array}{l} \dfrac{(\log_{\Omega_i} M_1^T)^2 + 1 - (\log_{\Omega_i} M_1^F)^2 + (\log_{\Omega_i}(M_1^{1-F}))^2 + 1 - (\log_{\Omega_i}({M_1}^{1-T}))^2}{2}\chi_1 \\ -\dfrac{(\log_{\Omega_i} M_2^T)^2 + 1 - (\log_{\Omega_i} M_2^F)^2 + (\log_{\Omega_i}(M_2^{1-F}))^2 + 1 - (\log_{\Omega_i}({M_2}^{1-T}))^2}{2}\chi_2 \end{array}\right]^2 \\ +\dfrac{1}{2}\left[\begin{array}{l} \dfrac{(\log_{\Omega_i} M_1^T)^2 + 1 - (\log_{\Omega_i} M_1^F)^2 + (\log_{\Omega_i}(M_1^{1-F}))^2 + 1 - (\log_{\Omega_i}({M_1}^{1-T}))^2}{2}\psi_1 \\ -\dfrac{(\log_{\Omega_i} M_2^T)^2 + 1 - (\log_{\Omega_i} M_2^F)^2 + (\log_{\Omega_i}(M_2^{1-F}))^2 + 1 - (\log_{\Omega_i}({M_2}^{1-T}))^2}{2}\psi_2 \end{array}\right]^2 \end{array}}$$

and HD between M_1 and M_2 is defined as

$$\Upsilon_H\left(M_1, M_2\right) = \frac{1}{2}\left[\left|\begin{array}{l}\frac{(\log_{\Omega_i} M_1^T)^2+1-(\log_{\Omega_i} M_1^F)^2+(\log_{\Omega_i}(M_1^{1-F}))^2+1-(\log_{\Omega_i}({M_1}^{1-T}))^2}{2}\chi_1 \\ -\frac{(\log_{\Omega_i} M_2^T)^2+1-(\log_{\Omega_i} M_2^F)^2+(\log_{\Omega_i}(M_2^{1-F}))^2+1-(\log_{\Omega_i}({M_2}^{1-T}))^2}{2}\chi_2\end{array}\right| + \frac{1}{2}\left|\begin{array}{l}\frac{(\log_{\Omega_i} M_1^T)^2+1-(\log_{\Omega_i} M_1^F)^2+(\log_{\Omega_i}(M_1^{1-F}))^2+1-(\log_{\Omega_i}({M_1}^{1-T}))^2}{2}\psi_1 \\ -\frac{(\log_{\Omega_i} M_2^T)^2+1-(\log_{\Omega_i} M_2^F)^2+(\log_{\Omega_i}(M_2^{1-F}))^2+1-(\log_{\Omega_i}({M_2}^{1-T}))^2}{2}\psi_2\end{array}\right|\right]$$

Theorem 4.1. Let $M_1 = \left\langle(\chi_1, \psi_1); [\log M_1^T, \log(M_1^{1-F})], [\log M_1^F, \log({M_1}^{1-T})]\right\rangle$,

$M_2 = \left\langle(\chi_2, \psi_2); [\log M_2^T, \log(M_2^{1-F})], [\log M_2^F, \log({M_2}^{1-T})]\right\rangle$ and

$M_3 = \left\langle(\chi_3, \psi_3); [\log M_3^T, \log({M_3}^{1-F})], [\log M_3^F, \log({M_T}^{1-F})]\right\rangle$ be any three log q-rung NVNs, then

1. $\Upsilon_E(M_1, M_2) = 0$, if and only if $M_1 = M_2$.
2. $\Upsilon_E(M_1, M_2)=\Upsilon_E(M_2, M_1)$.
3. $\Upsilon_E(M_1, M_3) \preceq \Upsilon_E(M_1, M_2) + \Upsilon_E(M_2, M_3)$.

Corollary 4.1. Let $M_1 = \left\langle(\chi_1, \psi_1); [\log M_1^T, \log(M_1^{1-F})], [\log M_1^F, \log({M_1}^{1-T})]\right\rangle$,

$M_2 = \left\langle(\chi_2, \psi_2); [\log M_2^T, \log(M_2^{1-F})], [\log M_2^F, \log({M_2}^{1-T})]\right\rangle$ and

$M_3 = \left\langle(\chi_3, \psi_3); [\log M_3^T, \log({M_3}^{1-F})], [\log M_3^F, \log({M_T}^{1-F})]\right\rangle$ be any three log q-rung NVNs. Then

1. $\Upsilon_H(M_1, M_2) = 0$ if and only if $M_1 = M_2$.
2. $\Upsilon_H(M_1, M_2) = \Upsilon_H(M_2, M_1)$.
3. $\Upsilon_H(M_1, M_3) \preceq \Upsilon_H(M_1, M_2) + \Upsilon_H(M_2, M_3)$.

5 log *q*-rung Normal Vague Number Using AOs

We will introduce the novel concepts of log q-rung NVWA, log q-rung NVWG, log G q-rung NVWA, and log G q-rung NVWG operators utilizing log q-rung NVN.

5.1 log q-rung NVWA Operator

Definition 5.1. Let $M_i = \left\langle(\chi_i, \psi_i); [\log {M_i}^T, \log M_i^{1-F})], [\log {M_i}^F, \log M_i^{1-T})]\right\rangle$ be the family of log q-rung NVNs, $\Psi = (\Psi_1, \Psi_2, ..., \Psi_n)$ be the weight of M_i, $\Psi_i \succeq 0$ and $\uplus_{i=1}^n \Psi_i = 1$ and $\Omega = \prod[T_{M_i}, M_i^{1-F}], [F_{M_i}, M_i^{1-T}]$, then log q-rung NVWA operator is log q-rung NVWA $(M_1, M_2, ..., M_n) = \uplus_{i=1}^n \Psi_i M_i$.

Theorem 5.1. Let $M_i = \left\langle (\chi_i, \psi_i); [\log M_i^{T}, \log M_i^{1-F})], [\log M_i^{F}, \log M_i^{1-T})] \right\rangle$ be the family of log q-rung NVNs, then log q-rung NVWA $(M_1, M_2, ..., M_n) =$

$$\left[\begin{array}{c} \left(\uplus_{i=1}^{n} \Psi_i \chi_i, \uplus_{i=1}^{n} \Psi_i \psi_i \right); \\ \left[\sqrt[q]{1 - \bigcirc_{i=1}^{n} \left(1 - (\log_{\Omega_i} M_i^{T})^q\right)^{\Psi_i}}, \sqrt[q]{1 - \bigcirc_{i=1}^{n} \left(1 - (\log_{\Omega_i} M_i^{1-F}))^q\right)^{\Psi_i}} \right], \\ \left[\bigcirc_{i=1}^{n} (\log_{\Omega_i} M_i^{F})^{\Psi_i}, \bigcirc_{i=1}^{n} (\log_{\Omega_i} M_i^{1-T}))^{\Psi_i} \right] \end{array}\right].$$

Theorem 5.2. (idempotency property) If all $M_i = \left\langle (\chi_i, \psi_i); [\log M_i^{T}, \log M_i^{1-F})], \right.$

$\left. [\log M_i^{F}, \log M_i^{1-T})] \right\rangle = M$, then log q-rung NVWA $(M_1, M_2, ..., M_n) = M$.

Theorem 5.3. (boundedness property) Let $M_i = \left\langle (\chi_{ij}, \psi_{ij}); [\log M_{ij}^{T}, \log (M_{ij}^{1-F})], \right.$

$\left. [\log M_{ij}^{F}, \log (M_{ij}^{1-T})] \right\rangle$ be the collection of log q-rung NVWA, where $\underbrace{\chi} = \min \chi_{ij}$, $\overbrace{\chi} = \max \chi_{ij}$, $\underbrace{\psi} = \max \psi_{ij}$, $\overbrace{\psi} = \min \psi_{ij}$,

$\underbrace{\log_{\Omega_i} M^{T}} = \min \log_{\Omega_i} M_{ij}^{T}$, $\overbrace{\log_{\Omega_i} M^{T}} = \max \log_{\Omega_i} M_{ij}^{T}$, $\underbrace{\log_{\Omega_i} (M^{1-F})} = \min \log_{\Omega_i} (M_{ij}^{1-F})$,

$\overbrace{\log_{\Omega_i} (M^{1-F})} = \max \log_{\Omega_i} (M_{ij}^{1-F})$, $\underbrace{\log_{\Omega_i} M^{F}} = \min \log_{\Omega_i} M_{ij}^{F}$, $\overbrace{\log_{\Omega_i} M^{F}} = \max \log_{\Omega_i} M_{ij}^{F}$,

$\underbrace{\log_{\Omega_i} (M^{1-T})} = \min \log_{\Omega_i} (M_{ij}^{1-T})$, $\overbrace{\log_{\Omega_i} (M^{1-T})} = \max \log_{\Omega_i} (M_{ij}^{1-T})$.

Then, $\left\langle (\underbrace{\chi}, \underbrace{\psi}); [\underbrace{\log_{\Omega_i} M^{T}}, \underbrace{\log_{\Omega_i} (M^{1-F})}], [\overbrace{\log_{\Omega_i} M^{F}}, \overbrace{\log_{\Omega_i} (M^{1-T})}] \right\rangle$

$\preceq logq - rungNVWA(M_1, M_2, \ldots, M_n)$

$\preceq \left\langle (\overbrace{\chi}, \overbrace{\psi}); [\overbrace{\log_{\Omega_i} M^{T}}, \overbrace{\log_{\Omega_i} (M^{1-F})}], [\underbrace{\log_{\Omega_i} M^{F}}, \underbrace{\log_{\Omega_i} (M^{1-T})}] \right\rangle.$

where $1 \preceq i \preceq n$ and $j = 1, 2, ..., i_j$.

Theorem 5.4. (monotonicity property) Let $M_i = \left\langle (\chi_{t_{ij}}, \psi_{t_{ij}}); [\log M_{t_{ij}}^{T}, \log (M_{t_{ij})}^{1-F}], \right.$

$\left. [\log M_{t_{ij}}^{F}, \log (M_{t_{ij}}^{1-T})] \right\rangle$ and $\Psi_i = \left\langle (\chi_{h_{ij}}, \psi_{h_{ij}}); [\log M_{h_{ij}}^{T}, \log (M_{h_{ij}}^{1-F})], \right.$

$\left. [\log M_{h_{ij}}^{F}, \log (M_{h_{ij}}^{1-T})] \right\rangle$ be the families of log q-rung NVWAs. For any i, if there is $\chi_{t_{ij}} \preceq \psi_{h_{ij}}$, $\left(\log_{\Omega_i} M_{t_{ij}}^{T}\right)^2 + \left(\log_{\Omega_i} M_{t_{ij}}^{1-F}\right)^2 \preceq \left(\log_{\Omega_i} M_{h_{ij}}^{T}\right)^2 + \left(\log_{\Omega_i} M_{h_{ij}}^{1-F}\right)^2$ $\left(\log_{\Omega_i} M_{t_{ij}}^{F}\right)^2 + \left(\log_{\Omega_i} M_{t_{ij}}^{1-T}\right)^2 \succeq \left(\log_{\Omega_i} M_{h_{ij}}^{F}\right)^2 + \left(\log_{\Omega_i} M_{h_{ij}}^{1-T}\right)^2$ or $M_i \preceq \mathscr{W}_i$, then log q-rung NVWA $(M_1, M_2, ..., M_n) \preceq q$-Rung log q-rung NVWA $(\mathscr{W}_1, \mathscr{W}_2, ..., \mathscr{W}_n)$.

5.2 log q-rung NVWG Operator

Definition 5.2. Let $M_i = \left\langle (\chi_i, \psi_i); [\log M_i^{T}, \log M_i^{1-F}], [\log M_i^{F}, \log M_i^{1-T}] \right\rangle$ be the family of log q-rung NVNs. Then log q-rung NVWG $(M_1, M_2, ..., M_n) = \bigcirc_{i=1}^{n} M_i^{\Psi_i}$.

Theorem 5.5. Let $M_i = \left\langle (\chi_i, \psi_i); [\log M_i^{T}, \log M_i^{1-F}], [\log M_i^{F}, \log M_i^{1-T}] \right\rangle$ be the family of log q-rung NVNs. Then log q-rung NVWG $(M_1, M_2, ..., M_n) =$

$$\left[\begin{array}{c} \left(\bigcirc_{i=1}^{n} \chi_i^{\Psi_i}, \bigcirc_{i=1}^{n} \psi_i^{\Psi_i}\right); \left[\bigcirc_{i=1}^{n} (\log_{\Omega_i} M_i^{T})^{\Psi_i}, \bigcirc_{i=1}^{n} (\log_{\Omega_i} M_i^{1-F}))^{\Psi_i}\right], \\ \left[\sqrt[q]{1 - \bigcirc_{i=1}^{n}\left(1 - (\log_{\Omega_i} M_i^{F})^q\right)^{\Psi_i}}, \sqrt[q]{1 - \bigcirc_{i=1}^{n}\left(1 - (\log_{\Omega_i} M_i^{1-T}))^q\right)^{\Psi_i}}\right] \end{array}\right].$$

Theorem 5.6. If all $M_i = \left\langle (\chi_i, \psi_i); [\log M_i^{T}, \log M_i^{1-F}][\log M_i^{F}, \log M_i^{1-T}] \right\rangle = M$, then log q-rung NVWG $(M_1, M_2, ..., M_n) = M$.

Corollary 5.1. Boundness and monotonicity properties can be satisfied using the log q-rung NVWG operator.

5.3 log G q-rung NVWA Operator

Definition 5.3. Let $M_i = \left\langle (\chi_i, \psi_i); [\log M_i^{T}, \log M_i^{1-F}], [\log M_i^{F}, \log M_i^{1-T}] \right\rangle$ be the family of log q-rung NVN, then log G q-rung NVWA $(M_1, M_2, ..., M_n) = \left(\uplus_{i=1}^{n} \Psi_i M_i^{\Theta}\right)^{1/\Theta}$. **Theorem 5.7.** Let $M_i = \left\langle (\chi_i, \psi_i); [\log M_i^{T}, \log M_i^{1-F}], [\log M_i^{F}, \log M_i^{1-T}] \right\rangle$ be the family of log q-rung NVNs. Then log G q-rung NVWA $(M_1, M_2, ..., M_n) =$

$$\left[\begin{array}{c} \left(\left(\uplus_{i=1}^{n} \Psi_i \chi_i^{\Theta}\right)^{1/\Theta}, \left(\uplus_{i=1}^{n} \Psi_i \psi_i^{\Theta}\right)^{1/\Theta}\right); \\ \left[\left(\sqrt[q]{1 - \bigcirc_{i=1}^{n}\left(1 - \left((\log_{\Omega_i} M_i^{T})^q\right)\right)^{\Psi_i}}\right)^{1/q}, \left(\sqrt[q]{1 - \bigcirc_{i=1}^{n}\left(1 - \left((\log_{\Omega_i} M_i^{1-F}))^q\right)\right)^{\Psi_i}}\right)^{1/q}\right], \\ \left[\begin{array}{c} \sqrt[q]{1 - \left(1 - \left(\bigcirc_{i=1}^{n}\left(\sqrt[q]{1 - \left(1 - (\log_{\Omega_i} M_i^{F})^q\right)}\right)^{\Psi_i}\right)\right)^{q}}^{\,1/q}, \\ \sqrt[q]{1 - \left(1 - \left(\bigcirc_{i=1}^{n}\left(\sqrt[q]{1 - \left(1 - (\log_{\Omega_i} M_i^{1-T}))^q\right)}\right)^{\Psi_i}\right)\right)^{q}}^{\,1/q} \end{array}\right] \end{array}\right].$$

Theorem 5.8. If all $M_i = \left\langle (\chi_i, \psi_i); [\log M_i^{T}, \log M_i^{1-F}], [\log M_i^{F}, \log M_i^{1-T}] \right\rangle = M$. Then log G q-rung NVWA $(M_1, M_2, ..., M_n) = M$.

Corollary 5.2. By using log G q-rung NVWA operators, it is possible to satisfy bounding and monotonicity properties.

5.4 log G q-rung NVWG Operator

Definition 5.4. Let $M_i = \left\langle (\chi_i, \psi_i); [\log M_i^{T}, \log M_i^{1-F}], \log \mathscr{I}_{Mi}], [\log M_i^{F}, \log M_i^{1-T}] \right\rangle$ be the family of log q-rung NVNs. Then log G q-rung NVWG $(M_1, M_2, ..., M_n) = \frac{1}{\Theta}\left(\bigcirc_{i=1}^{n} (\Theta M_i)^{\Psi_i}\right)$.

Theorem 5.9. Let $M_i = \left\langle (\chi_i, \psi_i); [\log M_i^{T}, \log M_i^{1-F}], [\log M_i^{F}, \log M_i^{1-T}] \right\rangle$ be the family of log q-rung NVNs. Then log G q-rung NVWG $(M_1, M_2, ..., M_n) =$

$$\left[\begin{array}{c} \left(\frac{1}{\Theta} \bigcirc_{i=1}^{n} (\Theta\chi_i)^{\Psi_i}, \frac{1}{\Theta} \bigcirc_{i=1}^{n} (\Theta\psi_i)^{\Psi_i} \right); \\ \left[\begin{array}{c} \sqrt[q]{1 - \left(1 - \left(\bigcirc_{i=1}^{n} \left(\sqrt[q]{1 - \left(1 - (\log_{\Omega_i} M_i^{T})^q\right)}\right)^{\Psi_i}\right)^{q}\right)}^{1/q}, \\ \sqrt[q]{1 - \left(1 - \left(\bigcirc_{i=1}^{n} \left(\sqrt[q]{1 - \left(1 - (\log_{\Omega_i} M_i^{1-F}))^q\right)}\right)^{\Psi_i}\right)^{q}\right)}^{1/q} \end{array} \right], \\ \left[\left(\sqrt[q]{1 - \bigcirc_{i=1}^{n} \left(1 - \left((\log_{\Omega_i} M_i^{F})^q\right)^{q}\right)^{\Psi_i}} \right)^{1/q}, \left(\sqrt[q]{1 - \bigcirc_{i=1}^{n} \left(1 - \left((\log_{\Omega_i} M_i^{1-T}))^q\right)^{q}\right)^{\Psi_i}} \right)^{1/q} \right] \end{array} \right].$$

The log G q-rung NVWG operations are modified to be log q-rung NVWG operations with $q = 1$.

Corollary 5.3. The log G q-rung NVWG operator is able to satisfy properties such as boundness and monotonicity.

Corollary 5.4. If all $M_i = \left\langle (\chi_i, \psi_i); [\log M_i^{T}, \log M_i^{1-F}], [\log M_i^{F}, \log M_i^{1-T}] \right\rangle = M$. Then log G q-rung NVWG$(M_1, M_2, ..., M_n) = M$.

6 log q-rung NVN Based on MADM

Let $M = \{M_1, M_2, ..., M_n\}$ be the set of n-alternatives, $\gamma = \{e_1, e_2, ..., \gamma_m\}$ be the set of m-attributes and $\Psi = \{\Psi_1, \Psi_2, ..., \Psi_m\}$ be the weights, where $\Psi_i \in [0, 1]$ and $\sum_i^m \Psi_i = 1$.

Let $M_{ij} = \left\langle (\chi_{ij}, \psi_{ij}); [\log_{\Omega_i} M_{ij}{}^{T}, \log_{\Omega_i} M_{ij}{}^{1-F}], [\log_{\Omega_i} M_{ij}^{F}, \log_{\Omega_i} M_{ij}{}^{1-T}] \right\rangle$ represent log q-rung NVN of alternative M_i in attribute γ_j, $i = 1, 2, ..., n$ and $j = 1, 2, ..., m$.

Since $\left[\log_{\Omega_i} M_{ij}{}^{T}, \log_{\Omega_i} M_{ij}{}^{1-F}\right], \left[\log_{\Omega_i} M_{ij}^{F}, \log_{\Omega_i} M_{ij}{}^{1-T}\right] \in [0, 1]$ and

$0 \preceq (\log_{\Omega_i} M_{ij}{}^{1-F}(\rho))^q + (\log_{\Omega_i} M_{ij}{}^{1-T}(\rho))^q \preceq 1$. Fig. 1 shows that an algorithm for the MADM process using log q-rung NVS is followed by a flowchart.

6.1 Algorithm

Step-1: The log q-rung NVN values that can be input.

Step-2: Calculate the normalized values for the DM. The decision matrix $n \times m$ as $\Upsilon = (\widetilde{\gamma}_{ij})_{n\times m}$ is normalized into $\overbrace{\Upsilon} = (\gamma_{ij})_{n\times m}$; where $\gamma_{ij} = \left\langle (\overbrace{\chi_{ij}}, \overbrace{\psi_{ij}}); [\overbrace{\log_{\Omega_i} M_{ij}{}^{T}}, \overbrace{\log_{\Omega_i} (M_{ij}{}^{1-F})}], [\overbrace{\log_{\Omega_i} M_{ij}^{F}}, \overbrace{\log_{\Omega_i} (M_{ij}{}^{1-T})}] \right\rangle$ and $\overbrace{\chi_{ij}} = \dfrac{\Omega_{ij}}{\max_i(\chi_{ij})}$, $\overbrace{\psi_{ij}} = \dfrac{\psi_{ij}}{\max_i(\psi_{ij})} \cdot \dfrac{\psi_{ij}}{\chi_{ij}}$, $\overbrace{\log_{\Omega_i} M_{ij}{}^{T}} = \log_{\Omega_i} M_{ij}{}^{T}$,

$\overbrace{\log_{\Omega_i} (M_{ij}{}^{1-F})} = \log_{\Omega_i} (M_{ij}{}^{1-F})$, where $\Omega_i = \prod [T_{M_i}, M_i^{1-F}], [F_{M_i}, M_i^{1-T}]$.

Step-3: By utilizing log q-rung NVN, you can determine the aggregate values for all alternative based on AOs, attribute γ_j in $\widetilde{\gamma}_i$, $\gamma_{ij} = \left\langle (\overbrace{\chi_{ij}}, \overbrace{\psi_{ij}}); [\overbrace{\log_{\Omega_i} M_{ij}^{T}, \log_{\Omega_i} (M_{ij}^{1-F})}], [\overbrace{\log_{\Omega_i} M_{ij}^{F}, \log_{\Omega_i} (M_{ij}^{1-T})}] \right\rangle$ is aggregated into

$$\gamma_i = \left\langle (\overbrace{\chi_i}, \overbrace{\psi_i}); [\overbrace{\log_{\Omega_i} M_i^{T}, \log_{\Omega_i} M_i^{1-F}}], [\overbrace{\log_{\Omega_i} M_i^{F}, \log_{\Omega_i} M_i^{1-T}}] \right\rangle.$$

Step-4: Calculate the positive and negative ideal values for each case as follows:

$$\gamma^{+} = \left[\begin{array}{c} \left(\max_{1 \le i \le n} (\overbrace{\chi_{ij}}), \min_{1 \le i \le n} (\overbrace{\psi_{ij}}) \right); \\ [1, 1], [0, 0] \end{array} \right] \text{ and } \gamma^{-} = \left[\begin{array}{c} \left(\min_{1 \le i \le n} (\overbrace{\chi_{ij}}), \max_{1 \le i \le n} (\overbrace{\psi_{ij}}) \right); \\ [0, 0], [1, 1] \end{array} \right]$$

Step-5: Determine the EDs between the alternatives with a positive and a negative ideal value so as to determine the value of each alternative as $\Upsilon_i^{+} = \Upsilon_E\left(\gamma_i, \gamma^{+}\right);\ \Upsilon_i^{-} = \Upsilon_E\left(\gamma_i, \gamma^{-}\right)$.

Step-6: It is possible to calculate the relative closeness values by using the following formula: $\Upsilon_i^{*} = \dfrac{\Upsilon_i^{-}}{\Upsilon_i^{+} + \Upsilon_i^{-}}$.

Step-7: The optimal value is max Υ_i^{*}.

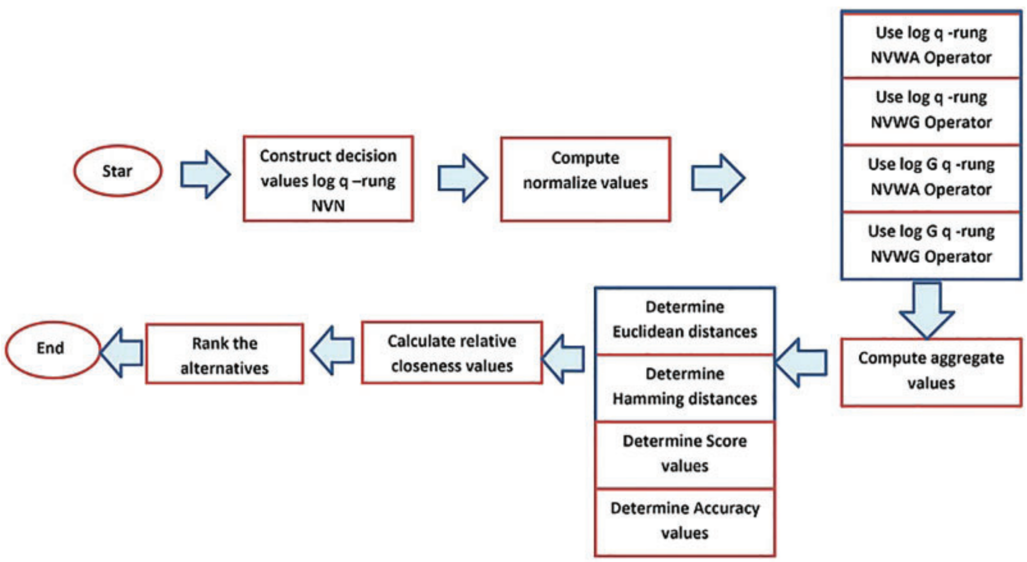

Figure 1: Flowchart of the MADM algorithm

6.2 Selection of Amnesia Patients

A person with anemia has insufficient or malfunctioning red blood cells. A man is diagnosed with anemia when his hemoglobin value is below 13.5 gm/dl, while a woman is diagnosed with anemia when her hemoglobin value is below 12.0 gm/dl. There are a variety of normal values for children depending on their age. Memory strategies are used to help deal with amnesia. Taking care of underlying diseases that cause amnesia is also important. An occupational therapist may help the person learn new information and replace what they have lost. Taking in new information may be based on intact memories. Additionally, memory training can help organize information to make it easier to remember and to better understand when you are speaking with others. Smartphones and hand held tablets are often used by people with amnesia. A simple electronic organizer can help even people who suffer from severe amnesia stay on top of their daily activities with a little training and practice. A person with amnesia may benefit from psychological therapy or cognitive behavioral therapy (CBT). When it comes to recalling forgotten memories, hypnosis can be very effective. It is important to retrieve memories and deal with psychological issues that may have contributed to amnesia as part of treatment for amnesia. A person may be able to retrieve forgotten memories through meditation and related mindfulness activities. It is also imperative to have your family's support. Playing familiar music, showing them photographs from the past, and exposing them to familiar scents may be helpful. Blood buildup in the brain may cause amnesia in people who have been injured in head trauma. Anti-inflammatory medications may be needed by people with encephalitis. If you cycle, skate, ski, or play contact sports, you may be at greater risk of developing amnesia due to headgear. It is important to consume a diet rich in leafy green vegetables and avoid saturated fats to prevent cardiovascular diseases that can negatively affect memory. Brain regulation is achieved through the Bilateral Sounds method. As well as relieving stress and symptoms of PCS and PTSD, it is excellent for reeducating the left and right hemispheres. There are some commonly used bilateral sounds available for this purpose through Psych Innovations, a web-based company.

1. Retrograde amnesia (A):
   A person suffering from retrograde amnesia is incapable of recalling past events. Memory loss usually affects memories made recently, not ones from years ago. You can experience amnesia if you lose the ability to make, store, and retrieve memories. Memory formation prior to amnesia onset is affected by retrograde amnesia. After a traumatic brain injury, a person may develop retrograde amnesia, which prevents him or her from remembering what happened decades earlier. A variety of brain regions can be damaged, causing retrograde amnesia to occur.
2. Anterograde amnesia (B):
   The type of amnesia that causes this is when you forget anything that has happened since your amnesia began. Even if you have a state of amnesia, you can still recall information you recall before the amnesia occurred. Unlike retrograde amnesia, this occurs more frequently. During an amnesia-inducing event, there is no memory creation after anterograde amnesia occurs. It is possible to suffer from anterograde amnesia either to the extent of being unable to remember events only partially or completely. In this case, a person with amnesia has retained long-term memories from the time before the incident occurred. In anterograde amnesia, new memories cannot be encoded (or possibly retrieved). As well as different severity levels of anterograde amnesia, some individuals forget recent events such as meals or phone numbers, while others forget what they were doing a few seconds ago. Memory is also affected by the difficulty of a task, with more complex tasks being harder to remember than simpler tasks that do not require as much mental energy.
3. Transient global amnesia (TGA) (C):
   It tends to resolve within 24 h if it is a temporary amnesia. Adults over the age of middle age and those who are older are more likely to experience it. It is rare for such amnesia to recur once it has resolved. Someone who is otherwise alert may experience transient global amnesia, which manifests itself suddenly as confusion. A person with transient global amnesia cannot create new memories, so the memory of recent events is lost. This condition is not caused by something more common, such as epilepsy or stroke. Neither you nor how you got here can be recalled. What's going on right now may not be clear to you. The answers you have just been given may not stick in your memory, so you keep repeating the same questions. Similarly, it is possible to lose track of events from a month ago if you are asked to recall them. People in their

middle and older years are most likely to suffer from this condition. Transient global amnesia also leaves you recognizing people you know and remembering who you are. There are always a few hours of recovery time after an episode of transient global amnesia. Your memory may begin to return during recovery. It is not dangerous, but transient global amnesia can still be frightening.

4. Post-traumatic (D):
   Amnesia can occur either anterogradely or retrogradely after an injury to the head. Post-traumatic amnesia is a type of memory loss that occurs immediately after a traumatic brain injury (TBI). This state is characterized by disorientation and inability to remember past events. An individual may be incapable of stating their name, location, and time. It is considered that PTA has been resolved when continuous memory returns. The memory is not able to store new events during PTA. The memory of some incidents is only recalled by one third of patients with mild head injuries. There is a "clouding" of consciousness experienced by the patient during PTA. It has been proposed as an alternative term for PTA since it includes confusion along with the memory loss typically associated with amnesia.
5. Infantile amnesia (E):
   Children often have difficulty recalling early childhood memories, which is referred to as childhood amnesia. The brains of young children are still developing, so they are incapable of consolidating memories. The ailment of being unable to recall episodic memories in adults younger than two to four years of age is known as childhood amnesia. During these years, the recollection of early childhood memories may also be scarce or fragmented, especially if they occurred between the ages of 2 and 6. Others believe that early memories are encoded and stored differently when a cognitive self is developed. The onset of childhood amnesia has differed between psychologists, but some research shows that children can recall things before they are two years old. As children grow, their memories may decline. An individual can recall their first memory at a certain age, according to some definitions. As a general rule, it occurs at the age of two to four, but this can vary from child to child.

The four factors are

1. Seizures (e_1):
   The underlying mechanisms of seizures are poorly understood, which leads to retrograde amnesia. It was determined whether seizures activate neurons that overlap with engrams of spatial memory and if seizures saturate LTP in engram cells. Retrograde amnesia was caused by a seizure for spatial memory tasks. Bilateral mesiotemporal lesions in humans can cause anterograde amnesia, a severely disabling state. An episode of retrograde and/or anterograde amnesia is characteristic of transient epileptic amnesia (TEA). In the event of a traumatic brain injury (TBI), posttraumatic amnesia may result in confusion and memory loss. This period can be characterized by seizures, but they are not common during this period. In general, seizures are more common after a TBI, during the acute phase immediately after the injury. This stage of the brain's development is when significant changes are occurring, and seizure activity is more likely to occur. Depending on the severity of the injury, seizures can also occur during the PTA phase. Infantile amnesia is currently not believed to be caused or contributed to by seizures during the period of infantile amnesia. The consequences of seizures on memory and cognition can be severe, especially if they occur at critical periods in a person's life. The consequences of repeated seizures on the brain include changes in the structure and function of the brain that can lead to mental impairments in the long run.

2. Emotional shock or hysteria (e_2):
   Patients who cannot recall particular past events or those that occur during a particular period of their lives suffer from one type of memory loss. There appears to be no connection between retrograde amnesia and any particular brain disorder, past or present. A triggering event for anterograde amnesia can be emotional shock or hysteria, which prevent the brain from processing or retaining memories of the event. A 'wandering womb' is thought to be responsible for hysteria, which is a set of symptoms common to women. Medical practitioners no longer use it as a diagnosis due to its discreditation. There is no connection between hysteria and PTA, which is caused by physical trauma to the brain. Hysteria and infantile amnesia do not have any direct connection. The terms refer to different phenomena within the brain, though they both involve the functioning of the brain. Hysteria is no longer used as a diagnosis because infantile amnesia is a normal development stage. Discussing mental health and neurological conditions requires accurate and current terminology.
3. Brain cause (e_3):
   Memory-storing areas of the brain in several brain regions can be damaged, resulting in retrograde amnesia. There are numerous different causes of this type of damage, including trauma, serious illnesses, seizures, strokes, and degenerative brain diseases. Alzheimer's disease and frontotemporal dementia are the two conditions that cause anterograde amnesia most often. When your brain deteriorates and stops functioning, memory loss is extremely common. An amnesia that lasts for several hours is known as transient global amnesia (TGA). A temporary disruption of blood flow and oxygen to certain parts of the brain, particularly the hippocampus, is thought to be the cause of TGA, but its exact cause is unknown. Located deep within the brain, the hippocampus is a small seahorse-shaped structure that plays a crucial role in forming new memories as well as retrieving old ones. There are several factors that can interfere with the flow of blood and oxygen to this area. Post-traumatic amnesia (PTA) can occur following trauma to the brain (TBI). When the brain is injured, it can disrupt the normal functioning of the brain cells, known as neurons. Damage to the brain can occur as a result of a direct impact on the head or from shaking the skull. During the processing of information within the brain, neurons are responsible for transmitting information, and damage to them causes the brain to lose function. Some people believe infantile amnesia is caused by the underdevelopment of the infant brain, which would make consolidation of memory impossible, or by memory retrieval deficits.
4. High fever (e_4):
   Memory loss and confusion are common symptoms of a high fever. Symptoms of high fever can include retrograde amnesia, which is the loss of memory of events that occurred before the fever began. High fevers, head trauma, strokes, and other medical conditions can cause retrograde amnesia, which is a condition caused by damage to the brain. As a result of an incident or injury, anterograde amnesia can occur. Confusion, delirium, and memory problems can all be caused by high fever. In contrast, there is no association between anterograde amnesia and this condition. A person experiencing anterograde amnesia is most likely suffering from damage to the brain regions involved in forming and consolidating new memories. Memory problems can occur as a result of high fevers, such as those caused by encephalitis or meningitis. TGA is not known to cause fever, but high fevers are sometimes experienced by people with TGA. It can take from minutes to weeks or even months to recover from a PTA injury, depending on the severity. An individual with a TBI may experience a fever during the acute phase. The body can fight off pathogens and promote healing when it experiences fever due to infection

or injury. In contrast, a high fever or prolonged fever can cause further damage to the brain and other complications if it is very high or prolonged. It is important to monitor infants and young children closely when they have a fever. Medical attention should be sought if the fever has an underlying cause. Infections caused by viruses or bacteria, teething, and immunizations all contribute to fever in infants. Medications such as acetaminophen and ibuprofen may be given to reduce fever, and hydration may be encouraged.

Suppose that five anemia patients as $\mathscr{P} = \{A, B, C, D, E\}$. Four factors are considered as $\gamma = \{e_1, e_2, e_3, e_4\}$ and their weights are $\Psi = \{0.4, 0.3, 0.2, 0.1\}$. First aid treatments should be selected for each alternative.

Step 1: Table 1 represents the DM values.

Table 1: DM values

	e_1	e_2	e_3	e_4
A	$\begin{bmatrix}(0.95, 0.75);\\ {[0.75, 0.95]},\\ {[0.05, 0.25]},\end{bmatrix}$	$\begin{bmatrix}(0.85, 0.8);\\ {[0.7, 0.9]},\\ {[0.1, 0.3]}\end{bmatrix}$	$\begin{bmatrix}(0.7, 0.65);\\ {[0.65, 0.85]},\\ {[0.15, 0.35]}\end{bmatrix}$	$\begin{bmatrix}(0.9, 0.8);\\ {[0.9, 0.95]},\\ {[0.05, 0.1]}\end{bmatrix}$
B	$\begin{bmatrix}(0.8, 0.65);\\ {[0.7, 0.85]},\\ {[0.15, 0.3]},\end{bmatrix}$	$\begin{bmatrix}(0.8, 0.7);\\ {[0.6, 0.7]},\\ {[0.3, 0.4]},\end{bmatrix}$	$\begin{bmatrix}(0.75, 0.7);\\ {[0.7, 0.8]},\\ {[0.2, 0.3]},\end{bmatrix}$	$\begin{bmatrix}(0.8, 0.6);\\ {[0.55, 0.6]},\\ {[0.4, 0.45]}\end{bmatrix}$
C	$\begin{bmatrix}(0.7, 0.6);\\ {[0.75, 0.8]},\\ {[0.2, 0.25]},\end{bmatrix}$	$\begin{bmatrix}(0.9, 0.7);\\ {[0.9, 0.95]},\\ {[0.05, 0.1]},\end{bmatrix}$	$\begin{bmatrix}(0.8, 0.7);\\ {[0.6, 0.65]},\\ {[0.35, 0.4]},\end{bmatrix}$	$\begin{bmatrix}(0.8, 0.65);\\ {[0.75, 0.8]},\\ {[0.2, 0.25]},\end{bmatrix}$
D	$\begin{bmatrix}(0.9, 0.85);\\ {[0.65, 0.8]},\\ {[0.2, 0.35]},\end{bmatrix}$	$\begin{bmatrix}(0.55, 0.5);\\ {[0.65, 0.75]},\\ {[0.25, 0.35]},\end{bmatrix}$	$\begin{bmatrix}(0.9, 0.85);\\ {[0.75, 0.8]},\\ {[0.2, 0.25]},\end{bmatrix}$	$\begin{bmatrix}(0.9, 0.75);\\ {[0.85, 0.9]},\\ {[0.1, 0.15]},\end{bmatrix}$
E	$\begin{bmatrix}(0.85, 0.75);\\ {[0.75, 0.9]},\\ {[0.1, 0.25]},\end{bmatrix}$	$\begin{bmatrix}(0.6, 0.55);\\ {[0.8, 0.85]},\\ {[0.15, 0.2]},\end{bmatrix}$	$\begin{bmatrix}(0.8, 0.75);\\ {[0.8, 0.9]},\\ {[0.1, 0.2]},\end{bmatrix}$	$\begin{bmatrix}(0.7, 0.65);\\ {[0.8, 0.85]},\\ {[0.15, 0.2]},\end{bmatrix}$

Step 2: Obtain normalized decision matrix: Table 2 represents the normalized decision values.

Step 3: For every alternative ($q = 1$), the aggregated information will be derived using log q-rung NVWA operators. Table 3 represents the weighted averaging values.

Step 4: Both of the ideal values for all alternative are as follows:

$$\mathscr{P}^{+} = \left[(0.9389, 0.6563); [1, 1], [0, 0]\right]$$

and

$$\mathscr{P}^{-} = \left[(0.8135, 0.8152); [0, 0], [1, 1]\right].$$

Table 2: Normalized decision values

	e_1	e_2	e_3	e_4
A	$\begin{bmatrix} (1, 0.6966); \\ 0.75, 0.95], \\ [0.05, 0.25], \end{bmatrix}$	$\begin{bmatrix} (0.9444, 0.9412); \\ [0.7, 0.9], \\ [0.1, 0.3] \end{bmatrix}$	$\begin{bmatrix} (0.7778, 0.7101); \\ [0.65, 0.85], \\ [0.15, 0.35] \end{bmatrix}$	$\begin{bmatrix} (1, 0.8889); \\ [0.9, 0.95], \\ [0.05, 0.1] \end{bmatrix}$
B	$\begin{bmatrix} (0.8421, 0.6213); \\ [0.7, 0.85], \\ [0.15, 0.3], \end{bmatrix}$	$\begin{bmatrix} (0.8889, 0.7656); \\ [0.6, 0.7], \\ [0.3, 0.4], \end{bmatrix}$	$\begin{bmatrix} (0.8333, 0.7686); \\ [0.7, 0.8], \\ [0.2, 0.3], \end{bmatrix}$	$\begin{bmatrix} (0.8889, 0.5625); \\ [0.55, 0.6], \\ [0.4, 0.45] \end{bmatrix}$
C	$\begin{bmatrix} (0.7368, 0.605); \\ [0.75, 0.8], \\ [0.2, 0.25], \end{bmatrix}$	$\begin{bmatrix} (1, 0.6806); \\ [0.9, 0.95], \\ [0.05, 0.1], \end{bmatrix}$	$\begin{bmatrix} (0.8889, 0.7206); \\ [0.6, 0.65], \\ [0.35, 0.4], \end{bmatrix}$	$\begin{bmatrix} (0.8889, 0.6602); \\ [0.75, 0.8], \\ [0.2, 0.25], \end{bmatrix}$
D	$\begin{bmatrix} (0.9474, 0.9444); \\ [0.65, 0.8], \\ [0.2, 0.35], \end{bmatrix}$	$\begin{bmatrix} (0.6111, 0.5682); \\ [0.65, 0.75], \\ [0.25, 0.35], \end{bmatrix}$	$\begin{bmatrix} (1, 0.9444); \\ [0.75, 0.8], \\ [0.2, 0.25], \end{bmatrix}$	$\begin{bmatrix} (1, 0.7813); \\ [0.85, 0.9], \\ [0.1, 0.15], \end{bmatrix}$
E	$\begin{bmatrix} (0.8947, 0.7785); \\ [0.75, 0.9], \\ [0.1, 0.25], \end{bmatrix}$	$\begin{bmatrix} (0.6667, 0.6302); \\ [0.8, 0.85], \\ [0.15, 0.2], \end{bmatrix}$	$\begin{bmatrix} (0.8889, 0.8272); \\ [0.8, 0.9], \\ [0.1, 0.2], \end{bmatrix}$	$\begin{bmatrix} (0.7778, 0.7545); \\ [0.8, 0.85], \\ [0.15, 0.2], \end{bmatrix}$

Table 3: Weighted averaging values

A	B	C	D	E
$\begin{bmatrix} (0.9389, 0.7919); \\ [0.2737, 0.2520], \\ [0.2479, 0.2225] \end{bmatrix}$	$\begin{bmatrix} (0.8591, 0.6882); \\ [0.2360, 0.2193], \\ [0.2647, 0.2582] \end{bmatrix}$	$\begin{bmatrix} (0.8614, 0.6563); \\ [0.2426, 0.2436], \\ [0.2451, 0.2480] \end{bmatrix}$	$\begin{bmatrix} (0.8623, 0.8152); \\ [0.2892, 0.2773], \\ [0.2309, 0.2187] \end{bmatrix}$	$\begin{bmatrix} (0.8135, 0.7414); \\ [0.2609, 0.2412], \\ [0.2537, 0.2440] \end{bmatrix}$

Step 5: The HD between for all alternative with a different ideal value is as follows:

$\Upsilon_1^+ = 0.5258, \Upsilon_2^+ = 0.5310, \Upsilon_3^+ = 0.5562, \Upsilon_4^+ = 0.5795, \Upsilon_5^+ = 0.5707,$

and

$\Upsilon_1^- = 0.5247, \Upsilon_2^- = 0.5179, \Upsilon_3^- = 0.4917, \Upsilon_4^- = 0.4751, \Upsilon_5^- = 0.4824.$

Step 6: Relative closeness values are

$\Upsilon_1^* = 0.4995, \Upsilon_2^* = 0.4937, \Upsilon_3^* = 0.4692, \Upsilon_4^* = 0.4505, \Upsilon_5^* = 0.4581.$

Step 7: Ranking of alternatives are

$A > B > C > E > D.$

Therefore A is a very urgent need for treatment.

6.3 Comparison of the Suggested Method with Existing Methods

Here, we demonstrate the effectiveness and superiority of new methods by comparing them with existing methods, using practical examples to analyze the results of the new proposed method. A Pythagorean neutrosophic interval valued weighted averaging using AOs was constructed by Yang et al. [26]. The following subsection is devoted to reviewing a few existing models and comparing them with the suggested models suggested in this section. In this way, it is demonstrated that it is valuable and advantageous. Based on ED and HD and score values, we calculate the log q-rung NVWA, log q-rung NVWG, log G q-rung NVWA, and log G q-rung NVWG. A list of the various distances is provided below: Tables 4 and 5 represent the comparison for the proposed and existing values.

Table 4: Proposed values

$q = 1$	log q-rung NVWA	log q-rung NVWG	log G q-rung NVWA	log G q-rung NVWG
ED	$A > B > C$	$A > C > B$	$A > B > C$	$A > C > B$
	$E > D$	$E > D$	$E > D$	$E > D$
HD	$A > B > C$	$A > C > B$	$A > B > C$	$A > C > B$
	$E > D$	$E > D$	$E > D$	$E > D$
Score	$A > E > D$	$A > E > D$	$A > E > D$	$A > E > D$
	$C > B$	$C > B$	$C > B$	$C > B$

Table 5: Existing values

$q = 1$	*IVPNVWA*	*IVPNVWG*	*GIVPNVWA*	*GIVPNVWG*
ED [26]	$A > C > E$	$A > E > C$	$A > C > E$	$A > E > C$
	$D > B$	$D > B$	$D > B$	$D > B$
HD [26]	$A > E > C$	$A > E > C$	$A > E > C$	$A > E > C$
	$D > B$	$D > B$	$D > B$	$D > B$
Score [26]	$A > E > D$	$A > E > D$	$A > E > D$	$A > E > D$
	$C > B$	$C > B$	$C > B$	$C > B$

As shown in Fig. 2, proposed and existing models are compared for ED.

As shown in Fig. 3, proposed and existing models are compared for HD.

MADM is compared to competing approaches for its merits. A log q-rung NVWA technique is used to derive the various values. Use a log q-rung NVWA operator to generate data for alternatives ($q = 2$).

Step 1: The aggregated data for each alternative is based on the log q-rung NVWA operators ($q = 2$). Table 6 represents log q-rung NVWA ($q = 2$).

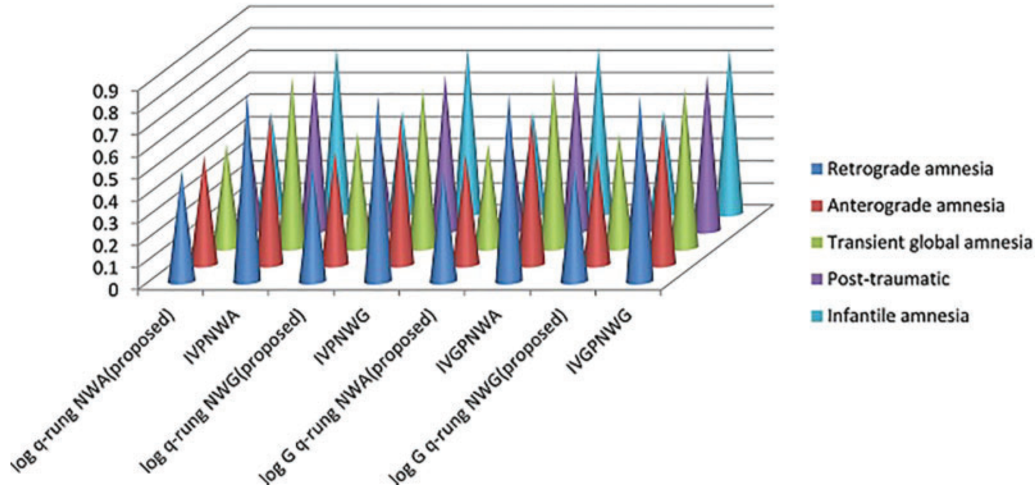

Figure 2: Different Euclidean distance

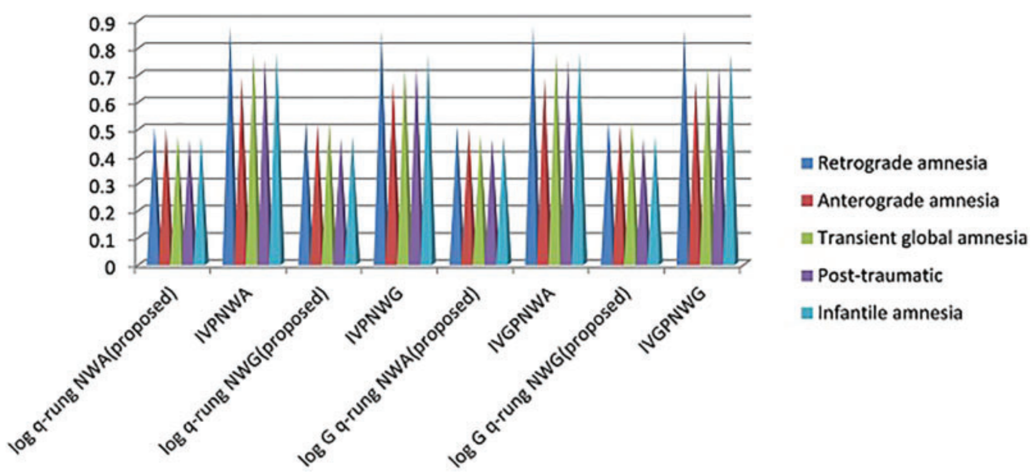

Figure 3: Different Hamming distance

Table 6: The values of q = 2

A	B	C	D	E
$\begin{bmatrix}(0.9389, 0.7919);\\ [0.2815, 0.2735],\\ [0.2479, 0.2225]\end{bmatrix}$	$\begin{bmatrix}(0.8591, 0.6882);\\ [0.2401, 0.2349],\\ [0.2647, 0.2582]\end{bmatrix}$	$\begin{bmatrix}(0.8614, 0.6563);\\ [0.2653, 0.2759],\\ [0.2451, 0.2480]\end{bmatrix}$	$\begin{bmatrix}(0.8623, 0.8152);\\ [0.2954, 0.2823],\\ [0.2309, 0.2187]\end{bmatrix}$	$\begin{bmatrix}(0.8135, 0.7414);\\ [0.2625, 0.2457],\\ [0.2537, 0.2440]\end{bmatrix}$

Step 2: In each alternative, the both ideal values are as follows:

$\mathscr{P}^{+} = \left[(0.9389, 0.6563); [1, 1], [0, 0]\right]$

and

$\mathscr{P}^{-} = \left[(0.8135, 0.8152); [0, 0], [1, 1]\right].$

Step 3: EDs between for all alternative with different ideal values are

$\Upsilon_1^+ = 0.5370, \Upsilon_2^+ = 0.5382, \Upsilon_3^+ = 0.5754, \Upsilon_4^+ = 0.5835, \Upsilon_5^+ = 0.5728,$

and

$\Upsilon_1^- = 0.5134, \Upsilon_2^- = 0.5107, \Upsilon_3^- = 0.4724, \Upsilon_4^- = 0.4711, \Upsilon_5^- = 0.4802.$

Step 4: Relative closeness values are

$\Upsilon_1^* = 0.4888, \Upsilon_2^* = 0.4869, \Upsilon_3^* = 0.4508, \Upsilon_4^* = 0.4467, \Upsilon_5^* = 0.4560.$

Step 5: Ranking of alternatives are

$A > B > E > C > D.$

Step 1: The aggregated data for each alternative is based on the log q-rung NVWA operators ($q = 3$).

Table 7 represents log q-rung NVWA ($q = 3$).

Table 7: The values of $q = 3$

A	B	C	D	E
$\begin{bmatrix}(0.9389, 0.7919);\\ [0.2883, 0.2948],\\ [0.2479, 0.2225]\end{bmatrix}$	$\begin{bmatrix}(0.8591, 0.6882);\\ [0.2449, 0.2514],\\ [0.2647, 0.2582]\end{bmatrix}$	$\begin{bmatrix}(0.8614, 0.6563);\\ [0.2861, 0.3026],\\ [0.2451, 0.2480]\end{bmatrix}$	$\begin{bmatrix}(0.8623, 0.8152);\\ [0.3007, 0.2869],\\ [0.2309, 0.2187]\end{bmatrix}$	$\begin{bmatrix}(0.8135, 0.7414);\\ [0.2644, 0.2508],\\ [0.2537, 0.2440]\end{bmatrix}$

Step 2: In each alternative, the both ideal values are as follows:

$\mathscr{P}^{+} = \left[(0.9389, 0.6563); [1, 1], [0, 0]\right]$

and

$\mathscr{P}^{-} = \left[(0.8135, 0.8152); [0, 0], [1, 1]\right].$

Step 3: EDs between for all alternative with different ideal values are

$\Upsilon_1^+ = 0.5476, \Upsilon_2^+ = 0.5458, \Upsilon_3^+ = 0.5916, \Upsilon_4^+ = 0.5869, \Upsilon_5^+ = 0.5752,$

and

$\Upsilon_1^- = 0.5028, \Upsilon_2^- = 0.5031, \Upsilon_3^- = 0.4562, \Upsilon_4^- = 0.4675, \Upsilon_5^- = 0.4778.$

Step 4: Relative closeness values are

$\Upsilon_1^* = 0.4787, \Upsilon_2^* = 0.4796, \Upsilon_3^* = 0.4354, \Upsilon_4^* = 0.4434, \Upsilon_5^* = 0.4537.$

Step 5: Ranking of alternatives are

$B > A > E > D > C.$

Fig. 4 shows that different q values for log q-rung NVWA.

6.4 Critical Analysis

On the basis of the log q-rung NVWA method, the alternative would rank as follows: $A > C > E > D > B$. Assuming that $q = 2$, then the ranking of alternatives would be $A > B > E > C > C$. In this case, $q = 3$, thus the ranking would be $B > A > E > D > C$. Due to this, the patient is in need of treatment B rather than A. In a similar way, log q-rung NVWGs, log G q-rung NVWAs, and log G q-rung NVWGs can be used.

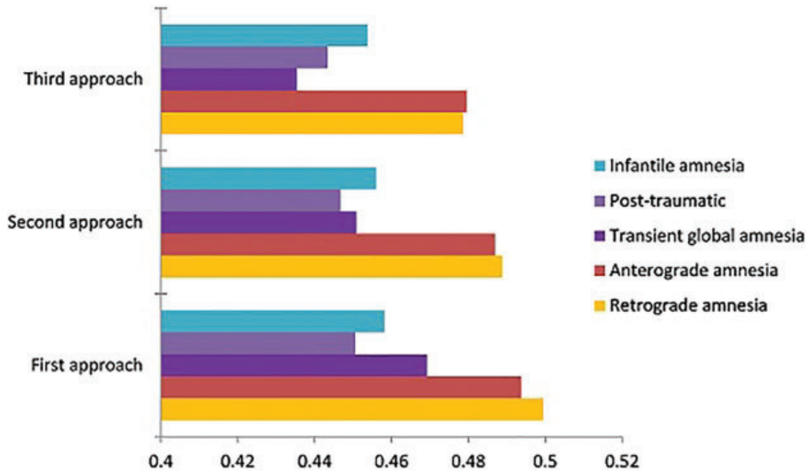

Figure 4: Different q values

6.5 Advantages

According to the study previously presented, the applications have numerous advantages. Our research presents the concept of VS and combines it with the concept of q-rung FS to develop a log q-rung VS. The log q-rung NVN analyzes human behaviors and natural events that follow a normal distribution in the real world. The total of its TMG, IMG and FMG exceeds one, but the square sum of those three is less than 1, and so on. A decision maker provides a number of options based on which the proposed log q-rung NVS is used to find the best alternative. As a result, the proposed MADM method that uses log q-rung NVS is another way to find the most effective DM alternative. The outcome of the alternatives based on q. Results of all alternatives obtained using log q-rung NVWAs, log q-rung NVWGs, and log G q-rung NVWAs.

7 Conclusion

This method is effective due to its ability to consider relationships between attributes. As a result, the proposed method produces more accurate ranking results. Considering the interrelationships between attributes, the proposed method is more efficient and superior to [26] in solving practical DM problems. In this article, we examined problems arising within DM domains using log q-rung NVS and MADM. Based on our discussion of log q-rung NVS, several AO reached a number of conclusions that were important to their log q-rung NVS. There should be a log q-rung NVWA and log q-rung NVWG, as well as a log G q-rung NVWA and log G q-rung NVWG. By applying log q-rung NVS based on the MADM methodology, individuals may be able to determine the appropriate action to take in scenarios with unclear and contradictory facts. We apply the operator representations of log q-rung NVWAs, log q-rung NVWGs, log G q-rung NVWAs and log G q-rung NVWGs to problems based on log q-rung NVS. We can estimate the different rankings using log q-rung NVWA, log q-rung NVWG, log G q-rung NVWA, and log G q-rung NVWG. As a final step, we have examined the values of q that affect alternative ranking most strongly. A decision-maker can select the most appropriate ranking based on a real-world scenario by adjusting q. Based on the actual values of q, the decision-maker can select a method. Finally, we compared the proposed models to a number of currently used models in order to demonstrate their applicability and benefits. In data analysis, HD and ED of neutrosophic sets are used in a number of practical applications. If further research shows that these operators are superior to others, such as power mean aggregation operators, Bonferroni mean operators, Heronian mean operators, etc., we may be able to extend new q-rung complex neutrosophic set to them. The following topics will be discussed in further detail:

(1) It is shown that expert sets and soft sets can be compared with log q-rung NVSs.

(2) The cubic NVS and spherical NVS are investigated on the basis of log q-rung NVSs.

(3) To solve problems with a generalized Fermatean NVS and a complex NVS.

Acknowledgement: Seifedine Kadry would like to thank Dr. Yunyoung Nam for his effort.

Funding Statement: This work was supported by the National Research Foundation of Korea (NRF) Grant funded by the Korea government (MSIT) (No. RS-2023-00218176), Korea Institute for Advancement of Technology (KIAT) Grant funded by the Korea government (MOTIE) (P0012724, The Competency Development Program for Industry Specialist) and the Soonchunhyang University Research Fund.

Author Contributions: The authors confirm contribution to the paper as follows: Conceptualization and methodology: M. Palanikumar; Writing original draft: Nasreen Kausar; Conceptualization: Dragan Pamucar and Chomyong Kim; Validation: Yunyoung Nam; Review and editing: S. Kadry.

Availability of Data and Materials: This statement should make clear how readers can access the data used in the study and explain why any unavailable data cannot be released.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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Appendix

Proof of the Theorem 4.1

Proof. It is clear that (1) and (2) can be proven. There is only one proof we provide for the last statement (3). Now,

$$\left(\Upsilon_E(M_1, M_2) + \Upsilon_E(M_2, M_3)\right)^2$$

$$
= \left[\begin{array}{l}
\dfrac{1}{2}\sqrt{\begin{array}{l}\left[\begin{array}{l}\dfrac{(\log_{\Omega_i} M_1^T)^2 + 1 - (\log_{\Omega_i} M_1^F)^2 + (\log_{\Omega_i}(M_1^{1-F}))^2 + 1 - (\log_{\Omega_i}(M_1{}^{1-T}))^2}{2}\chi_1 \\ -\dfrac{(\log_{\Omega_i} M_2^T)^2 + 1 - (\log_{\Omega_i} M_2^F)^2 + (\log_{\Omega_i}(M_2^{1-F}))^2 + 1 - (\log_{\Omega_i}(M_2{}^{1-T}))^2}{2}\chi_2\end{array}\right]^2 \\ +\dfrac{1}{2}\left[\begin{array}{l}\dfrac{(\log_{\Omega_i} M_1^T)^2 + 1 - (\log_{\Omega_i} M_1^F)^2 + (\log_{\Omega_i}(M_1^{1-F}))^2 + 1 - (\log_{\Omega_i}(M_1{}^{1-T}))^2}{2}\psi_1 \\ -\dfrac{(\log_{\Omega_i} M_2^T)^2 + 1 - (\log_{\Omega_i} M_2^F)^2 + (\log_{\Omega_i}(M_2^{1-F}))^2 + 1 - (\log_{\Omega_i}(M_2{}^{1-T}))^2}{2}\psi_2\end{array}\right]^2\end{array}} \\
+\dfrac{1}{2}\sqrt{\begin{array}{l}\left[\begin{array}{l}\dfrac{(\log_{\Omega_i} M_2^T)^2 + 1 - (\log_{\Omega_i} M_2^F)^2 + (\log_{\Omega_i}(M_2^{1-F}))^2 + 1 - (\log_{\Omega_i}(M_2{}^{1-T}))^2}{2}\chi_2 \\ -\dfrac{(\log_{\Omega_i} M_3^T)^2 + 1 - (\log_{\Omega_i} M_3^F)^2 + (\log_{\Omega_i}(M_3{}^{1-F}))^2 + 1 - (\log_{\Omega_i}(M_T{}^{1-F}))^2}{2}\chi_3\end{array}\right]^2 \\ +\dfrac{1}{2}\left[\begin{array}{l}\dfrac{(\log_{\Omega_i} M_2^T)^2 + 1 - (\log_{\Omega_i} M_2^F)^2 + (\log_{\Omega_i}(M_2^{1-F}))^2 + 1 - (\log_{\Omega_i}(M_2{}^{1-T}))^2}{2}\psi_2 \\ -\dfrac{(\log_{\Omega_i} M_3^T)^2 + 1 - (\log_{\Omega_i} M_3^F)^2 + (\log_{\Omega_i}(M_3{}^{1-F}))^2 + 1 - (\log_{\Omega_i}(M_T{}^{1-F}))^2}{2}\psi_3\end{array}\right]^2\end{array}}
\end{array} \right]^2
$$

$$
\begin{aligned}
&= \frac{1}{4}\left((\tau_1\chi_1 - \tau_2\chi_2)^2 + \frac{1}{2}(\tau_1\psi_1 - \tau_2\psi_2)^2\right) + \frac{1}{4}\left((\tau_2\chi_2 - \tau_3\chi_3)^2 + \frac{1}{2}(\tau_2\psi_2 - \tau_3\psi_3)^2\right) \\
&\quad + \frac{1}{2}\left(\sqrt{[(\tau_1\chi_1 - \tau_2\chi_2)^2 + \frac{1}{2}(\tau_1\psi_1 - \tau_2\psi_2)^2]} \times \sqrt{(\tau_2\chi_2 - \tau_3\chi_3)^2 + \frac{1}{2}(\tau_2\psi_2 - \tau_3\psi_3)^2}\right) \\
&\succeq \frac{1}{2}(\tau_1\chi_1 - \tau_2\chi_2 + \tau_2\chi_2 - \tau_3\chi_3)^2 + \frac{1}{4}(\tau_1\psi_1 - \tau_2\psi_2 + \tau_2\psi_2 - \tau_3\psi_3)^2 \\
&= \frac{1}{2}(\tau_1\chi_1 - \tau_3\chi_3)^2 + \frac{1}{4}(\tau_1\psi_1 - \tau_3\psi_3)^2 \\
&= \Upsilon_E(M_1, M_3)^2.
\end{aligned}
$$

where

$$\tau_1 = \frac{1 + (\log_{\Omega_i} M_1^T)^2 - (\log_{\Omega_i} M_1^F)^2 + 1 + (\log_{\Omega_i}(M_1^{1-F}))^2 - (\log_{\Omega_i}(M_1{}^{1-T}))^2}{2},$$

$$\tau_2 = \frac{1 + (\log_{\Omega_i} M_2^T)^2 - (\log_{\Omega_i} M_2^F)^2 + 1 + (\log_{\Omega_i}(M_2^{1-F}))^2 - (\log_{\Omega_i}(M_2{}^{1-T}))^2}{2},$$

$$\tau_3 = \frac{1 + (\log_{\Omega_i} M_3^T)^2 - (\log_{\Omega_i} M_3^F)^2 + 1 + (\log_{\Omega_i}(M_3{}^{1-F}))^2 - (\log_{\Omega_i}(M_T{}^{1-F}))^2}{2}.$$

Proof of the Theorem 5.1

Proof. This theorem is proven by using the induction method. If $n = 2$, then log q-rung NVWA $(M_1, M_2) = \Psi_1 M_1 \bigoplus \Psi_2 M_2$, where

$$\Psi_1 M_1 = \begin{bmatrix} \left(\Psi_1\chi_1, \Psi_1\psi_1\right); \\ \left[\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_1^T)^q\right)^{\Psi_1}}, \sqrt[q]{1-\left(1-(\log_{\Omega_i}(M_1^{1-F}))^q\right)^{\Psi_1}}\right], \\ \left[(\log_{\Omega_i} M_1^F)^{\Psi_1}, (\log_{\Omega_i}({M_1}^{1-T}))^{\Psi_1}\right] \end{bmatrix}$$

and

$$\Psi_2 M_2 = \begin{bmatrix} \left(\Psi_2\chi_2, \Psi_2\psi_2\right); \\ \left[\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_2^T)^q\right)^{\Psi_2}}, \sqrt[q]{1-\left(1-(\log_{\Omega_i}(M_2^{1-F}))^q\right)^{\Psi_2}}\right], \\ \left[(\log_{\Omega_i} M_2^F)^{\Psi_2}, (\log_{\Omega_i}({M_2}^{1-T}))^{\Psi_2}\right] \end{bmatrix}.$$

Hence,

$$\Psi_1 M_1 \bigoplus \Psi_2 M_2 = \begin{bmatrix} \left(\Psi_1\chi_1 + \Psi_2\chi_2, \Psi_1\psi_1 + \Psi_2\psi_2\right); \\ \left[\begin{array}{c} \sqrt[q]{\begin{array}{c}\left(1-\left(1-(\log_{\Omega_i} M_1^T)^q\right)^{\Psi_1}\right) + \left(1-\left(1-(\log_{\Omega_i} M_2^T)^q\right)^{\Psi_2}\right) \\ -\left(1-\left(1-(\log_{\Omega_i} M_1^T)^q\right)^{\Psi_1}\right)\cdot\left(1-\left(1-(\log_{\Omega_i} M_2^T)^q\right)^{\Psi_2}\right)\end{array}}, \\ \sqrt[q]{\begin{array}{c}\left(1-\left(1-(\log_{\Omega_i}(M_1^{1-F}))^q\right)^{\Psi_1}\right) + \left(1-\left(1-(\log_{\Omega_i}(M_2^{1-F}))^q\right)^{\Psi_2}\right) \\ -\left(1-\left(1-(\log_{\Omega_i}(M_1^{1-F}))^q\right)^{\Psi_1}\right)\cdot\left(1-\left(1-(\log_{\Omega_i}(M_2^{1-F}))^q\right)^{\Psi_2}\right)\end{array}} \end{array}\right], \\ \left[(\log_{\Omega_i} M_1^F)^{\Psi_1}\cdot(\log_{\Omega_i} M_2^F)^{\Psi_2}, (\log_{\Omega_i}({M_1}^{1-T}))^{\Psi_1}\cdot(\log_{\Omega_i}({M_2}^{1-T}))^{\Psi_2}\right] \end{bmatrix}$$

$$= \begin{bmatrix} \left(\Psi_1\chi_1 + \Psi_2\chi_2, \Psi_1\psi_1 + \Psi_2\psi_2\right); \\ \left[\begin{array}{c} \sqrt[q]{1-\left(1-(\log_{\Omega_i} M_1^T)^q\right)^{\Psi_1}\cdot\left(1-(\log_{\Omega_i} M_2^T)^q\right)^{\Psi_2}}, \\ \sqrt[q]{1-\left(1-(\log_{\Omega_i}(M_1^{1-F}))^q\right)^{\Psi_1}\cdot\left(1-(\log_{\Omega_i}(M_2^{1-F}))^q\right)^{\Psi_2}} \end{array}\right], \\ \left[(\log_{\Omega_i} M_1^F)^{\Psi_1}\cdot(\log_{\Omega_i} M_2^F)^{\Psi_2}, (\log_{\Omega_i}({M_1}^{1-T}))^{\Psi_1}\cdot(\log_{\Omega_i}({M_2}^{1-T}))^{\Psi_2}\right] \end{bmatrix}.$$

Thus, log q-rung NVWA $(M_1, M_2) =$

$$\begin{bmatrix} \left(\uplus_{i=1}^2 \Psi_i\chi_i, \uplus_{i=1}^2 \Psi_i\psi_i\right); \\ \left[\sqrt[q]{1-\bigcirc_{i=1}^2\left(1-(\log_{\Omega_i} {M_i}^T)^q\right)^{\Psi_i}}, \sqrt[q]{1-\bigcirc_{i=1}^2\left(1-(\log_{\Omega_i} M_i^{1-F}))^q\right)^{\Psi_i}}\right], \\ \left[\bigcirc_{i=1}^2 (\log_{\Omega_i} {M_i}^F)^{\Psi_i}, \bigcirc_{i=1}^2(\log_{\Omega_i} M_i^{1-T}))^{\Psi_i}\right] \end{bmatrix}.$$

It valid for $n = l$ and $l \succeq 3$. Hence, log q-rung NVWA $(M_1, M_2, \ldots, M_l) =$

$$\left[\begin{array}{c} \left(\uplus_{i=1}^{l} \Psi_i \chi_i, \uplus_{i=1}^{l} \Psi_i \psi_i\right); \\ \left[\sqrt[q]{1 - \bigcirc_{i=1}^{l}\left(1 - (\log_{\Omega_i} M_i^{T})^q\right)^{\Psi_i}}, \sqrt[q]{1 - \bigcirc_{i=1}^{l}\left(1 - (\log_{\Omega_i} M_i^{1-F}))^q\right)^{\Psi_i}}\right], \\ \left[\bigcirc_{i=1}^{l} (\log_{\Omega_i} M_i^{F})^{\Psi_i}, \bigcirc_{i=1}^{l} (\log_{\Omega_i} M_i^{1-T}))^{\Psi_i}\right] \end{array}\right].$$

If $n = l + 1$, then log q-rung NVWA $(M_1, M_2, ..., M_l, M_{l+1})$

$$= \left[\begin{array}{c} \left(\uplus_{i=1}^{l} \Psi_i \chi_i + \Psi_{l+1}\Omega_{l+1}, \uplus_{i=1}^{l} \Psi_i \psi_i + \Psi_{l+1}\psi_{l+1}\right); \\ \left[\begin{array}{c} \sqrt[q]{\begin{array}{c}\uplus_{i=1}^{l}\left(1 - \left(1 - (\log_{\Omega_i} M_i^{T})^q\right)^{\Psi_i}\right) + \left(1 - \left(1 - (\log_{\Omega_i} {M_{l+1}}^{T})^q\right)^{\Psi_{l+1}}\right) \\ - \bigcirc_{i=1}^{l}\left(1 - \left(1 - (\log_{\Omega_i} M_i^{T})^q\right)^{\Psi_i}\right) \cdot \left(1 - \left(1 - (\log_{\Omega_i} {M_{l+1}}^{T})^q\right)^{\Psi_{l+1}}\right)\end{array}}, \\ \sqrt[q]{\begin{array}{c}\uplus_{i=1}^{l}\left(1 - \left(1 - (\log_{\Omega_i} M_i^{1-F}))^q\right)^{\Psi_i}\right) + \left(1 - \left(1 - (\log_{\Omega_i} ({M_{l+1}}^{1-F}))^q\right)^{\Psi_{l+1}}\right) \\ - \bigcirc_{i=1}^{l}\left(1 - \left(1 - (\log_{\Omega_i} M_i^{1-F}))^q\right)^{\Psi_i}\right) \cdot \left(1 - \left(1 - (\log_{\Omega_i} ({M_{l+1}}^{1-F}))^q\right)^{\Psi_{l+1}}\right)\end{array}} \end{array}\right], \\ \left[\bigcirc_{i=1}^{l} (\log_{\Omega_i} M_i^{F})^{\Psi_i} \cdot (\log_{\Omega_i} M_{l+1}^{F})^{\Psi_{l+1}}, \bigcirc_{i=1}^{l} (\log_{\Omega_i} M_i^{1-T}))^{\Psi_i} \cdot (\log_{\Omega_i} ({M_{l+1}}^{1-T}))^{\Psi_{l+1}}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left(\uplus_{i=1}^{l+1} \Psi_i \chi_i, \uplus_{i=1}^{l+1} \Psi_i \psi_i\right); \\ \left[\sqrt[q]{1 - \bigcirc_{i=1}^{l+1}\left(1 - (\log_{\Omega_i} M_i^{T})^q\right)^{\Psi_i}}, \sqrt[q]{1 - \bigcirc_{i=1}^{l+1}\left(1 - (\log_{\Omega_i} M_i^{1-F}))^q\right)^{\Psi_i}}\right], \\ \left[\bigcirc_{i=1}^{l+1} (\log_{\Omega_i} M_i^{F})^{\Psi_i}, \bigcirc_{i=1}^{l+1} (\log_{\Omega_i} M_i^{1-T}))^{\Psi_i}\right] \end{array}\right].$$

Proof of the Theorem 5.2

Proof. Given that $(\chi_i, \psi_i) = (\chi, \psi)$, $[\log M_i^{T}, \log M_i^{1-F})] = [\log M^{T}, \log (M^{1-F})]$, and $[\log M_i^{F}, \log M_i^{1-T})] = [\log M^{F}, \log (M^{1-T})]$ and $\uplus_{i=1}^{n} \Psi_i = 1$. Now, log q-rung NVWA $(M_1, M_2, ..., M_n)$

$$= \left[\begin{array}{c} \left(\uplus_{i=1}^{n} \Psi_i \chi_i, \uplus_{i=1}^{n} \Psi_i \psi_i\right); \\ \left[\sqrt[q]{1 - \bigcirc_{i=1}^{n}\left(1 - (\log_{\Omega_i} M_i^{T})^q\right)^{\Psi_i}}, \sqrt[q]{1 - \bigcirc_{i=1}^{n}\left(1 - (\log_{\Omega_i} M_i^{1-F}))^q\right)^{\Psi_i}}\right], \\ \left[\bigcirc_{i=1}^{n} (\log_{\Omega_i} M_i^{F})^{\Psi_i}, \bigcirc_{i=1}^{n} (\log_{\Omega_i} M_i^{1-T}))^{\Psi_i}\right] \end{array}\right],$$

$$= \left[\begin{array}{c} \left(\chi \uplus_{i=1}^{n} \Psi_i, \psi \uplus_{i=1}^{n} \Psi_i\right); \\ \left[\sqrt[q]{1 - \left(1 - (\log_{\Omega_i} M^{T})^q\right)^{\uplus_{i=1}^{n}\Psi_i}}, \sqrt[q]{1\left(1 - (\log_{\Omega_i} (M^{1-F}))^q\right)^{\uplus_{i=1}^{n}\Psi_i}}\right], \\ \left[\log_{\Omega_i} M^{F})^{\uplus_{i=1}^{n}\Psi_i}, (\log_{\Omega_i} (M^{1-T}))^{\uplus_{i=1}^{n}\Psi_i}\right] \end{array}\right],$$

$$= \left[\begin{array}{c}(\chi,\psi);\\ \left[\sqrt[q]{1-\left(1-(\log_{\Omega_i} M^T)^q\right)}, \sqrt[q]{1-\left(1-(\log_{\Omega_i}(M^{1-F}))^q\right)}\right],\\ \left[(\log_{\Omega_i} M^F),(\log_{\Omega_i}(M^{1-T}))\right]\end{array}\right]$$
$$= M.$$

Proof of the Theorem 5.3

Proof. Since, $\underbrace{\log_{\Omega_i} M^T} = \min \log_{\Omega_i} {M_{ij}}^T, \overbrace{\log_{\Omega_i} M^T} = \max \log_{\Omega_i} {M_{ij}}^T$ $\underbrace{\log_{\Omega_i}(M^{1-F})} = \min \log_{\Omega_i}({M_{ij}}^{1-F})$, $\overbrace{\log_{\Omega_i}(M^{1-F})} = \max \log_{\Omega_i}({M_{ij}}^{1-F})$ and $\underbrace{\log_{\Omega_i} M^T} \preceq \log_{\Omega_i} {M_{ij}}^T \preceq \overbrace{\log_{\Omega_i} M^T}$ and $\underbrace{\log_{\Omega_i}(M^{1-F})} \preceq \log_{\Omega_i}(M^{1-F})_{ij} \preceq \overbrace{\log_{\Omega_i}(M^{1-F})}$. Now, $\underbrace{\log_{\Omega_i} M^T} + \underbrace{\log_{\Omega_i}(M^{1-F})}$

$$\begin{aligned}
&= \sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\underbrace{\log_{\Omega_i} M^T})^q\right)^{\Psi_i}} + \sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\underbrace{\log_{\Omega_i}(M^{1-F})})^q\right)^{\Psi_i}}\\
&\preceq \sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Omega_i} {M_{ij}}^T)^q\right)^{\Psi_i}} + \sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Omega_i}({M_{ij}}^{1-F}))^q\right)^{\Psi_i}}\\
&\preceq \sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\overbrace{\log_{\Omega_i} M^T})^q\right)^{\Psi_i}} + \sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\overbrace{\log_{\Omega_i}(M^{1-F})})^q\right)^{\Psi_i}}\\
&= \overbrace{\log_{\Omega_i} M^T} + \overbrace{\log_{\Omega_i}(M^{1-F})}.
\end{aligned}$$

Since, $\underbrace{\log_{\Omega_i} M^F} = \min \log_{\Omega_i} M_{ij}^F$, $\overbrace{\log_{\Omega_i} M^F} = \max \log_{\Omega_i} M_{ij}^F$ $\underbrace{\log_{\Omega_i}(M^{1-T})} = \min \log_{\Omega_i}({M_{ij}}^{1-T})$, $\overbrace{\log_{\Omega_i}(M^{1-T})} = \max \log_{\Omega_i}({M_{ij}}^{1-T})$ and $\underbrace{\log_{\Omega_i} M^F} \preceq \log_{\Omega_i} M_{ij}^F \preceq \overbrace{\log_{\Omega_i} M^F}$ and $\underbrace{\log_{\Omega_i}(M^{1-T})} \preceq \log_{\Omega_i}({M_{ij}}^{1-T}) \preceq \overbrace{\log_{\Omega_i}(M^{1-T})}$. Now,

$$\begin{aligned}
\underbrace{\log_{\Omega_i} M^F} + \underbrace{\log_{\Omega_i}(M^{1-T})} &= \bigcirc_{i=1}^{n}(\underbrace{\log_{\Omega_i} M^F})^{\Psi_i} + \bigcirc_{i=1}^{n}(\underbrace{\log_{\Omega_i}(M^{1-T})})^{\Psi_i}\\
&\preceq \bigcirc_{i=1}^{n}(\log_{\Omega_i} M_{ij}^F)^{\Psi_i} + \bigcirc_{i=1}^{n}(\log_{\Omega_i}({M_{ij}}^{1-T}))^{\Psi_i}\\
&\preceq \bigcirc_{i=1}^{n}(\overbrace{\log_{\Omega_i} M^F})^{\Psi_i} + \bigcirc_{i=1}^{n}(\overbrace{\log_{\Omega_i}(M^{1-T})})^{\Psi_i}\\
&= \overbrace{\log_{\Omega_i} M^F} + \overbrace{\log_{\Omega_i}(M^{1-T})}.
\end{aligned}$$

Since, $\underbrace{\chi} = \min \chi_{ij}$, $\overbrace{\chi} = \max \chi_{ij}$, $\underbrace{\psi} = \max \psi_{ij}$, $\overbrace{\psi} = \min \psi_{ij}$ and $\underbrace{\chi} \preceq \chi_{ij} \preceq \overbrace{\chi}$ and $\overbrace{\psi} \preceq \psi_{ij} \preceq \underbrace{\psi}$. Hence, $\uplus_{i=1}^{n}\Psi_i\underbrace{\chi} \preceq \uplus_{i=1}^{n}\Psi_i\chi_{ij} \preceq \uplus_{i=1}^{n}\Psi_i\overbrace{\chi}$ and $\uplus_{i=1}^{n}\Psi_i\overbrace{\psi} \preceq \uplus_{i=1}^{n}\Psi_i\psi_{ij} \preceq$

$\uplus_{i=1}^{n}\Psi_i \underbrace{\psi}$. Therefore,

$$\frac{\uplus_{i=1}^{n}\Psi_i \underbrace{\chi}}{2} \times \left[\frac{\left(\sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\underbrace{\log_{\Omega_i} M^T})^q\right)^{\Psi_i}}\right)^2 + \left(\sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\underbrace{\log_{\Omega_i}(M^{1-F})})^q\right)^{\Psi_i}}\right)^2}{2} + 1 - \frac{\left(\bigcirc_{i=1}^{n}\left(\overbrace{\log_{\Omega_i} M^F}\right)^{\Psi_i}\right)^2 + \left(\bigcirc_{i=1}^{n}\left(\overbrace{\log_{\Omega_i}(M^{1-T})}\right)^{\Psi_i}\right)^2}{2}\right]$$

$$\preceq \frac{\uplus_{i=1}^{n}\Psi_i\chi_{ij}}{2} \times \left[\frac{\left(\sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Omega_i} {M_{ij}}^T)^q\right)^{\Psi_i}}\right)^2 + \left(\sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Omega_i}({M_{ij}}^{1-F}))^q\right)^{\Psi_i}}\right)^2}{2} + 1 - \frac{\left(\bigcirc_{i=1}^{n}(\log_{\Omega_i} M_{ij}^F)^{\Psi_i}\right)^2 + \left(\bigcirc_{i=1}^{n}(\log_{\Omega_i}({M_{ij}}^{1-T}))^{\Psi_i}\right)^2}{2}\right]$$

$$\preceq \frac{\uplus_{i=1}^{n}\Psi_i \overbrace{\chi}}{2} \times \left[\frac{\left(\sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\overbrace{\log_{\Omega_i} M^T})^q\right)^{\Psi_i}}\right)^2 + \left(\sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\overbrace{\log_{\Omega_i}(M^{1-F})})^q\right)^{\Psi_i}}\right)^2}{2} + 1 - \frac{\left(\bigcirc_{i=1}^{n}(\underbrace{\log_{\Omega_i} M^F})^{\Psi_i}\right)^2 + \left(\bigcirc_{i=1}^{n}(\underbrace{\log_{\Omega_i}(M^{1-T})})^{\Psi_i}\right)^2}{2}\right].$$

$$\text{Hence, } \left\langle (\underbrace{\chi}, \underbrace{\psi}); [\underbrace{\log M^T}, \underbrace{\log(M^{1-F})}], [\overbrace{\log M^F}, \overbrace{\log(M^{1-T})}]\right\rangle$$
$$\preceq \ \text{<italic>} q \text{</italic>} -rungNVWA(M_1, M_2, \ldots, M_n)$$
$$\preceq \left\langle (\overbrace{\chi}, \overbrace{\psi}); [\overbrace{\log M^T}, \overbrace{\log(M^{1-F})}], [\underbrace{\log M^F}, \underbrace{\log(M^{1-T})}]\right\rangle.$$

Proof of the Theorem 5.4

Proof. For any i, $\chi_{t_{ij}} \preceq \psi_{h_{ij}}$. Therefore, $\uplus_{i=1}^{n}\chi_{t_{ij}} \preceq \uplus_{i=1}^{n}\psi_{h_{ij}}$. For any i, $\left(\log_{\Omega_i} M_{t_{ij}}^T\right)^2 + \left(\log_{\Omega_i} M_{t_{ij}}^{1-F}\right)^2 \preceq \left(\log_{\Omega_i} M_{h_{ij}}^T\right)^2 + \left(\log_{\Omega_i} M_{h_{ij}}^{1-F}\right)^2$. Therefore, $1-\left(\log_{\Omega_i} M_{t_i}^T\right)^2 + 1 - \left(\log_{\Omega_i} M_{t_i}^{1-F}\right)^2 \succeq 1-\left(\log_{\Omega_i} M_{h_i}^T\right)^2 + 1 - \left(\log_{\Omega_i} M_{h_i}^{1-F}\right)^2$. Hence, $\bigcirc_{i=1}^{n}\left(1-\left(\log_{\Omega_i} M_{t_i}^T\right)^2\right)^{\Psi_i} + \bigcirc_{i=1}^{n}\left(1-\left(\log_{\Omega_i} M_{t_i}^{1-F}\right)^2\right)^{\Psi_i} \succeq \bigcirc_{i=1}^{n}\left(1-\left(\log_{\Omega_i} M_{h_i}^T\right)^2\right)^{\Psi_i} + \bigcirc_{i=1}^{n}\left(1-\left(\log_{\Omega_i} M_{h_i}^{1-F}\right)^2\right)^{\Psi_i}$ and $\sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-\left(\log_{\Omega_i} M_{t_i}^T\right)^q\right)^{\Psi_i}} + \sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-\left(\log_{\Omega_i} M_{t_i}^{1-F}\right)^q\right)^{\Psi_i}} \preceq \sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-\left(\log_{\Omega_i} M_{h_i}^T\right)^q\right)^{\Psi_i}} + \sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-\left(\log_{\Omega_i} M_{h_i}^{1-F}\right)^q\right)^{\Psi_i}}$. For any i, $\left(\log_{\Omega_i} M_{t_{ij}}^F\right)^2 + \left(\log_{\Omega_i} M_{t_{ij}}^{1-T}\right)^2 \succeq \left(\log_{\Omega_i} M_{h_{ij}}^F\right)^2 + \left(\log_{\Omega_i} M_{h_{ij}}^{1-T}\right)^2$. Therefore,

$$1-\frac{\left(\bigcirc_{i=1}^{n}\log_{\Omega_i} M_{t_{ij}}^{F}\right)^2+\left(\bigcirc_{i=1}^{n}\log_{\Omega_i} M_{t_{ij}}^{1-T}\right)^2}{2}\preceq 1-\frac{\left(\bigcirc_{i=1}^{n}\log_{\Omega_i} M_{h_{ij}}^{F}\right)^2+\left(\bigcirc_{i=1}^{n}\log_{\Omega_i} M_{h_{ij}}^{1-T}\right)^2}{2}.$$

$$\frac{\uplus_{i=1}^{n}\chi_{tij}}{2}\times\left[\begin{array}{c}\frac{\left(\sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Omega_i} M_{ti}^{T})^q\right)^{\Psi_i}}\right)^2+\left(\sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Omega_i} M_{ti}^{1-F})^q\right)^{\Psi_i}}\right)^2}{2}\\ +1-\frac{\left(\bigcirc_{i=1}^{n}\log_{\Omega_i} M_{tij}^{F}\right)^2+\left(\bigcirc_{i=1}^{n}\log_{\Omega_i} M_{tij}^{1-T}\right)^2}{2}\end{array}\right]$$

$$\preceq\quad\frac{\uplus_{i=1}^{n}\chi_{hij}}{2}\times\left[\begin{array}{c}\frac{\left(\sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Omega_i} M_{hi}^{T})^q\right)^{\Psi_i}}\right)^2+\left(\sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-(\log_{\Omega_i}(M_{hi}^{1-F}))^q\right)^{\Psi_i}}\right)^2}{2}\\ +1-\frac{\left(\bigcirc_{i=1}^{n}(\log_{\Omega_i} M_{hij}^{F})\right)^2+\left(\bigcirc_{i=1}^{n}(\log_{\Omega_i}(M_{hij}^{1-T}))\right)^2}{2}\end{array}\right].$$

Hence, log q-rung NVWA $(M_1, M_2, \ldots, M_n) \preceq log\ q\text{-}rungNVWA\ (\mathscr{W}_1, \mathscr{W}_2, \ldots, \mathscr{W}_n)$.

Proof of the Theorem 5.7

Proof. We prove that, $\uplus_{i=1}^{n}\Psi_i M_i^{\Theta} =$

$$\left[\begin{array}{c}\left(\left(\uplus_{i=1}^{n}\ \Psi_i\chi_i^{\Theta}\right),\left(\uplus_{i=1}^{n}\ \Psi_i\psi_i^{\Theta}\right)\right);\\ \left[\sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-\left((\log_{\Omega_i} M_i^{T})^q\right)\right)^{\Psi_i}},\sqrt[q]{1-\bigcirc_{i=1}^{n}\left(1-\left((\log_{\Omega_i} M_i^{1-F}))^q\right)\right)^{\Psi_i}}\right],\\ \left[\bigcirc_{i=1}^{n}\left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_i^{F})^q\right)}\right)^{\Psi_i},\bigcirc_{i=1}^{n}\left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_i^{1-T}))^q\right)}\right)^{\Psi_i}\right]\end{array}\right].$$ Based on the

inductive approach, the proof can be made. If $n = 2$, then $\Psi_1 M_1 \bigoplus \Psi_2 M_2 =$

$$\left[\begin{array}{c} \left(\Psi_1\chi_1^{\Theta} + \Psi_2\chi_2^{\Theta}, \Psi_1\psi_1^{\Theta} + \Psi_2\psi_2^{\Theta}\right); \\ \left[\begin{array}{c} \sqrt[q]{\begin{array}{c}\left(\sqrt[q]{1-\left(1-\left((\log_{\Omega_i} M_1^T)^q\right)^q\right)^{\Psi_1}}\right)^q + \left(\sqrt[q]{1-\left(1-\left((\log_{\Omega_i} M_2^T)^q\right)^q\right)^{\Psi_1}}\right)^q, \\ -\left(\sqrt[q]{1-\left(1-\left((\log_{\Omega_i} M_1^T)^q\right)^q\right)^{\Psi_1}}\right)^q \cdot \left(\sqrt[q]{1-\left(1-\left((\log_{\Omega_i} M_2^T)^q\right)^q\right)^{\Psi_1}}\right)^q\end{array}} \\ \sqrt[q]{\begin{array}{c}\left(\sqrt[q]{1-\left(1-\left((\log_{\Omega_i} (M_1^{1-F}))^q\right)^q\right)^{\Psi_1}}\right)^q + \left(\sqrt[q]{1-\left(1-\left((\log_{\Omega_i} (M_2^{1-F}))^q\right)^q\right)^{\Psi_1}}\right)^q \\ -\left(\sqrt[q]{1-\left(1-\left((\log_{\Omega_i} (M_1^{1-F}))^q\right)^q\right)^{\Psi_1}}\right)^q \cdot \left(\sqrt[q]{1-\left(1-\left((\log_{\Omega_i} (M_2^{1-F}))^q\right)^q\right)^{\Psi_1}}\right)^q\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_1^F)^q\right)^q}\right)^{\Psi_1} \cdot \left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_2^F)^q\right)^q}\right)^{\Psi_1}, \\ \left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} ({M_1}^{1-T}))^q\right)^q}\right)^{\Psi_1} \cdot \left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} ({M_2}^{1-T}))^q\right)^q}\right)^{\Psi_1} \end{array}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left(\uplus_{i=1}^{2} \Psi_i\chi_i^{\Theta}, \uplus_{i=1}^{2}\Psi_i\psi_i^{\Theta}\right); \\ \left[\sqrt[q]{1-\bigcirc_{i=1}^{2}\left(1-\left((\log_{\Omega_i} M_1^T)^q\right)^q\right)^{\Psi_i}}, \sqrt[q]{1-\bigcirc_{i=1}^{2}\left(1-\left((\log_{\Omega_i} (M_1^{1-F}))^q\right)^q\right)^{\Psi_i}}\right], \\ \left[\bigcirc_{i=1}^{2}\left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_i^F)^q\right)^q}\right)^{\Psi_i}, \bigcirc_{i=1}^{2}\left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_i^{1-T}))^q\right)^q}\right)^{\Psi_i}\right] \end{array}\right].$$

It valid for $n = l$ and $l \succeq 3$. Thus, $\uplus_{i=1}^{l}\Psi_i M_i^{\Theta}$

$$= \left[\begin{array}{c} \left(\uplus_{i=1}^{l} \Psi_i\chi_i^{\Theta}, \uplus_{i=1}^{l}\Psi_i\psi_i^{\Theta}\right); \\ \left[\sqrt[q]{1-\bigcirc_{i=1}^{l}\left(1-\left((\log_{\Omega_i} M_1^T)^q\right)^q\right)^{\Psi_i}}, \sqrt[q]{1-\bigcirc_{i=1}^{l}\left(1-\left((\log_{\Omega_i} (M_1^{1-F}))^q\right)^q\right)^{\Psi_i}}\right], \\ \left[\bigcirc_{i=1}^{l}\left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} {M_i}^F)^q\right)^q}\right)^{\Psi_i}, \bigcirc_{i=1}^{l}\left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} {M_i}^F)^q\right)^q}\right)^{\Psi_i}\right] \end{array}\right].$$

If $n = l + 1$, then $\uplus_{i=1}^{l}\Psi_i M_i^{\Theta} + \Psi_{l+1} M_{l+1}^{\Theta} = \uplus_{i=1}^{l+1}\Psi_i M_i^{\Theta}$.

Now, $\uplus_{i=1}^{l}\Psi_i M_i^{\Theta} + \Psi_{l+1}M_{l+1}^{\Theta} = \Psi_1 M_1^{\Theta} \bigoplus \Psi_2 M_2^{\Theta} \bigoplus ... \bigoplus w_l M_l^{\Theta} \bigoplus \Psi_{l+1}M_{l+1}^{\Theta} =$

$$\left[\begin{array}{c} \left(\uplus_{i=1}^{l}\Psi_i\chi_i^{\Theta} + \Psi_{l+1}\Omega_{l+1}^{\Theta}, \uplus_{i=1}^{l}\Psi_i\psi_i^{\Theta} + \Psi_{l+1}\psi_{l+1}^{\Theta}\right); \\ \left[\begin{array}{c} \sqrt[q]{\begin{array}{c}\left(\sqrt[q]{1-\bigcirc_{i=1}^{l}\left(1-\left((\log_{\Omega_i} M_i^{T})^q\right)^q\right)^{\Psi_i}}\right)^q + \left(\sqrt[q]{1-\left(1-\left((\log_{\Omega_i} M_{l+1}{}^{T})^q\right)^q\right)^{\Psi_1}}\right)^q, \\ -\left(\sqrt[q]{1-\bigcirc_{i=1}^{l}\left(1-\left((\log_{\Omega_i} M_i^{T})^q\right)^q\right)^{\Psi_i}}\right)^q \cdot \left(\sqrt[q]{1-\left(1-\left((\log_{\Omega_i} M_{l+1}{}^{T})^q\right)^q\right)^{\Psi_1}}\right)^q\end{array}} \\ \sqrt[q]{\begin{array}{c}\left(\sqrt[q]{1-\bigcirc_{i=1}^{l}\left(1-\left((\log_{\Omega_i} M_i^{1-F}))^q\right)^q\right)^{\Psi_i}}\right)^q + \left(\sqrt[q]{1-\left(1-\left((\log_{\Omega_i} (M_{l+1}{}^{1-F}))^q\right)^q\right)^{\Psi_1}}\right)^q \\ -\left(\sqrt[q]{1-\bigcirc_{i=1}^{l}\left(1-\left((\log_{\Omega_i} M_i^{1-F}))^q\right)^q\right)^{\Psi_i}}\right)^q \cdot \left(\sqrt[q]{1-\left(1-\left((\log_{\Omega_i} (M_{l+1}{}^{1-F}))^q\right)^q\right)^{\Psi_1}}\right)^q\end{array}} \end{array}\right], \\ \left[\begin{array}{c} \bigcirc_{i=1}^{l}\left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_i^{F})^q\right)^q}\right)^{\Psi_i} \cdot \left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_{l+1}^{F})^q\right)^q}\right)^{\Psi_1}, \\ \bigcirc_{i=1}^{l}\left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_i^{1-T}))^q\right)^q}\right)^{\Psi_i} \cdot \left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} (M_{l+1}{}^{1-T}))^q\right)^q}\right)^{\Psi_1} \end{array}\right] \end{array}\right].$$

Thus,

$$\uplus_{i=1}^{l+1}\Psi_i M_i^{\Theta} = \left[\begin{array}{c} \left(\uplus_{i=1}^{l+1}\Psi_i\chi_i^{\Theta}, \uplus_{i=1}^{l+1}\Psi_i\psi_i^{\Theta}\right); \\ \left[\sqrt[q]{1-\bigcirc_{i=1}^{l+1}\left(1-\left((\log_{\Omega_i} M_1^{T})^q\right)^q\right)^{\Psi_i}}, \sqrt[q]{1-\bigcirc_{i=1}^{l+1}\left(1-\left((\log_{\Omega_i} (M_1^{1-F}))^q\right)^q\right)^{\Psi_i}}\right], \\ \left[\bigcirc_{i=1}^{l+1}\left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_i^{F})^q\right)^q}\right)^{\Psi_i}, \bigcirc_{i=1}^{l+1}\left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_i^{F})^q\right)^q}\right)^{\Psi_i}\right] \end{array}\right].$$

Hence, $\left(\uplus_{i=1}^{l+1}\Psi_i M_i^{\Theta}\right)^{1/\Theta}$

$$\left[\begin{array}{c} \left(\left(\uplus_{i=1}^{l+1}\Psi_i\chi_i^{\Theta}\right)^{1/\Theta}, \left(\uplus_{i=1}^{l+1}\Psi_i\psi_i^{\Theta}\right)^{1/\Theta}\right); \\ \left[\left(\sqrt[q]{1-\bigcirc_{i=1}^{l+1}\left(1-\left((\log_{\Omega_i} M_i^{T})^q\right)^q\right)^{\Psi_i}}\right)^{1/q}, \left(\sqrt[q]{1-\bigcirc_{i=1}^{l+1}\left(1-\left((\log_{\Omega_i} M_i^{1-F}))^q\right)^q\right)^{\Psi_i}}\right)^{1/q}\right], \\ \left[\begin{array}{c} \sqrt[q]{1-\left(1-\left(\bigcirc_{i=1}^{l+1}\left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_i^{F})^q\right)^q}\right)^{\Psi_i}\right)\right)^2}^{1/q}, \\ \sqrt[q]{1-\left(1-\left(\bigcirc_{i=1}^{l+1}\left(\sqrt[q]{1-\left(1-(\log_{\Omega_i} M_i^{1-T}))^q\right)^q}\right)^{\Psi_i}\right)\right)^2}^{1/q} \end{array}\right] \end{array}\right].$$

A log G q-rung NVWA operator is modified into a log G q-rung NVWA operator when $q = 1$ is specified.