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ARTICLE

Far and Near Optimization: A New Simple and Effective Metaphor-Less Optimization Algorithm for Solving Engineering Applications

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ABSTRACT

In this article, a novel metaheuristic technique named Far and Near Optimization (FNO) is introduced, offering versatile applications across various scientific domains for optimization tasks. The core concept behind FNO lies in integrating global and local search methodologies to update the algorithm population within the problem-solving space based on moving each member to the farthest and nearest member to itself. The paper delineates the theory of FNO, presenting a mathematical model in two phases: (i) exploration based on the simulation of the movement of a population member towards the farthest member from itself and (ii) exploitation based on simulating the movement of a population member towards the nearest member from itself. FNO's efficacy in tackling optimization challenges is assessed through its handling of the CEC 2017 test suite across problem dimensions of 10, 30, 50, and 100, as well as to address CEC 2020. The optimization results underscore FNO's adeptness in exploration, exploitation, and maintaining a balance between them throughout the search process to yield viable solutions. Comparative analysis against twelve established metaheuristic algorithms reveals FNO's superior performance. Simulation findings indicate FNO's outperformance of competitor algorithms, securing the top rank as the most effective optimizer across a majority of benchmark functions. Moreover, the outcomes derived by employing FNO on twenty-two constrained optimization challenges from the CEC 2011 test suite, alongside four engineering design dilemmas, showcase the effectiveness of the suggested method in tackling real-world scenarios.

KEYWORDS

Optimization; stochastic method; far; near; metaheuristic algorithm; exploration; exploitation

1 Introduction

Currently, numerous optimization challenges are prevalent in science, engineering, mathematics, and practical applications, requiring suitable methodologies [1,2]. Typically, each optimization

problem consists of three basic components: decision variables, constraints, and an objective function. The main goal in optimization is to find the optimal values for the decision variables, ensuring that the objective function is optimized within the constraints [3,4].

Metaheuristic algorithms are recognized as effective problem-solving tools designed to tackle optimization challenges. These algorithms employ random search techniques within the problem domain, along with stochastic operators, to provide feasible solutions for optimization problems [5]. They are valued for their simple conceptualization, ease of implementation, and ability to address various optimization problems, including non-convex, non-linear, non-derivative, discontinuous, and NP-hard problems, as well as navigating through discrete and unexplored search spaces [6]. The operational process of metaheuristic algorithms typically begins with randomly generating a set of feasible solutions, forming the algorithm population. Through iterative refinement processes, these solutions are improved via algorithmic update mechanisms. Upon completion of the algorithm's execution, the most optimal solution obtained during the iterations is presented as the solution to the problem [7].

For metaheuristic algorithms to conduct an efficient search in the problem-solving space, they need to effectively manage the search process at both global and local levels. Global search, achieved through exploration, allows the algorithm to avoid getting trapped in local optima by thoroughly scanning the problem-solving space to identify the main optimum region. On the other hand, local search, through exploitation, enables the algorithm to converge towards potentially better solutions by closely examining the vicinity of discovered solutions and promising areas within the problem-solving space. In addition to exploration and exploitation, the success of a metaheuristic algorithm in facilitating an efficient search relies on maintaining a fine balance between these two strategies throughout the search process [8].

Because of the random nature of stochastic search methods, metaheuristic algorithms cannot guarantee reaching the global optimum. Therefore, solutions obtained from these algorithms are commonly labeled as quasi-optimal. This built-in uncertainty, along with researchers' desire to attain quasi-optimal solutions closer to the global optimum, has driven the creation of various metaheuristic algorithms [9].

The main research question revolves around whether the existing metaheuristic algorithms adequately address the challenges in science, or if there's a necessity for the development of newer ones. The No Free Lunch (NFL) theorem, as posited in reference [10], provides insight into this question by stating that the effectiveness of a metaheuristic algorithm in solving a specific set of optimization problems doesn't guarantee similar success across all optimization problems. Hence, there's no assured success or failure of a metaheuristic algorithm on any given optimization problem. According to the NFL theorem, no single metaheuristic algorithm can be deemed as the ultimate optimizer for all optimization tasks. This theorem, by continually stimulating the exploration and advancement of metaheuristic algorithms, encourages researchers to seek more efficient solutions for optimization problems through the design of newer algorithms.

This paper presents a novel contribution by introducing the Far and Near Optimization (FNO) metaheuristic algorithm, aimed at addressing optimization challenges across various scientific disciplines. Literature review shows that many metaheuristic algorithms have been designed so far. In most of these algorithms, researchers have tried to manage the search process by improving the state of exploration and exploitation in such a way as to achieve more effective solutions for optimization problems. In fact, the proper management of local and global search is the main key to the success of metaheuristic algorithms. This challenge of balancing exploration and exploitation has been the

main motivation of the authors of this paper to design a dedicated algorithm that specifically deals with the design of the local and global search process. Therefore, it can be stated that the main source of inspiration for the authors in designing the proposed FNO approach was balancing exploration and exploitation.

In this new approach, the authors have used two basic mechanisms to change the position of the population members in the search space.

The first mechanism is the movement of each FNO member towards the farthest member of the population. In this mechanism, with the aim of increasing the discovery power of the algorithm in order to manage the global search, extensive changes are made in the position of each member of the population. These large displacements have a significant impact on the comprehensive scanning of the search space and prevent the algorithm from getting stuck in local optima. The important point in this process is that updating the population based on movement towards the best member is avoided. Because the excessive dependence of the update process based on the position of the best member can lead to the algorithm getting stuck early in inappropriate local optimal solutions.

The second mechanism is the movement of each FNO member towards the nearest member of the population. In this mechanism, small changes are made in the position of each population member with the aim of increasing the ability to exploit FNO in order to manage local search. These small displacements are effective in accurate scanning near the obtained solutions with the aim of obtaining possible better solutions and converging towards the global optimum.

Although based on the best knowledge obtained from the literature review, the originality of the proposed FNO approach is confirmed, however, the authors describe the differences of the proposed approach with one of the recently published algorithms called Giant Armadillo Optimization (GAO) [11]. GAO is a swarm-based algorithm inspired by the natural behavior of the giant armadillo in the wild. The main idea in FNO design is to move giant armadillo towards termite mounds and then dig in that position.

The first difference between proposed FNO approach and GAO is the source of design inspiration. As the title of the paper suggests, FNO is a population-based yet metaphor-free algorithm. But GAO is inspired by nature and wildlife in its design.

The second difference between FNO and GAO is in the design of the exploration phase. In the GAO design, each member of the population is directed to a position in the problem solving space that has a better objective function value compared to the corresponding member. Therefore, in GAO, it does not matter whether the location of the destination member is far or near. Meanwhile, in the design of the exploration phase in FNO, each member of the population moves towards the member farthest from itself. In this mechanism, the criterion for selecting the position of the destination member is the greatest distance to the corresponding member.

The third difference between FNO and GAO is related to the design of the exploitation phase. In GAO, in order to manage the local search, it is assumed that the movement range of each member becomes smaller and more limited during the iterations of the algorithm. This is while in FNO design, local search management is independent of algorithm iterations. The exploitation phase in FNO is designed in such a way that each member of the population moves towards the member closest to itself.

In general, it can be said that FNO is different from GAO from the point of view of source of inspiration in design, theory, mathematical model, exploration phase design, and exploitation phase design.

FNO has several advantages compared to other algorithms, which are described below:

I. **Balanced Exploration and Exploitation**
- **Enhanced Global Search:** FNO leverages the exploration phase by moving towards the farthest member, which helps in thoroughly exploring the search space and avoiding local optima.
- **Improved Local Search:** The exploitation phase involves moving towards the nearest member, which refines the search around promising areas to find the optimal solution.

II. **Dynamic Population Update**
- **Adaptive Strategy:** FNO dynamically updates population members based on their distances, allowing for a more adaptive and responsive search process.
- **Efficient Convergence:** The dual-phase approach ensures that the algorithm does not prematurely converge and maintains a good balance between exploration and exploitation throughout the optimization process.

III. **Mathematical Rigor**
- **Clear Theoretical Foundation:** The FNO algorithm is mathematically modeled and well-defined, providing a clear understanding of its working principles and mechanisms.
- **Predictable Behavior:** The mathematical formulation allows for predictable and consistent behavior of the algorithm across different optimization problems.

IV. **Performance on Benchmark Functions**
- **Superior Results:** Comparative analyses show that FNO often outperforms other well-known metaheuristic algorithms in solving benchmark optimization problems.
- **Statistical Significance:** The performance improvements offered by FNO are not only observed empirically but are also statistically significant, providing robust evidence of its effectiveness.

V. **Versatility and Applicability**
- **Wide Range of Problems:** FNO has demonstrated effectiveness across various problem domains, including both unconstrained and constrained optimization problems.
- **Real-World Applications:** The algorithm has been successfully applied to real-world engineering design problems, showcasing its practical utility.

VI. **Scalability**
- **Handling High-Dimensional Problems:** FNO is capable of handling optimization problems with a large number of decision variables, making it suitable for complex, high-dimensional tasks.
- **Scalable Complexity:** Despite its quadratic time complexity with respect to the number of population members, the algorithm remains computationally feasible for a wide range of practical applications.

VII. **Robustness and Reliability**
- **Consistent Performance:** FNO consistently provides high-quality solutions across different types of optimization problems, indicating its robustness.
- **Reliability:** The algorithm's ability to balance exploration and exploitation ensures that it reliably finds good solutions without getting stuck in suboptimal regions.

The FNO algorithm offers several advantages over other metaheuristic algorithms, including a balanced approach to exploration and exploitation, dynamic and adaptive population updates, strong

mathematical foundations, superior performance on benchmark functions, versatility in application, scalability to high-dimensional problems, and overall robustness and reliability. These advantages make FNO a powerful and effective tool for tackling a wide variety of optimization challenges.

The key contributions of this research are outlined as follows:

- The main idea in the design of FNO is to effectively update the population member of the algorithm in the problem-solving space by applying the concept of global search based on movement towards the farthest member and local search based on movement towards the nearest member.
- The theory of FNO is stated and its implementation steps are mathematically modeled in two phases of exploration and exploitation.
- **Novel Approach:** The paper introduces a new metaheuristic algorithm named Far and Near Optimization, designed to address various optimization challenges in scientific disciplines.
- **Balance between Exploration and Exploitation:** FNO emphasizes a balanced approach to global and local search by employing distinct mechanisms for exploration (movement towards the farthest member) and exploitation (movement towards the nearest member).
- **Exploration Phase:** The algorithm enhances global search capability by moving each population member towards the farthest member, which helps in avoiding local optima and ensures thorough exploration of the search space.
- **Exploitation Phase:** FNO improves local search by moving each population member towards the nearest member, enabling precise scanning near discovered solutions to find potentially better solutions and converge towards the global optimum.
- **Clear Theoretical Foundation:** The FNO algorithm is mathematically modeled, providing a well-defined and understandable framework for its working principles and mechanisms.
- **Mathematical Rigor:** The paper provides a detailed mathematical formulation of the FNO algorithm, ensuring predictable and consistent behavior across different optimization problems.
- **Consistent Performance:** FNO consistently provides high-quality solutions across different types of optimization problems, indicating its robustness.
- **Reliability:** The algorithm's balanced exploration and exploitation ensure that it reliably finds good solutions without getting stuck in suboptimal regions.

Below outlines the structure of this paper: Section 2 presents the review of relevant literature. In Section 3, we introduce and model the FNO approach. Section 4 covers simulation studies and their results. Section 5 examines the efficacy of FNO in addressing real-world applications. Finally, Section 6 offers conclusions and suggestions for future research.

2 Literature Review

Metaheuristic algorithms draw inspiration from a range of natural and evolutionary phenomena, including biology, genetics, physics, and human behaviors. As a result, these algorithms are categorized into five groups based on their underlying sources of inspiration: swarm-based, evolutionary-based, physics-based, and human-based.

Swarm-based metaheuristic algorithms take inspiration from the collective behaviors observed in diverse natural swarms, encompassing birds, animals, aquatic creatures, insects, and other organisms within their habitats. Notable examples include Particle Swarm Optimization (PSO) [12], Artificial

Bee Colony (ABC) [13], Ant Colony Optimization (ACO) [14], and Firefly Algorithm (FA) [15]. PSO mirrors the coordinated movements of bird and fish flocks during foraging. ABC replicates the collaborative endeavors of bee colonies in locating new food sources. ACO is grounded in the efficient pathfinding capabilities of ants between their nest and food sites. FA is crafted based on the light signaling interactions among fireflies for communication and mate attraction. In the realm of natural behaviors exhibited by wild organisms, strategies like hunting, foraging, migration, chasing, digging, and searching prominently feature and have inspired the design of swarm-based metaheuristic algorithms such as: Arctic Puffin Optimization (APO) [16], Remora Optimization Algorithm (ROA) [17], Sand Cat Swarm Optimization (SCSO) [18,19], Giant Armadillo Optimization (GAO) [11], African Vultures Optimization Algorithm (AVOA) [20], Golden Jackal Optimization (GJO) [21], White Shark Optimizer (WSO) [22], Tunicate Swarm Algorithm (TSA) [23], Whale Optimization Algorithm (WOA) [24], Grey Wolf Optimizer (GWO) [25], Marine Predator Algorithm (MPA) [26], Honey Badger Algorithm (HBA) [27], and Reptile Search Algorithm (RSA) [28].

Evolutionary-based metaheuristic algorithms find their inspiration in principles from biological sciences and genetics, with a particular emphasis on concepts like natural selection, survival of the fittest, and Darwin's evolutionary theory. Prominent examples include Genetic Algorithm (GA) [29] and Differential Evolution (DE) [30], which are deeply rooted in genetic principles, reproduction processes, and the application of evolutionary operators such as selection, mutation, and crossover. Furthermore, Artificial Immune System (AIS) [31] has been developed, drawing inspiration from the human body's defense mechanisms against pathogens and diseases.

Physics-based metaheuristic algorithms draw inspiration from a wide array of forces, laws, phenomena, transformations, and other fundamental concepts within physics. Simulated Annealing (SA) [32] is a notable example, inspired by the process of metal annealing, where metals are heated and gradually cooled to achieve an optimal crystal structure. Moreover, algorithms like Spring Search Algorithm (SSA) [33] Momentum Search Algorithm (MSA) [34] and Gravitational Search Algorithm (GSA) [35] have been developed based on the emulation of physical forces. SSA employs spring tensile force modeling and Hooke's law, MSA is founded on impulse force modeling, and GSA integrates gravitational attraction force modeling. Noteworthy physics-based methods include the Multi-Verse Optimizer (MVO) [36], Archimedes Optimization Algorithm (AOA) [37], Thermal Exchange Optimization (TEO) [38], Electro-Magnetism Optimization (EMO) [39], Water Cycle Algorithm (WCA) [40], Black Hole Algorithm (BHA) [41], Equilibrium Optimizer (EO) [42], and Lichtenberg Algorithm (LA) [43].

Human-based metaheuristic algorithms draw inspiration from diverse aspects of human behavior, encompassing communication, decision-making, thought processes, interactions, and choices observed in personal and social contexts. Teaching-Learning Based Optimization (TLBO) [44] serves as a prime example, inspired by the dynamics of education within classrooms, where knowledge exchange occurs among teachers and students as well as between students themselves. The Mother Optimization Algorithm (MOA) is introduced, drawing inspiration from the care provided by mothers to their children [45]. Poor and Rich Optimization (PRO) [46] is formulated based on economic activities observed among individuals of different socioeconomic backgrounds, with the aim of improving their financial situations. Conversely, Doctor and Patient Optimization (DPO) [47] simulates therapeutic interactions between healthcare providers and patients in hospital settings. Other notable human-based metaheuristic algorithms include War Strategy Optimization (WSO) [48], Ali Baba and the Forty Thieves (AFT) [49], Coronavirus Herd Immunity Optimizer (CHIO) [50], and Gaining Sharing Knowledge based Algorithm (GSK) [51].

In addition to the mentioned groupings, many researchers are interested in combining several algorithms and developing hybrid versions of them to benefit from the advantages of existing algorithms at the same time. Also, designing improved versions of existing algorithms has become a research motivation for researchers. WOA-SCSO is a hybrid metaheuristic algorithm that is derived from the combination of WOA and SCSO [52]. The nonlinear chaotic honey badger algorithm (NCHBA) is an improved version of the honey badger algorithm, which is designed to balance exploration and exploitation in this algorithm [53]. Some other hybrid metaheuristic algorithms are: hybrid PSO-GA [54], hybrid GWO-WOA [55], hybrid GA-PSO-TLBO [56], and hybrid TSA-PSO [57].

Table 1 lists an overview of metaheuristic algorithms and their grouping.

Table 1: Overview of metaheuristic algorithms

Row	Group	Algorithm	Description and source of inspiration	Year
1	Swarm-based	Particle Swarm Optimization (PSO) [12]	Inspired by the social behavior of birds flocking or fish schooling. Particles move through the solution space influenced by their own best position and the best positions of their neighbors.	1995
2		Ant Colony Optimization (ACO) [14]	Inspired by the foraging behavior of ants. Ants deposit pheromones to mark paths, influencing others to follow and reinforce optimal paths.	1996
3		Artificial Bee Colony (ABC) [13]	Modeled on the foraging behavior of honey bees. The algorithm uses employed bees, onlookers, and scouts to explore and exploit food sources, representing solutions.	2007
4		Firefly Algorithm (FA) [15]	Based on the flashing behavior of fireflies. Attracted to each other based on their brightness, which represents the objective function. Brighter fireflies attract others, guiding the search.	2009
5		Grey Wolf Optimizer (GWO) [25]	Simulates the leadership hierarchy and hunting mechanism of grey wolves. The search is guided by alpha, beta, delta, and omega wolves representing different hierarchical roles.	2014
6		Whale Optimization Algorithm (WOA) [24]	Inspired by the bubble-net hunting strategy of humpback whales. The algorithm mimics the behavior of whales encircling prey and using bubble nets to trap them.	2016
7		Tunicate Swarm Algorithm (TSA) [23]	Modeled on the swarming behavior of tunicates in the ocean. The algorithm uses unique movement patterns and swarming behavior to explore and exploit the search space.	2020

(Continued)

Table 1 (continued)

Row	Group	Algorithm	Description and source of inspiration	Year
8		Marine Predator Algorithm (MPA) [26]	Based on the behavior of marine predators such as sharks and tuna during the foraging process. It includes strategies like Brownian and Lévy flight motions to enhance exploration and exploitation.	2020
9		African Vultures Optimization Algorithm (AVOA) [20]	Inspired by the scavenging behavior of African vultures. The algorithm simulates the way vultures search for and locate carrion, representing optimal solutions.	2021
10		Remora Optimization Algorithm (ROA) [17]	Inspired by the symbiotic relationship between remoras and their host fish. The algorithm mimics the behavior of remoras attaching to and detaching from hosts to optimize the search process.	2021
11		Honey Badger Algorithm (HBA) [27]	Mimics the foraging behavior of honey badgers. The algorithm uses a unique approach to balance exploration and exploitation, inspired by the badger's ability to dig and hunt.	2022
12		Reptile Search Algorithm (RSA) [28]	Based on the hunting strategies of reptiles. It incorporates adaptive strategies to efficiently explore and exploit the search space, inspired by reptiles' predatory behavior.	2022
13		Golden Jackal Optimization (GJO) [21]	Based on the social hunting behavior of golden jackals. The algorithm mimics the cooperative hunting strategies of jackals to explore and exploit the search space.	2022
14		White Shark Optimizer (WSO) [22]	Inspired by the hunting strategies of white sharks. The algorithm incorporates the predatory behavior of sharks, using strategies like circling and attacking prey to find optimal solutions.	2022
15		Sand Cat Swarm Optimization (SCSO) [18,19]	Inspired by the hunting behavior of sand cats. The algorithm simulates their adaptive strategies for hunting and surviving in harsh desert environments.	2022
16		Crayfish Optimization Algorithm (COA) [16]	Based on the behavior of crayfish. The algorithm uses their movement patterns and social behavior to explore and exploit the solution space.	2023

(Continued)

Table 1 (continued)

Row	Group	Algorithm	Description and source of inspiration	Year
17	Evolutionary-based	Giant Armadillo Optimization (GAO) [11]	Modeled on the foraging behavior of giant armadillos. The algorithm uses a unique approach to digging and searching for food to optimize solutions.	2023
18		Genetic Algorithm (GA) [29]	Inspired by the process of natural selection and genetics. Uses operators like selection, crossover, and mutation to evolve solutions to optimization problems.	1988
19		Differential Evolution (DE) [30]	Based on the concept of differential mutation, crossover, and selection to optimize a problem. The algorithm relies on the differences between solution vectors to create new candidate solutions.	1997
20		Artificial Immune System (AIS) [31]	Inspired by the human immune system's ability to recognize and combat pathogens. Uses mechanisms like clonal selection, immune network theory, and negative selection for optimization.	2003
21		Simulated Annealing (SA) [32]	Inspired by the annealing process in metallurgy, which involves heating and controlled cooling of a material to increase the size of its crystals and reduce defects.	1983
22		Gravitational Search Algorithm (GSA) [35]	Based on the law of gravity and mass interactions. Agents are considered as objects, and their performance is measured by their masses. All objects attract each other by the gravity force, and this force causes a global movement of all objects towards the objects with heavier masses.	2009
23		Electro-Magnetism Optimization (EMO) [39]	Inspired by the attraction-repulsion mechanism of electromagnetism theory. Solutions are considered as charged particles that interact with each other based on their charge (quality).	2012
24		Water Cycle Algorithm (WCA) [40]	Inspired by the water cycle process and how rivers and streams flow towards the sea. It simulates the process of evaporation, precipitation, and surface runoff.	2012

(Continued)

Table 1 (continued)

Row	Group	Algorithm	Description and source of inspiration	Year
25	Physics-based	Black Hole Algorithm (BHA) [41]	Based on the concept of black holes in astronomy. The algorithm uses the attraction of black holes to pull solutions towards them, representing a convergence towards optimal solutions.	2013
26		Multi-Verse Optimizer (MVO) [36]	Inspired by the theory of multiple universes in physics. Solutions are considered as universes, and optimization is achieved by exchanging objects (parameters) between the universes.	2016
27		Thermal Exchange Optimization (TEO) [38]	Inspired by the thermal exchange process, which involves heat transfer principles. The algorithm uses concepts of thermal equilibrium to guide the search for optimal solutions.	2017
28		Equilibrium Optimizer (EO) [42]	Inspired by dynamic and equilibrium states in physics and chemistry. It mimics the process of reaching equilibrium in a system, balancing exploration and exploitation.	2020
29		Spring Search Algorithm (SSA) [33]	Based on the mechanical properties and behaviors of springs. It uses Hooke's law and the spring force to model the search process.	2020
30		Momentum Search Algorithm (MSA) [34]	Inspired by the momentum in physics. The algorithm uses concepts of velocity and momentum to guide the search process, balancing exploration and exploitation.	2020
31		Lichtenberg Algorithm (LA) [43]	Based on the principles of Lichtenberg figures formed by electrical discharges. The algorithm uses these patterns to explore and exploit the search space.	2021
32		Archimedes Optimization Algorithm (AOA) [37]	Inspired by Archimedes' principle in fluid mechanics. The algorithm simulates the buoyancy force and how it affects the objects submerged in fluid, guiding the search process.	2021
33		Teaching-Learning Based Optimization (TLBO) [44]	Inspired by the teaching-learning process in a classroom. It mimics the influence of a teacher on learners and the interactions among learners to achieve optimal solutions.	2011
34		Poor and Rich Optimization (PRO) [46]	Inspired by the socio-economic interactions between poor and rich individuals, focusing on wealth redistribution and resource optimization.	2019

(Continued)

Table 1 (continued)

Row	Group	Algorithm	Description and source of inspiration	Year
35	Human-based	Gaining Sharing Knowledge Based Algorithm (GSK) [51]	Inspired by the process of gaining and sharing knowledge among individuals, simulating learning and knowledge dissemination to solve optimization problems.	2020
36		Doctor and Patient Optimization (DPO) [47]	Based on the interactions between doctors and patients, aiming to simulate diagnostic and treatment processes to find optimal solutions.	2020
37		Coronavirus Herd Immunity Optimizer (CHIO) [50]	Based on the herd immunity concept used in epidemiology, simulating the spread and control of viruses to find optimal solutions.	2021
38		War Strategy Optimization (WSO) [48]	Inspired by military strategies and tactics used in warfare to achieve objectives, focusing on strategic planning and resource management.	2022
39		Ali Baba and the Forty Thieves (AFT) [49]	Inspired by the tale of Ali Baba and the Forty Thieves, using concepts of hidden treasures and strategic planning to solve optimization problems.	2022
40		Mother Optimization Algorithm (MOA) [45]	Inspired by the behavior and strategies of a mother to solve problems and optimize resources for the best outcomes.	2023
41		Hybrid PSO-GA [54]	A combination of Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) to leverage the strengths of both for better optimization.	2016
42		Hybrid GWO-WOA [55]	A hybrid of Grey Wolf Optimizer (GWO) and Whale Optimization Algorithm (WOA) to enhance search capabilities and convergence rates.	2020
43		Hybrid TSA-PSO [57]	Combines Tunicate Swarm Algorithm (TSA) with Particle Swarm Optimization (PSO) to improve exploration and exploitation in the search process.	2022
44		Hybrid GA-PSO-TLBO [56]	An integration of Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Teaching-Learning Based Optimization (TLBO) for robust optimization.	2023

(Continued)

Table 1 (continued)

Row	Group	Algorithm	Description and source of inspiration	Year
45	Hybrid and modified methods	WOA-SCSO [52]	A hybrid algorithm combining Whale Optimization Algorithm (WOA) and Sand Cat Swarm Optimization (SCSO) to enhance performance and convergence.	2023
46		Nonlinear Chaotic Honey Badger Algorithm (NCHBA) [53]	An improved version of the Honey Badger Algorithm (HBA) that incorporates nonlinear chaotic maps to balance exploration and exploitation.	2024

3 Far and Near Optimization

This section elucidates the theory behind the FNO approach and subsequently delineates its implementation steps through mathematical modeling.

3.1 Inspiration of FNO

Metaheuristic algorithms represent stochastic approaches to problem-solving within optimization tasks, primarily relying on random search techniques in the problem-solving domain. Two fundamental principles guiding this search process are exploration and exploitation. In the development of FNO, these principles are harnessed to update the algorithm's population position within the problem-solving space. Each member of the population undergoes identification of both the farthest and nearest members based on distance calculations. Steering a population member towards the farthest counterpart enhances the algorithm's exploration capacity for global search within the problem-solving space. Hence, within the FNO framework, a population update phase is dedicated to moving each member towards the farthest counterpart. Conversely, directing a population member towards the nearest counterpart enhances the algorithm's exploitation capacity for local search within the problem-solving space. Accordingly, within the FNO framework, a separate population update phase is allocated for moving each member towards its closest counterpart.

3.2 Algorithm Initialization

The proposed FNO methodology constitutes a population-based metaheuristic algorithm, leveraging the search capabilities of its members within the problem-solving space to attain viable solutions for optimization problems through an iterative process. Each member within the FNO framework derives values for decision variables based on its position in the problem-solving space. Consequently, each FNO member serves as a candidate solution to the problem, represented mathematically as a vector. As a result, the algorithm population, comprising these vectors, can be mathematically represented as a matrix, as indicated by Eq. (1). The initial positions of the population members within the problem-solving space are randomly initialized using Eq. (2).

$$X = \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_N \end{bmatrix}_{N\times m} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,d} & \cdots & x_{1,m} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{i,1} & \cdots & x_{i,d} & \cdots & x_{i,m} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ x_{N,1} & \cdots & x_{N,d} & \cdots & x_{N,m} \end{bmatrix}_{N\times m} \tag{1}$$

$$x_{i,d} = lb_d + r \cdot (ub_d - lb_d) \tag{2}$$

Here, X is the FNO population matrix, X_i is the ith FNO member (candidate solution), $x_{i,d}$ is its dth dimension in search space (decision variable), N is the number of population members, m is the number of decision variables, r is a random number in interval [0, 1], lb_d, and ub_d are the lower bound and upper bound of the dth. decision variable, respectively.

In alignment with each FNO member serving as a potential solution to the problem, the objective function of the problem can be assessed. Consequently, the array of evaluated objective function values can be depicted utilizing a matrix, as outlined in Eq. (3).

$$F = \begin{bmatrix} F_1 \\ \vdots \\ F_i \\ \vdots \\ F_N \end{bmatrix}_{N\times 1} = \begin{bmatrix} F(X_1) \\ \vdots \\ F(X_i) \\ \vdots \\ F(X_N) \end{bmatrix}_{N\times 1} \tag{3}$$

Here, F is the vector of evaluated objective function and F_i is the evaluated objective function based on the ith FNO member.

The assessed objective function values provide crucial insights into the quality of population members and potential solutions. The best evaluated objective function value corresponds to the top-performing member within the population, while the worst value corresponds to the least effective member. It should also be explained that since optimization problems fall into two groups of minimization and maximization, the concept of the best and worst value of the objective function is different. In minimization problems, the best value of the objective function corresponds to the lowest evaluated value for the objective function and the worst value of the objective function corresponds to the highest evaluated value for the objective function. On the other hand, in maximization problems, the meaning of the best value of the objective function is the highest value evaluated for the objective function, and the meaning of the worst value of the objective function is the lowest value evaluated for the objective function.

As the position of population members in the problem-solving space is updated in each iteration, new objective function values are computed accordingly. Consequently, throughout each iteration, the top-performing member must also be updated based on comparisons of objective function values. Upon completion of the algorithm's execution, the position of the top-performing member within the population is presented as the solution to the problem.

3.3 Mathematical Modelling of FNO

Within the FNO framework, the adjustment of each population member's position occurs by moving towards both the farthest and nearest member from itself. The distance between each member and another is computed using Eq. (4).

$$D_{i,j} = \sqrt{\sum_{d=1}^{m} \left(X_{i,d} - X_{j,d}\right)^2}, i = 1, 2, \ldots, N, \text{ and } i \neq j, \ d = 1, 2, 3, \ldots, m \tag{4}$$

Here, $D_{i,j}$ is the distance between the ith and jth population members from each other. According to Eq. (4), the highest calculated value for D_i corresponds to the farthest member (FM_i) and the lowest calculated value for D_i corresponds to the nearest member (NM_i) for the ith member.

3.3.1 Phase 1: Moving to the Farthest Member (Exploration)

In the initial phase of FNO, the adjustment of population member positions in the problem-solving space is orchestrated by directing each member towards the farthest counterpart. This modeling process instigates significant alterations in the positions of population members within the problem-solving space, thereby enhancing FNO's capacity for global search and exploration.

The calculation of a new position for each population member is based on the modeling of their movement towards the farthest counterpart, as outlined in Eq. (5). Subsequently, if an improvement in the objective function value is observed at this new position, it supersedes the previous position of the corresponding member, as per Eq. (6).

$$x_{i,d}^{P1} = x_{i,d} + r_{i,d} \cdot (FM_{i,d} - I \cdot x_{i,d}), \tag{5}$$

$$X_i = \begin{cases} X_i^{P1}, & F_i^{P1} \leq F_i, \text{ for minimization problems} \\ X_i^{P1}, & F_i^{P1} \geq F_i, \text{ for maximization problems} \\ X_i, & else \end{cases} \tag{6}$$

Here, FM_i is the farthest member for ith population member, $FM_{i,d}$ is the its dth dimension, X_i^{P1} is the new position calculated for the ith population member based on exploration phase of FNO, $x_{i,d}^{P1}$ is its dth dimension, F_i^{P1} is its objective function value, $r_{i,d}$ are random numbers from the interval [0, 1], and I is number which is randomly selected as 1 or 2.

3.3.2 Phase 2: Moving to the Nearest Member (Exploitation)

In the subsequent phase of FNO, adjustments to population member positions within the problem-solving space are made by guiding each member towards its nearest counterpart. This modeling procedure induces subtle alterations in the positions of population members, thereby enhancing FNO's capacity for local search management and exploitation.

Utilizing the modeling of each FNO member's movement towards its nearest counterpart, a fresh position for every population member is computed, as specified in Eq. (7). Subsequently, if an enhancement in the objective function value is observed at this new position, it supplants the previous position of the corresponding member, in accordance with Eq. (8).

$$x_{i,d}^{P2} = x_{i,d} + r_{i,d} \cdot (NM_{i,d} - I \cdot x_{i,d}), \tag{7}$$

$$X_i = \begin{cases} X_i^{P2}, & F_i^{P2} \leq F_i, \text{ for minimization problems} \\ X_i^{P2}, & F_i^{P2} \geq F_i, \text{ for maximization problems} \\ X_i, & else \end{cases} \tag{8}$$

Here, NM_i is the nearest member for ith population member, $NM_{i,d}$ is the its dth dimension, X_i^{P2} is the new position calculated for the ith population member based on exploitation phase of FNO, $x_{i,d}^{P2}$ is its dth dimension, F_i^{P2} is its objective function value, $r_{i,d}$ are random numbers from the interval [0, 1], and I is number which is randomly selected as 1 or 2.

3.4 Repetition Process, Pseudocode, and Flowchart of FNO

The initial iteration of FNO concludes with the updating of all population members through the first and second phases. Subsequently, based on the updated values for both the algorithm population and the objective function, the algorithm progresses to the next iteration, and the updating process persists using Eqs. (4) to (8) until the final iteration is reached. Throughout each iteration, the best solution attained is updated and preserved. Upon the culmination of the FNO execution, the best solution acquired during the algorithm's iterations is provided as the solution for the given problem. The procedural steps of FNO are depicted in a flowchart in Fig. 1, while its pseudocode is outlined in Algorithm 1.

3.5 Theoretical Analysis of Time Complexity

To understand the efficiency and scalability of the FNO algorithm, we need to analyze its time complexity. The analysis is broken down into the key steps involved in each iteration of the algorithm.

I) Initialization Phase

- **Population Initialization:** Generating the initial population of N members, each with m decision variables.
  - **Time Complexity:** $O(N \cdot m)$

II) Distance Calculation Phase

- **Distance Calculation:** For each member, distances to all other $N-1$ members need to be calculated. Each distance calculation involves m dimensions.
  - **Time Complexity:** $O(N^2.m)$

III) Exploration Phase (Moving to the Farthest Member)

- **Identify Farthest Member:** For each member, determine the farthest member from the distance matrix.
  - **Time Complexity:** $O(N^2)$
- **Update Position:** For each member, update its position towards the farthest member, which involves m dimensions.
  - **Time Complexity:** $O(N \cdot m)$
- **Evaluate Objective Function:** For each member's new position, compute the objective function.
  - **Time Complexity:** $O(N)$

IV) Exploitation Phase (Moving to the Nearest Member)

- **Identify Nearest Member:** For each member, determine the nearest member from the distance matrix.
  - **Time Complexity:** $O(N^2)$
- **Update Position**: For each member, update its position towards the nearest member, which involves m dimensions.
  - **Time Complexity:** $O(N \cdot m)$
- **Evaluate Objective Function:** For each member's new position, compute the objective function.
  - **Time Complexity:** $O(N)$

V) Iterative Process

The above phases are repeated for T iterations.

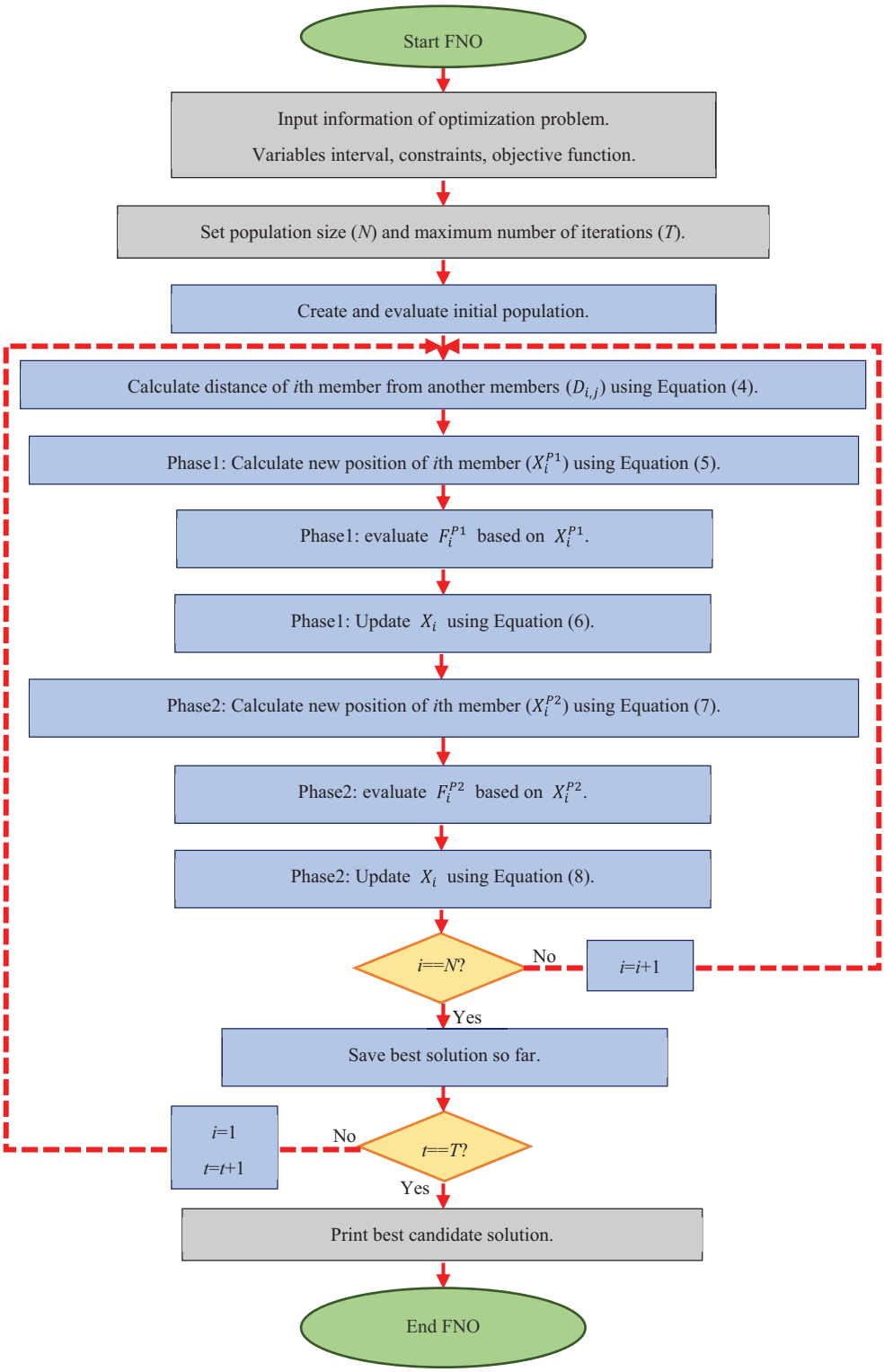

Figure 1: Flowchart of FNO

Algorithm 1: Pseudocode of FNO

Start FNO.

Input problem information: variables, objective function, and constraints.
Set population size (N) and iterations (T).
Generate the initial population matrix at random.
Evaluate the objective function.
  For $t = 1$ to T
  For $i = 1$ to N
    Calculate D_i using Eq. (4). $D_{i,j} \leftarrow \sqrt{\sum_{d=1}^{m} \left(X_{i,d} - X_{j,d}\right)^2}$
    Determine FM_i and NM_i.
    Phase 1: Moving to the farthest member (Exploration).
      Calculate new position of ith population member using Eq. (5).
      $x_{i,d}^{p1} \leftarrow x_{i,d} + r_{i,d} \cdot (FM_{i,d} - I \cdot x_{i,d})$
      Update ith population member using Eq. (6).

$$X_i \leftarrow \begin{cases} X_i^{P1}, & F_i^{P1} \le F_i, \textit{ for minimization problems} \\ X_i^{P1}, & F_i^{P1} \ge F_i, \textit{ for maximization problems} \\ X_i, & \textit{else}, \end{cases}$$

    Phase 2: Moving to the nearest member (Exploitation).
      Calculate new position of ith population member using Eq. (7).
      $x_{i,d}^{p2} \leftarrow x_{i,d} + r_{i,d} \cdot (NM_{i,d} - I \cdot x_{i,d})$
      Update ith population member using Eq. (8).

$$X_i \leftarrow \begin{cases} X_i^{P2}, & F_i^{P2} \le F_i, \textit{ for minimization problems} \\ X_i^{P2}, & F_i^{P2} \ge F_i, \textit{ for maximization problems} \\ X_i, & \textit{else}, \end{cases}$$

    end
    Save best candidate solution so far.
  end
Output best quasi-optimal solution obtained with the FNO.

End FNO.

3.6 Overall Time Complexity

Combining the complexities of each phase, the total time complexity per iteration is:

$$O(N.m) + O\left(N^2.m\right) + O\left(N^2\right) + O(N.m) + O(N) + O\left(N^2\right) + O(N.m) + O(N)$$

Simplifying, we get:

$$O\left(N^2 + m\right) + O\left(N^2\right)$$

Since $O(N^2 + m)$ dominates $O(N^2)$ when $m \geq 1$, the overall time complexity per iteration is:

$$O\left(N^2.m\right)$$

For T iterations, the overall time complexity becomes:

$$O\left(T.N^2.m\right)$$

The time complexity of the FNO algorithm per iteration is $(N^2.m)$, and for T iterations, it is $O(T.N^2.m)$. This analysis indicates that the algorithm scales quadratically with the number of population members N and linearly with the number of decision variables m. The number of iterations T also linearly affects the total computational cost.

3.7 Population Diversity, Exploration, and Exploitation Analysis

Population diversity in the FNO pertains to the spread of individuals within the solution space, which is crucial for tracking the algorithm's search dynamics. This metric reveals whether the algorithm's focus is on exploring new solutions (exploration) or refining existing ones (exploitation). Evaluating FNO's population diversity allows for an assessment and adjustment of the algorithm's balance between exploration and exploitation. Several researchers have proposed different ways to define diversity. For instance, Pant introduced a method to calculate diversity using the following equations [58]:

$$Diversity = \frac{1}{N}\sum_{i=1}^{N}\sqrt{\sum_{d=1}^{m}\left(x_{i,d}-\overline{x}_d\right)^2}, \tag{9}$$

$$\overline{x}_d = \frac{1}{N}\sum_{i=1}^{N} x_{i,d} \tag{10}$$

where N is the population size, m represents the number of dimensions in the problem space, and $x_{i,d}$ is the mean value of the population in the dth dimension. Consequently, the percentages of exploration and exploitation at each iteration can be determined using the following equations:

$$Exploration = \frac{Diversity}{Diversity_{max}}, \tag{11}$$

$$Exploitation = \frac{|Diversity - Diversity_{max}|}{Diversity_{max}} \tag{12}$$

This subsection delves into the analysis of population diversity, as well as exploration and exploitation, using twenty-three standard benchmarks. These include seven unimodal functions (F1 to F7) and sixteen multimodal functions (F8 to F23), with detailed descriptions available in [59].

Fig. 2 demonstrates the ratio of exploration to exploitation throughout the iterations of the FNO, providing visual insights into how the algorithm navigates between global and local search strategies. Additionally, Table 2 presents the outcomes of the population diversity analysis, alongside exploration and exploitation metrics.

The simulation results highlight that FNO maintains high population diversity at the initial iterations, which gradually decreases as the iterations progress. This pattern indicates a shift from exploration to exploitation over time. Furthermore, the exploration-exploitation ratio for FNO frequently aligns closely with 0.00% exploration and 100% exploitation by the final iterations. These findings validate that the FNO effectively manages the balance between exploration and exploitation through its diverse population, ensuring robust performance across the iterative search process.

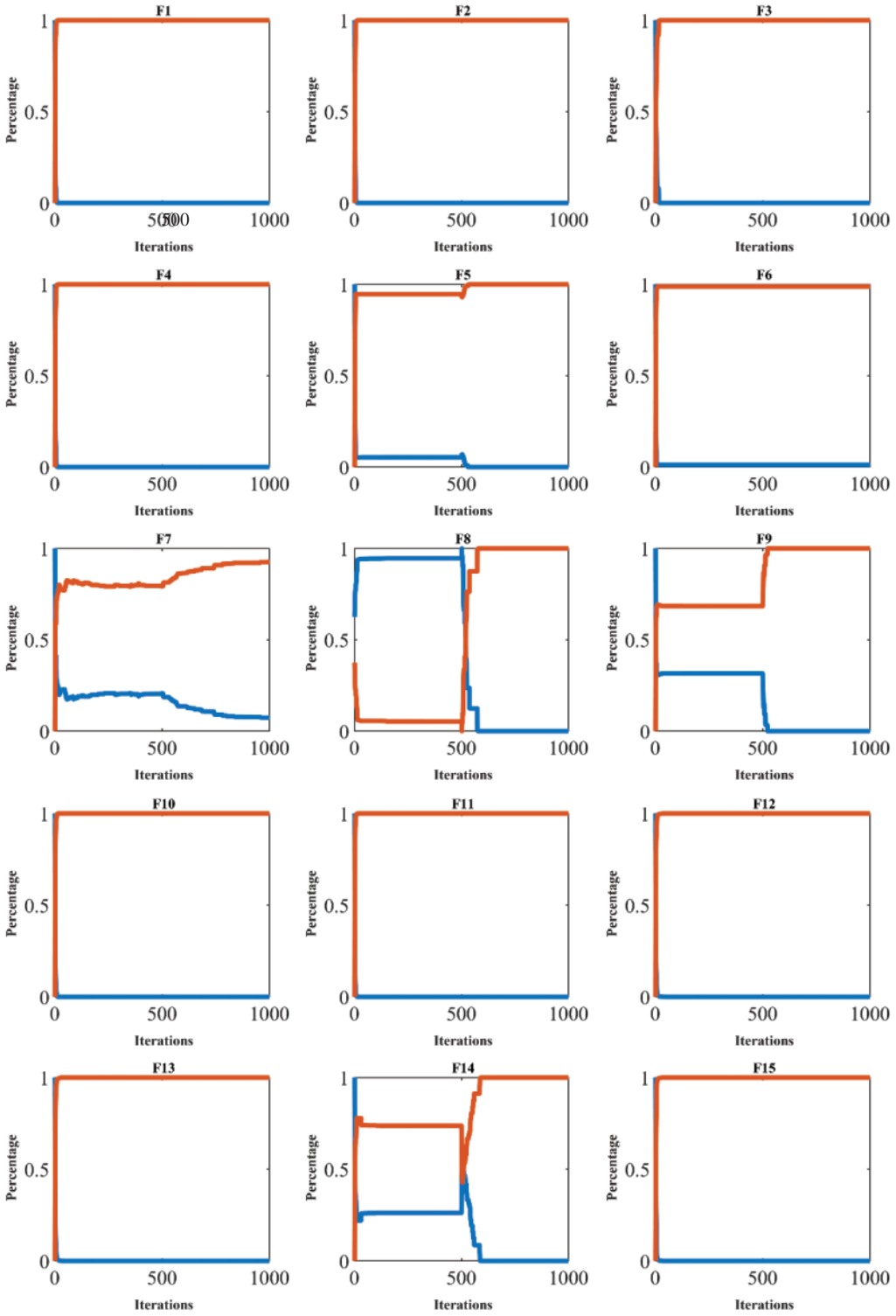

Figure 2: (Continued)

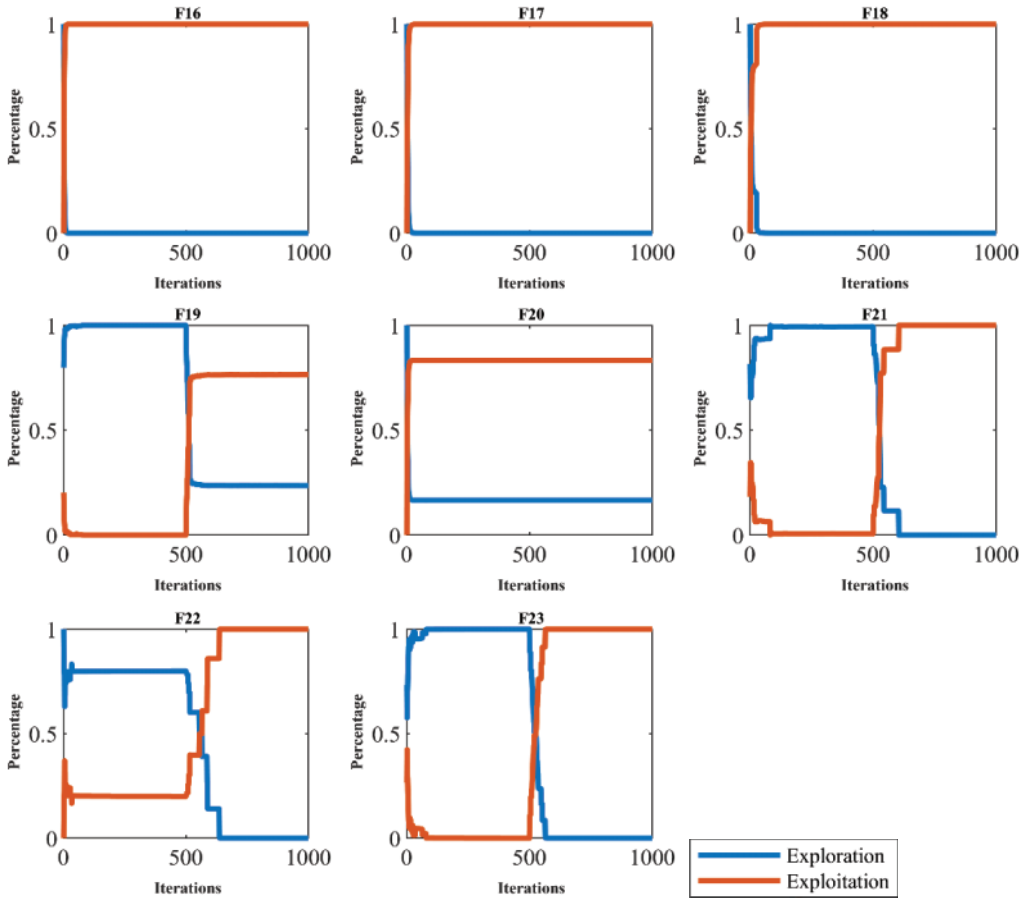

Figure 2: Exploration and exploitation of the FNO

Table 2: Population diversity, exploration, and exploitation percentage results

Function name	Exploration	Exploitation	Diversity	
			First iteration	Last iteration
F1	9.4E-165	1	129.6066	1.2E-162
F2	0	1	17.2539	0
F3	0	1	264.3119	0
F4	0	1	211.8951	0
F5	0	1	39.25608	0
F6	0.01236	0.98764	117.865	1.456773
F7	0.072464	0.927536	1.386376	0.100462
F8	5.96E-10	1	1290.08	1.23E-06
F9	4.04E-10	1	10.44909	4.23E-09
F10	1.70E-17	1	46.04662	7.82E-16
F11	3.61E-11	1	730.849	2.64E-08
F12	0	1	78.17809	0

(Continued)

Table 2 (continued)

Function name	Exploration	Exploitation	Diversity	
			First iteration	Last iteration
F13	0	1	83.89034	0
F14	2.32E-09	1	32.43996	7.54E-08
F15	4.37E-11	1	2.875723	1.26E-10
F16	0	1	1.629282	0
F17	1.28E-09	1	3.916857	5.01E-09
F18	2.9E-10	1	0.91886	2.67E-10
F19	0.23545	0.76455	0.378556	0.111708
F20	0.167065	0.832935	0.428116	0.071523
F21	8.75E-11	1	3.661421	3.92E-10
F22	2.79E-10	1	2.763374	7.70E-10
F23	5.79E-11	1	3.303417	2.61E-10

4 Simulation Studies and Results

In this section, the performance of the proposed FNO approach to solve optimization problems is evaluated. For this purpose, CEC 2017 test suite [59] for problem dimensions equal to 10, 30, 50, and 100, as well as to address CEC 2020 [60] have been selected.

4.1 Performance Comparison

The performance of FNO has been assessed by comparing its results with those of twelve established metaheuristic algorithms: GA, GSA, MVO, GWO, MPA, TSA, RSA, AVOA, and WSO. Table 3 specifies the parameter settings for each competing metaheuristic algorithm. To tackle the challenges posed by the CEC 2017 test suite, FNO and each competing algorithm were subjected to 51 independent runs, each consisting of 10,000·m (where m represents the number of variables) function evaluations (FEs). The optimization outcomes are evaluated using six statistical measures: mean, best, worst, standard deviation (std), median, and rank. The mean values serve as the basis for ranking the metaheuristic algorithms in their performance across the benchmark functions.

Table 3: Control parameters values

Algorithm	Parameter	Value
GA		
	Type	Real coded
	Selection	Roulette wheel (Proportionate)
	Crossover	Whole arithmetic (Probability = 0.8, $\alpha \in [-0.5, 1.5]$)
	Mutation	Gaussian (Probability = 0.05)
PSO		
	Topology	Fully connected
	Cognitive and social constant	$(C_1, C_2) = (2, 2)$

(Continued)

Table 3 (continued)

Algorithm	Parameter	Value
	Inertia weight	Linear reduction from 0.9 to 0.1
	Velocity limit	10% of dimension range
GSA		
	Alpha, G_0, R_{norm}, R_{power}	20, 100, 2, 1
TLBO		
	T_F: teaching factor	$T_F = \text{round}\,[(1 + rand)]$
	random number	*rand* is a random number between $[0 - 1]$.
GWO		
	Convergence parameter (a)	a: Linear reduction from 2 to 0.
MVO		
	Wormhole existence probability (WEP)	Min(WEP) = 0.2 and Max(WEP) = 1.
	Exploitation accuracy over the iterations (p)	$p = 6$.
WOA		
	Convergence parameter (a)	a: Linear reduction from 2 to 0.
	r is a random vector in $[0 - 1]$.	
	l is a random number in $[-1, 1]$.	
TSA		
	P_{min} and P_{max}	1, 4
	c1, *c2*, *c3*	Random numbers lie in the range of $[0 - 1]$.
MPA		
	Constant number	$P = 0.5$
	Random vector	R is a vector of uniform random numbers in $[0, 1]$.
	Fish Aggregating Devices (*FADs*)	$FAs = 0.2$
	Binary vector	$U = 0$ or 1
RSA		
	Sensitive parameter	$\beta = 0.01$
	Sensitive parameter	$\alpha = 0.1$
	Evolutionary Sense (ES)	ES: Randomly decreasing values between 2 and −2
AVOA		
	L_1, L_2	0.8, 0.2
	w	2.5
	P_1, P_2, P_3	0.6, 0.4, 0.6
WSO		
	F_{min} and F_{max}	0.07, 0.75
	τ, a_o, a_1, a_2	4.125, 6.25, 100, 0.0005

4.2 Evaluation CEC 2017 Test Suite

In this section, we assess the performance of FNO alongside competitor algorithms in addressing the CEC 2017 test suite across problem dimensions of 10, 30, 50, and 100. This test suite comprises thirty benchmark functions, encompassing unimodal, multimodal, hybrid, and composition functions. Specifically, it includes three unimodal functions (C17-F1 to C17-F3), seven multimodal functions (C17-F4 to C17-F10), ten hybrid functions (C17-F11 to C17-F20), and ten composition functions (C17-F21 to C17-F30). Notably, function C17-F2 is excluded from simulation studies due to its erratic behavior. Detailed information on the CEC 2017 test suite is accessible at [59]. The outcomes of applying metaheuristic algorithms to the CEC 2017 test suite are presented in Tables 4 to 7. Additionally, boxplot diagrams illustrating the performance of metaheuristic algorithms across the CEC 2017 test suite are depicted in Figs. 3 to 6.

According to the optimization outcomes, when tackling the CEC 2017 test suite with problem dimensions equal to 10 ($m = 10$), the proposed FNO approach emerges as the top-performing optimizer for functions C17-F1, C17-F3 to C17-F21, C17-F23, C17-F24, and C17-F26 to C17-F30. In the case of problem dimensions equal to 30 ($m = 30$), FNO proves to be the foremost optimizer for functions C17-F1, C17-F3 to C17-F22, C17-F24, C17-F25, and C17-F27 to C17-F30. Likewise, for problem dimensions of 50 ($m = 50$), FNO exhibits superiority as the primary optimizer for functions C17-F1, C17-F3 to C17-F25, and C17-F27 to C17-F30. Lastly, for problem dimensions of 100 ($m = 100$), FNO stands out as the premier optimizer for functions C17-F1, and C17-F3 to C17-F30.

The optimization outcomes highlight FNO's exceptional ability in exploration, exploitation, and maintain a harmonious balance between these strategies throughout the search procedure, resulting in the discovery of viable solutions for the benchmark functions. Examination of the simulation results reveals FNO's superior performance in tackling the challenges presented by the CEC 2017 test suite when compared to its competitors. FNO consistently outperforms other algorithms across a significant portion of the benchmark functions, earning the top rank as the most effective optimizer.

In addition to comparing the proposed approach of FNO and competing algorithms using statistical indicators and boxplot diagrams, it is valuable to present a comparison analysis of the computational cost and convergence speed between FNO and other competing metaheuristic algorithms. For this purpose, the results obtained from the analysis of the computational cost between FNO and competing algorithms in handling the CEC 2017 test suite for dimensions 10, 30, 50, and 100 are reported in Table 8. In order to analyze the computational cost of each algorithm in handling each benchmark function, the average execution time of each algorithm (in seconds) in different implementations on a benchmark function has been used. It should be mentioned that the experiments have been implemented on the software MATLAB R2022a using a 64-bit Core i7 processor with 3.20 GHz and 16 GB main memory. In fact, the results reported in Table 8 show that each algorithm needs a few seconds on average to run an implementation on each of the benchmark functions. The findings show that FNO has a lower computational cost compared to competing algorithms in most of the benchmark functions.

Also, with the aim of analyzing the convergence speed between FNO and competing algorithms, the convergence curves obtained from the performance of metaheuristic algorithms are drawn in Figs. 7 to 10. The findings show that FNO has different mechanisms to achieve the optimal solution during algorithm iterations. What is evident is that FNO, by balancing exploration and exploitation, has been able to handle most of the CEC 2017 test suite benchmark functions in a reasonable number of iterations.

Table 4: Optimization outcomes for the CEC 2017 test suite (dimension = 10)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F1	Mean	100	4.76E+09	5.66E+06	8.62E+09	3.54E+07	1.47E+09	1.11E+07	5661835.6	8.01E+07	1.30E+08	5.66E+06	5.66E+06	1.57E+07
	Best	100	3.95E+09	4.18E+03	7.47E+09	1.12E+04	3.15E+08	4.92E+06	9201.9453	2.52E+04	5.55E+07	1.88E+03	2.08E+03	7.19E+06
	Worst	100	6.09E+09	2.05E+07	1.03E+10	1.29E+08	3.20E+09	2.45E+07	20556443	2.91E+08	3.00E+08	2.05E+07	2.05E+07	3.17E+07
	Std	0.00E+00	9.78E+08	1.05E+07	1.33E+09	6.57E+07	1.35E+09	9.49E+06	10491366	1.49E+08	1.20E+08	1.05E+07	1.05E+07	1.17E+07
	Median	100	4.49E+09	1.04E+06	8.38E+09	6.49E+06	1.19E+09	7.48E+06	1040849.1	1.47E+07	8.22E+07	1.04E+06	1.04E+06	1.18E+07
	Rank	1	12	4	13	8	11	6	5	9	10	2	3	7
C17-F3	Mean	300	7420.6332	479.07361	8363.136	1411.7556	9674.4523	1683.6476	477.52226	2813.1747	837.06161	8877.6904	477.47621	12686.724
	Best	300	4047.3872	378.72942	4793.4295	793.1808	4003.6941	660.52489	378.7401	1414.8579	523.17669	5583.4037	378.72941	4074.2349
	Worst	300	9754.7015	661.5522	11027.212	2543.4504	13486.237	2935.2008	658.24031	5371.423	1100.3784	11990.252	658.13544	19926.992
	Std	0	2621.2477	136.59549	2995.4456	849.45664	4234.1132	1145.8495	135.64703	1920.2358	270.08019	2766.3585	135.60386	8740.8537
	Median	300	7940.2221	438.00641	8815.9512	1155.1956	10603.939	1569.4324	436.55432	2233.2089	862.34567	8968.5529	436.51999	13372.835
	Rank	1	9	4	10	6	12	7	3	8	5	11	2	13
C17-F4	Mean	400	851.10595	404.76442	1203.5993	406.43193	549.69733	421.98551	403.56824	410.66296	408.49557	404.59703	417.90241	413.17988
	Best	400	651.04451	401.44571	777.43767	402.45609	466.11707	407.25809	401.74359	405.53194	408.24557	403.4047	400.47988	411.51274
	Worst	400	1032.1503	405.9144	1621.7177	411.42675	646.50645	462.49338	404.5371	425.76359	408.89934	405.87998	459.81981	415.97178
	Std	0	182.90496	2.3279243	379.14117	4.4747736	93.308293	28.464626	1.3474457	10.590088	0.3171326	1.3582575	29.663767	2.186983
	Median	400	860.61449	405.84879	1207.6209	405.92243	543.0829	409.09529	403.99613	405.67816	408.41869	404.55171	405.65498	412.6175
	Rank	1	12	4	13	5	11	10	2	7	6	3	9	8
C17-F5	Mean	501.24636	555.44167	538.5083	563.03261	511.94581	555.82717	535.88604	521.16423	512.06635	529.99139	546.87355	524.747	524.84229
	Best	500.99515	542.86299	524.28716	550.49922	508.02119	537.72756	520.86599	509.58873	507.90451	524.99816	542.73113	510.14116	520.74543
	Worst	501.9917	563.45867	554.4502	575.74225	516.75578	583.08614	566.41609	533.28796	518.73037	533.45311	556.58047	545.00492	530.20678
	Std	0.522698	9.996335	16.711565	14.975468	4.803656	21.239825	22.489086	10.343269	4.900485	3.8382191	6.8854589	16.938782	4.4387094
	Median	500.99929	557.7225	537.64792	562.94449	511.50313	551.24749	528.13104	520.89011	510.81526	530.75714	544.0913	521.92097	524.20849
	Rank	1	11	9	13	2	12	8	4	3	7	10	5	6
C17-F6	Mean	600	627.83962	614.89965	634.91964	601.09529	621.32902	619.90548	601.91366	601.03807	605.94803	614.80197	606.43361	608.85652
	Best	600	624.46953	614.04332	632.135	600.6744	612.9704	606.50871	600.44288	600.54902	604.13958	602.6083	601.27146	605.97698
	Worst	600	630.70375	617.04936	638.55334	602.12967	634.7137	638.73196	603.7691	601.58336	608.75983	630.97865	616.56372	612.49364
	Std	0	3.0631164	1.51176	3.0217425	0.7279899	9.8689709	14.269137	1.5739663	0.4504082	2.2033685	13.816701	7.3085161	3.0264091
	Median	600	628.0926	614.25296	634.4951	600.78854	618.81598	617.19063	601.72132	601.00995	605.44636	612.81047	603.94963	608.47772
	Rank	1	12	9	13	3	11	10	4	2	5	8	6	7
C17-F7	Mean	711.12673	790.87231	758.69647	791.91696	723.66381	812.56483	755.72192	729.00651	724.8415	747.1352	717.22623	730.60269	734.13897
	Best	710.6726	774.70542	741.26705	781.70142	719.51427	779.35164	745.77109	716.90719	716.97424	744.14442	714.88378	724.08601	724.89382
	Worst	711.79949	804.92184	781.92139	802.57737	728.56053	847.89195	780.54267	746.61681	740.9555	753.90822	720.2424	740.31988	738.69955
	Std	0.538751	13.3631	19.939207	10.430973	4.0096476	31.385407	17.531888	13.208505	11.581012	4.8395511	2.5162099	7.4693104	6.684949
	Median	711.01742	791.93099	755.79872	791.69452	723.29023	811.50785	748.28695	726.25102	720.71813	745.24408	716.88936	729.00244	736.48125
	Rank	1	11	10	12	3	13	9	5	4	8	2	6	7

(Continued)

Table 4 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F8	Mean	801.49284	842.78691	827.81151	847.14615	812.00355	842.5128	832.29998	811.28633	814.72846	833.4538	818.16944	820.65722	815.53611
	Best	800.99496	835.88776	818.14478	837.51314	808.34501	828.61797	817.35463	807.7985	809.78114	827.82098	811.73209	814.88054	812.40758
	Worst	801.99121	849.58647	841.64225	851.2519	813.96612	858.65958	842.72897	815.00262	819.28739	839.91548	824.44168	826.30895	821.82891
	Std	0.6047211	6.5728026	10.403119	6.8325399	2.7714905	14.035102	11.481443	3.1089481	4.1863009	6.6741068	5.7336457	6.0656766	4.4817738
	Median	801.49259	842.83671	825.72951	849.90977	812.85153	841.38683	834.55815	811.1721	814.92264	833.03937	818.25199	820.71969	813.95398
	Rank	1	12	8	13	3	11	9	2	4	10	6	7	5
C17-F9	Mean	900	1348.6519	1147.5446	1386.6599	905.22904	1312.4997	1307.8602	901.46319	910.99909	910.9071	900.77674	904.41039	905.15499
	Best	900	1225.8335	946.05487	1303.8944	900.31783	1130.4121	1049.9878	900.06388	900.52832	908.35124	900.03731	900.87844	903.24706
	Worst	900	1468.9325	1554.1004	1505.7946	913.58498	1558.6494	1547.8733	903.51813	930.53534	917.19838	902.15635	910.61526	907.81342
	Std	0.00E+00	115.18992	296.38668	90.403133	6.3213379	195.12276	220.30744	1.7701066	14.808567	4.4564475	1.0457564	4.4872647	2.0242914
	Median	900	1349.9208	1045.0116	1368.4753	903.50668	1280.4687	1316.7899	901.13537	906.46634	909.03938	900.45664	903.07393	904.77974
	Rank	1	12	9	13	6	11	10	3	8	7	2	4	5
C17-F10	Mean	1006.179	2153.3678	1706.8564	2386.1904	1485.3041	1923.1247	1916.6759	1709.3747	1662.391	2041.4113	2131.4303	1849.4835	1654.1263
	Best	1000.2838	1931.3951	1445.7148	2241.2707	1367.416	1706.2859	1428.3869	1425.3817	1492.6378	1698.2813	1894.2644	1510.9421	1398.9345
	Worst	1012.6676	2304.0115	2246.6561	2686.2499	1565.7032	2125.5261	2378.2974	2129.834	1905.419	2302.6484	2216.081	2210.7234	2006.2522
	Std	7.0021346	174.80971	393.27276	215.14523	93.386807	237.50991	478.77417	353.81086	184.64503	268.32516	166.46054	302.11784	274.62709
	Median	1005.8824	2189.0322	1567.5274	2308.6204	1504.0486	1930.3435	1930.0096	1641.1416	1625.7536	2082.3577	2207.688	1838.1342	1605.6593
	Rank	1	12	5	13	2	9	8	6	4	10	11	7	3
C17-F11	Mean	1100	3442.4451	1144.6592	3547.5511	1126.4766	4797.7396	1146.7386	1126.8673	1150.3948	1146.6999	1136.7697	1140.4431	2190.5438
	Best	1100	2387.5952	1122.7124	1406.7662	1112.5779	4670.2627	1119.2473	1107.5907	1119.7139	1134.9488	1124.9118	1129.015	1121.0143
	Worst	1100	4459.2232	1187.6426	5659.4764	1158.0748	4864.4887	1164.8368	1142.8398	1217.0461	1162.6632	1159.8287	1163.3642	5237.4207
	Std	0	980.87612	31.04434	2011.8945	22.555088	91.554661	21.933729	17.382694	47.729054	12.391583	16.515448	16.358967	2137.1669
	Median	1100	3461.4811	1134.1409	3561.9809	1117.627	4828.1036	1151.4351	1128.5193	1132.4095	1144.5937	1131.169	1134.6967	1201.8702
	Rank	1	11	6	12	2	13	8	3	9	7	4	5	10
C17-F12	Mean	1352.9587	3.00E+08	1.03E+06	5.99E+08	5.74E+05	9.75E+05	2.09E+06	9.66E+05	1.29E+06	4.38E+06	9.58E+05	9.84E+04	6.06E+05
	Best	1318.6465	6.73E+07	4.24E+05	1.33E+08	1.99E+04	4.61E+05	2.44E+05	1.06E+05	4.17E+04	1.25E+06	5.25E+05	1.22E+04	2.70E+05
	Worst	1438.1762	5.25E+08	1.70E+06	1.05E+09	8.98E+05	1.20E+06	3.46E+06	2.75E+06	2.03E+06	7.74E+06	1.47E+06	1.55E+05	1.01E+06
	Std	60.273395	2.43E+08	6.48E+05	4.87E+08	406857.01	3.67E+05	1.54E+06	1.27E+06	9.20E+05	3.58E+06	4.24E+05	6.48E+04	3.31E+05
	Median	1327.506	3.05E+08	9.92E+05	6.08E+08	6.89E+05	1.12E+06	2.33E+06	5.04E+05	1.55E+06	4.27E+06	9.20E+05	1.13E+05	5.73E+05
	Rank	1	12	8	13	3	7	10	6	9	11	5	2	4
C17-F13	Mean	1305.324	1.46E+07	16351.25	2.92E+07	5393.7543	1.16E+04	7.22E+03	6493.1508	9.53E+03	1.50E+04	9.33E+03	6.40E+03	4.70E+04
	Best	1303.1138	1.22E+06	3354.2064	2.43E+06	3693.0197	7.23E+03	3.57E+03	2218.3639	6060.3467	1.39E+04	5.04E+03	3.06E+03	8.04E+03
	Worst	1308.5079	4.85E+07	27217.996	9.70E+07	6687.0615	1.77E+04	1.36E+04	11302.106	1.33E+04	1.69E+04	1.31E+04	1.50E+04	1.54E+05
	Std	2.3907745	2.38E+07	13090.479	4.76E+07	1442.5993	4.69E+03	4.76E+03	5069.6506	3.11E+03	1.39E+03	3.53E+03	6.06E+03	75055.794
	Median	1304.8371	4.36E+06	17416.399	8.72E+06	5597.4679	1.07E+04	5.83E+03	6226.0667	9.39E+03	1.45E+04	9.60E+03	3.78E+03	1.30E+04
	Rank	1	12	10	13	2	8	5	4	7	9	6	3	11

(Continued)

Table 4 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F14	Mean	1400.7462	3666.7795	1990.1139	4818.0713	1920.6143	3150.8267	1562.4793	1607.447	2266.1706	1623.5515	5003.9687	2818.3796	11294.412
	Best	1400	2897.6488	1694.9945	4194.1896	1434.4845	1480.5258	1473.7195	1425.3931	1457.1239	1504.519	4127.3818	1432.3512	3383.6173
	Worst	1400.995	4836.903	2620.3688	6080.2525	2909.0859	4962.7505	1755.1245	2135.2382	4661.1276	1818.4585	6864.1109	6034.232	22182.724
	Std	0.5233088	893.59526	448.17093	911.81891	727.89443	1884.0922	138.61727	370.16242	1679.565	142.5087	1351.6216	2291.0016	8348.5021
	Median	1400.995	3466.2832	1822.5462	4498.9215	1669.4434	3080.0152	1510.5367	1434.5783	1473.2155	1585.6142	4512.1911	1903.4675	9805.6528
	Rank	1	10	6	11	5	9	2	3	7	4	12	8	13
C17-F15	Mean	1500.3314	9.25E+03	5008.9488	1.23E+04	3884.4658	6.46E+03	5791.6279	1814.4186	5.45E+03	1.96E+03	2.08E+04	8.15E+03	4372.7758
	Best	1500.0007	3.08E+03	2120.6899	2.85E+03	3099.4047	2.50E+03	2286.5546	1655.8502	3394.4168	1.77E+03	1.01E+04	3015.4991	2165.2967
	Worst	1500.5	1.59E+04	11312.08	2.64E+04	4717.945	1.10E+04	1.20E+04	1894.8258	6.44E+03	2.10E+03	3.10E+04	1.31E+04	7339.412
	Std	0.2476483	5.65E+03	4465.54	1.08E+04	697.42725	3.86E+03	4479.5662	113.62289	1.47E+03	1.58E+02	1.05E+04	4.43E+03	2669.2889
	Median	1500.4125	9.00E+03	3301.5127	9.98E+03	3860.2567	6.16E+03	4443.0871	1853.4991	5.98E+03	1.98E+03	2.11E+04	8232.3278	3993.1973
	Rank	1	11	6	12	4	9	8	2	7	3	13	10	5
C17-F16	Mean	1600.7598	1959.1164	1786.5679	1962.4531	1679.9885	1988.7089	1906.3858	1792.2412	1717.6287	1673.7548	2010.7068	1883.5623	1780.4292
	Best	1600.3563	1897.2686	1650.5592	1801.1325	1636.6174	1824.2438	1749.9153	1708.7038	1614.5751	1651.5043	1896.2632	1798.6825	1715.5748
	Worst	1601.1203	2092.2505	1878.5166	2196.4551	1712.2718	2146.7995	2008.6371	1850.9513	1806.3395	1726.1708	2182.6426	2019.2319	1806.6717
	Std	0.3323135	94.264386	101.52865	176.48528	33.687183	154.44341	132.44575	63.003168	83.228217	37.071147	135.40165	105.70158	45.79671
	Median	1600.7812	1923.4733	1808.5979	1926.1125	1685.5323	1991.8962	1933.4954	1804.6548	1724.8	1658.672	1981.9607	1858.1674	1799.7351
	Rank	1	10	6	11	3	12	9	7	4	2	13	8	5
C17-F17	Mean	1700.0992	1805.0034	1747.8063	1805.1308	1734.8402	1791.3669	1825.1395	1825.8888	1762.734	1754.1053	1829.2845	1748.9843	1752.0637
	Best	1700.0202	1795.3463	1730.8768	1797.4	1721.078	1775.6183	1765.2283	1768.3914	1722.4174	1743.4809	1743.2184	1740.5002	1747.5758
	Worst	1700.3319	1812.321	1783.2541	1810.9758	1774.8212	1798.8067	1872.1579	1924.1523	1857.0616	1766.7008	1943.4015	1757.3931	1760.0577
	Std	0.1632193	7.508255	25.241524	5.9769223	28.040026	11.267494	47.777177	77.082108	66.487125	11.988861	106.39812	7.9036705	5.9769379
	Median	1700.0223	1806.1731	1738.5472	1806.0737	1721.7308	1795.5213	1831.5858	1805.5057	1735.7284	1753.1197	1815.2591	1749.0219	1750.3107
	Rank	1	9	3	10	2	8	11	12	7	6	13	4	5
C17-F18	Mean	1805.3596	2425001.8	11498.264	4834517.4	10813.884	11670.033	21210.616	19208.262	18325.247	26467.103	9679.6755	19996.51	12310.609
	Best	1800.0028	126560.07	6431.6407	241579.24	4092.6721	8655.306	6261.9389	8172.5536	5929.291	20914.706	6214.6236	4765.8125	5236.974
	Worst	1820.4506	7025868	15314.063	1.40E+07	16096.374	14381.1	31617.441	30679.343	30813.877	32092.843	12379.334	35347.946	17762.149
	Std	10.585845	3361898.8	3945.5162	6721890.8	5904.2647	2466.1447	12996.961	10652.537	13273.875	5255.1353	2774.0167	17342.485	5546.9078
	Median	1800.4924	1273789.7	12123.675	2532128.6	11533.245	11821.863	23481.542	18990.577	18278.91	26430.432	10062.372	19936.141	13121.657
	Rank	1	12	4	13	3	5	10	8	7	11	2	9	6
C17-F19	Mean	1900.4447	3.37E+05	6203.3703	5.98E+05	5261.7861	1.07E+05	3.00E+04	2137.1057	5.08E+03	4.50E+03	3.48E+04	2.17E+04	5757.4089
	Best	1900.0392	2.16E+04	2144.7479	3.93E+04	2264.3986	2.10E+03	6903.4991	1916.4041	1940.9054	2.02E+03	1.05E+04	2632.5024	2933.2963
	Worst	1901.5593	7.11E+05	12289.95	1.28E+06	8278.5987	2.13E+05	5.51E+04	2675.5494	1.28E+04	1.09E+04	5.01E+04	6.55E+04	8674.2095
	Std	0.7832733	3.13E+05	5145.6529	5.90E+05	3387.5079	1.27E+05	20825.106	382.35928	5.45E+03	4.50E+03	1.87E+04	3.11E+04	2503.294
	Median	1900.0902	3.08E+05	5189.3919	5.34E+05	5252.0736	1.06E+05	2.91E+04	1978.2347	2.80E+03	2.54E+03	3.93E+04	9.28E+03	5711.065
	Rank	1	12	7	13	5	11	9	2	4	3	10	8	6

(Continued)

Table 4 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F20	Mean	2000.3122	2193.3805	2155.6474	2200.1488	2089.2879	2186.8782	2186.2131	2129.352	2155.0998	2072.0504	2226.1675	2154.3058	2053.5693
	Best	2000.3122	2143.6293	2042.4575	2149.7325	2070.1713	2099.8897	2099.2859	2049.9965	2119.5065	2060.184	2169.4557	2133.0467	2040.5576
	Worst	2000.3122	2250.4329	2258.2612	2252.0631	2120.0696	2280.6586	2252.6492	2218.3412	2224.5441	2080.0928	2310.0411	2178.8048	2058.5614
	Std	0	46.269775	102.50676	51.845353	22.640443	79.824499	78.07532	72.579052	49.808882	9.9783738	71.823513	22.507088	9.1631705
	Median	2000.3122	2189.7299	2160.9355	2199.3999	2083.4553	2183.4823	2196.4586	2124.5352	2138.1743	2073.9624	2212.5866	2152.6858	2057.579
	Rank	1	11	8	12	4	10	9	5	7	3	13	6	2
C17-F21	Mean	2200	2286.3188	2219.027	2264.2827	2255.8574	2313.5537	2300.5565	2252.4169	2303.4731	2291.9248	2350.1747	2308.1456	2290.6345
	Best	2200	2246.0953	2211.1318	2227.6262	2253.473	2225.2939	2222.8853	2207.0423	2299.6298	2210.4489	2335.0681	2301.0321	2230.1704
	Worst	2200	2308.241	2240.4068	2284.8449	2258.331	2353.3982	2337.7968	2298.616	2308.012	2324.4928	2364.8482	2314.5411	2319.771
	Std	0	30.60076	15.023608	26.722181	2.1478202	62.994544	55.126044	54.764002	3.6261999	57.518479	13.059611	7.0419485	42.930214
	Median	2200	2295.4694	2212.2847	2272.3299	2255.8127	2337.7613	2320.7719	2252.0047	2303.1254	2316.3788	2350.3912	2308.5046	2306.2983
	Rank	1	6	2	5	4	12	9	3	10	8	13	11	7
C17-F22	Mean	2300.0725	2671.6169	2308.1915	2823.6641	2304.8121	2652.0979	2320.7772	2288.4799	2307.8667	2317.1867	2300.5656	2311.8331	2315.79
	Best	2300	2565.0503	2305.1527	2646.2284	2300.8833	2426.7632	2316.404	2240.147	2301.1577	2311.8731	2300.0818	2300.6237	2312.8473
	Worst	2300.2902	2790.6254	2309.5498	2954.7721	2309.3983	2829.5451	2326.7908	2305.0614	2320.4814	2328.0419	2301.4464	2338.7724	2319.5903
	Std	0.1526151	109.42018	2.1466184	136.53847	3.7261119	188.88785	4.8595222	33.897008	9.3491972	7.9192899	0.6602797	18.97897	3.0344778
	Median	2300	2665.3959	2309.0318	2846.828	2304.4835	2676.0417	2319.957	2304.3556	2304.9138	2314.416	2300.3671	2303.9681	2315.3613
	Rank	2	12	6	13	4	11	10	1	5	9	3	7	8
C17-F23	Mean	2600.9194	2683.5874	2636.8034	2686.5847	2613.1558	2706.0115	2642.4583	2618.2085	2612.6594	2637.2078	2764.1977	2638.6902	2648.7811
	Best	2600.0029	2648.1609	2626.7669	2661.7054	2610.8654	2630.489	2627.6187	2607.3298	2607.3883	2628.3281	2709.1096	2632.3491	2632.1783
	Worst	2602.8695	2704.5253	2652.2916	2721.4998	2615.8269	2743.4006	2659.4206	2628.4298	2618.7466	2644.9101	2881.5475	2648.4222	2655.6101
	Std	1.3888855	27.52311	12.486654	29.241572	2.5543836	53.776504	18.013295	9.6077211	6.2223052	7.8882033	85.342398	7.5422556	11.78494
	Median	2600.4025	2690.8317	2634.0776	2681.5667	2612.9654	2725.0781	2641.397	2618.5372	2612.2513	2637.7965	2733.0668	2636.9947	2653.668
	Rank	1	10	5	11	3	12	8	4	2	6	13	7	9
C17-F24	Mean	2630.4876	2763.2738	2754.8101	2824.8243	2638.2762	2670.3005	2748.8576	2683.0874	2738.7893	2744.7922	2737.6729	2753.0706	2716.9546
	Best	2516.677	2707.9507	2720.7595	2796.5348	2615.172	2555.4111	2715.2998	2517.064	2706.9475	2723.2862	2523.3395	2739.3286	2568.7324
	Worst	2732.3195	2839.7566	2778.2909	2885.6158	2653.6271	2800.816	2783.5195	2756.1684	2757.6746	2762.2237	2874.7403	2778.9987	2800.0354
	Std	122.54978	62.590977	25.634392	43.197622	18.587946	138.58396	30.415546	117.7466	23.162993	20.387004	158.54861	19.519292	106.9475
	Median	2636.4769	2752.6939	2760.095	2808.5732	2642.1528	2662.4874	2748.3054	2729.5587	2745.2676	2746.8295	2776.3058	2746.9776	2749.5253
	Rank	1	12	11	13	2	3	9	4	7	8	6	10	5
C17-F25	Mean	2932.6395	3126.3222	2916.7491	3225.6256	2920.4965	3103.9039	2911.691	2924.0714	2938.1804	2933.7873	2924.2164	2925.1207	2949.6981
	Best	2898.0475	3048.7781	2901.8847	3168.6477	2916.4387	2911.6293	2790.2716	2906.2796	2923.3805	2916.3464	2907.655	2901.5215	2935.1339
	Worst	2945.7929	3298.4504	2948.351	3287.6299	2926.5165	3546.8314	2954.8275	2943.8258	2945.6736	2951.007	2943.4285	2945.3138	2960.0259
	Std	24.288727	121.91252	22.397416	51.844628	5.0051593	314.00578	85.191096	21.078025	10.699238	19.613827	19.576532	24.450935	11.29632
	Median	2943.3588	3079.0301	2908.3804	3223.1123	2919.5154	2978.5775	2950.8324	2923.0901	2941.8338	2933.8979	2922.891	2926.8237	2951.8163
	Rank	7	12	2	13	3	11	1	4	9	8	5	6	10

(Continued)

Table 4 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F26	Mean	2900	3519.927	2991.5403	3652.0496	3018.597	3536.7641	3164.3649	2923.7389	3234.3753	3184.4516	3741.6631	2927.0664	2921.2454
	Best	2900	3268.7401	2833.0076	3418.0377	2897.7673	3112.2192	2988.2513	2904.639	2963.3632	2922.3718	2833.0075	2885.9911	2740.6481
	Worst	2900	3717.1511	3183.5625	3927.2774	3299.8995	4077.8509	3494.9418	2965.1976	3821.8745	3795.145	4197.7742	2997.3981	3091.0648
	Std	3.906E-13	233.15046	194.35472	236.07564	198.30374	481.02655	241.66989	29.339685	415.79741	430.48989	651.23366	51.036749	172.70929
	Median	2900	3546.9084	2974.7955	3631.4416	2938.3605	3478.4932	3087.1332	2912.5594	3076.1317	3010.1449	3967.9354	2912.4381	2926.6343
	Rank	1	10	5	12	6	11	7	3	9	8	13	4	2
C17-F27	Mean	3089.518	3192.3719	3117.1701	3211.6981	3104.1448	3167.807	3180.9036	3093.0319	3113.8587	3112.9921	3207.3632	3130.842	3151.1992
	Best	3089.518	3149.4565	3095.0187	3121.9033	3092.1545	3101.0776	3171.3047	3090.0749	3094.0208	3095.0853	3195.6391	3096.2821	3115.2083
	Worst	3089.518	3253.6555	3172.9159	3373.951	3132.8375	3202.3755	3189.5088	3095.322	3169.3913	3159.3994	3224.2975	3175.0172	3200.1882
	Std	2.762E-13	46.122103	39.183452	116.90737	20.243137	49.442778	7.8477851	2.4854051	38.982802	32.648572	12.707645	34.9409	37.339828
	Median	3089.518	3183.1878	3100.373	3175.4691	3095.7935	3183.8875	3181.4005	3093.3653	3096.0114	3098.7419	3204.7582	3126.0344	3144.7
	Rank	1	11	6	13	3	9	10	2	5	4	12	7	8
C17-F28	Mean	3100	3560.0545	3231.458	3692.9092	3216.5335	3528.9783	3274.532	3233.6835	3323.9279	3307.0585	3413.8264	3290.5561	3240.1611
	Best	3100	3520.6036	3118.7891	3625.9508	3162.9819	3378.5307	3162.9764	3106.2187	3186.5686	3216.9812	3404.952	3184.3081	3144.2455
	Worst	3100	3599.0674	3352.7959	3745.7833	3242.0142	3709.7028	3365.906	3365.4723	3385.2633	3365.6769	3432.4598	3352.9785	3470.0257
	Std	0	36.051086	109.17755	55.633329	38.366679	181.80825	105.72227	147.77663	97.083903	71.523896	13.223228	82.542029	162.3039
	Median	3100	3560.2735	3227.1236	3699.9513	3230.5689	3513.8399	3284.6228	3231.5214	3361.9399	3322.788	3408.9468	3312.4689	3173.1867
	Rank	1	12	3	13	2	11	6	4	9	8	10	7	5
C17-F29	Mean	3132.2407	3307.6202	3270.4257	3347.5646	3201.1466	3229.3398	3325.1795	3200.791	3253.967	3209.2618	3322.6187	3254.7089	3230.1883
	Best	3130.0764	3288.2063	3202.7142	3282.2068	3164.9039	3164.9358	3230.6385	3144.9381	3185.1782	3171.0395	3228.5479	3166.5459	3183.9606
	Worst	3134.8406	3331.411	3334.4362	3406.4848	3244.032	3284.3083	3445.476	3273.8423	3358.7537	3236.1876	3570.3166	3320.4689	3267.328
	Std	2.6112316	22.52269	72.885701	69.440415	37.027887	51.594863	93.984831	56.390324	86.896578	29.649665	174.17385	73.331447	37.989204
	Median	3132.0229	3305.4317	3272.2761	3350.7835	3197.8253	3234.0576	3312.3017	3192.1919	3235.9681	3214.9101	3245.8052	3265.9104	3234.7323
	Rank	1	10	9	13	3	5	12	2	7	4	11	8	6
C17-F30	Mean	3418.7336	1.94E+06	310177.96	3.17E+06	411838.42	5.81E+05	9.01E+05	317081.66	8.53E+05	1.12E+05	7.23E+05	3.89E+05	1.35E+06
	Best	3394.6821	1.23E+06	136759.81	7.88E+05	15956.556	1.81E+05	6.98E+04	8764.3179	3.09E+04	2.73E+04	5.97E+05	9.22E+03	5.32E+05
	Worst	3442.9073	2.83E+06	716359.94	4.92E+06	604678.05	1.10E+06	3.26E+06	1.07E+06	1.23E+06	1.54E+05	8.49E+05	7.36E+05	2.95E+06
	Std	29.212532	6.95E+05	286649.53	1.83E+06	282891.92	4.13E+05	1.66E+06	527102.97	5.95E+05	6.24E+04	1.12E+05	4.12E+05	1198764.2
	Median	3418.6725	1.85E+06	193796.04	3.49E+06	513359.55	5.20E+05	1.37E+05	96977.566	1.07E+06	1.33E+05	7.24E+05	4.05E+05	9.68E+05
	Rank	1	12	3	13	6	7	10	4	9	2	8	5	11
Sum rank		36	320	178	352	107	285	240	117	189	192	240	184	199
Mean rank		1.2413793	11.034483	6.137931	12.137931	3.6896552	9.8275862	8.2758621	4.0344828	6.5172414	6.6206897	8.2758621	6.3448276	6.862069
Total rank		1	11	4	12	2	10	9	3	6	7	9	5	8

Table 5: Optimization outcomes for the CEC 2017 test suite (dimension = 30)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F1	Mean	100	2.23E+10	1.07E+08	3.48E+10	1.07E+08	1.52E+10	1.54E+09	1.07E+08	1.51E+09	5.31E+09	1.16E+08	1.29E+09	2.57E+08
	Best	100	1.91E+10	17600000	3.09E+10	17600000	9.54E+09	1.15E+09	18189683	2.49E+08	3.34E+09	17600000	17700000	1.46E+08
	Worst	100	2.78E+10	3.22E+08	4.27E+10	3.22E+08	2.06E+10	2.1E+09	3.22E+08	4.56E+09	7.77E+09	3.26E+08	4.77E+09	4.52E+08
	Std	8.63E-15	4.25E+09	1.52E+08	5.66E+09	1.52E+08	5.52E+09	4.75E+08	1.52E+08	2.15E+09	1.93E+09	1.51E+08	2.44E+09	1.41E+08
	Median	100	2.11E+10	44100000	3.27E+10	44100000	1.54E+10	1.45E+09	44437917	6.24E+08	5.06E+09	59500000	1.91E+08	2.15E+08
	Rank	1	12	2	13	3	11	9	4	8	10	5	7	6
C17-F3	Mean	300	84750.61	40376.27	64739.13	3625.891	42481.21	198183.4	4193.116	37817.86	31930.31	83495.36	29578.43	143656.5
	Best	300	77668.66	23140.75	50782.13	3384.862	40702.15	164739.7	3858.621	33041.33	27571.3	72571.02	21862.74	109023.7
	Worst	300	92774.38	51735.07	70079.09	3838.437	44279.7	227261.9	4466.153	42232.35	34669.78	91714.8	37534.37	198536.4
	Std	0	7700.572	12819.59	9806.415	252.9757	2021.714	27317.34	310.4686	3966.306	3246.36	9119.259	7411.365	44640.38
	Median	300	84279.7	43314.64	69047.65	3640.133	42471.5	200366	4223.844	37998.88	32740.08	84847.8	29458.3	133533
	Rank	1	11	7	9	2	8	13	3	6	5	10	4	12
C17-F4	Mean	458.5616	5514.996	513.1326	8362.45	494.9801	3911.395	801.7626	498.0427	561.3617	844.6749	580.6721	605.4398	763.8951
	Best	458.5616	3132.861	494.7781	5388.965	488.1533	962.903	747.54	493.4245	510.8723	670.7046	564.4267	514.6421	720.829
	Worst	458.5616	7439.73	529.0903	11659.91	509.7619	6450.92	872.2656	505.8455	589.8111	1178.504	601.0365	765.3713	784.5743
	Std	0	1877.646	14.8523	2737.94	10.57664	2438.42	60.80812	5.884109	36.48582	238.8188	17.8775	122.1118	30.70809
	Median	458.5616	5743.697	514.3309	8200.463	491.0026	4115.879	793.6224	496.4504	572.3818	764.7457	578.6126	570.8729	775.0885
	Rank	1	12	4	13	2	11	9	3	5	10	6	7	8
C17-F5	Mean	502.4874	799.4111	698.497	832.5222	578.375	756.4103	781.1346	608.9617	611.0662	736.4978	696.3058	620.0834	679.0406
	Best	500.995	781.5667	666.3582	809.5298	561.1385	731.2881	758.4144	593.6933	574.1608	719.1954	681.6443	596.7078	634.9207
	Worst	503.9798	819.2639	745.4427	863.1647	595.1056	786.6623	790.1124	640.445	637.2454	756.0871	717.5099	660.8697	734.0958
	Std	1.351177	16.2677	37.1795	25.80494	15.42662	28.11175	15.98737	22.36284	32.78049	18.73322	17.98576	29.65458	43.17018
	Median	502.4874	798.4069	691.0935	828.6972	578.6279	753.8454	788.0058	600.8543	616.4294	735.3544	693.0346	611.3781	673.573
	Rank	1	12	8	13	2	10	11	3	4	9	7	5	6
C17-F6	Mean	600	667.9712	640.0658	670.6346	603.5126	665.5659	664.9208	621.2911	610.7685	637.2181	648.316	640.2583	626.1605
	Best	600	666.8004	637.9333	666.6184	602.5929	652.1961	655.6832	611.336	604.2382	631.6804	647.8989	630.4598	619.7779
	Worst	600	669.1425	642.6248	675.7851	604.7565	673.1688	669.0234	632.4298	617.1937	646.5213	649.2118	648.8411	629.9802
	Std	6.9E-14	1.136042	2.034682	4.578038	1.026945	10.45666	6.564253	10.50084	5.609068	6.910974	0.636862	8.645146	4.715387
	Median	600	667.971	639.8525	670.0674	603.3504	668.4493	667.4884	620.6993	610.8211	635.3354	648.0766	640.8662	627.4421
	Rank	1	12	7	13	2	11	10	4	3	6	9	8	5
C17-F7	Mean	733.478	1219.29	1091.623	1253.658	839.0586	1156.961	1226.86	845.2525	871.9205	1031.863	943.9246	865.7492	940.7737
	Best	732.8186	1174.469	994.7341	1241.523	818.8062	1029.695	1191.149	802.7711	808.0542	958.3184	904.9486	850.421	909.8576
	Worst	734.5199	1251.416	1230.863	1274.244	885.4101	1284.733	1297.729	903.1913	908.5046	1097.755	998.8954	884.294	987.8676
	Std	0.793172	34.9786	109.401	15.50187	33.08332	115.8256	52.06295	45.25291	46.244	78.41539	42.95978	15.65768	35.03757
	Median	733.2867	1225.637	1070.447	1249.433	826.009	1156.709	1209.282	837.5238	885.5616	1035.689	935.9273	864.1409	932.6847
	Rank	1	11	9	13	2	10	12	3	5	8	7	4	6

(Continued)

Table 5 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F8	Mean	803.3298	1046.685	932.7806	1078.193	883.0652	1024.811	1000.726	885.379	884.2528	994.1707	942.7767	910.855	963.5851
	Best	801.2023	1033.646	907.0712	1060.65	877.565	987.1286	953.1646	859.0602	877.9547	977.7503	922.0261	900.5087	950.1541
	Worst	804.1574	1063.417	951.4572	1101.524	889.8239	1112.305	1035.907	910.7902	891.4972	1021.979	965.6326	924.7244	980.5287
	Std	1.494596	14.36074	21.0321	21.8794	5.319828	61.97885	37.30537	23.7039	6.199516	20.22222	20.10029	11.32729	16.37696
	Median	803.9798	1044.838	936.2969	1075.298	882.436	999.9053	1006.917	885.8327	883.7797	988.4766	941.724	909.0934	961.8287
	Rank	1	12	6	13	2	11	10	4	3	9	7	5	8
C17-F9	Mean	900	9461.472	4283.897	9175.401	1131.329	9903.382	9513.784	4815.659	1968.546	5083.842	3656.238	3207.913	1307.076
	Best	900	8166.16	3239.019	8920.075	1051.039	6168.601	7339.868	3929.968	1479.72	3783.026	3170.149	1967.19	1095.069
	Worst	900	10694.1	4818.392	9287.292	1227.411	13267.34	11269.31	7266.008	2683.006	7532.341	4319.993	4827.528	1450.387
	Std	6.9E-14	1103.407	747.843	180.2673	84.8897	3087.953	2069.596	1720.807	614.1961	1791.942	519.4671	1271.33	174.0566
	Median	900	9492.814	4539.089	9247.118	1123.433	10088.8	9722.979	4033.331	1855.728	4510	3567.405	3018.468	1341.425
	Rank	1	11	7	10	2	13	12	8	4	9	6	5	3
C17-F10	Mean	2293.267	6617.502	5124.519	7195.948	3888.845	6060.538	6006.836	4446.157	4563.693	7212.543	4613.82	4776.043	5708.204
	Best	1851.756	6108.489	4529.878	6452.183	3597.103	4864.181	5231.715	4208.484	4105.32	6914.032	4395.01	4544.5	5275.655
	Worst	2525.027	6861.894	5538.909	7704.967	4250.525	6543.559	7115.713	4800.919	4825.055	7338.563	4939.68	5194.373	6169.657
	Std	315.9698	362.4263	515.0489	560.0898	309.6836	842.0577	870.4231	301.3097	335.4876	211.5766	272.3037	303.5602	437.3793
	Median	2398.142	6749.814	5214.646	7313.321	3853.877	6417.205	5839.958	4387.612	4662.199	7298.789	4560.295	4682.649	5693.752
	Rank	1	11	7	12	2	10	9	3	4	13	5	6	8
C17-F11	Mean	1102.987	6582.375	1304.218	7680.113	1229.656	4576.876	6846.279	1351.868	2096.195	1921.005	2690.562	1297.058	7990.261
	Best	1100.995	5419.241	1248.01	6257.126	1149.5	3455.679	5125.362	1266.483	1363.82	1721.828	2096.592	1220.331	3037.177
	Worst	1105.977	7458.404	1385.226	8751.453	1395.854	6740.9	9967.603	1524.43	4043.944	2490.803	3395.82	1419.804	14745.55
	Std	2.264257	926.6255	65.02055	1173.611	118.4	1588.081	2250.131	126.0088	1367.808	399.7072	628.0357	90.1088	5250.521
	Median	1102.487	6725.927	1291.818	7855.936	1186.634	4055.463	6146.076	1308.28	1488.508	1735.694	2634.918	1274.048	7089.158
	Rank	1	10	4	12	2	9	11	5	7	6	8	3	13
C17-F12	Mean	1744.553	5.94E+09	21000000	9.23E+09	3400000	4.29E+09	2.13E+08	12900000	47800000	2.59E+08	1.72E+08	5550000	9880000
	Best	1721.81	4.91E+09	6710000	8.23E+09	351000	2.21E+09	57800000	7500000	4640000	1.68E+08	32900000	1710000	6370000
	Worst	1764.937	7.55E+09	44900000	1.16E+10	7090000	5.61E+09	4.2E+08	23300000	1E+08	4.45E+08	5.4E+08	9820000	14500000
	Std	21.19911	1.19E+09	17400000	1.7E+09	3092785	1.55E+09	1.77E+08	7590000	43900000	1.33E+08	2.58E+08	4450000	3600000
	Median	1745.733	5.66E+09	16200000	8.53E+09	3070000	4.67E+09	1.87E+08	10300000	43200000	2.11E+08	57300000	5320000	9290000
	Rank	1	12	6	13	2	11	9	5	7	10	8	3	4
C17-F13	Mean	1315.791	4.83E+09	175118.5	8.92E+09	50217.03	1.24E+09	813000	125529.5	687000	74600000	79500	76000	10100000
	Best	1314.587	2.35E+09	76007.4	4.68E+09	8027.036	16700000	369000	39921.68	83038.86	51800000	32800	24200	2740000
	Worst	1318.646	6.77E+09	269199.3	1.1E+10	152150.9	4.29E+09	1280000	184625.3	2130000	1.1E+08	176000	162000	21800000
	Std	2.036813	1.92E+09	94504.53	3.01E+09	72389.4	2.16E+09	477000	73902.61	1030000	26500000	68500	63200	8606555
	Median	1314.967	5.1E+09	177633.7	1E+10	20345.07	3.19E+08	802000	138785.5	267000	68300000	54700	58900	7970000
	Rank	1	12	6	13	2	11	8	5	7	10	4	3	9

(Continued)

Table 5 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F14	Mean	1423.017	1634752	263366.9	1888993	35552.08	1026183	1912000	51507.84	484503.9	152453.4	1000197	50179.14	1729492
	Best	1422.014	1060359	37015.7	1005611	3549.286	766299.7	86898.11	24116.25	31353.6	73631.86	681834.3	5015.86	285566.3
	Worst	1423.993	2028265	603690.1	2766761	74662.43	1474661	5738512	77656.23	1038274	207296.8	1514900	102374.1	2914484
	Std	0.850137	474385.1	255762.2	913836.5	37974.42	351058.9	2729671	27671.05	537723.5	63352.08	415047.3	49656.46	1284368
	Median	1423.03	1725192	206381	1891800	31998.3	931886.7	911295.1	52129.44	434194.2	164442.4	902026.1	46663.32	1858958
	Rank	1	10	6	12	2	9	13	4	7	5	8	3	11
C17-F15	Mean	1503.129	2.58E+08	1046649	5.05E+08	1016409	13100000	5272313	1051149	14400000	5350000	1030000	1020000	1821548
	Best	1502.462	2.26E+08	44622.61	4.39E+08	7813.061	7470000	306514.5	41863.7	89358.33	1130000	24800	8956.889	402511.6
	Worst	1504.265	2.85E+08	3827791	5.57E+08	3801484	28400000	17600000	3821003	53800000	12000000	3810000	3800000	3948137
	Std	0.899978	31000000	1951161	61300000	1954080	10700000	8725229	1943692	27700000	4870000	1950000	1950000	1635485
	Median	1502.893	2.6E+08	157091.8	5.13E+08	128169.8	8380000	1579864	170865.1	1790000	4150000	139000	131371.7	1467773
	Rank	1	12	5	13	2	10	8	6	11	9	4	3	7
C17-F16	Mean	1663.469	3961.074	2849.212	4515.955	2027.496	3077.947	3898.56	2500.936	2463.631	3240.441	3410.79	2790.136	2806.118
	Best	1614.72	3677.659	2476.504	3855.314	1764.597	2707.522	3265.791	2299.515	2323.967	3064.406	3240.178	2581.133	2501.688
	Worst	1744.118	4201.708	3298.531	5115.564	2246.754	3302.559	4621.969	2712.367	2571.478	3443.361	3556.238	3034.575	3106.607
	Std	65.1896	247.91	356.8489	702.0488	228.7552	275.4284	588.9608	186.9542	132.7904	175.877	148.535	235.2757	309.1403
	Median	1647.519	3982.464	2810.906	4546.472	2049.316	3150.854	3853.24	2495.931	2479.539	3226.999	3423.372	2772.417	2808.088
	Rank	1	12	7	13	2	8	11	4	3	9	10	5	6
C17-F17	Mean	1728.099	3163.114	2374.139	3420.292	1857.178	3049.381	2690.766	2042.076	1917.07	2138.156	2416.428	2257.343	2105.831
	Best	1718.761	2648.213	2255.375	3092.244	1754.337	2154.186	2279.176	1990.285	1798.503	1949.912	2322.314	2061.615	2065.827
	Worst	1733.659	3784.672	2463.238	3995.396	1919.354	5370.143	2957.458	2174.366	2053.107	2380.795	2549.195	2601.013	2170.049
	Std	7.056339	507.3959	94.96416	431.1526	75.62203	1630.205	307.3274	93.28991	127.3899	190.9237	117.1204	256.8487	49.57056
	Median	1729.987	3109.785	2388.972	3296.763	1877.51	2336.599	2763.215	2001.827	1908.335	2110.958	2397.102	2183.372	2093.725
	Rank	1	12	8	13	2	11	10	4	3	6	9	7	5
C17-F18	Mean	1825.696	24010735	2262488	27603268	28667.7	30692338	5006997	567089.3	381073.5	1432871	461586.4	142846.3	3103457
	Best	1822.524	6978171	307329.9	8984696	6797.463	1129661	1683388	141343.3	71377.77	721909.7	249063	110706.7	2471012
	Worst	1828.42	46583044	4465379	54200000	70864.66	58117699	10284009	1490237	978877.4	1772554	915305.6	184528.4	4537750
	Std	2.842212	18427398	2053628	20164677	31714.99	33278329	3879872	653493.7	449261	510714.7	323244	32458.41	1013281
	Median	1825.92	21240861	2138621	23623254	18504.34	31760996	4030295	318388.4	237019.5	1618509	340988.5	138075	2702534
	Rank	1	11	8	12	2	13	10	6	4	7	5	3	9
C17-F19	Mean	1910.989	4.9E+08	315910.4	8.26E+08	260410	2.49E+08	12300000	1051513	3660000	5110000	328000	296000	1626779
	Best	1908.84	3.67E+08	37353.84	5.97E+08	6343.378	3190000	1681847	242120.3	64465.94	2520000	78500	17479.07	545242
	Worst	1913.095	6.39E+08	846283.4	1.25E+09	835727.6	6.88E+08	21000000	1870060	11800000	7290000	872000	852000	2538842
	Std	2.03232	1.44E+08	380170.1	3.06E+08	406209.7	3.33E+08	9128459	718481.9	5750000	2510000	385000	399000	878084.5
	Median	1911.01	4.78E+08	190002.2	7.28E+08	99784.5	1.52E+08	13400000	1046937	1390000	5320000	181000	158000	1711516
	Rank	1	12	4	13	2	11	10	6	8	9	5	3	7

(Continued)

Table 5 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F20	Mean	2065.787	2794.033	2570.338	2839.824	2180.062	2752.443	2741.942	2544.446	2348.901	2708.446	2887.979	2495.248	2434.172
	Best	2029.521	2724.894	2432.237	2696.952	2067.884	2627.215	2576.548	2334.178	2187.258	2640.185	2567.061	2450.476	2379.227
	Worst	2161.126	2877.872	2756.83	2920.153	2263.673	2867.962	2893.318	2897.838	2502.758	2822.006	3303.539	2604.654	2469.3
	Std	66.95098	66.35057	147.2205	105.3712	85.94739	105.5067	140.171	257.4623	135.7586	89.57131	323.8441	76.9979	40.56966
	Median	2036.25	2786.684	2546.142	2871.095	2194.346	2757.296	2748.951	2472.884	2352.794	2685.796	2840.658	2462.931	2444.081
	Rank	1	11	7	12	2	10	9	6	3	8	13	5	4
C17-F21	Mean	2308.456	2581.072	2426.648	2629.866	2363.126	2506.634	2570.745	2396.176	2383.24	2472.976	2536.603	2421.174	2470.527
	Best	2304.034	2501.02	2234.454	2563.105	2354.875	2313.541	2502.956	2366.469	2351.179	2462.634	2521.697	2405.49	2443.098
	Worst	2312.987	2635.249	2561.544	2707.006	2374.773	2621.898	2627.027	2421.872	2397.203	2483.079	2568.411	2432.981	2511.61
	Std	4.690555	65.79854	144.9785	65.94695	9.025478	142.6196	64.13207	24.40785	22.9312	9.764223	22.54543	13.53134	30.59232
	Median	2308.402	2594.01	2455.297	2624.676	2361.427	2545.549	2576.499	2398.182	2392.29	2473.095	2528.151	2423.113	2463.7
	Rank	1	12	6	13	2	9	11	4	3	8	10	5	7
C17-F22	Mean	2300	7162.287	5280.339	6955.703	2328.61	7834.465	6670.479	3733.822	2669.869	5206.244	5752.875	4525.201	2668.312
	Best	2300	6870.709	2320.359	6076.275	2319.631	7629.459	5848.004	2324.847	2551.481	2683.847	3765.296	2477.509	2598.517
	Worst	2300	7630.107	6417.946	7853.968	2344.931	7944.492	7393.994	5492.296	2913.245	7991.475	6619.919	6515.202	2734.765
	Std	0	343.238	2078.969	802.5975	11.77052	150.2962	674.6946	1735.435	174.0463	3047.671	1402.711	1956.452	65.708
	Median	2300	7074.165	6191.526	6946.285	2324.939	7881.954	6719.959	3559.072	2607.376	5074.827	6313.141	4554.047	2669.984
	Rank	1	12	8	11	2	13	10	5	4	7	9	6	3
C17-F23	Mean	2655.081	3120.621	2894.091	3167.58	2653.629	3124.79	2996.974	2731.957	2743.71	2874.386	3613.467	2871.495	2935.55
	Best	2653.745	3047.576	2800.027	3122.593	2503.876	3023.532	2845.78	2692.6	2725.441	2856.16	3521.287	2842.408	2909.968
	Worst	2657.377	3189.22	3042.441	3234.388	2710.94	3292.322	3081.08	2756.299	2762.972	2915.986	3704.465	2915.759	2990.302
	Std	1.739034	69.8468	111.8945	51.22572	105.2408	124.4942	110.0882	28.99956	17.02262	29.74436	101.5155	35.06682	38.7881
	Median	2654.6	3122.845	2866.949	3156.669	2699.851	3091.652	3030.518	2739.464	2743.214	2862.698	3614.058	2863.906	2920.966
	Rank	2	10	7	12	1	11	9	3	4	6	13	5	8
C17-F24	Mean	2831.409	3251.741	3128.122	3337.445	2881.944	3222.413	3081.473	2900.991	2913.982	3017.974	3293.713	3094.043	3175.646
	Best	2829.992	3219.322	3009.504	3260.959	2868.63	3129.054	3025.536	2858.848	2902.383	2997.75	3262.042	3028.998	3094.958
	Worst	2832.366	3318.283	3261.316	3469.713	2887.389	3266.805	3104.275	2920.647	2920.445	3048.333	3326.556	3192.255	3243.627
	Std	1.205041	47.28937	116.2057	101.4474	9.414219	67.32037	39.42918	29.91368	8.562528	22.57761	30.1828	73.57656	72.82889
	Median	2831.64	3234.68	3120.834	3309.554	2885.878	3246.897	3098.041	2912.233	2916.551	3012.907	3293.127	3077.459	3181.999
	Rank	1	11	8	13	2	10	6	3	4	5	12	7	9
C17-F25	Mean	2886.698	3795.058	2912.454	4330.432	2897.554	3392.39	3060.626	2913.157	2984.862	3054.509	2986.69	2900.698	3082.708
	Best	2886.691	3477.227	2897.797	3817.652	2889.3	3066.182	3026.898	2891.875	2949.8	2955.656	2974.308	2891.93	3066.093
	Worst	2886.707	4034.019	2943.728	5017.569	2905.01	3726.284	3081.356	2965.62	3050.147	3169.692	2997.536	2915.561	3093.099
	Std	0.008001	244.3369	22.28778	526.0797	8.014929	338.2477	25.55259	37.18806	48.94882	109.2882	10.85877	11.19416	12.21118
	Median	2886.698	3834.494	2904.146	4243.254	2897.953	3388.547	3067.125	2897.566	2969.751	3046.344	2987.458	2897.651	3085.819
	Rank	1	12	4	13	2	11	9	5	6	8	7	3	10

(Continued)

Table 5 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F26	Mean	3578.65	8427.399	6846.745	8929.917	3106.09	8047.021	7741.671	4675.154	4485.424	5646.393	6971.007	4727.61	4339.296
	Best	3559.841	8084.62	5740.92	8207.081	3077.579	7520.993	7104.988	4372.919	4120.433	4456.012	6063.07	3654.509	3983.145
	Worst	3607.686	9056.172	7534.017	10172.57	3147.082	8396.486	8512.776	5255.964	5049.506	6742.773	7479.708	6046.245	4734.68
	Std	23.9591	478.2391	821.2362	959.3764	30.92081	391.5256	610.0976	436.6787	416.9052	1114.625	679.6726	1186.512	326.029
	Median	3573.536	8284.402	7056.021	8670.008	3099.849	8135.303	7674.46	4535.866	4385.879	5693.393	7170.626	4604.844	4319.679
	Rank	2	12	8	13	1	11	10	5	4	7	9	6	3
C17-F27	Mean	3207.018	3555.358	3336.878	3688.367	3216.307	3438.126	3398.685	3230.689	3246.474	3304.809	4721.009	3271.16	3426.23
	Best	3200.749	3505.772	3263.936	3447.52	3205.531	3322.404	3253.301	3214.567	3237.797	3237.528	4335.987	3237.518	3359.991
	Worst	3210.656	3640.619	3401.573	3933.77	3232.666	3651.717	3506.54	3253.049	3260.534	3366.91	5001.347	3307.613	3466.384
	Std	4.889133	63.51793	76.12302	219.1523	13.37553	154.1578	114.5444	16.96452	10.30028	56.45407	343.1252	32.15411	48.49125
	Median	3208.335	3537.521	3341.002	3686.088	3213.514	3389.191	3417.451	3227.571	3243.781	3307.398	4773.351	3269.755	3439.272
	Rank	1	11	7	12	2	10	8	3	4	6	13	5	9
C17-F28	Mean	3100	4571.492	3274.378	5350.776	3229.928	4038.758	3422.379	3266.493	3558.309	3620.368	3493.381	3328.645	3546.101
	Best	3100	4357.75	3261.3	5073.8	3209.397	3547.01	3357.018	3222.483	3374.319	3478.399	3419.218	3200.17	3505.195
	Worst	3100	4783.471	3294.513	5621.595	3248.433	4558.9	3458.389	3296.655	4007.822	3947.304	3610.44	3498.118	3587.261
	Std	2.76E-13	198.0358	16.68602	265.2672	21.99204	489.1332	47.05939	34.06323	317.1353	231.7292	87.1701	135.5622	36.94125
	Median	3100	4572.373	3270.849	5353.855	3230.942	4024.562	3437.055	3273.417	3425.548	3527.885	3471.933	3308.145	3545.973
	Rank	1	12	4	13	2	11	6	3	9	10	7	5	8
C17-F29	Mean	3353.75	5134.859	4219.761	5323.349	3642.492	4999.19	4867.14	3801.405	3756.488	4373.415	4846.548	4080.165	4182.013
	Best	3325.385	4749.269	3922.493	4783.376	3505.854	4538.943	4630.085	3697.795	3680.25	4096.975	4607.025	3906.717	3859.084
	Worst	3370.797	5542.685	4405.449	6073.321	3759.512	5756.815	5012.457	3900.874	3865.582	4789.895	5069.516	4292.148	4483.304
	Std	20.71115	405.6956	224.2443	667.7618	117.3124	600.4622	173.4909	90.23052	86.93062	312.7738	261.4941	167.2987	293.9022
	Median	3359.41	5123.742	4275.55	5218.349	3652.3	4850.501	4913.009	3803.476	3740.06	4303.395	4854.825	4060.897	4192.832
	Rank	1	12	7	13	2	11	10	4	3	8	9	5	6
C17-F30	Mean	5007.854	1.22E+09	1621953	2.4E+09	418257.8	33000000	33700000	3036637	5820000	32500000	2330000	643000	1010000
	Best	4955.449	8.95E+08	671084.5	1.72E+09	98713	12300000	6880000	670571.7	1300000	17300000	1920000	207000	257000
	Worst	5086.396	1.34E+09	2235273	2.65E+09	1116520	76400000	53500000	4870000	15700000	67600000	2930000	1120000	1340000
	Std	62.03637	2.25E+08	747641.2	4.74E+08	494240.5	30800000	20500000	1835030	7010000	24800000	508000	491000	540838.7
	Median	4994.785	1.31E+09	1790728	2.61E+09	228898.9	21700000	37200000	3303337	3140000	22600000	2240000	624000	1210000
	Rank	1	12	5	13	2	10	11	7	8	9	6	3	4
Sum rank		31	334	182	361	57	305	284	128	151	232	231	139	204
Mean rank		1.068966	11.51724	6.275862	12.44828	1.965517	10.51724	9.793103	4.413793	5.206897	8	7.965517	4.793103	7.034483
Total rank		1	12	6	13	2	11	10	3	5	9	8	4	7

Table 6: Optimization outcomes for the CEC 2017 test suite (dimension = 50)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F1	Mean	100	5.09E+10	6.06E+08	7.94E+10	6.03E+08	3.26E+10	7.07E+09	6.01E+08	8.47E+09	1.8E+10	1.5E+10	2.73E+09	9.34E+09
	Best	100	4.55E+10	4.52E+08	6.95E+10	4.34E+08	3.03E+10	4.25E+09	4.34E+08	6.1E+09	1.24E+10	1.23E+10	1.43E+09	8.76E+09
	Worst	100	5.47E+10	8.23E+08	8.69E+10	8.3E+08	3.49E+10	1.05E+10	8.22E+08	1.16E+10	2.44E+10	1.78E+10	3.28E+09	9.98E+09
	Std	0	4.2E+09	1.64E+08	7.95E+09	1.74E+08	2E+09	3.07E+09	1.7E+08	2.41E+09	6.08E+09	2.37E+09	9.24E+08	6.14E+08
	Median	100	5.17E+10	5.74E+08	8.06E+10	5.73E+08	3.27E+10	6.77E+09	5.74E+08	8.09E+09	1.77E+10	1.5E+10	3.1E+09	9.32E+09
	Rank	1	12	4	13	3	11	6	2	7	10	9	5	8
C17-F3	Mean	300	141527.3	131525.3	141034.2	23403.48	100043.7	204985.5	47205.31	117567.3	90951.38	157919.5	129996.3	229748.6
	Best	300	121563.3	103028.2	127704	21390.34	87899.5	156596.8	38144.74	103289.3	70765.19	142410.3	99809.81	192833
	Worst	300	162532.8	158172.8	153027	27137.16	107129.6	309337.8	57697.29	131966.1	102641.1	177381.3	166822.8	261745.4
	Std	0	18102.03	25848.1	11613.55	2714.699	9130.536	75905.38	8548.613	12326.71	15071.96	17773.61	30859.99	29744.5
	Median	300	141006.6	132450.2	141703	22543.21	102572.8	177003.7	46489.6	117506.9	95199.63	155943.2	126676.3	232208.1
	Rank	1	10	8	9	2	5	12	3	6	4	11	7	13
C17-F4	Mean	470.3679	12542.52	722.9987	20104.5	583.0711	7122.787	1767.146	609.9954	1343.592	2485.619	2711.873	990.6342	1418.679
	Best	428.5127	9765.637	684.1463	13308.16	530.698	5751.399	1184.253	551.7917	1007.244	1437.509	2260.271	684.1991	1217.201
	Worst	525.7252	14277.93	763.208	24002.36	643.5572	9154.095	2063.632	692.8009	1632.317	4152.242	2895.976	1680.062	1517.505
	Std	52.14539	2125.239	37.61654	5136.568	49.2991	1515.247	421.9472	62.38522	296.6327	1247.318	319.3414	486.8769	145.573
	Median	463.6168	13063.25	722.3202	21553.75	579.0147	6792.828	1910.35	597.6944	1367.404	2176.363	2845.623	799.1376	1470.005
	Rank	1	12	4	13	2	11	8	3	6	9	10	5	7
C17-F5	Mean	504.7261	1018.134	815.083	1042.679	712.9127	1057.679	897.4574	715.0726	703.9083	933.4549	771.6502	757.314	843.6417
	Best	503.9798	989.1953	787.7418	1028.692	642.629	936.5436	864.8231	651.5795	679.1571	896.4815	725.7214	709.7572	816.6328
	Worst	505.9698	1050.26	851.704	1051.881	768.4815	1151.525	917.0944	811.8154	729.5904	958.033	800.8448	812.7633	860.8587
	Std	1.00206	30.61951	29.21637	11.08712	55.65075	110.1758	25.19615	75.57236	28.35438	28.33957	37.77926	44.50367	22.19255
	Median	504.4773	1016.541	810.443	1045.07	720.27	1071.323	903.956	698.4477	703.4429	939.6525	780.0173	753.3677	848.5377
	Rank	1	11	7	12	3	13	9	4	2	10	6	5	8
C17-F6	Mean	600	681.0686	651.7962	682.8024	610.9442	676.6633	683.342	632.9514	620.5286	655.1734	649.9146	646.2755	642.0595
	Best	600	678.1986	647.5107	681.3601	609.2425	659.2651	679.4761	624.1956	615.416	644.5228	646.4357	644.3562	630.9974
	Worst	600	684.9664	656.3546	684.8934	613.7656	690.3687	689.8617	652.695	629.4142	662.1647	652.0972	648.8926	653.2569
	Std	0	3.289016	4.470527	1.768662	2.102176	14.68595	4.874904	14.18613	6.60734	8.072209	2.579837	2.029196	9.74892
	Median	600	680.5547	651.6598	682.478	610.3843	678.5097	682.0152	627.4576	618.6421	657.0031	650.5628	645.9266	641.9919
	Rank	1	11	8	12	2	10	13	4	3	9	7	6	5
C17-F7	Mean	756.7298	1644.188	1538.138	1727.768	1004.287	1552.107	1573.357	1025.318	1035.219	1381.144	1324.448	1147.271	1237.482
	Best	754.7543	1624.439	1479.328	1661.121	955.0207	1425.341	1521.481	993.0339	1014.131	1274.033	1181.567	1014.313	1172.963
	Worst	758.3522	1669.262	1595.085	1815.38	1045.879	1675.914	1645.534	1052.14	1052.398	1431.386	1434.044	1340.702	1278.86
	Std	1.633977	19.58956	51.54233	69.9839	46.19779	123.5724	60.54072	25.99127	18.79231	75.88935	119.2501	148.394	49.20122
	Median	756.9065	1641.525	1539.07	1717.286	1008.124	1553.587	1563.206	1028.05	1037.173	1409.579	1341.09	1117.033	1249.052
	Rank	1	12	9	13	2	10	11	3	4	8	7	5	6

(Continued)

Table 6 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F8	Mean	805.721	1335.022	1087.274	1358.176	991.921	1349.604	1255.265	1001.485	1011.543	1253.122	1100.663	1030.468	1199.121
	Best	802.9849	1289.435	1045.995	1333.933	963.6715	1263.856	1145.543	969.2718	980.1505	1204.615	1092.332	992.3313	1165.187
	Worst	810.9445	1370.621	1128.433	1374.243	1020.792	1461.103	1348.702	1059.897	1046.142	1301.303	1114.109	1088.202	1217.854
	Std	3.761519	39.34634	49.10596	18.02852	30.90425	88.88505	87.91615	42.07854	31.04849	41.79327	10.66891	46.80177	24.52874
	Median	804.4773	1340.016	1087.334	1362.263	991.6103	1336.728	1263.407	988.3863	1009.94	1253.285	1098.105	1020.67	1206.722
	Rank	1	11	6	13	2	12	10	3	4	9	7	5	8
C17-F9	Mean	900	30772.55	11638.79	30933.8	3307.786	32254.31	28146.79	16978.94	6276.016	20640.78	9452.087	9133.861	11230.66
	Best	900	29589.87	11068.87	29155.41	2204.308	29714.04	26184.67	9372.494	5467.319	15959.14	8609.989	8544.643	9392.944
	Worst	900	33603.47	12424.93	32478.36	4604.404	35961.33	32879.46	22239.73	7136.336	24255.87	10210.24	10261.73	12857.08
	Std	9.76E-14	2011.086	604.8312	1648.716	1038.164	2833.419	3331.318	6400.267	911.7464	3623.59	708.5189	829.514	1956.424
	Median	900	29948.43	11530.67	31050.72	3211.217	31670.93	26761.51	18151.76	6250.205	21174.05	9494.061	8864.535	11336.31
	Rank	1	11	7	12	2	13	10	8	3	9	5	4	6
C17-F10	Mean	4347.157	11885.23	7971.508	12907.47	6470.665	10860.4	10866.9	7413.879	8256.326	12729.66	8200.247	7522.295	10801.01
	Best	3555.132	11265.41	7838.321	12472.74	5600.523	10351.78	9617.516	6113.926	6409.333	11934.6	7802.553	7184.992	10207.11
	Worst	5099.795	12500.49	8226.909	13225.36	6977.814	11746.61	11819.14	8210.155	12933.87	13537.33	9113.913	8288.141	11725.77
	Std	678.2324	535.8009	189.0794	338.5087	639.5372	695.6393	974.722	1033.641	3307.629	755.9321	645.8184	541.3844	686.608
	Median	4366.851	11887.52	7910.4	12965.9	6652.161	10671.6	11015.47	7665.717	6841.048	12723.35	7942.26	7308.023	10635.57
	Rank	1	11	5	13	2	9	10	3	7	12	6	4	8
C17-F11	Mean	1128.435	13428.73	1848.699	18128.57	1554.192	11361.14	4780.523	1817.763	5648.534	4793.872	12425.7	1904.268	20669.07
	Best	1121.25	12255.22	1591.828	16024.93	1375.253	10122.77	4255.55	1613.982	3434.277	4370.147	11546.64	1653.027	12131.06
	Worst	1133.132	13936.53	2065.526	19612.36	1868.384	13401.8	6145.706	2152.267	9729.479	5567.932	13861.21	2040.1	27411.01
	Std	5.725574	830.7024	206.4517	1619.792	240.2695	1519.338	961.3885	249.0541	3058.853	580.8578	1052.101	180.4036	6665.257
	Median	1129.678	13761.59	1868.721	18438.49	1486.566	10959.99	4360.417	1752.403	4715.189	4618.704	12147.47	1961.973	21567.1
	Rank	1	11	4	12	2	9	6	3	8	7	10	5	13
C17-F12	Mean	2905.102	3.68E+10	1.23E+08	5.99E+10	73347319	2.18E+10	1.17E+09	1.28E+08	8.67E+08	4.31E+09	1.89E+09	1.41E+09	2.33E+08
	Best	2527.376	3.09E+10	35784212	4.38E+10	21043151	9.19E+09	9.28E+08	78021765	1.36E+08	2.51E+09	6.25E+08	1.24E+08	1.52E+08
	Worst	3168.37	4.4E+10	1.93E+08	8.21E+10	1.26E+08	3.67E+10	1.54E+09	1.7E+08	1.61E+09	8.46E+09	3.29E+09	3.91E+09	3.09E+08
	Std	287.9189	6.25E+09	69519444	1.86E+10	54844761	1.2E+10	2.75E+08	39799181	7.74E+08	2.96E+09	1.16E+09	1.88E+09	68262316
	Median	2962.331	3.61E+10	1.32E+08	5.69E+10	73044386	2.07E+10	1.11E+09	1.32E+08	8.61E+08	3.14E+09	1.82E+09	8.05E+08	2.36E+08
	Rank	1	12	3	13	2	11	7	4	6	10	9	8	5
C17-F13	Mean	1340.1	2.07E+10	22967563	3.63E+10	22855892	8.51E+09	1.03E+08	23045160	3.23E+08	5.15E+08	38436442	4.24E+08	57781476
	Best	1333.781	1.19E+10	10465749	1.83E+10	10380127	4.52E+09	70415791	10491036	1.47E+08	4.12E+08	10390647	10407244	33139515
	Worst	1343.015	2.83E+10	57463352	5.22E+10	57450695	1.32E+10	1.42E+08	57627665	8.13E+08	6.85E+08	67190110	1.03E+09	97872469
	Std	4.504617	7.54E+09	24207126	1.49E+10	24276893	3.87E+09	31368815	24268771	3.44E+08	1.27E+08	32865138	5.08E+08	30094534
	Median	1341.801	2.13E+10	11970575	3.73E+10	11796374	8.15E+09	99086365	12030970	1.67E+08	4.82E+08	38082505	3.31E+08	50056960
	Rank	1	12	3	13	2	11	7	4	8	10	5	9	6

(Continued)

Table 6 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F14	Mean	1429.458	21935048	1103685	40832413	75140.78	2336716	4089133	234560.7	1043637	802696.4	12831098	557162.3	9515152
	Best	1425.995	7214597	461328.4	12574637	7187.684	671663.7	3628898	175729.6	81381.62	647883.2	3034735	247531.8	4719792
	Worst	1431.939	42938777	2526862	82665781	143642	3662323	4844757	317872.2	2013790	973866.9	21019463	780001	16391720
	Std	2.757393	15864301	1009890	31368165	58601.46	1310208	554053.4	67117.14	829888	173927.3	8566000	238718.4	5194623
	Median	1429.95	18793408	713275.6	34044616	74866.7	2506438	3941438	222320.6	1039689	794517.8	13635096	600558.1	8474547
	Rank	1	12	7	13	2	8	9	3	6	5	11	4	10
C17-F15	Mean	1530.66	2.2E+09	412884.8	3.52E+09	382787.4	1.44E+09	8737447	483112.1	5391658	59814905	1.67E+08	390025.5	7604082
	Best	1526.359	1.55E+09	61784.01	2.75E+09	4653.511	4.94E+08	773089.7	155334.7	38503.39	35033078	200784.3	20819.02	3457491
	Worst	1532.953	2.88E+09	1029399	4.18E+09	1004987	3.13E+09	15785805	1045281	14200642	77699681	6.45E+08	1005296	15679424
	Std	3.086361	6.54E+08	450699.7	6.63E+08	459134.1	1.29E+09	7059025	410424.3	6500456	18795844	3.36E+08	452809.6	5807911
	Median	1531.664	2.18E+09	280178.3	3.58E+09	260754.6	1.06E+09	9195447	365916.5	3663743	63263429	10726021	266993.4	5639706
	Rank	1	12	4	13	2	11	8	5	6	9	10	3	7
C17-F16	Mean	2062.891	5699.523	4074.397	6795.786	2725.273	4316.641	5033.187	3209.482	3207.036	4234.201	3738.721	3220.766	3703.39
	Best	1728.6	4991.884	3783.19	5235.585	2578.603	3828.207	4171.31	3059.139	2858.826	3933.037	3497.091	2857.985	3172.11
	Worst	2242.663	7151.479	4473.244	9942.431	2930.036	4561.304	5641.014	3403.622	3747.804	4476.671	4067.87	3579.751	4137.484
	Std	245.0055	1056.398	343.4174	2274.21	180.3051	350.3472	673.6919	154.9074	453.2542	237.3669	298.7512	392.506	455.3884
	Median	2140.15	5327.365	4020.577	6002.565	2696.226	4438.528	5160.212	3187.583	3110.756	4263.548	3694.961	3222.665	3751.983
	Rank	1	12	8	13	2	10	11	4	3	9	7	5	6
C17-F17	Mean	2021.151	6799.16	3376.901	9690.992	2534.684	3713.112	4198.649	2965.845	2879.747	3874.469	3597.558	3202.179	3399.034
	Best	1900.43	5247.89	2986.733	7158.665	2460.411	3036.733	3787.688	2486.158	2747.158	3321.536	3202.359	3008.377	3190.795
	Worst	2138.267	8241.347	3822.006	12468.86	2608.129	4126.772	4413.815	3372.908	3137.724	4195.892	3855.625	3509.988	3594.281
	Std	141.197	1299.605	408.6522	2301.189	65.01964	495.3898	301.7025	387.896	184.7869	412.1108	297.0178	249.3597	193.4599
	Median	2022.954	6853.701	3349.433	9568.22	2535.098	3844.472	4296.546	3002.157	2817.053	3990.224	3666.124	3145.176	3405.53
	Rank	1	12	6	13	2	9	11	4	3	10	8	5	7
C17-F18	Mean	1830.62	64170678	2398969	95016900	388871.4	29910759	38445004	2592349	5191221	7279008	7454786	1061435	8349872
	Best	1822.239	51126603	854818.5	43147219	77122.78	2729594	10386403	1385179	989904.6	4827815	3425281	370055.8	3591932
	Worst	1841.673	75968343	3792345	1.31E+08	756875.1	84998459	69519156	4195966	10355161	10199877	13837502	1870582	19261497
	Std	8.567484	11186140	1501254	45876193	367878.7	39748350	30518666	1268250	5140465	2331445	4852989	649626.7	7698194
	Median	1829.285	64793884	2474356	1.03E+08	360744	15957491	36937229	2394125	4709909	7044171	6278180	1002551	5273029
	Rank	1	12	4	13	2	10	11	5	6	7	8	3	9
C17-F19	Mean	1925.185	2.3E+09	293564	3.24E+09	76414.37	2.25E+09	5841277	4394130	1054868	42804093	455626	406423.6	910549.9
	Best	1924.437	1.09E+09	141977	2.18E+09	38408.47	8278426	949492.4	3324575	516413.6	36312372	267716.5	84602.72	731142.5
	Worst	1926.121	3.83E+09	533634.9	4E+09	116458.8	6.58E+09	13706250	5422001	1621820	54343028	949546.7	865286.8	1168957
	Std	0.832428	1.21E+09	184100.1	8.51E+08	34320.55	3.09E+09	5774541	901093.5	486160.9	8436335	346811	404082.5	213076.8
	Median	1925.091	2.13E+09	249322.1	3.38E+09	75395.1	1.21E+09	4354684	4414972	1040620	40280485	302620.4	337902.4	871050.2
	Rank	1	12	3	13	2	11	9	8	7	10	5	4	6

(Continued)

Table 6 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F20	Mean	2160.172	3596.52	3121.432	3822.334	2610.593	3264.264	3530.367	3133.198	2580.432	3551.812	3776.141	3140.935	3038.794
	Best	2104.423	3294.344	2644.519	3567.011	2353.967	2862.637	3260.237	2919.468	2391.942	3451.526	3560.555	2802.434	2964.285
	Worst	2323.891	3739.246	3557.368	3991.747	2855.842	3470.266	4010.517	3534.865	2782.628	3679.721	4018.795	3275.322	3132.011
	Std	114.8219	219.8205	409.6954	191.3138	227.0213	287.0385	354.336	296.4387	211.1917	109.2904	198.2779	237.9579	75.44452
	Median	2106.186	3676.245	3141.92	3865.289	2616.281	3362.078	3425.356	3039.23	2573.579	3538.001	3762.607	3242.991	3029.441
	Rank	1	11	5	13	3	8	9	6	2	10	12	7	4
C17-F21	Mean	2314.895	2901.352	2700.793	2933.962	2441.558	2871.823	2864.119	2546.686	2502.41	2756.99	2774.088	2618.818	2695.968
	Best	2309.045	2873.079	2593.82	2841.206	2421.936	2783.96	2763.306	2517.667	2452.306	2734.665	2716.293	2554.049	2672.139
	Worst	2329.683	2931.93	2860.697	3007.919	2462.981	3011.56	2947.563	2580.988	2541.145	2795.345	2807.956	2710.19	2712.863
	Std	10.40007	32.35623	120.2411	82.81497	22.12068	103.152	82.9268	33.343	39.62193	29.74565	43.62064	71.98111	20.23425
	Median	2310.426	2900.199	2674.329	2943.362	2440.657	2845.885	2872.803	2544.045	2508.094	2748.976	2786.051	2605.518	2699.436
	Rank	1	12	7	13	2	11	10	4	3	8	9	5	6
C17-F22	Mean	3095.169	13517.11	10270.09	14590.28	5375.373	12468.72	12406.76	8454.284	8348.814	14123.76	10516.29	9096.576	8314.903
	Best	2300	13223.15	8237.641	14346.73	2766.066	11968.01	11833.33	6990.985	7321.24	13671.59	10201.28	8365.926	4210.276
	Worst	5480.678	13859.02	11642.45	15005.66	7931.411	12950.1	12761.38	9504.468	8943.026	14569.17	11046.42	9620.424	12277.72
	Std	1672.91	294.0908	1740.035	316.3445	2899.597	455.9694	440.4211	1117.818	748.7427	406.5688	386.128	603.4836	4779.261
	Median	2300	13493.14	10600.13	14504.38	5402.007	12478.39	12516.17	8660.842	8565.494	14127.15	10408.73	9199.976	8385.808
	Rank	1	11	7	13	2	10	9	5	4	12	8	6	3
C17-F23	Mean	2743.354	3678.351	3227.937	3743.013	2885.902	3612.825	3614.985	2970.319	2996.687	3219.329	4474.671	3300.686	3288.313
	Best	2729.988	3608.53	3150.285	3711.87	2875.409	3430.968	3456.415	2930.449	2922.703	3141.649	4302.852	3240.987	3174.615
	Worst	2752.657	3771.603	3295.439	3775.913	2901.01	3911.468	3711.137	3030.987	3125.731	3272.81	4630.927	3358.456	3404.945
	Std	10.53657	74.72065	74.92236	28.64086	11.36073	240.0266	117.6759	50.85003	93.49217	58.29603	141.2965	62.70506	98.93037
	Median	2745.387	3666.635	3233.012	3742.134	2883.595	3554.432	3646.194	2959.919	2969.158	3231.428	4482.452	3301.651	3286.846
	Rank	1	11	6	12	2	9	10	3	4	5	13	8	7
C17-F24	Mean	2919.043	4040.878	3444.876	4276.356	3062.307	3865.277	3715.582	3122	3176.363	3389.432	4187.516	3402.075	3574.021
	Best	2909.046	3830.395	3348.258	3867.116	3035.341	3776.542	3618.722	3082.915	3085.5	3321.364	4155.978	3263.134	3537.835
	Worst	2924.412	4523.088	3597.584	5292.026	3088.615	3984.917	3769.272	3150.67	3295.284	3433.696	4240.84	3536.062	3652.351
	Std	7.178381	340.6766	112.5232	719.1763	26.85258	97.6386	71.20958	32.49476	92.03854	55.18308	42.62542	130.3534	55.47785
	Median	2921.358	3905.013	3416.831	3973.141	3062.637	3849.824	3737.168	3127.207	3162.334	3401.334	4176.623	3404.552	3552.948
	Rank	1	11	7	13	2	10	9	3	4	5	12	6	8
C17-F25	Mean	2983.145	7838.176	3217.399	10680.59	3123.502	5627.323	4048.426	3112.341	3945.876	4235.21	4153.106	3169.032	3958.382
	Best	2980.235	6553.487	3197.082	8685.566	3095.221	4678.998	3712.934	3092.796	3765.987	3810.052	3851.107	3125.101	3856.029
	Worst	2991.831	8670.519	3246.205	11924.9	3149.163	6551.606	4300.854	3141.903	4132.868	4750.528	4729	3221.047	4076.149
	Std	6.091109	983.0776	23.32322	1596.928	26.5223	845.9211	261.7568	22.85313	201.1658	489.5289	431.76	45.2997	94.9917
	Median	2980.257	8064.348	3213.155	11055.94	3124.812	5639.344	4089.958	3107.332	3942.324	4190.13	4016.159	3164.989	3950.674
	Rank	1	12	5	13	3	11	8	2	6	10	9	4	7

(Continued)

Table 6 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F26	Mean	3776.432	12756.29	10100.84	13596.61	3571.107	11509.16	12526.2	5673.764	6292.395	9031.66	10588.37	7665.832	8404.252
	Best	3748.807	12574.2	9638.146	13042.07	3372.993	9721.084	11715.13	5229.332	5932.46	8361.025	10310.84	7155.502	6827.484
	Worst	3793.643	12902.04	10559.79	14430.52	3800.907	12574.27	13975.15	5921.823	6629.391	9657.959	10907.47	8167.723	10456.84
	Std	20.46024	154.1112	397.8401	627.9618	199.5097	1305.321	1045.028	330.7332	382.6795	568.6506	266.1505	466.3633	1811.848
	Median	3781.639	12774.46	10102.72	13456.92	3555.264	11870.64	12207.26	5771.951	6303.864	9053.828	10567.59	7670.051	8166.342
	Rank	2	12	8	13	1	10	11	3	4	7	9	5	6
C17-F27	Mean	3251.26	4597.307	3787.875	4758.891	3389.548	4520.769	4308.156	3371.267	3608.452	3771.179	7405.516	3613.129	4294.924
	Best	3227.701	4321.079	3743.694	4433.711	3297.904	3912.01	3812.743	3332.574	3565.304	3607.839	7189.611	3383.523	4195.937
	Worst	3313.631	4783.86	3845.23	4989.516	3477.239	4945.493	4801.97	3432.577	3652.8	3917.286	7709.005	3823.361	4421.116
	Std	43.8751	214.7351	47.96221	278.8706	77.51737	473.3302	488.8805	45.81443	46.98394	144.1234	264.5516	209.3332	100.0853
	Median	3231.854	4642.145	3781.289	4806.168	3391.525	4612.786	4308.956	3359.958	3607.851	3779.794	7361.724	3622.817	4281.322
	Rank	1	11	7	12	3	10	9	2	4	6	13	5	8
C17-F28	Mean	3258.849	7999.693	3618.551	10087.69	3412.849	6741.734	4666.63	3356.222	4309.486	5031.277	4870.076	3856.588	4853.245
	Best	3258.849	7286.35	3530.54	8976.483	3363.858	5572.379	4128.84	3324.771	4060.516	4508.556	4819.887	3592.894	4623.979
	Worst	3258.849	9829.865	3687.232	12979.58	3473.523	7940.82	4887.833	3380.646	4622.49	5520.56	4954.975	4284.702	5021.755
	Std	0	1292.085	82.77575	2032.374	48.77597	1256.929	379.6752	25.40913	275.7401	436.0028	65.74432	313.8239	192.5004
	Median	3258.849	7441.278	3628.217	9197.354	3407.007	6726.869	4824.923	3359.736	4277.469	5047.996	4852.721	3774.378	4883.623
	Rank	1	12	4	13	3	11	7	2	6	10	9	5	8
C17-F29	Mean	3263.038	12211.53	5280.264	17218.32	4078.035	6473.44	8301.687	4712.762	4744.794	6161.912	7563.355	4715.582	5832.508
	Best	3247.132	8272.219	5176.352	9397.87	3762.061	6111.434	5772.278	4350.998	4554.733	5377.936	6337.615	4507.282	5567.211
	Worst	3278.787	16541.08	5389.738	26882.5	4272.789	6929.208	10702.86	5211.296	5021.55	7014.496	9721.849	4796.785	6336.51
	Std	18.36308	4010.782	92.65268	8198.624	249.787	359.2502	2138.111	379.9115	225.4195	806.0822	1606.279	146.3749	380.5694
	Median	3263.116	12016.41	5277.482	16296.46	4138.645	6426.558	8365.808	4644.378	4701.447	6127.608	7096.977	4779.13	5713.156
	Rank	1	12	6	13	2	9	11	3	5	8	10	4	7
C17-F30	Mean	623575.2	2.77E+09	27440265	4.64E+09	10394169	1.41E+09	1.43E+08	68465410	1.27E+08	2.62E+08	1.65E+08	13055313	58302603
	Best	582411.6	2.14E+09	15652470	2.85E+09	5568300	1.84E+08	96319490	65109024	61308682	1.82E+08	1.33E+08	8841014	45665936
	Worst	655637.4	3.76E+09	37658691	7.28E+09	15612508	2.84E+09	1.97E+08	72793179	1.88E+08	3.34E+08	2.16E+08	17182093	75102726
	Std	34361.91	7.45E+08	11930648	2.01E+09	5137494	1.44E+09	54229872	3441313	67249195	66105583	38154677	4447187	13355368
	Median	628125.9	2.59E+09	28224950	4.22E+09	10197934	1.3E+09	1.39E+08	67979719	1.29E+08	2.67E+08	1.55E+08	13099073	56220875
	Rank	1	12	4	13	2	11	8	6	7	10	9	3	5
Sum rank		30	335	166	367	63	294	269	112	144	248	254	150	207
Mean rank		1.034483	11.55172	5.724138	12.65517	2.172414	10.13793	9.275862	3.862069	4.965517	8.551724	8.758621	5.172414	7.137931
Total rank		1	12	6	13	2	11	10	3	4	8	9	5	7

Table 7: Optimization outcomes for the CEC 2017 test suite (dimension = 100)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F1	Mean	100	1.44E+11	6.88E+09	2E+11	4.1E+09	1.1E+11	5.65E+10	3.71E+09	5.18E+10	8.05E+10	1.19E+11	2.05E+10	5.09E+10
	Best	100	1.41E+11	4.74E+09	1.96E+11	3.53E+09	9.69E+10	5.41E+10	3.23E+09	4.49E+10	7.7E+10	1.09E+11	1.45E+10	4.79E+10
	Worst	100	1.48E+11	8.01E+09	2.02E+11	4.64E+09	1.23E+11	6.23E+10	4.19E+09	5.86E+10	8.86E+10	1.27E+11	2.69E+10	5.76E+10
	Std	1.22E-14	3.45E+09	1.54E+09	2.67E+09	5E+08	1.14E+10	4.13E+09	4.75E+08	6.85E+09	5.77E+09	8.04E+09	6.74E+09	4.69E+09
	Median	100	1.44E+11	7.39E+09	2.01E+11	4.1E+09	1.1E+11	5.48E+10	3.72E+09	5.18E+10	7.82E+10	1.19E+11	2.04E+10	4.91E+10
	Rank	1	12	4	13	3	10	8	2	7	9	11	5	6
C17-F3	Mean	300	384186	298493.9	295264.1	156523.7	329756.2	688135.8	416072.2	333655.9	272979.9	312759.1	479049.1	509734.2
	Best	300	350275.4	290066.2	287319.9	123438.6	267026.7	603313.2	347605	305316.2	259758.2	292908.6	366718.8	487879.4
	Worst	300	402400.3	306321.3	302732	186111.5	371238	794850.4	495687.3	365383.7	285568.6	342328.3	660960.7	527209.5
	Std	0	24852.19	7696.093	7277.639	27938.99	47057.12	86182.54	77637.33	33804.21	11196.94	22074.12	142523.5	18726.87
	Median	300	392034.2	298794.1	295502.2	158272.3	340380	677189.7	410498.3	331961.9	273296.4	307899.8	444258.5	511923.9
	Rank	1	9	5	4	2	7	13	10	8	3	6	11	12
C17-F4	Mean	602.1722	37839.72	1651.056	63568.58	1200.02	13816.82	9551.933	982.5642	4102.694	9383.969	29054.11	2423.824	8091.054
	Best	592.0676	34785.2	1585.253	57587.84	1064.838	9120.618	8121.545	897.6428	3164.425	8892.001	23317.13	1745.081	7624.647
	Worst	612.2769	41414.12	1744.126	70750.3	1443.04	18403.11	10414.38	1114.888	6139.211	10264.06	32797.07	2928.695	8719.858
	Std	12.27362	3012.543	82.17368	5724.531	179.8372	4030.024	1043.687	101.9348	1441.19	675.2766	4831.003	532.1992	529.5449
	Median	602.1722	37579.78	1637.422	62968.09	1146.102	13871.78	9835.905	958.8629	3553.569	9189.906	30051.13	2510.759	8009.856
	rank	1	12	4	13	3	10	9	2	6	8	11	5	7
C17-F5	Mean	512.9345	1767.627	1206.576	1742.292	1132.853	1894.7	1640.354	1141.542	1097.76	1669.283	1224.8	1289.127	1428.609
	Best	510.9445	1750.961	1192.889	1715.996	1028.606	1876.915	1561.127	1047.197	1046.913	1642.942	1197.928	1207.159	1303.202
	Worst	514.9244	1774.985	1215.696	1767.463	1203.278	1920.43	1769.561	1202.99	1141.191	1695.079	1250.498	1433.941	1500.84
	Std	1.910853	11.82202	10.56836	29.24215	87.4949	22.32009	95.72828	72.99895	43.28702	22.48711	30.99835	112.5505	94.36175
	Median	512.9345	1772.281	1208.859	1742.854	1149.764	1890.728	1615.365	1157.991	1101.468	1669.555	1225.388	1257.704	1455.197
	Rank	1	12	5	11	3	13	9	4	2	10	6	7	8
C17-F6	Mean	600	689.6817	653.1467	688.2584	633.0622	693.4257	687.6238	663.6087	635.4128	669.048	654.8867	652.7851	654.1288
	Best	600	687.8542	649.6552	684.5988	629.7664	683.6872	679.4285	658.4562	630.9869	661.4931	652.5965	647.2492	647.7613
	Worst	600	691.4492	656.7168	690.7339	638.0694	700.095	702.2429	668.6172	641.0052	673.8035	658.7463	657.4664	658.6304
	Std	0	1.725645	3.034378	2.803539	4.024104	8.374682	10.74671	4.560116	4.570866	6.124423	2.901033	5.078164	5.657866
	Median	600	689.7117	653.1075	688.8504	632.2065	694.9603	684.4119	663.6807	634.8296	670.4477	654.102	653.2124	655.0618
	Rank	1	12	5	11	2	13	10	8	3	9	7	4	6
C17-F7	Mean	811.392	3177.953	2742.663	3273.316	1726.896	3034.827	3154.236	1862.1	1874.806	2756.23	2777.075	2246.796	2327.245
	Best	810.0205	3114.71	2599.214	3206.928	1666.865	2889.325	3055.234	1730.272	1706.931	2645.585	2657.242	2028.783	2245.133
	Worst	813.1726	3250.547	2853.958	3339.395	1796.125	3162.623	3287.682	1951.931	1998.794	2855.117	2958.074	2353.691	2496.857
	Std	1.537009	58.61408	134.2186	58.32358	58.13667	129.8823	110.9943	98.81629	128.4496	90.26558	135.1524	159.3284	120.8128
	Median	811.1874	3173.278	2758.74	3273.47	1722.298	3043.679	3137.014	1883.098	1896.75	2762.109	2746.493	2302.355	2283.495
	Rank	1	12	7	13	2	10	11	3	4	8	9	5	6

(Continued)

Table 7 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F8	Mean	812.437	2169.05	1610.61	2213.877	1360.243	2150.434	2084.906	1379.637	1429.943	2031.524	1682.836	1584.837	1851.062
	Best	808.9546	2120.901	1558.193	2187.13	1215.212	2096.215	1915.556	1262.946	1336.635	1986.99	1610.983	1544.083	1804.472
	Worst	816.9143	2227.386	1638.175	2227.9	1449.629	2231.086	2220.551	1531.47	1553.933	2068.457	1798.92	1665.095	1902.25
	Std	3.57488	48.29645	39.03698	19.22737	107.2002	69.0738	163.5185	117.7724	102.9739	36.9778	89.25118	57.45093	44.60344
	Median	811.9395	2163.956	1623.037	2220.238	1388.065	2137.218	2101.759	1362.066	1414.602	2035.325	1660.721	1565.085	1848.763
	Rank	1	12	6	13	2	11	10	3	4	9	7	5	8
C17-F9	Mean	900	75606.64	23848.01	65144.33	20550.14	100282.5	64748.58	50474.58	31545.16	62845.3	21473.32	29036.77	39702.93
	Best	900	68540.34	19720.36	62200.57	18438.24	83422.25	51420.5	43719.63	19544.48	59438.49	19629.93	25707.2	36736.3
	Worst	900	86473.41	27118.34	66555.95	21702.74	124006.2	80493.64	56579.01	42874.22	65006.06	23064.5	31565.89	45234.24
	Std	9.76E-14	8136.098	3314.988	2122.257	1619.198	17950.48	15062.24	5658.25	11980.49	2508.21	1703.869	3108.69	4011.37
	Median	900	73706.4	24276.66	65910.41	21029.78	96850.83	63540.09	50799.84	31880.97	63468.32	21599.42	29437	38420.59
	Rank	1	12	4	11	2	13	10	8	6	9	3	5	7
C17-F10	Mean	11023.04	27081	15321.59	28169.92	13607.5	26334.05	25479.1	16159.95	14682.62	28177.85	16351.9	16226.86	23647.68
	Best	9625.608	26811.66	13217.89	27488.87	13078.74	25800.97	24818.57	15684.11	13576.61	26967.2	14905.38	14803.74	23040.65
	Worst	11858.81	27403.11	17252.62	28636.67	14375.92	27161.91	26709.72	16706.78	15222.35	29116.84	17249.06	17112.11	24175.73
	Std	1019.169	290.9045	1887.648	566.6808	615.592	691.8126	897.8666	489.5861	794.5095	950.4212	1134.619	1047.92	490.1947
	Median	11303.87	27054.61	15407.93	28277.08	13487.67	26186.66	25194.05	16124.46	14965.75	28313.69	16626.58	16495.79	23687.17
	Rank	1	11	4	12	2	10	9	5	3	13	7	6	8
C17-F11	Mean	1162.329	141439.6	58535.87	176075.5	9554.956	59564.29	177662.8	9387.922	77580.04	64849.26	148239.2	48569.92	120600.9
	Best	1139.568	110092	52232.38	135088.3	8926.633	30106.52	106407.9	7967.33	64456.83	55491.73	123527.8	24204.24	93273.55
	Worst	1220.662	164393.4	69553.09	249209.4	10644.08	83129.01	282965.5	10241.63	87404.87	80216.59	172330.4	93379.56	164482.5
	Std	41.06257	24479.09	8172.572	53862.66	791.8939	23060.55	86798.28	1058.982	10329.7	11254.24	21147.87	32228.44	32704.7
	Median	1144.542	145636.4	56179.01	160002.1	9324.554	62510.82	160638.9	9671.365	79229.23	61844.35	148549.2	38347.95	112323.7
	Rank	1	10	5	12	3	6	13	2	8	7	11	4	9
C17-F12	Mean	5974.805	8.79E+10	1.26E+09	1.42E+11	9.34E+08	4.77E+10	1.16E+10	9.93E+08	1.02E+10	1.88E+10	5.59E+10	9.06E+09	1.09E+10
	Best	5383.905	6.27E+10	1.06E+09	1.07E+11	7.55E+08	2.48E+10	9.59E+09	9.25E+08	7.04E+09	1.5E+10	4.86E+10	1.58E+09	1E+10
	Worst	6570.199	9.79E+10	1.45E+09	1.66E+11	1.09E+09	7.87E+10	1.33E+10	1.13E+09	1.21E+10	2.55E+10	6.58E+10	1.66E+10	1.25E+10
	Std	520.1462	1.77E+10	1.87E+08	2.82E+10	1.53E+08	2.37E+10	1.64E+09	1.03E+08	2.3E+09	5.01E+09	7.53E+09	7.19E+09	1.18E+09
	Median	5972.559	9.54E+10	1.27E+09	1.49E+11	9.42E+08	4.35E+10	1.18E+10	9.56E+08	1.08E+10	1.74E+10	5.46E+10	9.02E+09	1.05E+10
	Rank	1	12	4	13	2	10	8	3	6	9	11	5	7
C17-F13	Mean	1407.28	2.31E+10	59560970	3.53E+10	59559855	1.77E+10	4.92E+08	59772432	8.42E+08	2.39E+09	7.28E+09	1.52E+09	2.04E+08
	Best	1371.145	2.01E+10	5240419	2.74E+10	5182405	1.27E+10	3.25E+08	5387611	72638228	1.61E+09	4.59E+09	3.18E+08	1.74E+08
	Worst	1439.935	2.55E+10	1.57E+08	4E+10	1.57E+08	2.12E+10	6.8E+08	1.58E+08	2.23E+09	2.88E+09	9.28E+09	2.65E+09	2.79E+08
	Std	36.55441	2.96E+09	73888065	6.14E+09	73970080	3.77E+09	1.79E+08	73932725	1.05E+09	6.34E+08	2.06E+09	1.24E+09	52968231
	Median	1409.02	2.33E+10	37850377	3.7E+10	37813725	1.85E+10	4.8E+08	38065123	5.35E+08	2.53E+09	7.62E+09	1.55E+09	1.81E+08
	Rank	1	12	3	13	2	11	6	4	7	9	10	8	5

(Continued)

Table 7 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F14	Mean	1467.509	38201209	6130231	66560108	680421.3	7969907	12650912	3116933	8567929	12120333	10124288	1279978	9300795
	Best	1458.803	33118529	3830170	60805305	464925.2	3996189	7585099	1140868	5419411	8962357	7816864	830032.6	5248219
	Worst	1472.733	43330494	9554680	72582739	928675.2	14752220	17373173	4364593	12844546	15194947	15183338	1880808	13289419
	Std	6.359294	4805838	2618143	6085202	201246.1	4944907	4213852	1468003	3423689	3337191	3579954	502025.8	3537634
	Median	1469.25	38177905	5568037	66426194	664042.5	6565611	12822688	3481135	8003879	12162014	8748475	1204536	9332772
	Rank	1	12	5	13	2	6	11	4	7	10	9	3	8
C17-F15	Mean	1609.893	1.28E+10	31617831	1.95E+10	31594360	1E+10	89585483	31652734	4.47E+08	1.02E+09	1.06E+09	3.08E+08	42053666
	Best	1551.154	1.19E+10	2144784	1.39E+10	2123802	3.02E+08	34393185	2177506	29295145	3.32E+08	4.14E+08	2119821	8842930
	Worst	1652.294	1.43E+10	94565759	2.43E+10	94553853	1.87E+10	1.36E+08	94661194	1.34E+09	2.13E+09	1.35E+09	1.18E+09	1.02E+08
	Std	46.45507	1.14E+09	44996355	5.43E+09	44998644	8.41E+09	55363513	45033640	6.37E+08	8.25E+08	4.64E+08	6.15E+08	43471904
	Median	1618.063	1.25E+10	14880390	1.99E+10	14849892	1.05E+10	94043556	14886117	2.1E+08	8.06E+08	1.24E+09	21955245	28501401
	Rank	1	12	3	13	2	11	6	4	8	9	10	7	5
C17-F16	Mean	2711.795	16369.04	6592.343	19438.38	5259.269	12745.91	14130.39	6147.395	5737.624	10203.94	9840.933	6054.665	9415.349
	Best	2171.69	15349.77	5682.333	15480.72	5142.58	10630.79	11597.34	5486.457	5194.048	9721.722	8561.5	5892.124	8624.359
	Worst	3397.326	16864.19	7158.504	21677.54	5338.165	15225.35	15534.26	6517.712	6317.253	11120.18	11297.05	6252.058	10041.05
	Std	536.4345	726.6692	681.9347	2945.635	97.52369	1990.874	1877.149	504.3495	623.8206	692.1885	1298.565	156.3137	695.3848
	Median	2639.081	16631.1	6764.267	20297.64	5278.166	12563.74	14694.98	6292.706	5719.598	9986.928	9752.591	6037.238	9497.994
	Rank	1	12	6	13	2	10	11	5	3	9	8	4	7
C17-F17	Mean	2716.564	3497941	5481.873	6880833	4490.27	181909.1	14759.07	4755.924	5200.646	7885.493	39106.67	5691.714	6568.89
	Best	2275.021	1025540	5231.872	1865463	4229.64	9025.842	9292.5	4370.788	4241.736	7719.051	25995.03	5399.483	6474.75
	Worst	3429.127	7957748	5802.97	15832349	4769.554	482199.4	24700.64	5169.876	6710.02	8173.115	63170.41	5826.365	6706.455
	Std	541.1628	3437545	281.4699	6912225	279.5708	217461.6	7311.942	397.6002	1155.152	214.537	17350.2	207.4288	103.6696
	Median	2581.054	2504239	5446.324	4912761	4480.944	118205.6	12521.58	4741.517	4925.414	7824.903	33630.61	5770.504	6547.178
	Rank	1	12	5	13	2	11	9	3	4	8	10	6	7
C17-F18	Mean	1903.746	49098228	3025157	86114835	882863.4	13043036	10639498	4759167	9775293	14122618	10435430	6026153	5695366
	Best	1881.15	22599719	1920448	33825726	564261.1	5742485	8069360	3230760	3078416	10657313	5252169	4054730	4767197
	Worst	1919.921	87758368	3954017	1.56E+08	1249819	25459826	12901749	7503672	15796532	19655583	21881730	8356217	8361237
	Std	20.3854	29199061	1114076	54288953	297587.5	9402383	2338341	1999687	5504577	4130297	8230532	2324072	1870331
	Median	1906.955	43017412	3113081	77107955	858686.9	10484916	10793441	4151119	10113112	13088787	7303911	5846833	4826515
	Rank	1	12	3	13	2	10	9	4	7	11	8	6	5
C17-F19	Mean	1972.839	1.06E+10	25103411	1.86E+10	22948166	4.21E+09	1.34E+08	36516864	3.22E+08	5.78E+08	1.33E+09	2.46E+08	33331507
	Best	1967.139	9.37E+09	2452035	1.36E+10	451315	1.88E+09	45635836	8380169	2554023	2.62E+08	2.37E+08	54035041	7405979
	Worst	1977.869	1.24E+10	70713595	2.31E+10	68367939	8.39E+09	2.13E+08	90255473	9.67E+08	1.34E+09	2.47E+09	5.52E+08	73738269
	Std	4.772401	1.45E+09	33379074	4.11E+09	33344392	3.03E+09	93655716	40315077	4.74E+08	5.42E+08	1.16E+09	2.52E+08	30165409
	Median	1973.174	1.03E+10	13624008	1.88E+10	11486704	3.29E+09	1.38E+08	23715906	1.58E+08	3.52E+08	1.31E+09	1.9E+08	26090890
	Rank	1	12	3	13	2	11	6	5	8	9	10	7	4

(Continued)

Table 7 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F20	Mean	3192.04	6786.56	5862.115	6997.391	4461.887	6571.54	6582.012	5558.61	5777.372	6752.353	5982.709	5192.615	5942.198
	Best	2806.762	6653.788	5645.851	6832.146	4382.099	6077.651	6298.833	5229.125	4674.732	6105.004	5692.855	4515.708	5459.311
	Worst	3662.121	7030.393	6042.001	7154.899	4566.542	7310.59	6840.553	5950.009	6630.726	7070.085	6169.483	6009.876	6286.234
	Std	462.1717	178.6222	192.5978	146.4021	80.77073	570.1676	243.4149	312.7201	1013.239	464.0537	232.9712	659.0846	400.3169
	Median	3149.639	6731.029	5880.305	7001.259	4449.453	6448.959	6594.331	5527.653	5902.014	6917.161	6034.25	5122.437	6011.624
	Rank	1	12	6	13	2	9	10	4	5	11	8	3	7
C17-F21	Mean	2342.155	3993.262	3479.831	4095.943	2788.949	3856.443	3941.788	3123.929	2907.373	3513.908	4347.53	3409.695	3273.544
	Best	2338.689	3951.57	3303.936	4035.382	2752.489	3730.444	3691.765	3067	2833.288	3382.693	3878.657	3253.382	3238.239
	Worst	2346.015	4053.164	3598.823	4144.363	2818.995	3941.854	4136.567	3227.704	2956.224	3667.875	4721.159	3710.202	3316.122
	Std	3.54374	51.52115	132.4952	48.78381	28.92288	108.0884	210.7737	75.48026	54.98993	127.4221	370.0533	219.1612	34.93936
	Median	2341.959	3984.157	3508.283	4102.012	2792.156	3876.737	3969.409	3100.505	2919.991	3502.532	4395.153	3337.597	3269.907
	Rank	1	11	7	12	2	9	10	4	3	8	13	6	5
C17-F22	Mean	11739	29080.23	19541.57	30460.41	18247.88	28243.97	26884.17	17076.2	22096.66	30358.4	20292.51	20926.96	26602.71
	Best	11119.08	28903.56	18102.09	29922.55	16873.58	27921.91	26278.75	15966.08	17858.91	30187.28	19389.43	19449.65	25592.13
	Worst	12601.6	29194.49	21769.04	30835.49	19511.99	28931.49	27660.32	17517.06	32076.89	30493.67	21165.68	22120.46	26992.6
	Std	686.6098	140.794	1698.94	425.363	1300.083	487.2466	679.3289	780.5536	7110.969	134.9744	763.3099	1318.171	709.8329
	Median	11617.67	29111.43	19147.58	30541.79	18302.97	28061.24	26798.8	17410.82	19225.42	30376.33	20307.47	21068.87	26913.04
	Rank	1	11	4	13	3	10	9	2	7	12	5	6	8
C17-F23	Mean	2877.697	4978.106	3955.4	4979.918	3275.195	5079.131	4822.038	3431.943	3543.955	4039.899	7116.307	4588.425	4083.7
	Best	2872.107	4763.585	3890.138	4751.6	3262.976	4441.438	4699.796	3352.089	3514.192	3992.441	6614.835	4146.553	4030.291
	Worst	2884.013	5225.31	4025.858	5160.213	3300.136	5945.907	4941.281	3530.708	3585.504	4108.926	7470.267	4822.508	4136.13
	Std	5.486704	216.7082	67.17814	177.8853	17.84688	709.191	121.2573	79.212	34.30244	51.81721	407.8773	319.938	62.56197
	Median	2877.334	4961.764	3952.802	5003.929	3268.834	4964.59	4823.538	3422.488	3538.061	4029.115	7190.063	4692.32	4084.189
	Rank	1	10	5	11	2	12	9	3	4	6	13	8	7
C17-F24	Mean	3327.407	7762.246	5108.848	9421.599	3715.752	6200.975	5955.771	3928.173	4202.363	4587.434	9685.991	5603.383	5111.337
	Best	3295.518	6188.448	4910.294	6507.595	3661.71	5797.666	5587.4	3881.303	3987.354	4402.673	9145.69	5299.36	5021.174
	Worst	3357.991	8852.361	5251.505	11357.15	3784.675	6464.177	6511.428	4010.43	4395.986	4794.583	11124.92	5995.987	5267.189
	Std	31.15784	1337.708	161.4569	2478.136	55.17383	300.6364	424.3766	62.676	223.1889	169.6631	1010.363	333.603	113.4499
	Median	3328.059	8004.088	5136.795	9910.828	3708.311	6271.028	5862.128	3910.479	4213.056	4576.24	9236.68	5559.093	5078.492
	Rank	1	11	6	12	2	10	9	3	4	5	13	8	7
C17-F25	Mean	3185.232	13549.67	4195.488	18643.97	3813.876	9521.712	6855.864	3580.533	6122.821	8204.563	9990.768	4196.381	7347.412
	Best	3137.371	12906.96	3866.822	17330.56	3687.409	8991.715	6321.705	3548.261	5983.044	7186.91	9291.77	3960.044	6736.166
	Worst	3261.571	15027.29	4519.012	21554.91	3914.156	9871.17	7203.615	3630.205	6487.094	9589.049	11270.11	4549.077	7986.061
	Std	63.01686	1049.542	282.3588	2090.283	98.70775	420.3253	413.4837	38.62969	256.4221	1160.468	927.654	288.2573	675.5625
	Median	3170.992	13132.21	4198.059	17845.2	3826.97	9611.981	6949.068	3571.833	6010.574	8021.147	9700.597	4138.202	7333.711
	Rank	1	12	4	13	3	10	7	2	6	9	11	5	8

(Continued)

Table 7 (continued)

		FNO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F26	Mean	5757.621	34824.54	22332.59	39818.21	11406.35	29620	30147.17	11581.86	15804.84	21687.1	30064.86	19032.81	20960.61
	Best	5645.905	34335.8	19804.5	37641.14	10904.61	28595.15	27300.88	10409.61	14089.45	17906.8	28925.2	17189.52	19696.16
	Worst	5844.642	35125.8	24904.48	41259.34	12080.26	30263.45	32684.34	13477.27	17259.74	26307.69	31482.31	20596.5	21766.88
	Std	88.27609	373.8791	2293.116	1767.9	607.5807	753.6511	2825.833	1394.721	1406.436	3653.115	1117.408	1512.922	956.8509
	Median	5769.969	34918.28	22310.7	40186.18	11320.27	29810.7	30301.74	11220.27	15935.09	21266.95	29925.98	19172.62	21189.7
	Rank	1	12	8	13	2	9	11	3	4	7	10	5	6
C17-F27	Mean	3309.493	8431.117	4079.44	10925.66	3549.154	6137.902	5634.58	3624.834	4011.29	4219.569	12415.63	4004.856	5195.937
	Best	3278.01	7193.617	3920.903	8328.655	3526.795	5893.316	5032.975	3578.077	3854.802	3991.86	12117.8	3836.301	4982.324
	Worst	3344.5	9697.866	4330.141	13644.12	3566.024	6447.772	6306.475	3699.561	4135.916	4596.338	12658.07	4171.816	5523.266
	Std	29.83731	1433.942	184.7193	3016.573	17.79592	256.779	713.5143	55.66379	145.5687	281.6278	251.8467	202.401	245.9069
	Median	3307.732	8416.493	4033.359	10864.93	3551.899	6105.259	5599.435	3610.849	4027.222	4145.039	12443.32	4005.654	5139.079
	Rank	1	11	6	12	2	10	9	3	5	7	13	4	8
C17-F28	Mean	3322.242	18529.7	4871.811	24712.73	4082.497	14143.31	9651.713	3818.942	8720.66	10325.03	16757.36	7349.623	10589.75
	Best	3318.742	17221.14	4746.14	22200.13	3949.672	11194.77	8344.877	3719.588	7427.774	8162.806	14548.96	5240.782	9676.89
	Worst	3327.816	20913.76	5033.137	27953.61	4234.788	16439.41	10477.89	3878.271	10565.86	12273.48	18311.19	10788.9	11683.62
	Std	4.609511	1757.062	125.1428	2550.493	123.1342	2607.096	968.2252	79.32491	1392.117	1986.002	1679.512	2624.736	1102.466
	Median	3321.205	17991.95	4853.983	24348.59	4072.764	14469.52	9892.042	3838.955	8444.501	10431.91	17084.65	6684.406	10499.25
	Rank	1	12	4	13	3	10	7	2	6	8	11	5	9
C17-F29	Mean	4450.696	155369.8	9044.079	294907.2	6738.552	16472.6	14872.16	8252.835	7934.814	11403.39	21937.29	8222.392	10904.45
	Best	4169.151	88915.29	7974.815	158677.9	6012.683	12863.77	12524.18	7480.357	7756.966	10648.75	18265.13	7675.744	10729.29
	Worst	4829.521	211686.9	9659.388	409063.8	7395.178	20638.08	16924.57	8810.071	8207.92	11920.19	28467.68	8950.622	11303.46
	Std	297.0014	54987.89	775.3449	112240.6	596.7075	3414.595	2274.751	600.637	206.4733	571.55	4993.753	635.7098	284.6627
	Median	4402.056	160438.5	9271.057	305943.5	6773.173	16194.27	15019.95	8360.456	7887.186	11522.31	20508.17	8131.601	10792.53
	Rank	1	12	6	13	2	10	9	5	3	8	11	4	7
C17-F30	Mean	5407.166	1.96E+10	1.4E+08	3.18E+10	1.21E+08	1.14E+10	1.38E+09	2.04E+08	1.66E+09	3.3E+09	6.29E+09	6.25E+08	6.76E+08
	Best	5337.48	1.72E+10	64490286	2.97E+10	53136534	7E+09	1.08E+09	1.34E+08	6.81E+08	1.35E+09	4.54E+09	1.72E+08	6.06E+08
	Worst	5557.155	2.12E+10	1.94E+08	3.43E+10	1.55E+08	1.41E+10	1.83E+09	2.59E+08	2.17E+09	5.94E+09	7.62E+09	1.7E+09	7.52E+08
	Std	106.4092	1.77E+09	58117376	2.06E+09	48957562	3.24E+09	3.38E+08	56208885	7.04E+08	2.44E+09	1.37E+09	7.58E+08	73496112
	Median	5367.014	1.99E+10	1.52E+08	3.15E+10	1.38E+08	1.22E+10	1.3E+09	2.1E+08	1.89E+09	2.95E+09	6.49E+09	3.14E+08	6.73E+08
	Rank	1	12	3	13	2	11	7	4	8	9	10	5	6
Sum rank		29	336	140	355	65	293	265	114	156	249	272	162	203
Mean rank		1	11.58621	4.827586	12.24138	2.241379	10.10345	9.137931	3.931034	5.37931	8.586207	9.37931	5.586207	7
Total rank		1	12	4	13	2	11	9	3	5	8	10	6	7

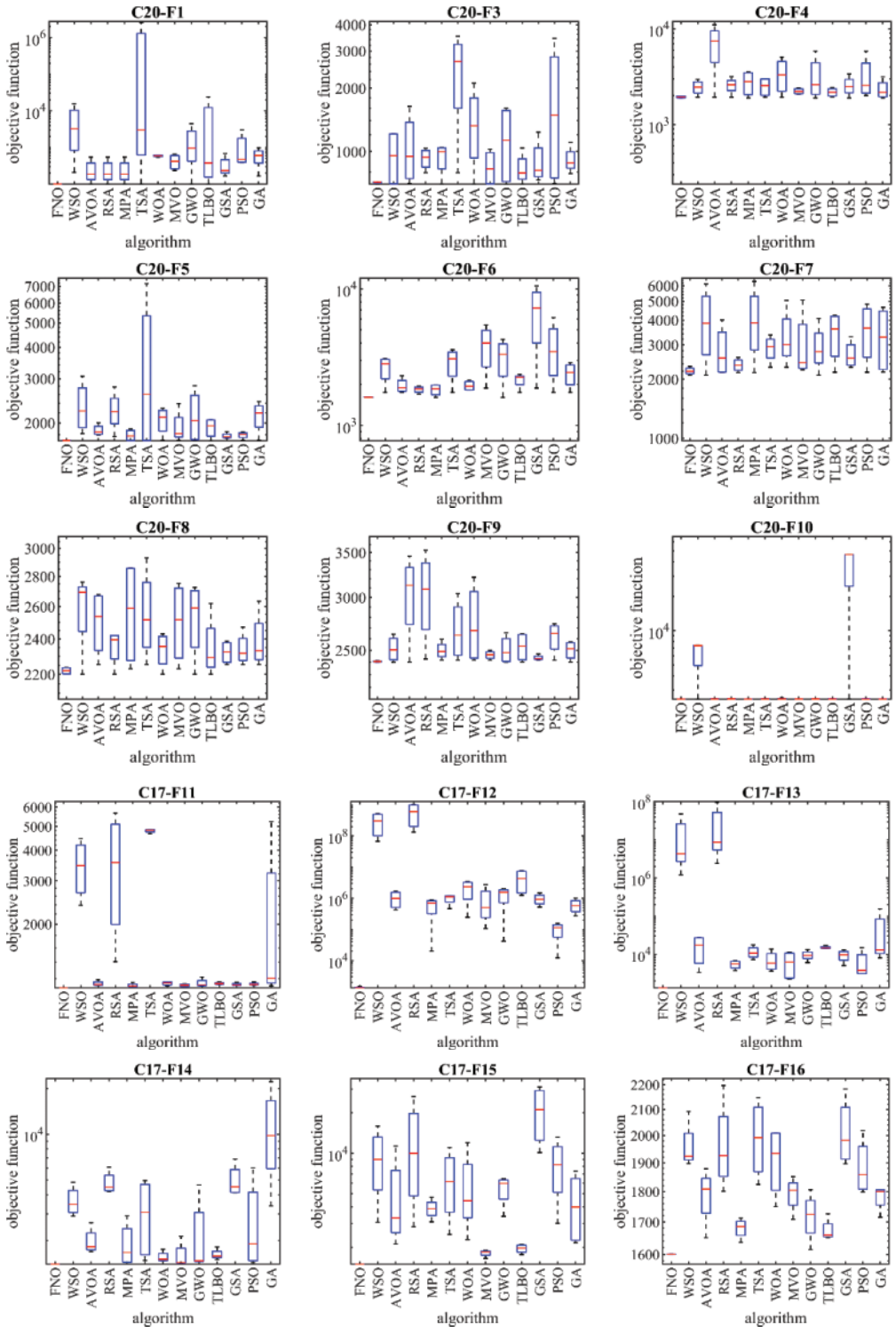

Figure 3: (Continued)

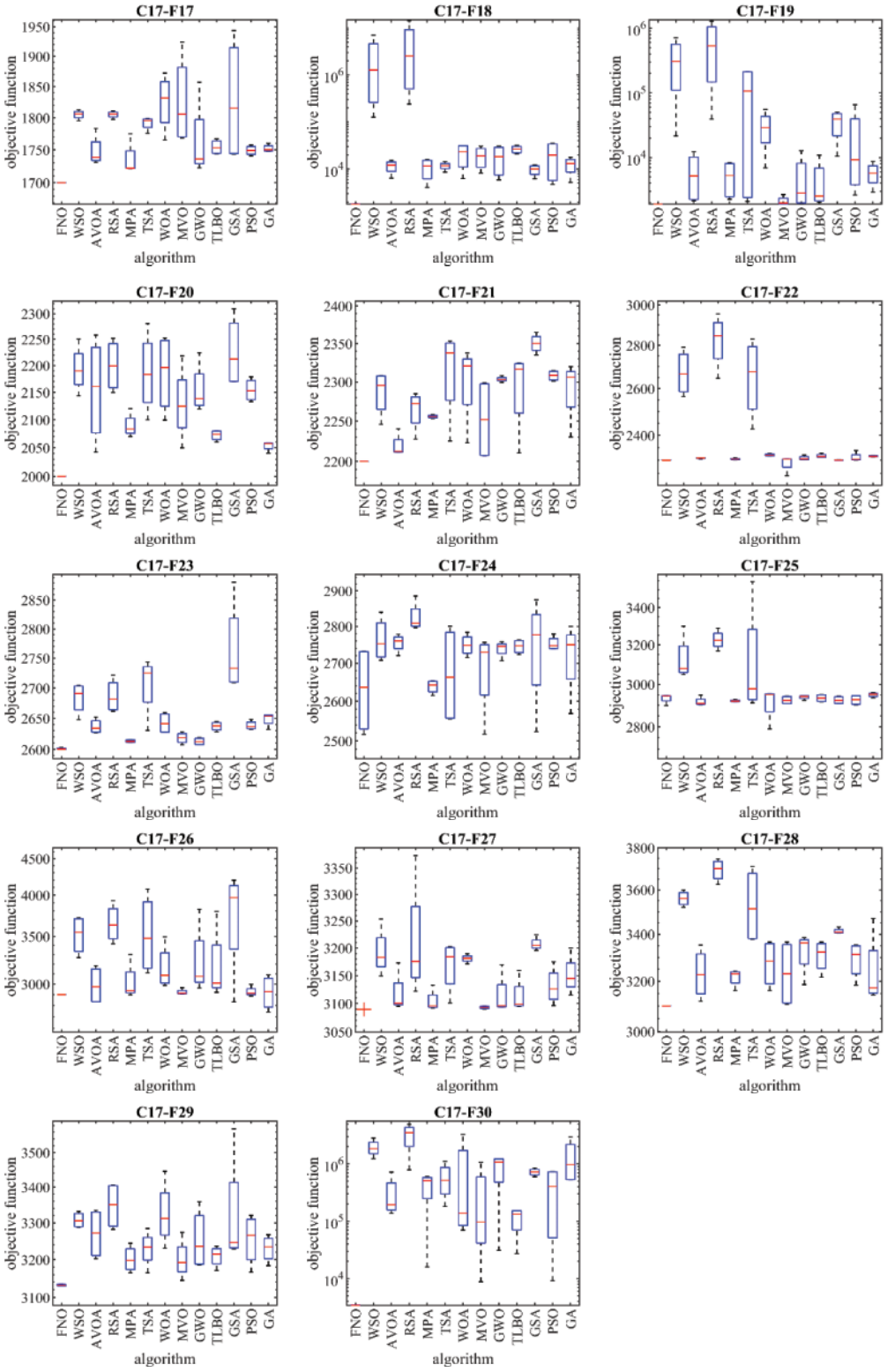

Figure 3: Boxplot diagrams of FNO and competitor algorithms performances on CEC 2017 test suite (dimension = 10)

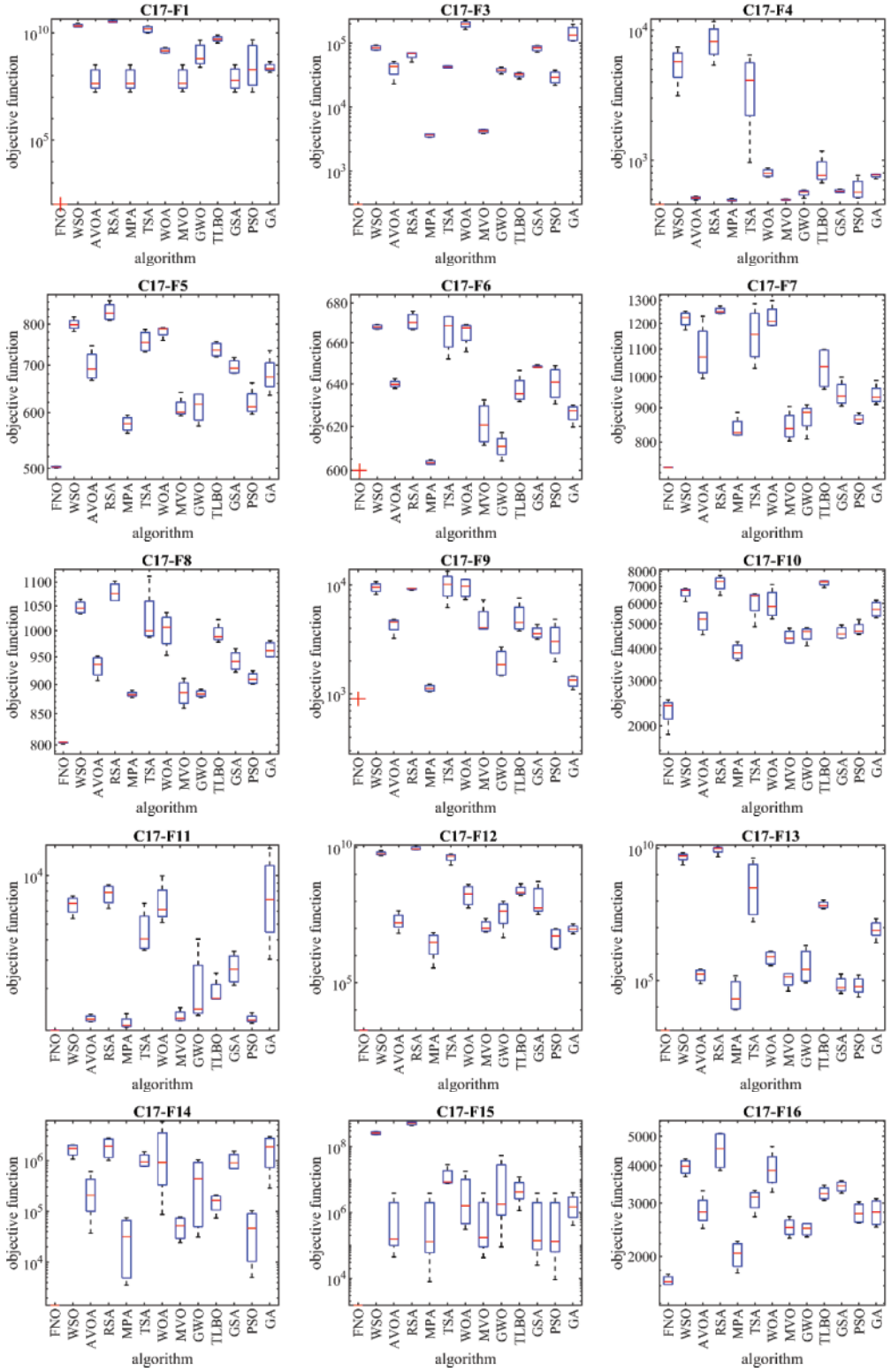

Figure 4: (Continued)

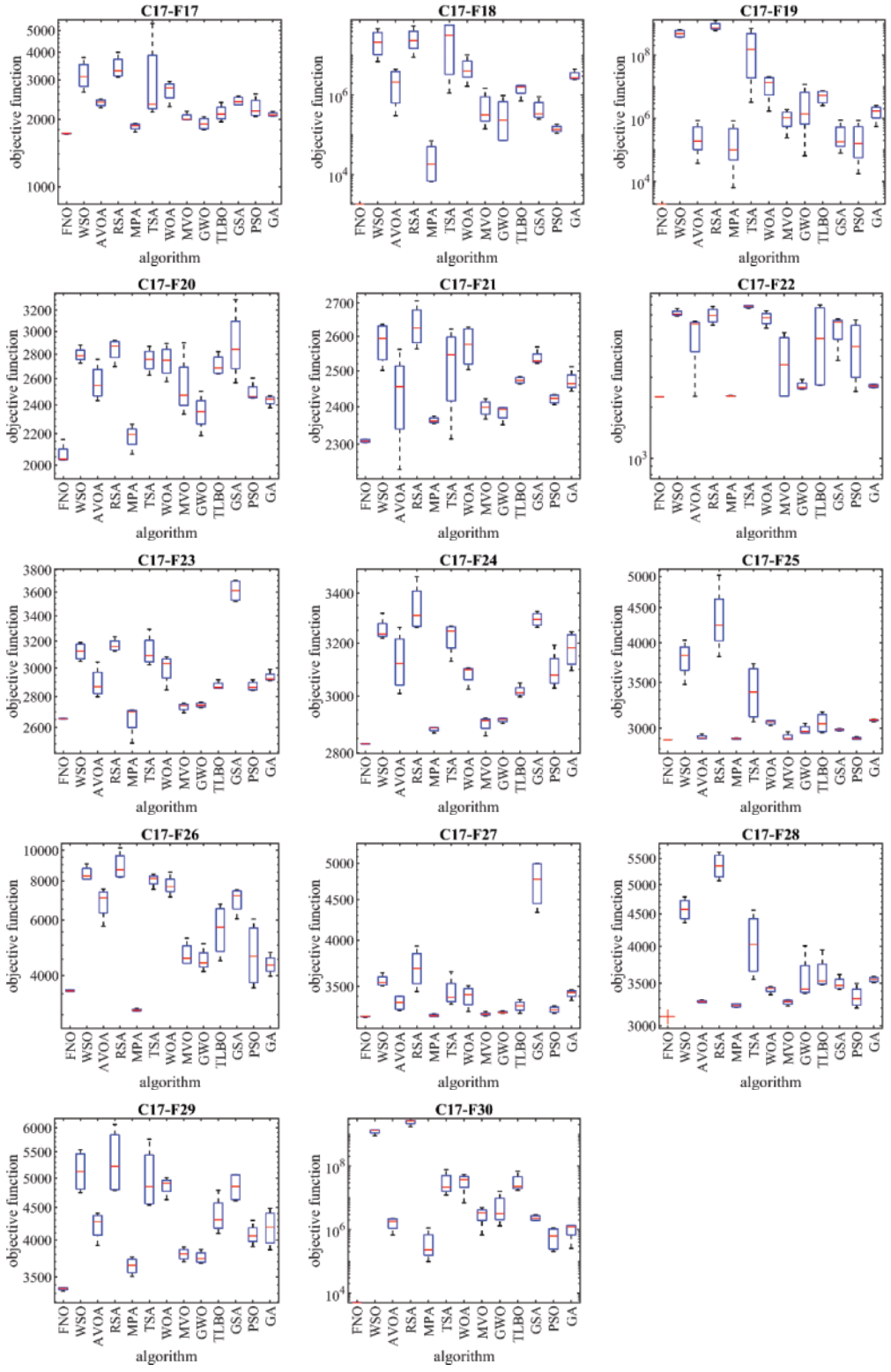

Figure 4: Boxplot diagrams of FNO and competitor algorithms performances on CEC 2017 test suite (dimension = 30)

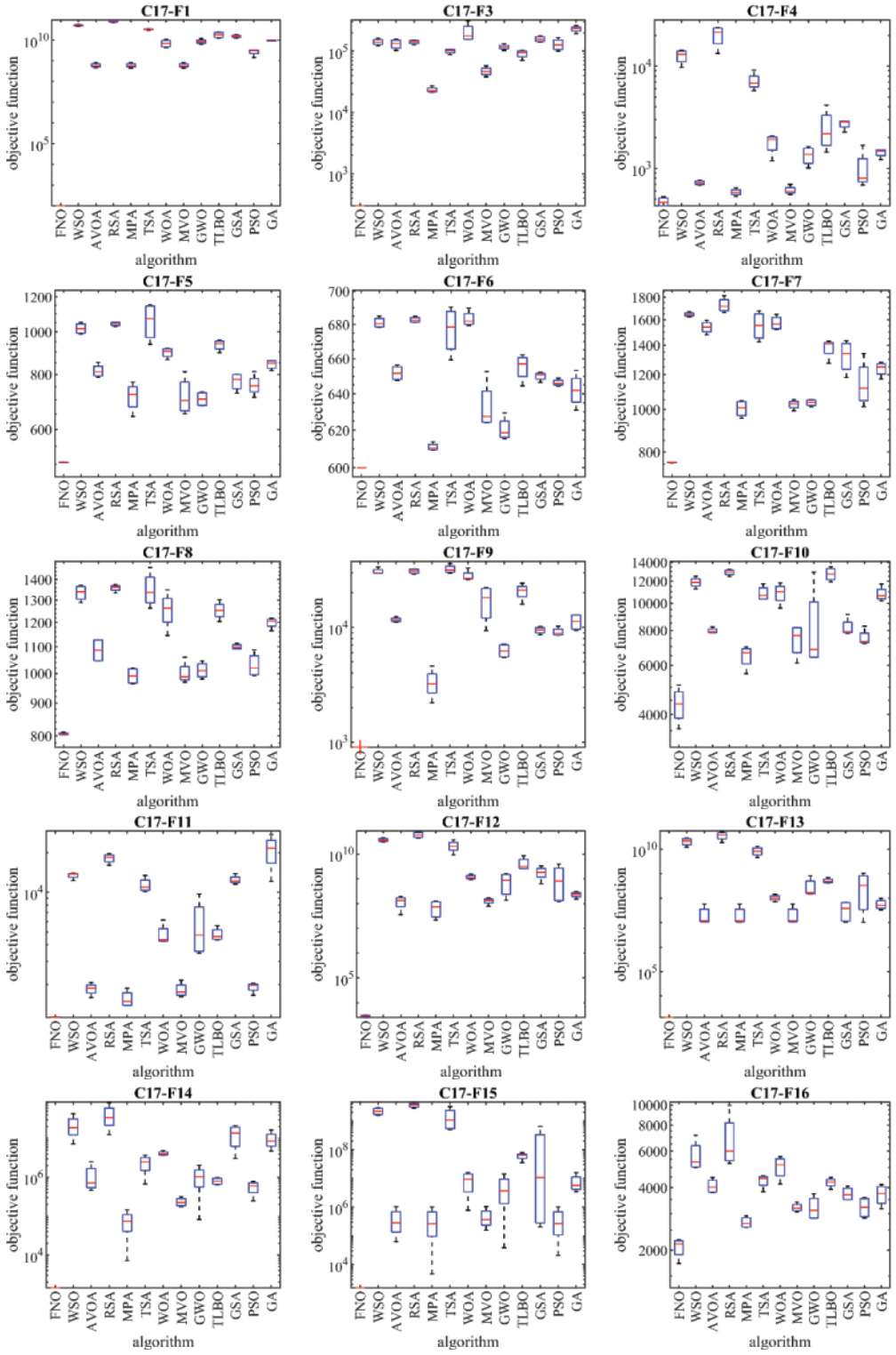

Figure 5: (Continued)

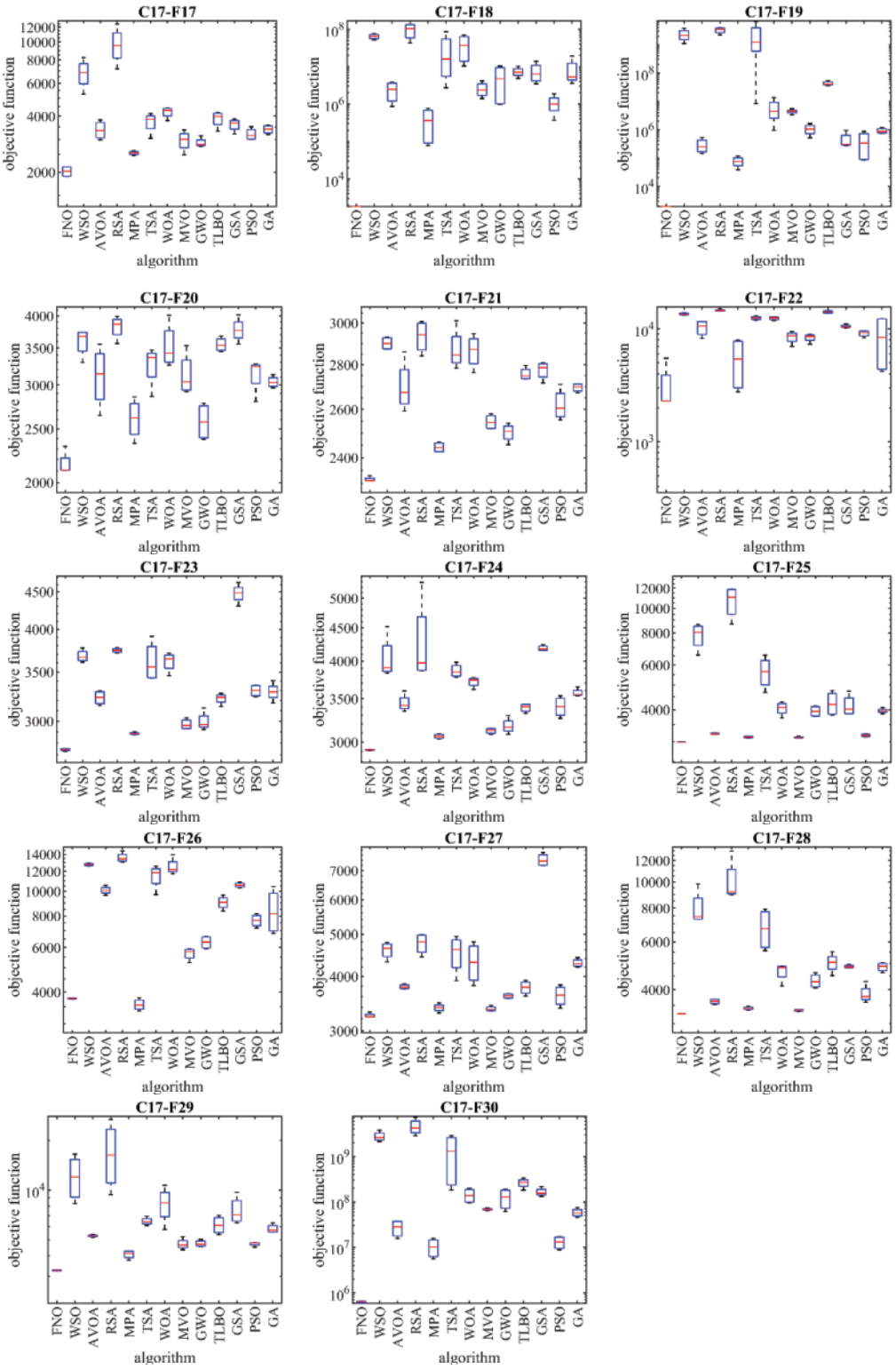

Figure 5: Boxplot diagrams of FNO and competitor algorithms performances on CEC 2017 test suite (dimension = 50)

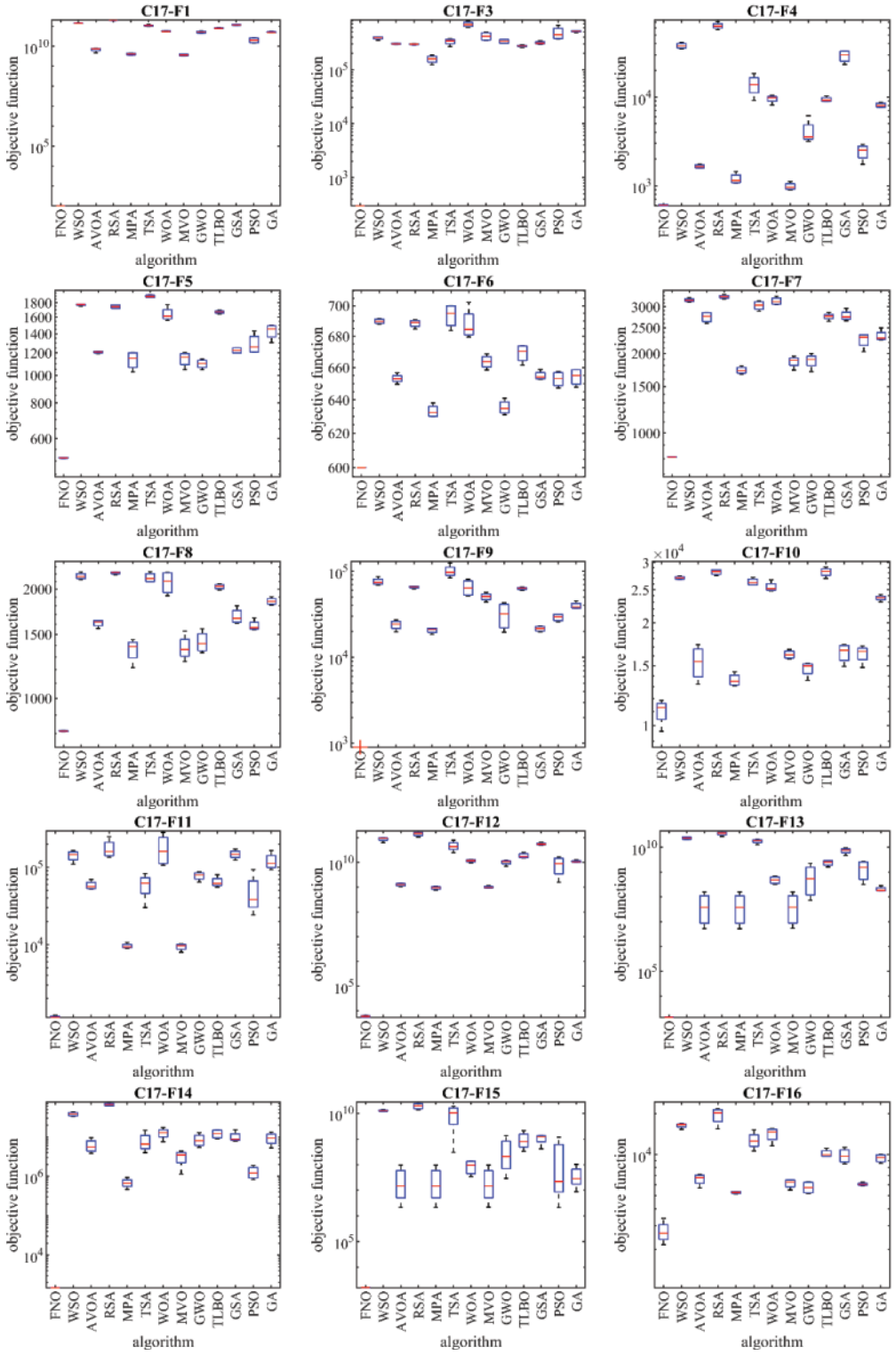

Figure 6: (Continued)

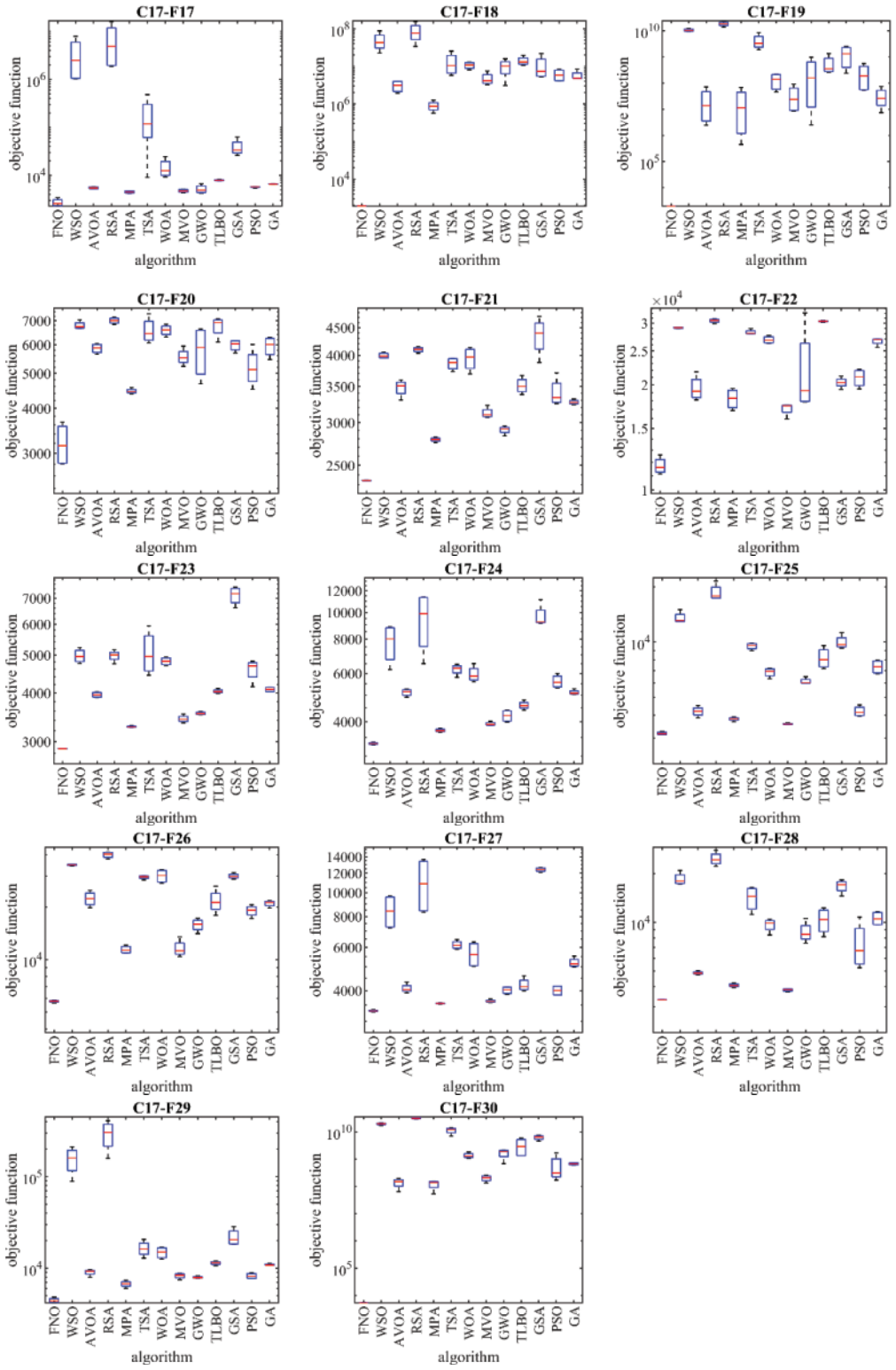

Figure 6: Boxplot diagrams of FNO and competitor algorithms performances on CEC 2017 test suite (dimension = 100)

Table 8: Computational cost (in seconds) results of CEC 2017 test suite

F	D	FNO	AVOA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F1	10	**0.943638**	1.583096	4.024191	6.405592	3.911916	1.303047	1.162363	2.05708	1.372409	4.741238	3.213548	1.517339	2.037542
	30	**1.265738**	1.413261	1.846767	16.48146	4.009595	1.943186	1.343995	2.564344	2.286843	5.168074	9.150027	1.627347	1.886391
	50	**1.99737**	2.089604	2.211443	28.36328	5.273381	2.952156	2.034547	3.944988	3.213452	6.750656	6.569285	2.657902	2.457211
	100	4.51712	4.579711	**4.271255**	56.39666	11.53558	6.761015	4.556632	8.365783	7.24913	14.96532	13.36212	4.896176	5.327995
C17-F3	10	**0.929089**	1.04286	1.383004	6.16697	2.937675	1.256226	1.088733	1.522373	1.285341	4.677143	2.703576	1.259734	1.515914
	30	**1.225901**	1.391617	1.902824	16.514	3.922917	1.97258	1.305722	2.236516	2.229595	5.213546	4.488007	1.593242	1.849118
	50	**1.859628**	1.994077	2.301813	30.99464	5.280301	2.970793	1.870235	3.278145	3.204619	6.799276	6.592748	2.194544	2.317156
	100	4.414052	4.482771	**4.211579**	54.33139	11.43952	6.747196	4.550615	7.193599	7.37274	14.72922	13.19979	4.901213	5.146623
C17-F4	10	**0.915769**	1.032237	1.384822	6.208042	2.976414	1.237309	1.08464	1.630819	1.291011	4.61799	2.657587	1.327702	1.472259
	30	**1.185222**	1.349146	1.858214	16.34037	3.842284	1.959314	1.291446	2.585777	2.137474	5.276378	4.353586	1.637337	1.748408
	50	1.870192	2.035837	2.467055	27.98853	5.349814	3.049928	**1.863984**	3.835268	3.194857	6.76349	6.626663	2.163599	2.298814
	100	**4.405351**	4.519008	4.428645	55.26566	11.48673	6.761333	4.611475	8.24082	7.379116	14.90168	13.33273	5.161936	5.048013
C17-F5	10	**0.933385**	1.04602	1.381088	6.273573	3.015628	1.323984	1.152157	1.683115	1.379372	4.862399	2.760354	1.345287	1.582547
	30	**1.377501**	1.523034	1.934754	16.30239	4.219738	2.135077	1.482399	2.743176	2.414575	5.709299	4.668204	1.765746	1.928986
	50	2.301316	2.413232	2.5762	27.62056	5.81124	3.37138	**2.246185**	4.144463	3.478511	7.676686	6.844373	2.4271	2.607201
	100	5.030091	5.138763	**4.951171**	56.21442	12.58825	7.288918	5.108913	8.77272	7.909036	16.22265	14.20052	5.458182	5.584455
C17-F6	10	**1.126398**	1.294872	1.829421	6.466014	3.375569	1.507921	1.328946	1.897401	1.59733	5.499674	2.886424	1.536365	1.738574
	30	**1.912489**	2.10214	2.624145	16.83683	5.491943	2.717874	2.070499	3.427208	2.927015	7.549411	5.089443	2.314395	2.519413
	50	3.288581	3.346758	3.172633	28.89353	7.869561	4.273647	**3.156906**	5.138662	4.504524	10.65626	7.780686	3.491476	3.640336
	100	6.786919	6.796686	**5.996134**	58.8152	16.60051	9.297162	7.13046	10.77563	9.977606	22.16249	15.7976	7.380006	7.599076
C17-F7	10	**1.012659**	1.156141	1.604793	6.439517	3.212544	1.371907	1.213753	1.751752	1.444604	5.031541	2.8185	1.333314	1.606219
	30	**1.525872**	1.707508	2.245283	16.51654	4.44808	2.201025	1.568279	3.016487	2.40867	5.808502	4.540416	1.904321	1.982735
	50	2.335053	2.437017	2.555998	27.61007	6.153847	3.377765	**2.333907**	4.45428	3.612121	7.827493	6.877402	2.499854	2.656767
	100	5.194727	5.224492	**4.700902**	57.56033	12.73284	7.407609	5.195627	8.873078	7.999521	16.81509	13.96065	5.501317	5.710446
C17-F8	10	**0.992334**	1.20789	1.947352	6.283318	3.087241	1.317752	1.171157	1.698311	1.397969	5.10731	2.722496	1.318552	1.641561
	30	**1.430777**	1.705661	2.628195	16.44981	4.313289	2.171037	1.536237	2.78404	2.318408	5.805171	4.630729	1.834799	1.966238
	50	**2.220374**	2.331793	2.502783	27.35975	6.141061	3.30834	2.329043	4.238619	3.583135	7.823168	6.965961	2.488757	2.776774
	100	5.140905	5.165027	**4.625524**	57.47095	12.79873	7.492479	5.23961	8.872122	7.956285	16.68273	13.92364	5.600977	5.804097
C17-F9	10	**1.031931**	1.161275	1.550988	6.311677	3.158561	1.38957	1.236032	1.957807	1.41982	4.984022	2.833075	1.323421	1.669605
	30	**1.39125**	1.73815	2.953635	16.50098	4.362066	2.203161	1.566925	2.798778	2.39086	5.963327	4.527136	1.796131	1.967704
	50	**2.147766**	2.310723	2.696847	27.50515	6.138456	3.380792	2.297848	4.362373	3.646226	7.813568	6.872742	2.473953	2.66579
	100	4.996202	5.033284	**4.563522**	58.32623	12.75466	7.426297	5.195485	8.842833	7.982558	16.55273	14.04435	5.491215	5.636551
C17-F10	10	**0.961296**	1.08633	1.467542	6.264981	3.145646	1.375752	1.164882	2.280881	1.429785	5.111629	2.787736	1.357167	1.617746
	30	**1.393677**	1.614085	2.323308	16.43479	4.532878	2.317967	1.623748	2.898713	2.461401	6.234178	4.604056	2.097938	2.083909
	50	**2.190106**	2.33686	2.652936	27.66271	6.422074	3.509917	2.410516	4.683054	3.761256	8.420045	7.178027	2.707651	2.9727
	100	**4.637676**	4.834292	5.047025	61.24723	13.7027	7.979256	5.674736	9.529146	8.458111	18.24622	14.36243	5.994234	6.183812

(Continued)

Table 8 (continued)

F	D	FNO	AVOA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F11	10	**0.952038**	1.066553	1.406832	6.37464	3.175761	1.320965	1.136778	1.697331	1.360972	4.933614	2.734688	1.363847	1.632728
	30	**1.31211**	1.529566	2.237064	16.33456	4.090852	2.100505	1.409705	2.371073	2.316734	5.643277	4.387027	1.736832	1.899494
	50	**1.94532**	2.083396	2.395045	27.85837	5.617448	3.130142	2.038194	3.732305	3.37602	7.292931	6.67387	2.368867	2.68521
	100	4.482023	4.58193	**4.427082**	55.82035	12.13102	7.270974	4.877049	7.426876	7.748179	15.63841	13.65734	5.176525	5.608402
C17-F12	10	**1.030992**	1.123171	1.364338	6.209154	3.06695	1.315664	1.125769	1.800704	1.385309	4.994331	2.748337	1.39018	1.621577
	30	**1.364377**	1.602179	2.384595	16.48504	4.331025	2.173524	1.515228	2.748809	2.311146	5.874035	4.615434	1.906525	1.960998
	50	**2.194549**	2.32554	2.578012	27.51985	6.062683	3.312017	2.223965	4.210764	3.558431	7.861787	6.9194	2.654364	2.802153
	100	4.967953	5.028629	**4.656741**	58.48422	12.85632	7.538511	5.260312	9.031602	8.065777	16.70683	13.88842	5.618708	5.96717
C17-F13	10	**1.014724**	1.13616	1.49637	6.281446	3.215371	1.383178	1.193222	1.937149	1.418381	5.075544	2.789648	1.404257	1.615838
	30	**1.270571**	1.435383	1.937448	16.42227	4.225391	2.156006	1.479912	2.704063	2.392655	5.71325	4.523365	1.798766	1.922749
	50	2.375259	2.449219	2.451216	27.28862	5.763	3.217029	**2.114405**	4.242541	3.474091	7.502669	6.748075	2.515707	2.58898
	100	4.511082	4.617708	**4.486141**	56.90503	12.27883	7.24645	5.015775	8.769536	7.846623	15.83409	13.85038	5.246276	5.386116
C17-F14	10	**1.073627**	1.182944	1.487391	6.304518	3.135322	1.391854	1.196807	1.848184	1.409921	5.105715	2.801147	1.424332	1.705614
	30	**1.519454**	1.63361	1.902259	16.5747	4.695981	2.306656	1.653657	2.725056	2.794987	6.228647	4.771402	2.025846	2.116047
	50	**2.215228**	2.361414	2.672111	27.36955	6.409105	3.549084	2.426521	4.2535	3.838332	8.41478	7.047098	2.838305	2.888709
	100	**4.993744**	5.133095	5.072713	57.31802	13.72988	7.933819	5.692025	9.126907	8.546136	18.14142	14.38219	6.094356	6.209456
C17-F15	10	**0.96446**	1.061595	1.330819	6.31015	3.058498	1.313433	1.155879	2.205131	1.345113	4.878095	2.754639	1.35315	1.533785
	30	**1.336478**	1.448251	1.730002	16.25109	4.082402	2.056317	1.513005	2.611331	2.442912	5.436479	4.393152	1.74878	1.871535
	50	**1.94656**	2.083208	2.388985	26.89541	5.60423	3.125149	2.026338	4.000995	3.399974	7.227762	6.804952	2.329487	2.568064
	100	**4.367917**	4.495948	4.467709	56.88235	11.92176	7.097925	4.797472	8.441369	7.671604	15.40191	13.51557	5.141028	5.269202
C17-F16	10	**0.982363**	1.092563	1.411833	6.354393	3.09145	1.345011	1.152327	2.155294	1.374953	4.979615	2.775434	1.379423	1.580402
	30	**1.429814**	1.534563	1.776673	16.5913	4.26574	2.150607	1.613688	2.761878	2.422901	5.790602	4.47891	1.835975	1.976617
	50	**2.102965**	2.229841	2.477186	27.28086	5.838524	3.253143	2.164184	4.341446	3.481773	8.050005	7.06325	2.472514	2.643974
	100	4.820672	4.908906	**4.665472**	55.32919	12.51733	7.465189	5.14283	8.809536	7.929018	16.17491	13.82993	5.445556	5.693101
C17-F17	10	**1.221679**	1.317902	1.551661	6.445206	3.482107	1.566034	1.415488	2.318922	1.587765	5.741628	2.99679	1.547552	1.794696
	30	**2.015923**	2.107086	2.222342	17.03146	5.538777	2.733683	2.095749	3.3172	2.935717	7.503896	5.352274	2.447915	2.534663
	50	3.181131	3.264275	3.203312	28.97495	7.712769	4.210812	**3.104564**	5.18683	4.411553	10.48736	8.238313	3.405478	3.57624
	100	6.367798	6.423244	**5.85726**	58.48177	16.31319	10.68463	6.987761	10.45312	9.780839	21.86651	15.70102	7.306115	7.456471
C17-F18	10	**0.990391**	1.14438	1.637812	6.31286	3.039392	1.37345	1.18141	2.114435	1.432993	5.049462	2.774735	1.400416	1.592656
	30	**1.344953**	1.476479	1.836197	16.50794	4.439156	2.169175	1.520467	2.629998	2.305029	5.727457	4.481437	1.855193	1.974733
	50	**1.98622**	2.101119	2.314997	27.16928	5.922221	3.258925	2.147991	4.263039	3.466858	7.574409	6.982974	2.565446	2.65009
	100	4.623392	4.726155	**4.565243**	56.72284	12.53637	7.645209	5.255334	8.743703	7.952184	16.20433	13.72077	5.467938	5.664581
C17-F19	10	**1.955417**	2.095637	2.414612	7.333061	5.116793	2.408426	2.185875	3.148839	2.444147	8.19639	3.898278	2.445585	2.6079
	30	4.239491	4.27608	**3.897961**	19.42097	10.65424	5.221357	4.589672	5.773174	5.404539	14.96195	7.703112	4.994248	5.057154
	50	6.900162	6.896193	**6.026687**	32.21721	16.04018	8.341345	7.271696	9.803923	8.6811	24.11799	12.13466	7.579072	7.912256
	100	13.34602	13.34456	**11.68769**	68.29124	33.09738	17.48625	15.39419	18.96095	18.26364	46.70063	24.32194	15.61414	15.88095

(Continued)

Table 8 (continued)

F	D	FNO	AVOA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F20	10	**1.213736**	1.37911	1.890453	6.551243	3.476617	1.576621	1.443784	2.250428	1.671454	5.755097	3.013205	1.594801	1.821807
	30	**1.943917**	2.067675	2.322222	17.23006	5.746742	2.850805	2.23519	3.468045	3.209074	7.935644	5.209856	2.463929	2.661997
	50	**3.122607**	3.330494	3.775742	28.25861	8.188441	4.448716	3.327005	5.605411	4.676373	11.69909	8.00799	3.614669	3.802783
	100	6.496966	6.646131	**6.439043**	60.20271	17.2582	9.685805	7.542347	11.13922	10.36337	23.18623	16.36276	7.950868	7.962873
C17-F21	10	**1.246397**	1.365075	1.685593	6.509065	3.67821	1.571497	1.383807	2.096905	1.619051	5.661371	3.025062	1.52408	1.808687
	30	**2.207122**	2.293717	2.367049	17.31182	6.164841	3.042045	2.413057	3.652132	3.205279	8.44079	5.405171	2.760043	2.857065
	50	4.045295	4.134449	**3.990613**	29.33839	10.19363	5.345634	4.241401	6.332394	5.69715	14.44177	8.949361	4.481806	4.821397
	100	11.43088	11.40177	**9.871171**	65.44038	28.04148	15.01686	12.88325	16.61197	15.65352	39.34907	21.54734	13.16289	13.4443
C17-F22	10	**1.358797**	1.488432	1.838874	6.626836	3.684655	1.772597	1.497466	2.13106	1.726702	5.981163	3.148835	1.693244	1.901028
	30	**2.465815**	2.544686	2.555118	17.58437	6.58743	3.251819	2.626755	3.880485	3.404116	9.152478	5.615678	2.885135	3.054332
	50	4.276386	4.348363	**4.107236**	29.66646	10.83081	5.685521	4.609728	6.547461	5.968059	15.27601	9.208929	4.874647	5.142972
	100	12.5568	12.44629	**10.45083**	65.52973	29.34387	15.71686	13.49964	17.35543	16.31821	41.40921	22.23516	13.77283	14.15693
C17-F23	10	**1.310664**	1.440466	1.797534	6.619369	3.832611	1.771668	1.532794	2.16684	1.771407	6.15411	3.16642	1.730208	1.93059
	30	**2.542504**	2.626794	2.649416	17.69085	6.897547	3.458376	2.866457	4.104512	3.657416	9.707524	5.821798	3.089022	3.298424
	50	**4.547107**	4.772215	5.110118	30.57443	12.09766	6.401262	5.302998	7.469741	6.647262	17.54353	10.04646	5.525315	5.679863
	100	**10.41426**	11.07139	12.41156	66.17142	35.51952	18.66019	16.57562	20.25858	19.37997	1326.56	25.21985	16.77714	16.84864
C17-F24	10	**1.362291**	1.490608	1.835344	6.672468	4.01277	1.750173	1.593779	2.191942	1.941247	6.276043	3.1959	1.831297	2.006232
	30	**2.725426**	2.825318	2.887719	17.86842	7.378802	3.654539	3.042872	4.275243	3.846216	10.42713	6.222771	3.2371	3.457234
	50	**5.076273**	5.339674	5.765289	30.62493	12.74456	6.690299	5.620654	7.720118	6.944134	19.02458	10.27779	5.857397	6.289918
	100	**12.99194**	13.39079	13.37895	67.70093	36.73724	19.35246	17.22645	20.81343	19.80846	52.53937	26.01052	17.42242	17.61044
C17-F25	10	**1.257575**	1.39261	1.777173	6.583291	3.712224	1.751549	1.459254	2.094637	1.714968	5.942937	3.088336	1.63839	1.884882
	30	**2.578471**	2.660071	2.667484	17.86822	6.815261	3.390773	2.765187	4.046097	3.565367	9.740856	5.808113	2.964355	3.242644
	50	5.230465	5.347034	**5.16624**	30.53703	12.60585	6.584492	5.493316	7.485713	6.778405	18.16804	10.14696	5.779912	5.918255
	100	17.85481	17.54259	**14.08484**	68.49214	39.34139	20.6831	18.54472	22.25741	21.42674	56.27984	27.35005	18.87771	18.95531
C17-F26	10	**1.442191**	1.562987	1.867755	6.79877	4.273244	1.919745	1.651185	2.237936	1.87562	6.472393	3.240714	1.812288	2.125065
	30	**3.086272**	3.199675	3.271482	18.20593	7.909905	3.931616	3.485475	4.563493	4.192108	11.37455	6.425178	3.491959	4.0234
	50	6.212109	6.188404	**5.325075**	31.61121	14.37638	7.502224	6.384111	8.41198	7.73557	20.81697	11.0707	6.623204	6.873124
	100	19.1232	18.78656	**15.07423**	74.59587	42.9371	22.54353	20.36853	24.20819	23.06341	62.13359	31.59323	1266.628	20.82038
C17-F27	10	**1.392044**	1.528941	1.904317	6.868196	4.134728	1.867463	1.694939	2.283099	1.930563	6.715525	3.329278	1.842092	2.078829
	30	**3.035675**	3.149578	3.229646	18.85547	8.60177	4.290813	3.762545	4.927789	4.498036	12.8161	6.693306	3.975765	4.528151
	50	**6.226264**	6.56681	7.158735	751.4049	16.45292	8.501632	7.367071	9.311285	8.67136	23.61473	12.10706	7.552317	8.20048
	100	19.43779	19.48405	**17.26439**	75.07449	50.08541	25.96096	23.95681	27.61783	26.66497	73.20217	35.49329	25.63387	24.56765
C17-F28	10	**1.318753**	1.467862	1.901153	6.806764	3.979335	1.768222	1.598092	2.209172	1.823445	6.356397	3.247905	1.729504	2.013281
	30	**2.974985**	3.0557	3.010519	18.28915	7.850114	3.871084	3.309712	4.519823	4.079317	11.3554	6.226844	3.509667	3.948564
	50	**6.361014**	6.528713	6.412581	31.50602	462.7972	7.808279	6.632258	8.691254	7.967651	21.51879	11.34388	6.840787	7.841928
	100	21.30956	20.8315	**16.28302**	74.30412	47.22546	24.64309	22.39027	26.21548	25.22822	68.47812	33.33656	22.66888	22.7536

(Continued)

Table 8 (continued)

F	D	FNO	AVOA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F29	10	**1.492716**	1.690406	2.296503	6.831717	4.04011	1.837098	1.635945	2.236849	1.847688	6.525595	3.281418	1.820307	2.044941
	30	**2.734513**	2.843583	2.941571	17.98335	7.133749	3.572183	3.105965	4.303773	3.728723	10.63919	6.304657	3.10057	3.456765
	50	**4.591509**	4.765972	4.895802	30.34651	14.81977	6.187883	5.028518	7.096739	6.355616	16.67771	9.757751	5.312877	5.610104
	100	14.01545	13.7303	**10.85583**	65.29449	30.96446	16.63042	14.33984	17.74328	17.11014	43.88943	24.59769	14.58347	14.78207
C17-F30	10	**2.028156**	2.207273	2.672835	7.659937	5.684242	2.689453	2.51453	3.216611	2.727053	9.068879	4.098566	2.715899	2.887235
	30	**4.705207**	4.82594	4.726783	20.51795	12.15729	6.131017	5.462438	6.672822	6.23661	18.04703	8.628704	5.622371	6.123689
	50	**7.736195**	8.591965	11.05799	34.26105	19.98942	10.3383	9.198505	11.26854	10.54938	29.16824	14.0752	9.443989	9.82425
	100	19.25983	19.20305	**16.59325**	73.79778	47.71889	24.95313	22.64243	26.40534	25.54835	68.85316	33.08901	22.9091	23.04624

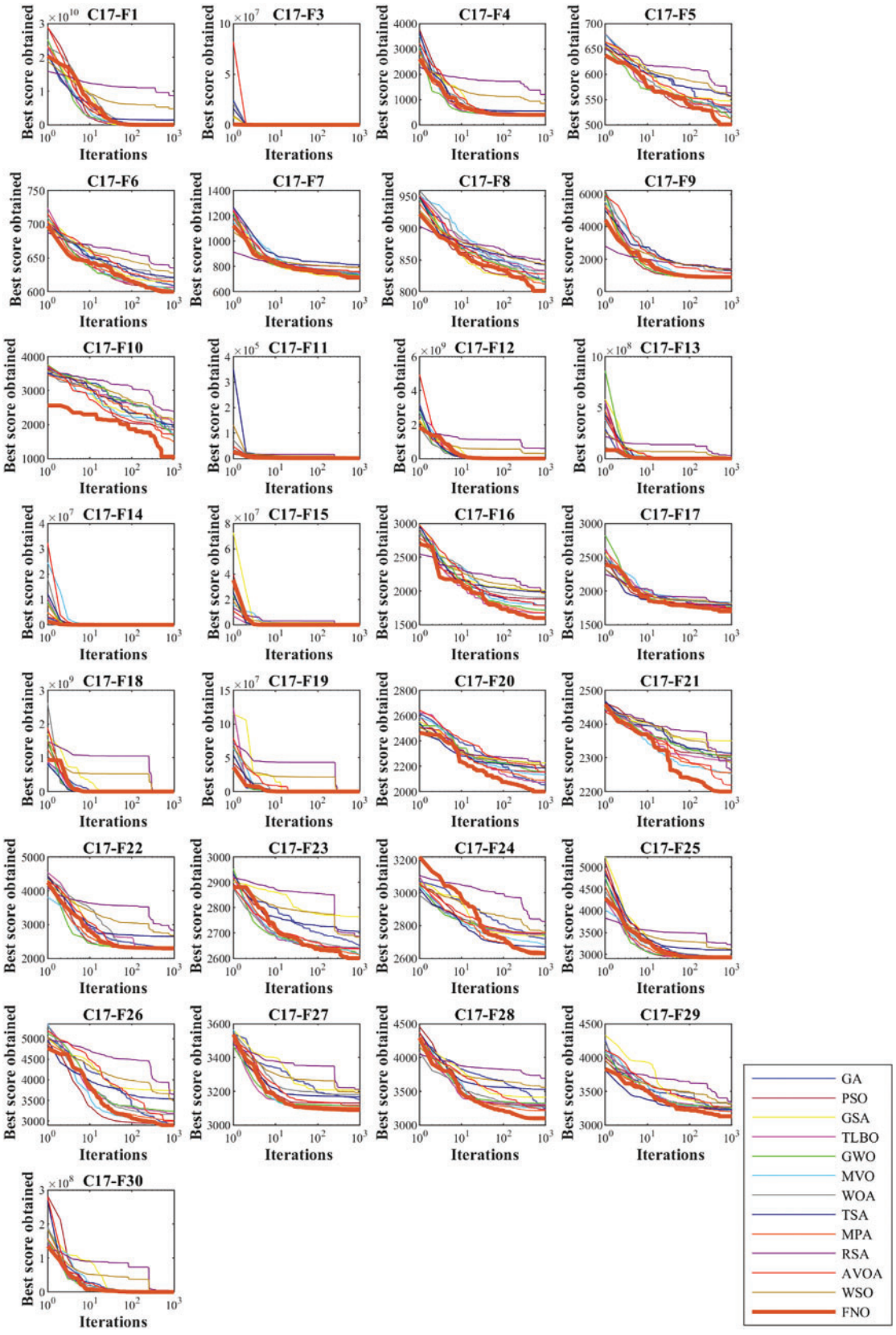

Figure 7: Convergence curves of FNO and competitor algorithms performances on CEC 2017 test suite (dimension = 10)

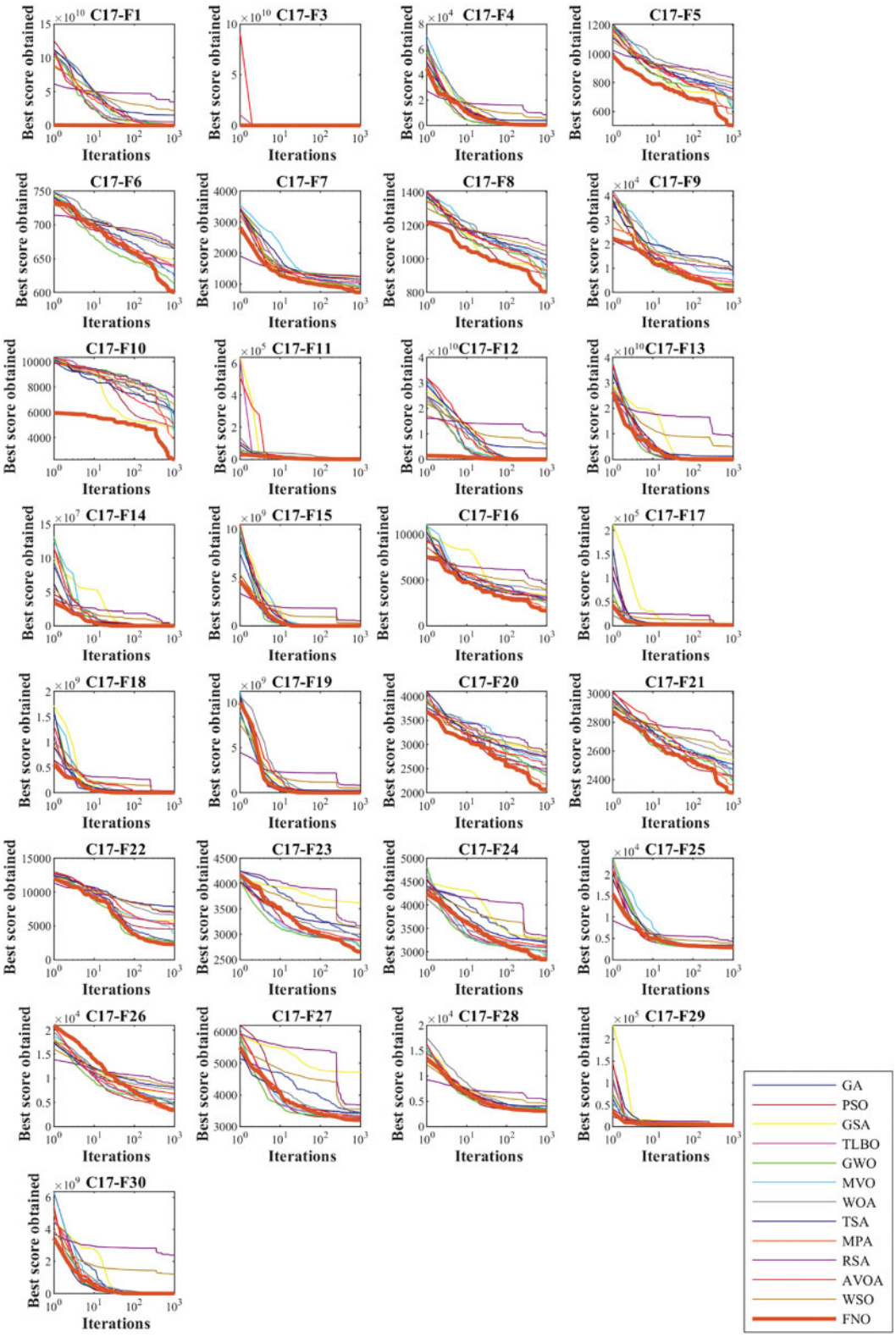

Figure 8: Convergence curves of FNO and competitor algorithms performances on CEC 2017 test suite (dimension = 30)

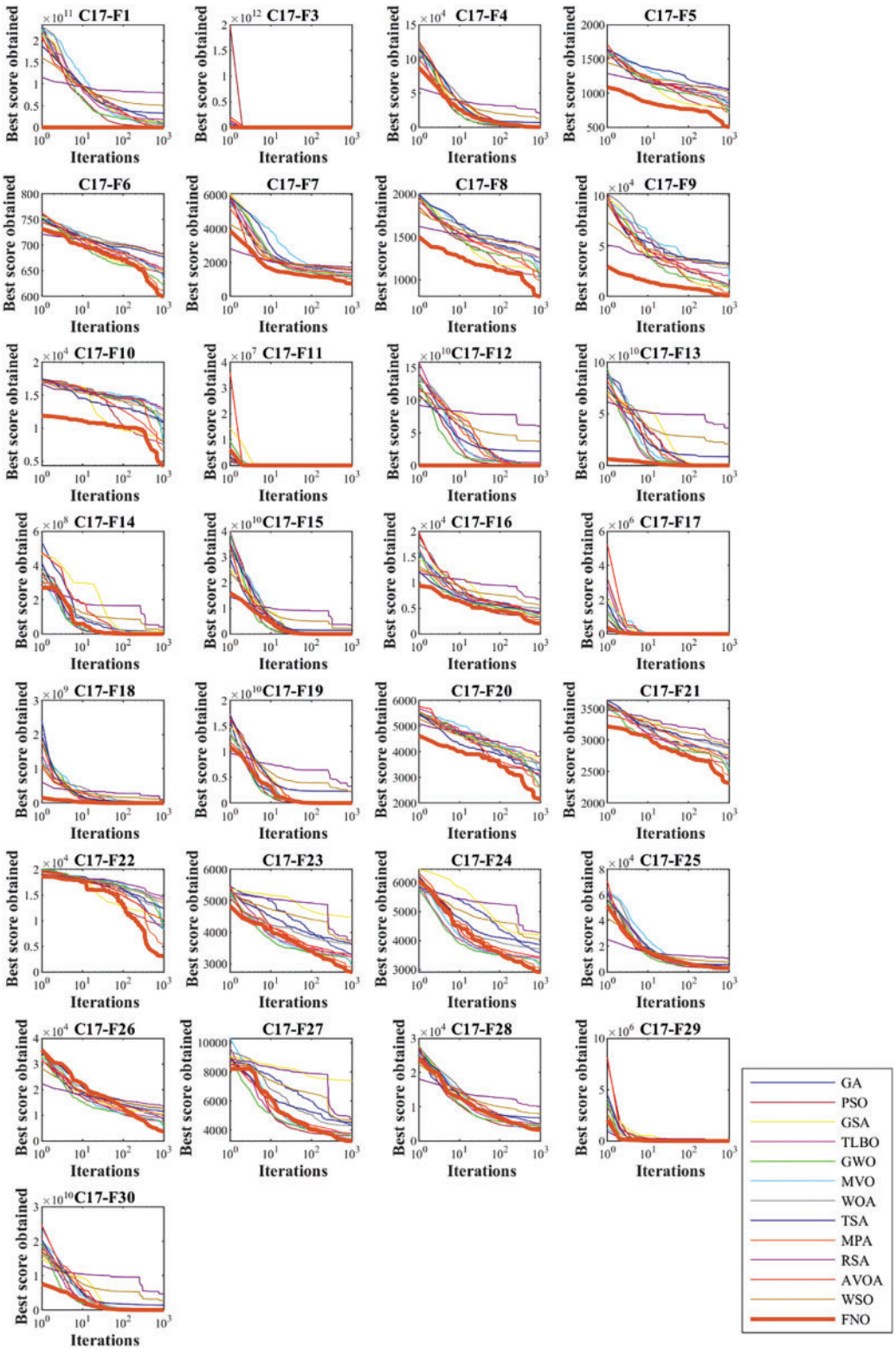

Figure 9: Convergence curves of FNO and competitor algorithms performances on CEC 2017 test suite (dimension = 50)

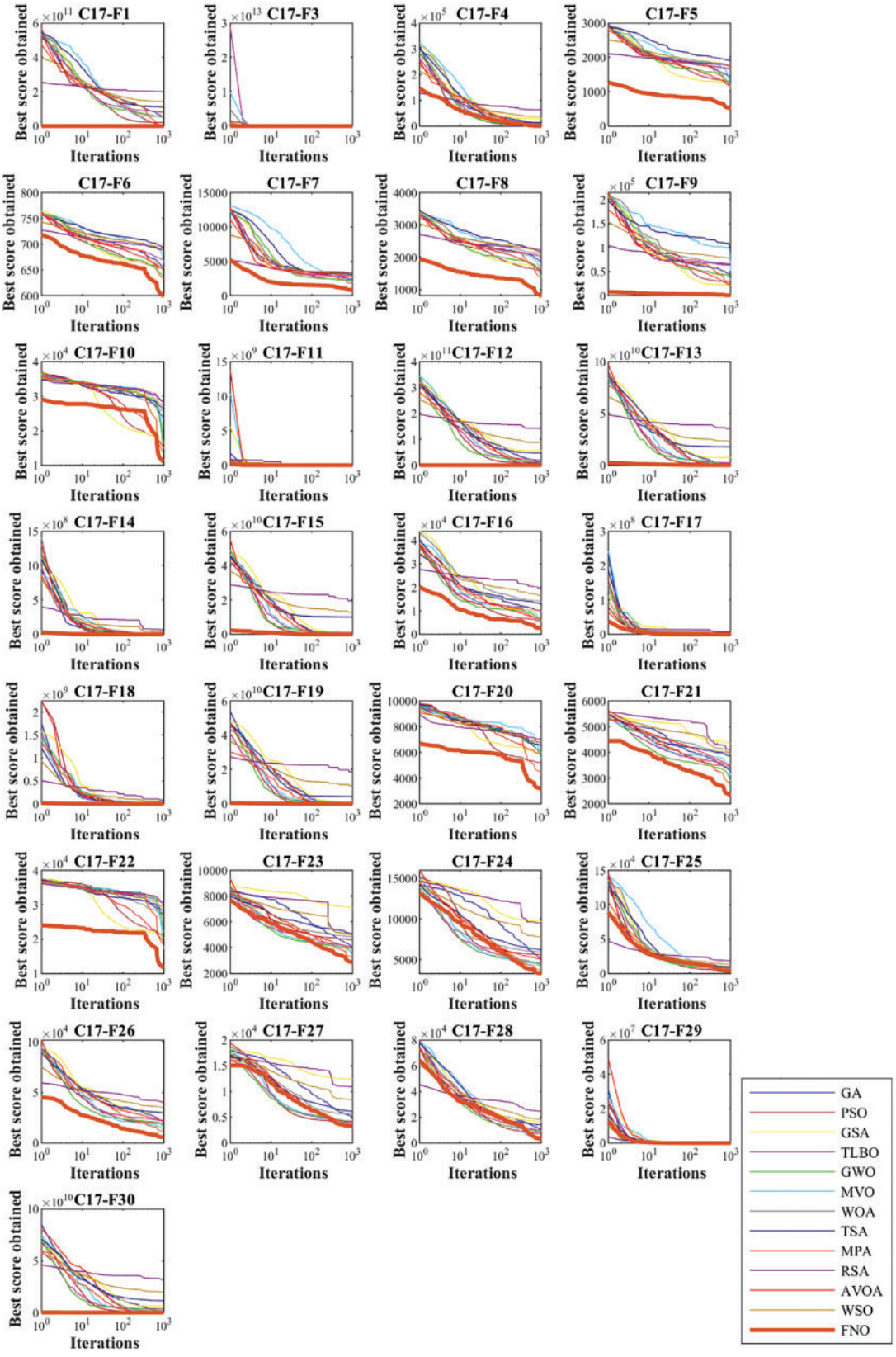

Figure 10: Convergence curves of FNO and competitor algorithms performances on CEC 2017 test suite (dimension = 100)

4.3 Evaluation of CEC 2020 Test Suite

In this sub-section, the effectiveness of the proposed approach to address CEC 2020 is challenged. This test suite consists of ten bound-constrained numerical optimization benchmark functions. Among these, C20-F1 is unimodal, C20-F2 to C20-F4 is basic, C20-F5 to C20-F7 is hybrid, and C20-F8 to C20-F10 is composition. Complete information of CEC 2020 is provided in detail on [60].

The implementation results of the proposed FNO approach and competing algorithms are reported in Table 9. Boxplot diagrams obtained from metaheuristic algorithms are drawn in Fig. 11. Based on the obtained results, FNO has been the first best optimizer to handle all 10 functions C20-F1 to C20-F10. The simulation findings show that FNO has provided superior performance for handling CEC 2020 by providing better results in each benchmark function compared to competing algorithms.

4.4 Statistical Analysis

In this section, a statistical analysis is conducted to determine whether FNO exhibits a significant advantage over competitor algorithms. To address this inquiry, the Wilcoxon rank sum test [61] is utilized, a non-parametric test renowned for discerning significant differences between the means of two data samples. In the Wilcoxon rank sum test, the presence or absence of a noteworthy discrepancy in performance between two metaheuristic algorithms is gauged using a parameter known as the *p*-value. The results of applying the Wilcoxon rank sum test to evaluate FNO's performance against each competitor algorithm are documented in Table 10.

Based on the outcomes of the statistical analysis, instances where the *p*-value falls below 0.05 indicate that FNO demonstrates a statistically significant superiority over the corresponding competitor algorithm. Consequently, it is evident that FNO outperforms all twelve competitor algorithms significantly across problem dimensions equal to 10, 30, 50, and 100 when handling the CEC 2017 test suite. Also, the findings indicate that FNO has a significant statistical advantage compared to competing algorithms for handling the CEC 2020 test suite.

Table 9: Optimization results of CEC 2020 test suite

		FNO	AVOA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C20-F1	Mean	100	5612.07	254.0577	254.0577	254.0578	646429.2	585.798	431.1935	1609.957	6189.223	329.3244	1083.229	586.4009
	Best	100	208.0469	100	100	100	100	542.7263	233.3933	100	100	165.4577	384.7248	165.4577
	Worst	100	15672.09	542.7263	542.7263	542.7263	2579671	611.8795	655.0074	4439.268	23903.82	686.4838	2994.488	974.7303
	Std	9.99E-07	7020.009	197.5098	197.5098	197.5098	1288829	32.63734	200.4796	1935.841	11811.23	240.2066	1276.046	331.5199
	Median	100	3284.07	186.7523	186.7523	186.7523	2972.937	594.2932	418.1867	950.2805	376.5343	232.6781	476.8524	602.7078
	Rank	1	10	2	2	3	12	6	5	9	11	4	8	7
C20-F2	Mean	1109.997	2434.894	1794.357	1706.476	1882.635	1970.708	2783.519	2643.909	7042.562	116943.3	3463.76	2292.66	2603.58
	Best	1100	1605.608	1256.933	1100.166	1255.67	1574.792	1803.602	1729.792	1100.166	1803.602	1100.166	1500.962	2192.343
	Worst	1120.638	3453.718	2192.343	2192.343	2192.343	2576.384	3851.913	3668.555	11806.32	207485.4	4904.348	3162.643	3069.593
	Std	8.462024	910.8041	398.1137	452.2742	426.1382	440.6826	1020.862	969.4117	4433.318	93546.47	1712.811	681.7148	445.9755
	Median	1109.674	2340.125	1864.076	1766.697	2041.263	1865.827	2739.28	2588.644	7631.881	129242.1	3925.263	2253.518	2576.193
	Rank	1	7	3	2	4	5	10	9	12	13	11	6	8
C20-F3	Mean	710.7399	956.2997	1059.434	925.9339	934.5255	2414.686	1364.274	845.1602	1141.895	829.5544	898.7981	1781.918	913.868
	Best	700.0001	701.2474	703.7808	792.4092	700	792.4092	700	700.0213	700	700	732.22	704.0253	784.6324
	Worst	714.3198	1215.565	1638.705	1035.392	1044.754	3525.084	2109.735	1024.31	1603.359	1037.98	1235.284	3447.834	1103.032
	Std	7.159875	292.9431	424.3349	107.5292	161.3755	1173.015	592.2339	167.8835	490.0681	145.7985	228.8461	1300.955	134.5699
	Median	714.3198	954.1933	947.6255	937.967	996.6739	2670.625	1323.681	828.1546	1132.111	790.1189	813.8445	1487.906	883.9038
	Rank	1	8	9	6	7	13	11	3	10	2	4	12	5
C20-F4	Mean	1941.536	2457.537	6964.756	2579.226	2770.2	2508.438	3397.925	2233.294	3248.065	2178.674	2558.146	3255.209	2343.38
	Best	1900	1936.524	1936.524	1936.524	1900	1936.524	1936.524	2084.561	1900	1940.98	1900	2011.54	1900
	Worst	1983.073	2967.722	11009.4	3163.728	3559.686	2995.979	5056.833	2425.333	5856.74	2440.504	3370.068	5855.145	3148.67
	Std	41.6773	439.4324	3769.793	502.3301	832.6108	565.4491	1423.251	160.5537	1797.781	233.5194	610.9464	1775.042	566.0155
	Median	1941.536	2462.952	7456.547	2608.327	2810.557	2550.625	3299.172	2211.64	2617.759	2166.606	2481.257	2577.076	2162.425
	Rank	1	5	13	8	9	6	12	3	10	2	7	11	4
C20-F5	Mean	1702.285	2337.247	1871.272	2245.405	1786.11	3520.324	2051.627	1930.957	2148.966	1913.67	1777.165	1788.926	2131.297
	Best	1700	1813.561	1793.517	1765.604	1700	1700	1704.217	1704.217	1700	1700	1736.904	1707.399	1704.217
	Worst	1704.57	3070.717	2004.301	2781.858	1894.087	7174.555	2288.053	2390.347	2812.285	2059.271	1846.726	1840.713	2431.711
	Std	2.410464	554.9085	95.00707	416.2292	95.59094	2578.037	258.8121	310.6519	530.7737	174.17	47.98654	57.62389	309.1618
	Median	1702.285	2232.355	1843.636	2217.079	1775.176	2603.369	2107.118	1814.632	2041.79	1947.704	1762.515	1803.796	2194.629
	Rank	1	12	5	11	3	13	8	7	10	6	2	4	9

(Continued)

Table 9 (continued)

		FNO	AVOA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C20-F6	Mean	1603.727	2615.996	1953.889	1827.439	1820.596	2864.624	1951.399	3820.267	3117.815	2143.747	6701.285	3699.206	2371.762
	Best	1600	1741.01	1741.01	1690.778	1600	1741.01	1808.386	1870.293	1600	1741.01	1870.293	1741.01	1741.01
	Worst	1609.322	3070.568	2305.198	1934.076	1972.651	3570.869	2131.085	5414.023	4248.29	2353.274	10489.21	6143.423	2875.183
	Std	4.558769	613.3696	249.9763	104.593	182.5059	806.9465	165.1641	1519.262	1136.337	273.7522	3676.626	1877.921	499.5352
	Median	1602.793	2826.203	1884.674	1842.451	1854.868	3073.308	1933.063	3998.376	3311.486	2240.352	7222.817	3456.197	2435.427
	Rank	1	8	5	3	2	9	4	12	10	6	13	11	7
C20-F7	Mean	2206.269	3982.005	2828.481	2366.614	4058.79	2883.029	3342.977	3039.795	2926.368	3404.023	2678.63	3572.791	3348.542
	Best	2100	2102.328	2165.527	2165.527	2165.527	2305.424	2305.424	2238.232	2102.328	2171.14	2305.424	2171.14	2177.255
	Worst	2327.209	6110.685	4014.801	2585.222	6309.339	3367.293	5045.988	5057.327	4075.858	4225.813	3290.587	4832.517	4651.728
	Std	95.35991	1723.317	869.6533	182.0925	1736.29	444.4175	1183.745	1352.058	827.495	979.0758	442.0911	1209.711	1289.525
	Median	2198.933	3857.505	2566.798	2357.854	3880.147	2929.699	3010.248	2431.81	2763.643	3609.569	2559.253	3643.754	3282.592
	Rank	1	12	4	2	13	5	8	7	6	10	3	11	9
C20-F8	Mean	2218.874	2586.823	2500.863	2353.317	2566.249	2554.43	2335.656	2504.404	2526.738	2351.101	2322.496	2339.469	2387.394
	Best	2200	2200.701	2253.626	2200.701	2230.552	2253.626	2200.701	2230.552	2200.701	2200.701	2253.626	2253.626	2253.626
	Worst	2237.305	2761.507	2678.263	2421.571	2858.097	2931.105	2430.407	2751.89	2724.962	2619.704	2386.932	2470.734	2634.236
	Std	19.65987	259.4729	204.0465	104.4793	336.9903	285.6439	103.0891	254.8663	238.3239	184.5883	65.90148	93.6301	169.462
	Median	2219.095	2692.542	2535.782	2395.499	2588.173	2516.495	2355.759	2517.586	2590.646	2291.999	2324.713	2316.758	2330.858
	Rank	1	13	8	6	12	11	3	9	10	5	2	4	7
C20-F9	Mean	2404.914	2511.727	3028.135	3030.942	2497.218	2680.029	2745.351	2458.897	2503.327	2529.488	2435.639	2612.916	2500.01
	Best	2400	2400.11	2401.226	2425.972	2415.193	2415.193	2415.193	2415.193	2400.11	2401.226	2417.49	2415.193	2401.226
	Worst	2411.095	2641.865	3457.152	3529.858	2596.745	3040.057	3214.858	2498.374	2654.112	2643.281	2467.442	2738.017	2572.971
	Std	4.761701	111.6805	448.8483	468.9578	75.74189	282.4686	383.5691	35.93406	121.3041	128.5102	21.98644	143.527	80.96721
	Median	2404.28	2502.467	3127.081	3083.968	2488.468	2632.433	2675.676	2461.011	2479.543	2536.723	2428.811	2649.227	2512.923
	Rank	1	7	12	13	4	10	11	3	6	8	2	9	5
C20-F10	Mean	2500.005	6167.644	2507.443	2510.208	2500.733	2506.725	2517.898	2502.757	2506.417	2503.958	35525.22	2504.073	2501.125
	Best	2500	2500	2501.329	2500.539	2500.019	2500	2500.751	2500.539	2500	2500.539	2501.329	2500.027	2500.539
	Worst	2500.017	7436.058	2521.801	2517.744	2501.329	2520.226	2557.207	2503.921	2513.03	2511.398	46553.02	2514.363	2501.839
	Std	0.007955	2445.313	9.622326	7.156732	0.539975	9.173942	26.40593	1.517281	5.387741	5.021045	22015.94	6.882091	0.548087
	Median	2500.001	7367.259	2503.32	2511.274	2500.791	2503.336	2506.817	2503.283	2506.32	2501.947	46523.27	2500.951	2501.06
	Rank	1	12	9	10	2	8	11	4	7	5	13	6	3
Sum rank		10	94	70	63	59	92	84	62	90	68	61	82	64
Mean rank		1	9.4	7	6.3	5.9	9.2	8.4	6.2	9	6.8	6.1	8.2	6.4
Total rank		1	13	8	5	2	12	10	4	11	7	3	9	6

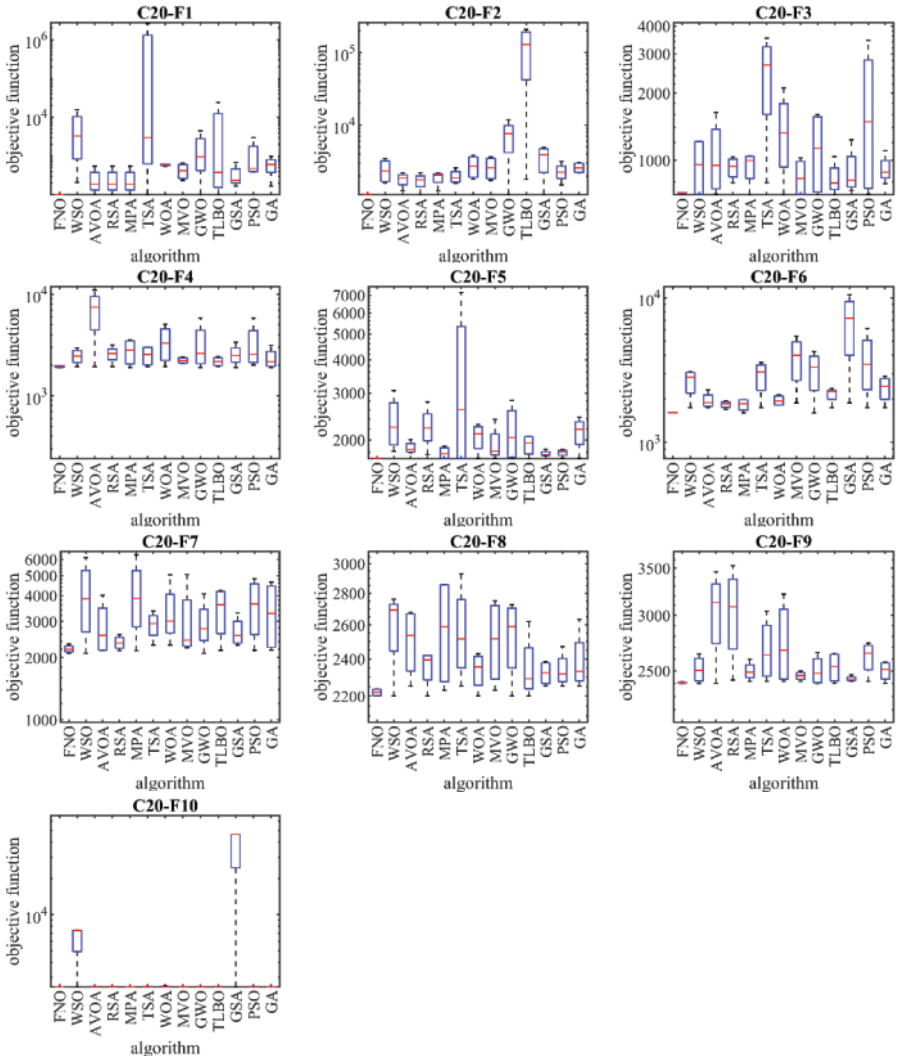

Figure 11: Boxplot diagrams of FNO and competitor algorithms performances on CEC 2020 test suite

Table 10: Wilcoxon rank sum test results

Compared algorithm	Objective function type				
	CEC 2017				CEC 2020
	D = 10	D = 30	D = 50	D = 100	
FNO *vs*. WSO	6.32E-25	6.32E-25	8.84E-25	c	3.50E-07
FNO *vs*. AVOA	1.69E-22	1.36E-24	8.84E-25	8.84E-25	1.18E-07
FNO *vs*. RSA	8.84E-25	6.32E-25	6.32E-25	8.84E-25	1.71E-07
FNO *vs*. MPA	8.99E-22	6.99E-20	2.97E-21	8.84E-25	1.10E-07
FNO *vs*. TSA	4.26E-24	8.84E-25	8.84E-25	8.84E-25	2.28E-07
FNO *vs*. WOA	4.26E-24	8.84E-25	8.84E-25	8.84E-25	2.45E-07

(Continued)

Table 10 (continued)

Compared algorithm	Objective function type				
	CEC 2017				CEC 2020
	D = 10	D = 30	D = 50	D = 100	
FNO *vs*. MVO	4.05E-22	9.51E-25	8.84E-25	8.84E-25	1.27E-07
FNO *vs*. GWO	2.34E-24	8.84E-25	8.84E-25	8.84E-25	8.10E-06
FNO *vs*. TLBO	1.66E-24	8.84E-25	8.84E-25	8.84E-25	8.68E-07
FNO *vs*. GSA	7.19E-22	9.06E-25	8.84E-25	8.84E-25	7.59E-08
FNO *vs*. PSO	6.94E-23	1.06E-24	8.84E-25	8.84E-25	7.59E-08
FNO *vs*. GA	1.21E-22	8.84E-25	8.84E-25	8.84E-25	1.27E-07

5 FNO for Real-World Applications

In this section, we examine the efficacy of the proposed FNO methodology in tackling real-world optimization challenges. To achieve this, we have selected a subset of twenty-two constrained optimization problems from the CEC 2011 test suite, along with four engineering design problems, for assessment. As in the previous section, here the simulation results are reported using six statistical indicators: mean, best, worst, std, median, and rank. It should be noted that the values of the "mean" index are the criteria used to rank the algorithms in handling each of the CEC 2011 test suite problems as well as each of the engineering design problems.

CEC 2011 test suite has been used in many algorithms recently published in similar articles, in order to judge the efficiency of designed metaheuristic algorithms. This test suite consists of twenty-two constrained optimization problems from real world applications. For this reason, in this paper, CEC 2011 test suite has been chosen to evaluate the effectiveness of the proposed FNO approach in handling optimization problems in real world applications.

In order to adapt the proposed approach of FNO to deal with constrained optimization problems, there are different solutions. In this study, the method of penalty coefficient is used. In this case, for a candidate FNO solution, a penalty value is added to the objective function for each of the constraints that are not satisfied. As a result, automatically this inappropriate corresponding solution will not be placed as a solution in the output. However, it is possible that during the update process in the next iteration, a new solution will be produced that satisfies the constraints.

5.1 Evaluation of CEC 2011 Test Suite

In this section, we assess the performance of FNO and competitor algorithms in tackling the CEC 2011 test suite, comprising twenty-two constrained optimization problems derived from real-world applications. A comprehensive description and detailed information on the CEC 2011 test suite can be found in [62]. The proposed FNO approach and each of the competitor algorithms is implemented on the CEC 2011 functions in twenty-five independent implementations where each implementation contains 150,000 FEs. The implementation outcomes of FNO and competitor algorithms on the CEC 2011 test suite are documented in Table 11, while boxplot diagrams illustrating the performance of metaheuristic algorithms are presented in Fig. 12.

Table 11: Optimization results of CEC 2011 test suite

		FNO	AVOA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C11-F1	Mean	5.920103	12.52841	16.74213	20.55445	7.723161	17.39382	12.78836	13.45957	10.65985	17.42229	20.30587	16.98281	21.8149
	Best	2E-10	8.402211	13.72119	17.98675	0.405713	16.51963	7.388265	11.2761	1.066509	16.51282	17.52035	10.47591	20.46457
	Worst	12.30606	16.34896	19.54639	23.07193	12.72065	18.96086	16.7112	15.47524	16.6408	18.92175	21.98956	23.05791	24.2157
	Std	7.196379	4.623049	2.891741	2.46555	5.891302	1.155486	4.518685	2.427897	7.078568	1.177854	2.032557	5.830417	1.750385
	Median	5.687176	12.68124	16.85048	20.57957	8.883141	17.04739	13.527	13.54346	12.46605	17.12729	20.85678	17.1987	21.28967
	Rank	1	4	7	12	2	9	5	6	3	10	11	8	13
C11-F2	Mean	−26.3179	−21.4236	−15.5557	−13.0722	−24.9902	−12.8247	−19.289	−10.6335	−22.8276	−12.4781	−16.5731	−22.8708	−14.2704
	Best	−27.0676	−21.9622	−16.824	−13.4768	−25.6486	−16.2028	−22.3841	−12.5221	−24.7628	−13.5921	−21.0914	−24.1606	−16.4087
	Worst	−25.4328	−20.8936	−14.5993	−12.7739	−23.5778	−10.997	−15.8323	−9.03411	−19.4166	−11.639	−12.7353	−20.839	−12.8069
	Std	0.738935	0.556485	1.134413	0.347572	1.012612	2.57848	3.448741	1.558767	2.495606	0.902004	3.950344	1.504127	1.850662
	Median	−26.3856	−21.4193	−15.3998	−13.019	−25.3672	−12.0494	−19.4698	−10.4888	−23.5656	−12.3407	−16.2328	−23.2418	−13.933
	Rank	1	5	8	10	2	11	6	13	4	12	7	3	9
C11-F3	Mean	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05
	Best	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05
	Worst	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05
	Std	2E-19	2.21E-09	1.93E-11	4.35E-11	1.33E-15	2.06E-14	2.47E-16	8.67E-13	3.49E-15	6.84E-14	2.47E-16	2.46E-16	2.46E-16
	Median	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05	1.16E-05
	Rank	1	13	11	12	6	8	4	10	7	9	3	2	5
C11-F4	Mean	13.68174	15.43223	14.39174	17.84035	15.43663	14.88466	15.08699	17.05755	14.48897	19.65085	15.96374	15.73561	13.78715
	Best	13.61536	13.67784	13.72765	13.66863	13.72765	13.72765	13.72765	13.72596	13.64036	13.72596	13.64036	13.72765	13.63191
	Worst	13.73188	16.52614	15.29246	19.85408	16.73196	15.57639	16.10149	19.20792	15.20547	22.70577	18.55033	17.02715	13.91219
	Std	0.058052	1.323076	0.756844	2.966604	1.48494	0.907301	1.108411	2.666134	0.821883	4.247393	2.113695	1.595204	0.12558
	Median	13.68986	15.76248	14.27343	18.91935	15.64345	15.1173	15.2594	17.64815	14.55502	21.08584	15.83213	16.09382	13.80225
	Rank	1	7	3	12	8	5	6	11	4	13	10	9	2
C11-F5	Mean	−34.1274	−28.7037	−25.8146	−21.5693	−33.2154	−27.8494	−28.2859	−27.727	−31.73	−13.5176	−28.0391	−11.621	−12.3745
	Best	−34.7494	−29.7163	−26.8808	−23.5116	−33.7977	−31.8374	−28.4882	−31.6034	−34.1261	−15.5107	−31.4293	−14.4729	−13.3683
	Worst	−33.3862	−28.033	−24.7368	−19.2408	−31.7926	−23.257	−28.0604	−25.707	−27.9425	−12.1616	−25.4024	−10.1843	−10.9842
	Std	0.589989	0.755729	0.964252	2.29988	1.000596	3.70703	0.230197	2.909826	2.788723	1.499243	2.784274	2.122759	1.13996
	Median	−34.1871	−28.5328	−25.8204	−21.7625	−33.6357	−28.1515	−28.2975	−26.7987	−32.4256	−13.199	−27.6623	−10.9135	−12.5727
	Rank	1	4	9	10	2	7	5	8	3	11	6	13	12

(Continued)

Table 11 (continued)

		FNO	AVOA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C11-F6	Mean	−24.1119	−19.3783	−15.0036	−14.1328	−22.5108	−9.32911	−20.1856	−11.0554	−19.9032	−4.73973	−21.876	−5.49833	−6.29263
	Best	−27.4298	−20.9932	−15.3343	−14.6856	−25.6345	−17.0202	−22.803	−17.9435	−22.7104	−5.39677	−25.8224	−8.43116	−10.6843
	Worst	−23.0059	−17.783	−14.63	−13.0677	−21.2106	−6.29264	−13.8854	−4.47264	−18.2866	−4.47264	−18.6799	−4.47264	−4.47264
	Std	2.324951	1.497394	0.345464	0.761753	2.222427	5.422546	4.458259	7.468412	2.212179	0.465924	3.29799	2.056378	3.106794
	Median	−23.0059	−19.3684	−15.0251	−14.389	−21.5991	−7.00181	−22.0269	−10.9027	−19.3078	−4.54476	−21.5009	−4.54476	−5.00682
	Rank	1	6	7	8	2	10	4	9	5	13	3	12	11
C11-F7	Mean	0.860699	1.242726	1.522903	1.795953	0.934354	1.258251	1.64259	0.892095	1.054319	1.620895	1.064804	1.103022	1.639835
	Best	0.582266	1.1199	1.454609	1.592827	0.769266	1.092462	1.555277	0.841362	0.82802	1.461789	0.879847	0.85696	1.28447
	Worst	1.025027	1.37333	1.612629	1.951345	1.012985	1.582892	1.787355	0.970732	1.264946	1.738519	1.253393	1.327696	1.82533
	Std	0.211503	0.141593	0.070239	0.156563	0.119362	0.231495	0.105518	0.064912	0.187998	0.131701	0.177573	0.251075	0.25626
	Median	0.91775	1.238838	1.512188	1.81982	0.977583	1.178825	1.613864	0.878143	1.062156	1.641636	1.062989	1.113716	1.72477
	Rank	1	7	9	13	3	8	12	2	4	10	5	6	11
C11-F8	Mean	220	238.3658	277.2247	312.5117	222.6229	253.0605	260.7144	224.0469	226.8948	224.0469	243.4968	438.3804	222.6624
	Best	220	224.1382	254.5798	277.2211	220	220	242.0717	220	220	220	220	245.4979	220
	Worst	220	253.5672	308.0582	351.0064	225.2457	338.4523	301.4288	234.2398	233.7896	235.2136	285.053	526.6478	229.6759
	Std	0	13.32447	24.72279	31.85048	3.183398	60.22576	28.78087	7.158832	8.368285	7.83979	32.45774	139.596	4.938214
	Median	220	237.8789	273.1304	310.9098	222.6229	226.8948	249.6785	220.9738	226.8948	220.4869	234.4671	490.6879	220.4869
	Rank	1	6	10	11	2	8	9	4	5	4	7	12	3
C11-F9	Mean	8789.286	334088.5	490433.2	931595.7	21036.05	61266.01	330884.1	119957.3	40982.68	360564.8	722998.3	949481.7	1701352
	Best	5457.674	297764.7	329399.1	609975.6	11289.81	46871.22	182907.1	68778.55	17767.84	298249.2	619636.7	761083.4	1631054
	Worst	14042.29	357645.2	564925.4	1094288	30402.18	76236.36	557575.5	180347.9	71029.98	462098.6	776344.7	1162652	1800150
	Std	3889.181	27700.03	115985.9	230040.8	8813.927	13560.96	179294.8	48505.11	23657.09	75392.44	73781.12	225753.1	86839.5
	Median	7828.591	340472.1	533704.1	1011060	21226.1	60978.23	291526.9	115351.4	37566.45	340955.7	748005.9	937095.6	1687102
	Rank	1	7	9	11	2	4	6	5	3	8	10	12	13
C11-F10	Mean	−21.4889	−17.0126	−14.4367	−12.9595	−18.8505	−14.8029	−13.4786	−15.0751	−14.5533	−12.0938	−13.7234	−12.1809	−11.9229
	Best	−21.8299	−17.2235	−15.4235	−13.3512	−19.2436	−18.6357	−14.0346	−20.7755	−15.0311	−12.2211	−14.2247	−12.2692	−12.0177
	Worst	−20.7878	−16.6527	−13.9493	−12.7677	−18.4764	−12.7956	−13.1018	−12.176	−13.4717	−11.981	−12.9762	−12.1131	−11.8264
	Std	0.498616	0.281136	0.705055	0.282262	0.424606	2.765963	0.415821	4.063559	0.772761	0.127544	0.641605	0.083879	0.092564
	Median	−21.669	−17.0871	−14.1869	−12.8596	−18.8411	−13.8901	−13.389	−13.6745	−14.8551	−12.0866	−13.8464	−12.1707	−11.9237
	Rank	1	3	7	10	2	5	9	4	6	12	8	11	13

(Continued)

Table 11 (continued)

		FNO	AVOA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C11-F11	Mean	571712.3	1153936	5354192	8022913	1737036	5478490	1349788	1430873	3635030	4836889	1520671	4846559	5633837
	Best	260837.9	967939	5101288	7769164	1619920	4591451	1236097	827905.4	3449232	4813404	1375620	4813404	5600872
	Worst	828560.9	1319938	5679544	8197048	1863179	6566972	1499541	2656953	3952245	4866281	1678745	4870909	5679144
	Std	260922.1	163646.1	279127.2	190070.7	119477.7	856487.1	116048.5	871229.6	230816.3	27507.81	130353.3	28465.49	36072.03
	Median	598725.2	1163934	5317967	8062720	1732523	5377768	1331757	1119316	3569322	4833935	1514160	4850962	5627666
	Rank	1	2	10	13	6	11	3	4	7	8	5	9	12
C11-F12	Mean	1199805	3113654	7491258	11720134	1279917	4557354	5242788	1326078	1410399	12674132	5220823	2187024	12814586
	Best	1155937	3020903	7175822	10885229	1201037	4334324	4881360	1197606	1254352	11951819	4972156	2050733	12699967
	Worst	1249353	3185139	7772240	12455343	1361256	4669878	5412120	1448642	1543516	13230852	5389417	2365904	12932976
	Std	47157.58	75270.06	259442.9	676945.3	73312.95	165786.6	259966.3	108704.4	126165.3	564168.1	189579.1	137821	101134.8
	Median	1196965	3124288	7508485	11769981	1278688	4612607	5338837	1329032	1421863	12756929	5260860	2165729	12812701
	Rank	1	6	10	11	2	7	9	3	4	12	8	5	13
C11-F13	Mean	15444.2	15451.29	15809.07	16204.44	15464.54	15488.14	15527.69	15503.61	15497.68	15875.05	114049.1	15488.7	28163.32
	Best	15444.19	15450.37	15646.6	15840.7	15462.07	15479.02	15490.32	15486.73	15491.17	15607.62	82990.68	15473.2	15461.89
	Worst	15444.21	15451.82	16202.36	17108.23	15468.82	15499.11	15578.85	15536.95	15508.94	16366.82	156175.3	15521.39	65959.56
	Std	0.009091	0.717424	278.2839	638.5421	3.147698	10.09012	43.54899	24.66549	8.276395	360.9329	34633.02	23.16551	26485.21
	Median	15444.2	15451.49	15693.65	15934.42	15463.64	15487.22	15520.79	15495.38	15495.3	15762.88	108515.2	15480.11	15615.92
	Rank	1	2	9	11	3	4	8	7	6	10	13	5	12
C11-F14	Mean	18295.35	18553.26	100636.2	202511.6	18630.5	19437.26	19170.31	19339.48	19176.37	273893.4	19054.11	19082.55	19071.41
	Best	18241.58	18461.39	77122.08	149760.8	18543.49	19208.23	19022.67	19237.88	19035.22	28811.09	18796.85	18934.7	18818.16
	Worst	18388.08	18647.79	139802.1	290790.4	18705.61	19925.54	19284.21	19419.49	19347.24	526464.6	19224.42	19219.06	19341.93
	Std	71.59938	93.57007	29465.91	66396.98	74.25737	345.7248	127.1688	81.53748	144.974	251121.5	197.5448	122.9683	225.598
	Median	18275.87	18551.94	92810.28	184747.5	18636.45	19307.63	19187.18	19350.27	19161.52	270149	19097.58	19088.23	19062.78
	Rank	1	2	11	12	3	10	7	9	8	13	4	6	5
C11-F15	Mean	32883.58	98422.07	797983	1678033	32953.1	51838.95	195252.6	33085.66	33066.31	13484325	266202.6	33250.03	6941821
	Best	32782.17	41898.3	331339	704158.7	32876.29	33048.61	32998.24	33003.58	33029.58	2828717	236067.3	33239.7	3161932
	Worst	32956.46	161675.2	2000291	4373616	33023.23	107987	277680.7	33137.91	33131.97	20106167	286828.8	33268.81	11893981
	Std	76.94696	67668.53	845546.8	1891824	63.23147	39344.94	116109.2	61.75003	49.5816	8257502	24822.65	13.9656	4208351
	Median	32897.86	95057.39	430151	817178.8	32956.44	33160.1	235165.7	33100.58	33051.85	15501209	270957.1	33245.8	6355685
	Rank	1	7	10	11	2	6	8	4	3	13	9	5	12

(Continued)

Table 11 (continued)

		FNO	AVOA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C11-F16	Mean	133550	135790.5	843849.2	1721618	137958.6	144486.8	141897	141586.3	145138.9	77731378	16384089	69575107	66804386
	Best	131374.2	134202.1	270072.4	431634.4	135881.6	141864.4	136687.2	133962.1	142825.4	75747443	8330119	57554826	53995364
	Worst	136310.8	136628.2	1970043	4252988	141711.4	146029.6	147084.5	149014.9	150507.4	79968786	29628046	83137356	85442852
	Std	2392.2	1136.541	803717.8	1806507	2739.482	2070.447	4549.789	6581.747	3796.672	1859489	9679765	11590320	14041017
	Median	133257.5	136165.9	567640.5	1100924	137120.6	145026.7	141908.2	141684.1	143611.4	77604641	13789096	68804123	63889663
	Rank	1	2	8	9	3	6	5	4	7	13	10	12	11
C11-F17	Mean	1926615	2.02E+09	7.83E+09	1.36E+10	2317696	1.12E+09	8.48E+09	3035513	2954660	1.95E+10	9.8E+09	1.82E+10	1.91E+10
	Best	1916953	1.84E+09	6.68E+09	9.74E+09	1960318	9.24E+08	6.05E+09	2309786	2030949	1.88E+10	8.63E+09	1.61E+10	1.79E+10
	Worst	1942685	2.22E+09	8.69E+09	1.66E+10	2979619	1.28E+09	1.13E+10	3542354	4704927	2.04E+10	1.04E+10	2.1E+10	2.16E+10
	Std	12003.53	1.74E+08	9.35E+08	3.08E+09	480715.1	1.93E+08	2.31E+09	576727.1	1265977	6.92E+08	8.4E+08	2.36E+09	1.78E+09
	Median	1923412	2.02E+09	7.99E+09	1.4E+10	2165424	1.14E+09	8.29E+09	3144956	2541382	1.94E+10	1.01E+10	1.79E+10	1.85E+10
	Rank	1	6	7	10	2	5	8	4	3	13	9	11	12
C11-F18	Mean	942057.5	5855154	48192826	1.04E+08	973927.2	1916670	8498968	988235.3	1025834	27226207	9852615	1.18E+08	1E+08
	Best	938416.2	3566023	33180053	71603383	950609.3	1687657	3723128	978489.2	965531.5	21601124	7381068	99118324	96585265
	Worst	944706.9	9964895	54798683	1.18E+08	1036253	2212191	14837763	993919.4	1185358	29452742	12407872	1.31E+08	1.04E+08
	Std	2774.139	3132056	10642560	22977772	43805.75	261885.6	4933562	7290.961	112018	3958415	2359599	15025192	3163274
	Median	942553.5	4944849	52396283	1.12E+08	954422.9	1883415	7717491	990266.4	976223.6	28925481	9810760	1.21E+08	1E+08
	Rank	1	6	10	12	2	5	7	3	4	9	8	13	11
C11-F19	Mean	1025341	5970619	47466451	1.02E+08	1146090	2304929	9084371	1442306	1342758	31266255	5630384	1.51E+08	1.01E+08
	Best	967927.7	5484323	40513198	87698402	1073916	2084415	1946393	1132085	1217571	21929855	2258733	1.37E+08	98112621
	Worst	1167142	7196938	60305322	1.28E+08	1302136	2688025	16329519	1860579	1522004	38959353	7342291	1.75E+08	1.04E+08
	Std	99675.04	863968.5	9380732	19523560	110512.2	277372.4	7123152	319822.8	134824.1	7750765	2423854	17128570	2383291
	Median	983146.6	5600607	44523642	95411344	1104154	2223638	9030786	1388279	1315728	32087906	6460255	1.46E+08	1E+08
	Rank	1	7	10	12	2	5	8	4	3	9	6	13	11
C11-F20	Mean	941250.4	5273531	50431412	1.1E+08	961749.9	1718994	6487484	972648.7	995138.1	30345716	12598168	1.39E+08	1.01E+08
	Best	936143.2	4664825	44384145	95906503	957836.9	1558156	6119757	963856.1	975704.5	29681866	8409912	1.27E+08	96060897
	Worst	946866.6	5925538	59698100	1.3E+08	964325.5	1985270	6978686	982895	1010227	31062861	19434543	1.51E+08	1.05E+08
	Std	5013.552	549976.2	6858771	15393470	2927.238	213153.7	386197.8	9087.642	15634.21	603666.9	5064395	14022604	3800527
	Median	940995.9	5251881	48821700	1.06E+08	962418.6	1666275	6425745	971921.8	997310.5	30319069	11274109	1.39E+08	1.01E+08
	Rank	1	6	10	12	2	5	7	3	4	9	8	13	11

(Continued)

Table 11 (continued)

		FNO	AVOA	WSO	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C11-F21	Mean	12.71443	21.16485	46.17786	69.10479	16.17329	28.31979	36.16095	26.31967	21.80684	90.29177	37.83232	94.6948	91.95816
	Best	9.974206	19.75353	38.68815	52.27901	14.03749	25.1995	33.5643	23.56607	20.05703	44.63123	33.85471	82.26917	53.75319
	Worst	14.97499	23.05336	54.27946	85.96943	18.46504	29.56548	39.51651	29.26324	24.14938	131.7641	40.51045	104.8618	111.555
	Std	2.412667	1.496981	7.047461	15.62425	2.166017	2.198137	2.760114	3.244344	1.936904	37.53317	3.07227	11.86593	28.27838
	Median	12.95425	20.92626	45.87191	69.08536	16.09532	29.25709	35.7815	26.22468	21.51047	92.3859	38.48206	95.82412	101.2622
	Rank	1	3	9	10	2	6	7	5	4	11	8	13	12
C11-F22	Mean	16.12513	26.61591	43.55688	58.13767	19.30005	30.72	43.13672	30.85655	24.47113	92.35837	43.44915	95.88593	83.58583
	Best	11.50133	21.77054	38.35566	43.02007	16.53439	26.94007	37.38023	24.20705	23.65659	60.61512	36.84021	80.57012	82.50637
	Worst	19.55286	31.36502	48.54746	66.67674	21.38723	33.13118	47.4267	35.44095	25.12117	109.1037	51.18962	105.8063	85.16528
	Std	4.197797	4.774252	4.606404	10.95576	2.422862	2.794893	4.789876	5.183775	0.654518	22.87719	6.228956	11.89961	1.183616
	Median	16.72317	26.66405	43.6622	61.42693	19.6393	31.40438	43.86997	31.8891	24.55339	99.8573	42.88339	98.58364	83.33583
	Rank	1	4	9	10	2	5	7	6	3	12	8	13	11
Sum rank		22	197	111	242	62	150	150	128	100	234	166	206	225
Mean rank		1.00E+00	8.95E+00	5.05E+00	1.10E+01	2.82E+00	6.82E+00	6.82E+00	5.82E+00	4.55E+00	1.06E+01	7.55E+00	9.36E+00	1.02E+01
Total rank		1	2	12	4	13	3	11	9	6	7	10	5	8
Wilcoxon: *p*-value			3.20E-18	1.80E-15	6.43E-19	7.23E-07	2.01E-18	2.17E-18	6.61E-15	7.92E-16	1.38E-18	3.31E-18	6.43E-19	9.48E-19

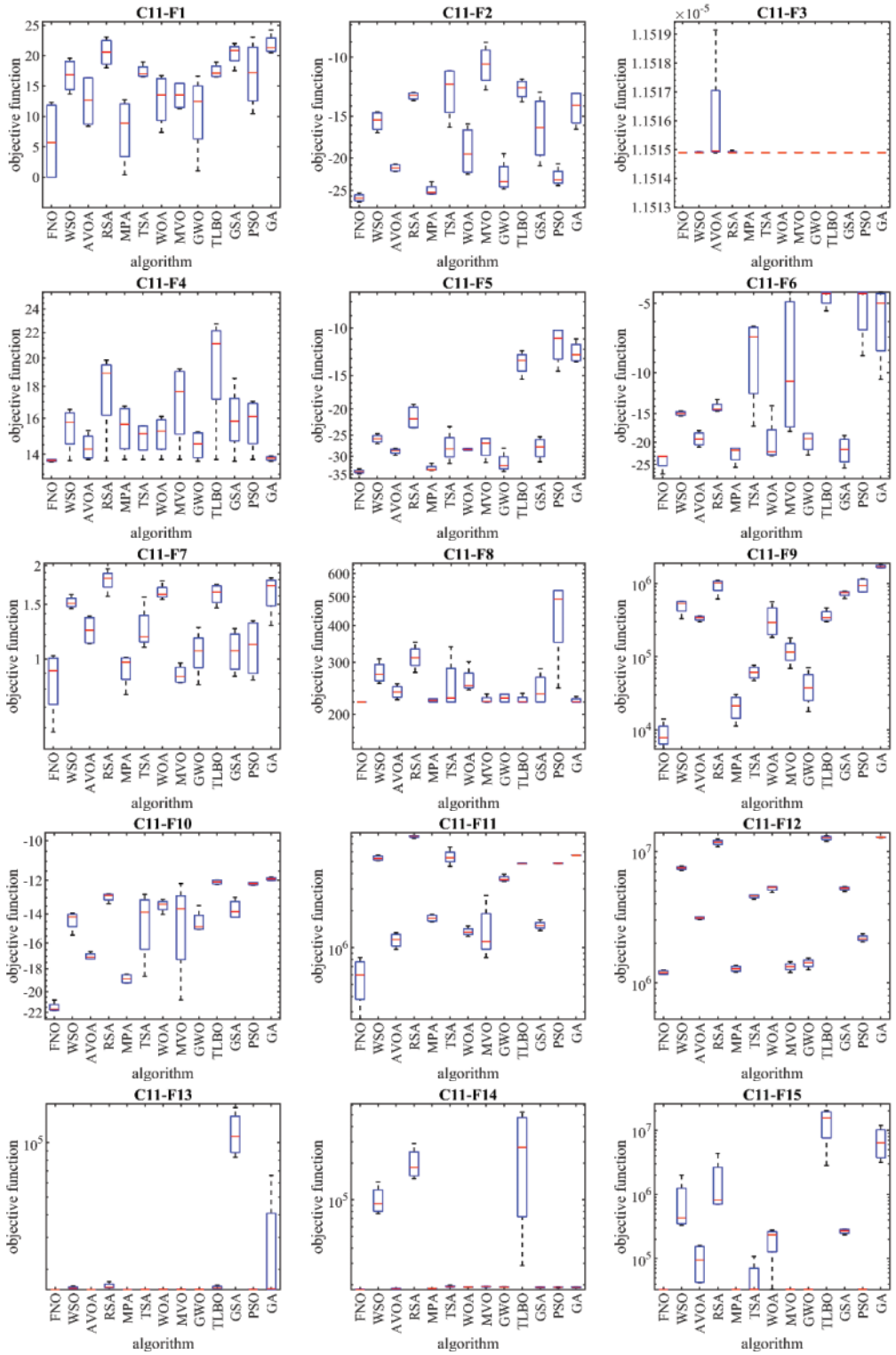

Figure 12: (Continued)

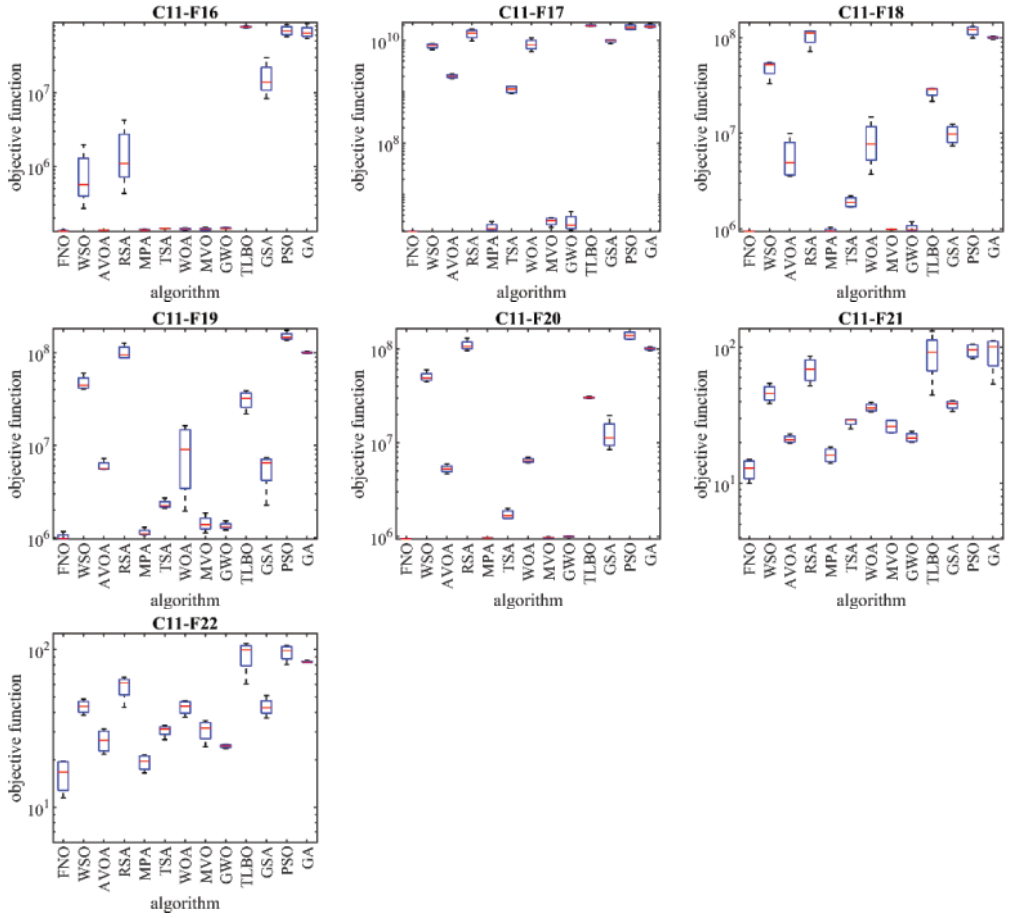

Figure 12: Boxplot diagrams of FNO and competitor algorithms performances on CEC 2011 test suite

Based on the optimization results, FNO emerges as the top-performing optimizer for optimization problems C11-F1 to C11-F22, demonstrating its adeptness in exploration, exploitation, and maintaining a balance between the two throughout the search process. The simulation results indicate FNO's superior performance in handling the CEC 2011 test suite, achieving superior results across most optimization problems and securing the top rank as the first best optimizer compared to competitor algorithms. Additionally, the *p*-value results obtained using the Wilcoxon rank sum test affirm that FNO exhibits a statistically significant superiority over all twelve competitor algorithms in handling the CEC 2011 test suite.

5.2 Pressure Vessel Design Problem

Designing pressure vessels is a pertinent optimization task in real-world applications, as illustrated in Fig. 13, with the objective of minimizing construction costs. The mathematical model for this design is outlined as follows [63]:

Consider: $X = [x_1, x_2, x_3, x_4] = [T_s, T_h, R, L]$.

Minimize: $f(x) = 0.6224x_1x_3x_4 + 1.778x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$.

Subject to:

$$g_1(x) = -x_1 + 0.0193x_3 \le 0, \quad g_2(x) = -x_2 + 0.00954x_3 \le 0,$$

$$g_3(x) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0, \quad g_4(x) = x_4 - 240 \le 0.$$

With

$$0 \le x_1, x_2 \le 100 \text{ and } 10 \le x_3, x_4 \le 200.$$

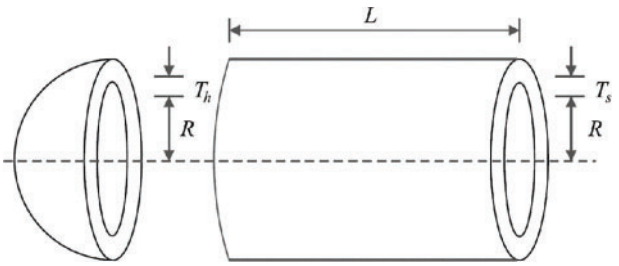

Figure 13: Schematic of pressure vessel design

The implementation outcomes of FNO and rival algorithms in addressing pressure vessel design are detailed in Tables 12 and 13. Fig. 14 illustrates the convergence curve of FNO during the attainment of the optimal design for this problem.

Table 12: Performance of optimization algorithms on speed reducer design problem

Algorithm	Optimum variables				Optimum cost
	T_s	T_h	R	L	
FNO	0.7781686	0.3846492	40.319619	200	5885.3269
WSO	0.7782309	0.38468	40.322845	200	5885.3325
AVOA	0.7782526	0.3846907	40.323967	199.94929	5885.34
RSA	0.8539516	0.4168657	40.388059	200	8086.5619
MPA	0.7782309	0.38468	40.322845	200	5885.3325
TSA	0.7798201	0.3858965	40.399775	200	5916.3803
WOA	0.8129108	0.5410562	40.39966	198.94944	6340.3278
MVO	0.8182678	0.4062317	42.356098	173.54905	6027.1791
GWO	0.7785163	0.3856561	40.33039	199.9589	5893.9046
TLBO	1.1979804	1.2640954	61.061039	91.748927	11660.68
GSA	0.9570947	0.4737652	49.585704	145.01146	13041.109
PSO	1.2768702	2.3223385	50.651074	110.16225	10712.216
GA	1.1435231	0.780001	54.789155	96.522722	11793.11

Table 13: Statistical results of optimization algorithms on speed reducer design problem

Algorithm	Mean	Best	Worst	Std	Median	Rank
FNO	5885.3269	5885.3269	5885.3269	2.06E-12	5885.3269	1
WSO	5895.0956	5885.3325	5981.6596	28.716785	5885.3329	3
AVOA	6280.1363	5885.34	7249.7752	455.40456	6078.6009	5
RSA	13539.742	8086.5619	22432.026	4041.5649	12359.627	9
MPA	5885.3325	5885.3325	5885.3325	4.76E-06	5885.3325	2
TSA	6340.6443	5916.3803	7134.9108	430.58947	6191.0944	6
WOA	8366.6396	6340.3278	14003.934	2173.7628	7875.2374	8
MVO	6630.2874	6027.1791	7254.5665	413.99521	6693.7212	7
GWO	6037.1687	5893.9046	6809.5978	309.39299	5903.6843	4
TLBO	32144.537	11660.68	69718.639	17830.143	28276.87	12
GSA	23196.473	13041.109	36638.786	8674.4726	22242.768	10
PSO	33803.139	10712.216	58460.668	16692.36	37347.025	13
GA	28807.292	11793.11	52384.008	13995.666	25433.758	11

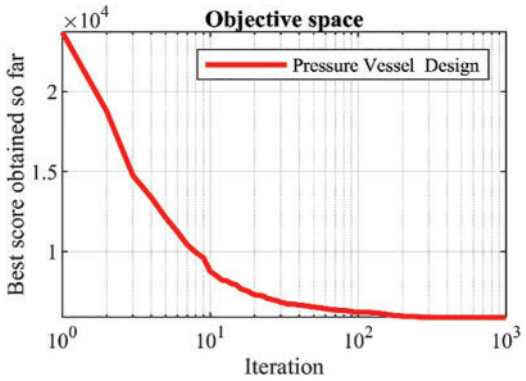

Figure 14: FNO's performance convergence curve on pressure vessel design

According to the findings, FNO has delivered the optimal design with design variable values of (0.7781686, 0.3846492, 40.319619, 200), and an objective function value of 5885.3269. Examination of the simulation outcomes highlights the superior performance of FNO in handling pressure vessel design compared to rival algorithms.

5.3 Speed Reducer Design Problem

The optimization task of speed reducer design finds practical application in real-world scenarios, as depicted in Fig. 15, with the objective of minimizing the weight of the speed reducer. The mathematical model for this design is outlined as follows [64,65]:

Consider: $X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7] = [b, m, p, l_1, l_2, d_1, d_2]$.

Minimize: $f(x) = 0.7854x_1x_2^2\left(3.3333x_3^2 + 14.9334x_3 - 43.0934\right) - 1.508x_1\left(x_6^2 + x_7^2\right) + 7.4777\left(x_6^3 + x_7^3\right) + 0.7854(x_4x_6^2 + x_5x_7^2)$.

Subject to:

$$g_1(x) = \frac{27}{x_1 x_2^2 x_3} - 1 \le 0, \quad g_2(x) = \frac{397.5}{x_1 x_2^2 x_3^2} - 1 \le 0,$$

$$g_3(x) = \frac{1.93 x_4^3}{x_2 x_3 x_6^4} - 1 \le 0, \quad g_4(x) = \frac{1.93 x_5^3}{x_2 x_3 x_7^4} - 1 \le 0,$$

$$g_5(x) = \frac{1}{110 x_6^3} \sqrt{\left(\frac{745 x_4}{x_2 x_3}\right)^2 + 16.9 \cdot 10^6} - 1 \le 0,$$

$$g_6(x) = \frac{1}{85 x_7^3} \sqrt{\left(\frac{745 x_5}{x_2 x_3}\right)^2 + 157.5 \cdot 10^6} - 1 \le 0,$$

$$g_7(x) = \frac{x_2 x_3}{40} - 1 \le 0, \quad g_8(x) = \frac{5 x_2}{x_1} - 1 \le 0,$$

$$g_9(x) = \frac{x_1}{12 x_2} - 1 \le 0, \quad g_{10}(x) = \frac{1.5 x_6 + 1.9}{x_4} - 1 \le 0,$$

$$g_{11}(x) = \frac{1.1 x_7 + 1.9}{x_5} - 1 \le 0.$$

With

$2.6 \le x_1 \le 3.6, 0.7 \le x_2 \le 0.8, 17 \le x_3 \le 28, 7.3 \le x_4 \le 8.3, 7.8 \le x_5 \le 8.3, 2.9 \le x_6 \le 3.9$, and $5 \le x_7 \le 5.5$.

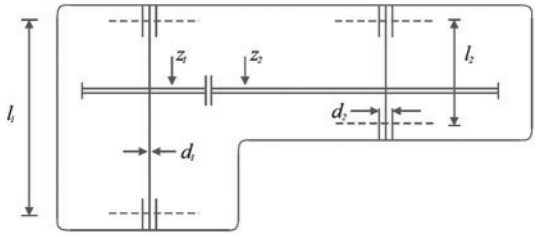

Figure 15: Schematic of speed reducer design

The outcomes of employing FNO and rival algorithms to optimize speed reducer design are presented in Tables 14 and 15. Fig. 16 illustrates the convergence curve of FNO during the attainment of the optimal design for this problem.

According to the findings, FNO has yielded the optimal design with design variable values of (3.5, 0.7, 17, 7.3, 7.8, 3.3502147, 5.2866832), and an objective function value of 2996.348165. Analysis of the simulation results underscores the superior performance of FNO in addressing speed reducer design when compared to rival algorithms.

Table 14: Performance of optimization algorithms on speed reducer design problem

Algorithm	Optimum variables							Optimum cost
	b	M	p	l_1	l_2	d_1	d_2	
FNO	3.5	0.7	17	7.3	7.8	3.3502147	5.2866832	2996.348165
WSO	3.5002244	0.7	17.000002	7.4136937	7.8000061	3.3518435	5.2873567	2998.2849
AVOA	3.5000092	0.7	17.000629	7.4084741	7.8136361	3.3523231	5.2868134	2998.3407
RSA	3.5531625	0.7	17	7.9230907	8.076683	3.3562084	5.3978997	3104.2319
MPA	3.5002366	0.7	17	7.3018979	7.8	3.3552821	5.287468	2998.2543
TSA	3.5089329	0.7	17	7.3851797	8.0837291	3.3509261	5.2895882	3008.8748
WOA	3.5500856	0.7	17.002304	7.3248187	7.9504512	3.3629435	5.2871019	3023.4616
MVO	3.5015097	0.7	17	7.3018979	7.9521085	3.3662389	5.2875803	3004.9744
GWO	3.5005991	0.7	17	7.3048057	7.8	3.3630462	5.2886704	3001.1749
TLBO	3.532811	0.70226	22.271331	7.9247095	8.0775428	3.5287298	5.3165585	4286.1391
GSA	3.5131765	0.7015566	17.208703	7.7076307	7.8506631	3.3849498	5.3434699	3096.3026
PSO	3.5047988	0.7000407	17.619454	7.3913565	7.9493891	3.4917496	5.3195215	3173.2805
GA	3.5441317	0.7031465	17.460113	7.6348954	7.8478162	3.5515621	5.3215997	3197.0824

Table 15: Statistical results of optimization algorithms on speed reducer design problem

Algorithm	Mean	Best	Worst	Std	Median	Rank
FNO	2996.348165	2996.348165	2996.348165	1.03E-12	2996.348165	1
WSO	2999.5614	2998.2849	3001.6023	1.01E+00	2999.5448	3
AVOA	3001.9497	2998.3407	3007.4644	2.8969711	3001.046	4
RSA	3157.9524	3104.2319	3192.6152	36.072081	3166.2611	9
MPA	2999.4011	2998.2543	3001.6022	1.0196094	2999.21	2
TSA	3019.6328	3008.8748	3026.7945	6.23825	3020.9355	7
WOA	3086.3025	3023.4616	3252.0021	65.814052	3067.2525	8
MVO	3018.3269	3004.9744	3040.5666	8.312502	3018.4292	6
GWO	3004.0787	3001.1749	3009.6522	2.5818026	3003.5947	5
TLBO	3.93E+13	4286.1391	2.85E+14	7.21E+13	1.54E+13	12
GSA	3258.5133	3096.3026	3611.8294	163.63956	3185.717	10
PSO	5.80E+13	3173.2805	2.94E+14	7.72E+13	4.15E+13	13
GA	2.79E+13	3197.0824	1.80E+14	4.85E+13	1.12E+13	11

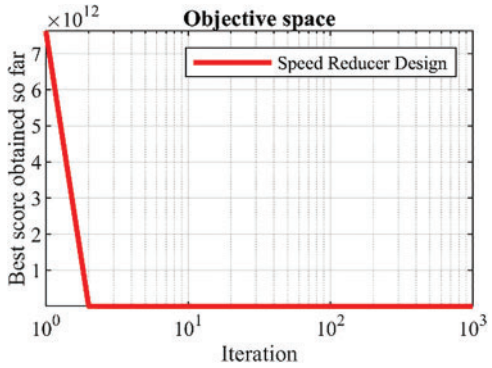

Figure 16: FNO's performance convergence curve on speed reducer design

5.4 Welded Beam Design

The optimization challenge of welded beam design holds significance in real-world applications, as depicted in Fig. 17, with the aim of minimizing the fabrication cost of the welded beam. The mathematical model for this design is articulated as follows [24]:

Consider: $X = [x_1, x_2, x_3, x_4] = [h, l, t, b]$.

Minimize: $f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2)$.

Subject to:

$$g_1(x) = \tau(x) - 13600 \leq 0, \quad g_2(x) = \sigma(x) - 30000 \leq 0,$$

$$g_3(x) = x_1 - x_4 \leq 0, \quad g_4(x) = 0.10471x_1^2 + 0.04811x_3x_4(14 + x_2) - 5.0 \leq 0,$$

$$g_5(x) = 0.125 - x_1 \leq 0, \quad g_6(x) = \delta(x) - 0.25 \leq 0,$$

$$g_7(x) = 6000 - p_c(x) \leq 0.$$

where

$$\tau(x) = \sqrt{(\tau')^2 + (2\tau\tau')\frac{x_2}{2R} + (\tau'')^2}, \quad \tau' = \frac{6000}{\sqrt{2}x_1x_2}, \quad \tau'' = \frac{MR}{J},$$

$$M = 6000\left(14 + \frac{x_2}{2}\right), \quad R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2},$$

$$J = 2x_1x_2\sqrt{2}\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right], \quad \sigma(x) = \frac{504000}{x_4x_3^2},$$

$$\delta(x) = \frac{65856000}{(30 \cdot 10^6)x_4x_3^3}, \quad p_c(x) = \frac{4.013(30 \cdot 10^6)x_3x_4^3}{1176}\left(1 - \frac{x_3}{112}\right).$$

With

$$0.1 \leq x_1, x_4 \leq 2 \text{ and } 0.1 \leq x_2, x_3 \leq 10.$$

Tables 16 and 17 present the outcomes of addressing welded beam design using FNO alongside rival algorithms. Fig. 18 illustrates the convergence curve of FNO as it achieves the optimal design for this problem.

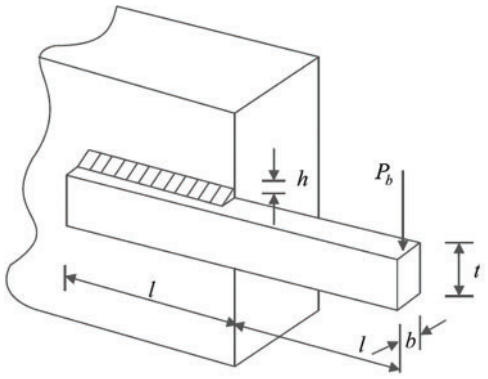

Figure 17: Schematic of welded beam design

Table 16: Performance of optimization algorithms on welded beam design problem

Algorithm	Optimum variables				Optimum cost
	h	*l*	*t*	*b*	
FNO	0.2057296	3.4704887	9.0366239	0.2057296	1.7246798
WSO	0.2056788	3.4716602	9.0364826	0.2057551	1.7251006
AVOA	0.2049682	3.4877457	9.0372012	0.2057455	1.7262173
RSA	0.2004819	3.5100311	9.5392895	0.2125585	1.8668773
MPA	0.2056789	3.471653	9.0364823	0.2057551	1.7250999
TSA	0.2047591	3.4897943	9.0521996	0.2059867	1.7306038
WOA	0.2097327	3.4067416	9.0014555	0.2143872	1.7809749
MVO	0.2050577	3.4851199	9.0428067	0.2059109	1.7282199
GWO	0.2056011	3.473437	9.0362654	0.2057942	1.7254793
TLBO	0.2675139	4.0119186	7.7707622	0.3297252	2.4592782
GSA	0.2554716	3.0475935	8.1263568	0.2634856	1.9284377
PSO	0.2996899	3.4546043	8.0800161	0.413847	3.0245026
GA	0.215722	5.4287681	8.3186692	0.2614805	2.3115269

Table 17: Statistical results of optimization algorithms on welded beam design problem

Algorithm	Mean	Best	Worst	Std	Median	Rank
FNO	1.7246798	1.7246798	1.7246798	2.51E-16	1.7246798	1
WSO	1.7257138	1.7250766	1.7272081	5.54E-04	1.7256231	3
AVOA	1.7462601	1.7261934	1.791771	0.0226569	1.7383159	7
RSA	1.983814	1.8668534	2.1817364	0.0900392	1.9701757	8
MPA	1.7257136	1.725076	1.727205	0.0005539	1.7256231	2
TSA	1.7360523	1.7305798	1.7423464	0.0037193	1.736282	6

(Continued)

Table 17 (continued)

Algorithm	Mean	Best	Worst	Std	Median	Rank
WOA	2.0567587	1.7809509	3.0388629	0.3998829	1.9294051	9
MVO	1.7349638	1.7281959	1.7555686	0.0088849	1.7323746	5
GWO	1.7270698	1.7254554	1.7308463	0.0014025	1.7268408	4
TLBO	1.88E+13	2.4592542	1.81E+14	5.05E+13	3.966695	12
GSA	2.1319031	1.9284138	2.3060492	0.1190564	2.1487315	10
PSO	2.59E+13	3.0244787	1.57E+14	5.45E+13	4.55E+00	13
GA	6.36E+12	2.311503	6.89E+13	2.15E+13	3.9480164	11

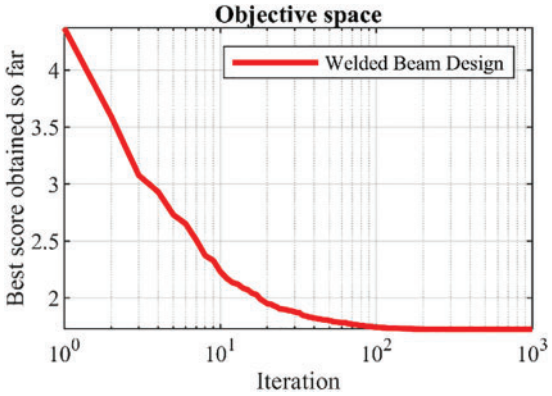

Figure 18: FNO's performance convergence curve on welded beam design

As per the results obtained, FNO has delivered the optimal design with design variable values of (0.2057296, 3.4704887, 9.0366239, 0.2057296), and an objective function value of 1.7246798. Analysis of the simulation outcomes reveals the superior performance of FNO in optimizing welded beam design compared to rival algorithms.

5.5 Tension/Compression Spring Design

The optimization challenge of tension/compression spring design holds practical relevance in real-world scenarios, as depicted in Fig. 19, with the goal of minimizing the weight of the tension/compression spring. The mathematical model for this design is articulated as follows [24]:

Consider: $X = [x_1, x_2, x_3] = [d, D, P]$.

Minimize: $f(x) = (x_3 + 2)\, x_2 x_1^2$.

Subject to:

$$g_1(x) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \leq 0, \quad g_2(x) = \frac{4x_2^2 - x_1 x_2}{12566\left(x_2 x_1^3\right)} + \frac{1}{5108 x_1^2} - 1 \leq 0,$$

$$g_3(x) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \leq 0, \quad g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \leq 0.$$

With

$$0.05 \leq x_1 \leq 2, 0.25 \leq x_2 \leq 1.3 \quad \text{and} \quad 2 \leq x_3 \leq 15.$$

Figure 19: Schematic of tension/compression spring design

Tables 18 and 19 depict the outcomes of employing FNO and competitor algorithms to address tension/compression spring design. Fig. 20 showcases the convergence curve of FNO as it attains the optimal design for this problem.

Table 18: Performance of optimization algorithms on tension/compression spring design problem

Algorithm	Optimum variables			Optimum cost
	d	*D*	*P*	
FNO	0.0516891	0.3567177	11.288966	0.0126019
WSO	0.0517981	0.3593483	11.140038	0.0126672
AVOA	0.050839	0.336765	12.627845	0.0126879
RSA	0.0508249	0.3330204	13.213587	0.012955
MPA	0.0517857	0.3590499	11.157505	0.0126672
TSA	0.0507372	0.3343296	12.794302	0.0126975
WOA	0.0508192	0.3363009	12.661049	0.0126892
MVO	0.0502892	0.3238205	13.591003	0.0127312
GWO	0.0519385	0.3627343	10.94956	0.0126703
TLBO	0.0601787	0.6456368	7.3809506	0.0154035
GSA	0.0530532	0.3911432	10.25442	0.0129158
PSO	0.0608052	0.6596202	6.3144359	0.015329
GA	0.0624287	0.7002184	4.9706342	0.0157093

Table 19: Statistical results of optimization algorithms on tension/compression spring design problem

Algorithm	Mean	Best	Worst	Std	Median	Rank
FNO	0.0126019	0.0126019	0.0126019	7.58E-18	0.0126019	1
WSO	0.0126837	0.0126585	0.012766	3.05E-05	0.0126775	3
AVOA	0.0130553	0.0126792	0.013507	0.0003454	0.0130109	8
RSA	0.0130014	0.0129462	0.0130806	4.525E-05	0.0129959	6
MPA	0.0126775	0.0126585	0.0127587	2.218E-05	0.0126765	2
TSA	0.0128432	0.0126887	0.0131486	0.0001479	0.0128097	5
WOA	0.013016	0.0126804	0.0137001	0.0003669	0.0129008	7
MVO	0.0148014	0.0127224	0.0155865	0.0010085	0.0153132	9

(Continued)

Table 19 (continued)

Algorithm	Mean	Best	Worst	Std	Median	Rank
GWO	0.0127096	0.0126615	0.0129154	5.616E-05	0.0127073	4
TLBO	1.57E-02	0.0153948	1.61E-02	2.26E-04	0.0156604	10
GSA	0.0164459	0.012907	0.0234955	0.0026141	0.0162001	11
PSO	1.17E+13	0.0153202	2.07E+14	5.10E+13	1.53E-02	13
GA	9.11E+11	0.0157005	9.43E+12	3.00E+12	0.0198638	12

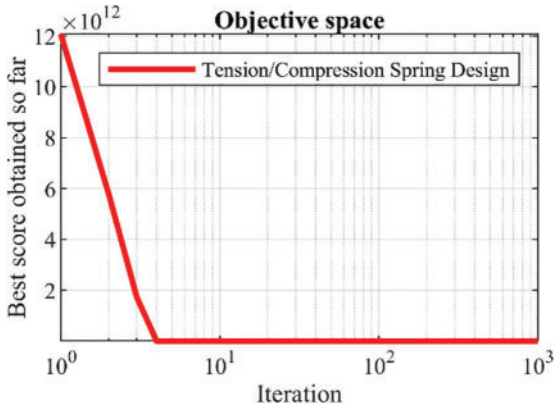

Figure 20: FNO's performance convergence curve on tension/compression spring

According to the findings, FNO has achieved the optimal design with design variable values of (0.0516891, 0.3567177, 11.288966), and an objective function value of 0.0126019. Analysis of the simulation results underscores the superior performance of FNO in handling tension/compression spring design compared to competitor algorithms.

6 Conclusions and Future Works

This paper introduced a novel metaheuristic algorithm called Far and Near Optimization, tailored for addressing optimization challenges across various scientific domains. FNO's core concept drew inspiration from global and local search strategies, dynamically updating population members within the problem-solving space by navigating towards both the farthest and nearest members, respectively. The theoretical underpinnings of FNO were elucidated and mathematically formulated in two distinct phases: (i) exploration based on the simulation of the movement of the population member towards the farthest member from itself and (ii) exploitation based on simulating the movement of the population member towards nearest member from itself. FNO's performance was rigorously assessed on the CEC 2017 test suite across problem dimensions of 10, 30, 50, and 100, as well as to address CEC 2020. The optimization results underscored FNO's adeptness in exploration, exploitation, and maintaining a delicate balance between the two throughout the search process, yielding effective solutions across various benchmark functions. The effectiveness of FNO in addressing optimization problems was assessed by comparing it with twelve widely recognized metaheuristic algorithms. The results of the simulation and statistical analysis indicated that FNO consistently outperformed most

competitor algorithms across various benchmark functions, securing the top rank as the premier optimizer. This superiority is statistically significant and underscores FNO's exceptional performance. Furthermore, FNO's efficacy in real-world applications was evaluated through the resolution of twenty-two constrained optimization problems from the CEC 2011 test suite and four engineering design problems. The results demonstrated FNO's effectiveness in tackling optimization tasks in practical scenarios.

While the FNO algorithm has shown promise in balancing exploration and exploitation, it has several shortcomings, including computational complexity, parameter sensitivity, risk of premature convergence, complexity in mathematical modeling, and limited experimental validation. Potential improvements include enhancing computational efficiency, adopting adaptive parameter tuning, mitigating premature convergence through hybridization and diversification strategies, simplifying the mathematical modeling, and conducting extensive experimental validation. These enhancements can help make FNO a more robust, efficient, and widely applicable optimization algorithm.

The introduction of the proposed FNO approach presents numerous avenues for future research endeavors. One particularly noteworthy proposal is the development of binary and multi-objective versions of FNO. Additionally, exploring the utilization of FNO for solving optimization problems across various scientific disciplines and real-world applications stands as another promising research direction.

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