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An Improved Wheeler Residual Stress Model For Remaining Life Assessment of Cracked Plate Panels

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Abstract: In this paper an improved Wheeler residual stress model has been proposed for remaining life assessment of cracked plate panels under variable amplitude loading (VAL). The improvement to the Wheeler residual stress model is in terms of the expressions for the shaping exponent, which is generally obtained through experiments. Simple expressions for the computation of shaping exponent have been proposed for compact tension (CT) specimen and plate panels with a center crack or an edge crack. The remaining life assessment has been carried out by employing linear elastic fracture mechanics (LEFM) principles. In the present study, the degree of influence of overload ratio (OLR) and the shape factor (β) on the shaping exponent have been investigated for remaining life assessment of cracked plate panels under tensile overload. It is observed that the parameters OLR and β have differing influences on the shaping exponent. Crack growth studies have been conducted on CT specimen and plate panels with a center crack or an edge crack subjected to tensile overloads for validating the proposed expressions. It is observed from the studies that the remaining life predicted using the improved Wheeler model for these plate panels are in close agreement with the experimental values reported in literature.

keyword: Fracture mechanics, overload, crack growth, remaining life

1 Introduction

Structural components are generally subjected to a wide spectrum of stresses over their lifetime. The fracture behaviour of these structural components under fatigue loading can be estimated through LEFM principles and SIF is the influencing design parameter. A detailed review of fatigue and fracture behaviour of structural components has been presented by Schijve (2003). Using the finite element method (FEM) for basic stress analysis, SIF can be computed through post-processing of finite element analysis (FEA) results reported by Owen and Fawkes (1982), Anderson (1995), Han and Atluri (2003, 2004). Further, it is observed that in general fatigue loading is treated as ‘constant amplitude loading’. However, these components rarely experience loading amplitude that remains constant during length of the service. A major influencing parameter to be considered is the load history, which is usually variable. In most of the structural components of offshore, airplanes, bridges and ships, load sequence is not completely random in nature but rather semi-random. The assessment of the behaviour of structure subjected to VAL is more complex as compared to constant amplitude loading (CAL). Crack growth under VAL involves the sequence effects or interaction effects. Load interaction or sequence effects significantly affect the fatigue crack growth rate and consequently fatigue life. Reliable prediction of fatigue life requires appropriate representation of load interaction effects. The crack length increase observed in each cycle under VAL will not be identical with that resulting from CAL of the same magnitude. Such an increase in crack length will depend not only on the maximum and minimum values for the loading cycle and crack length, but also on the previous loading history.

The effect of tensile overload has been reported by many investigators such as Yaha and Oktern (1996), Sheu, Song and Hwang (1995), Shin and Knott (1987), Robin, Louah and Pluvinage (1983), Stephens, Sheets and Njus, (1977), Taheri, Trask and Pegg (2003), Kim and Shim (2003), Dawicke (1997), Ramos, Pereira, Darwish, Motta and Carneiro (2003), Morman and Dubensky (1976). Single tensile overload introduce significant crack growth delay depending on the OLR. In general, longer delays are obtained by increasing the magnitude of overload, repeating the overload during the crack propagation and application of blocks of overloads instead of single overload.

A superimposed single overload during CAL is the sim-

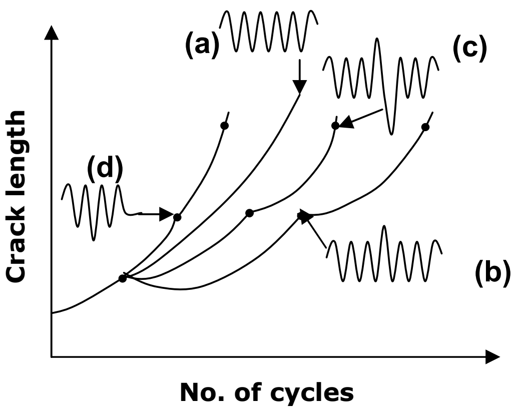

Figure 1 : Decrease in the rate of crack growth due to the overload followed by CAL

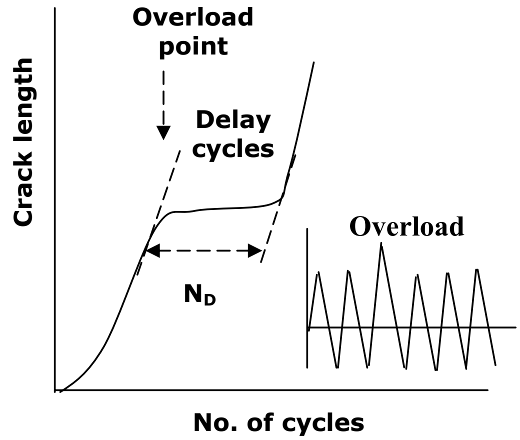

Figure 2 : Crack growth produced by (a) CAL, (b) single overload, (c) single overload-underloadand (d) single underload

plest case of VAL. The application of single overload will cause significant decrease in the crack growth rate for a large number of cycles subsequent to the overload as shown in Fig. 1. This phenomenon is referred to as crack retardation. Further, it is observed that higher OLRs can arrest crack growth. Increase in OLR value results in increase in number of delay cycles, N_D. Application of fatigue underloads [negative overloads] has the detrimental effect on fatigue crack initiation and crack growth. The crack growth rate is augmented and fatigue life will be reduced as reported by Dawicke (1997), Stephens, Chen and Hom (1976), Marissen, Trautmann and Nowack (1984), Carlson and Kardomateas (1994). The combination of overloads and underloads, singly or in blocks, produces much more complex situation. The application of an underload immediately following overloading diminishes the effect of the latter depending on their relative values, reported by Dawicke (1997), Carlson and Kardomateas (1994), Jaime (1994). It was observed that the retardation effect is lesser due to a tensile overload followed by a subsequent higher compressive overload, which can even lead to an acceleration of the crack growth. Also, application of an underload prior to overloading may have no influence or even decrease the retardation effect of the overload, depending on the particular loading conditions. Jaime (1994) explained some of the transient effects on crack growth and is illustrated in Fig. 2.

Several theories are available in the literature, reported by Shin and Knott (1987), Kim and Shim (2003), Ramos, Pereira, Darwish, Motta and Carneiro (2003), Wheeler (1972), Elber (1971), Newman (1997) to explain crack retardation, including crack tip blunting, crack closure effects and compressive residual stresses at the crack tip. Some investigators such as Shin and Knott (1987) and Elber (1971) reported that plasticity induced closure is the major cause for retardation. Many others, namely, Kim and shim (2003) and Ramos, Pereira, Darwish, Motta and Carneiro (2003) and Wheeler (1972) reported that compressive residual stresses are the primary cause for retardation. Since most of the retardation models are based on compressive residual stress concept, in the present studies it is proposed to use the concept of residual stress to represent the retardation effect due to overloads.

Retardation due to an overload is a complex phenomenon. There are number of empirical models for retardation, which contain one or more curve fitting parameters that are to be obtained experimentally. The widely used models are crack tip plasticity models (yield zone models), crack closure models and statistical models. Two primary influences on crack growth behavior are retardation following overloads and acceleration following underloads. Many models do not consider crack growth acceleration due to underload because of dominance of retardation due to overload. Yield zone models such as Wheeler (1972) and Generalised Willenborg model proposed by Gallagher and Hughes (1974) assume retardation persist as long as the crack tip plastic zones for subsequent load cycles are within the overload plastic zone. Crack closure models such as Elber (1971) and Newman

(1997) are based on the assumption that crack growth is controlled not only by the behaviour of the plastic zone but also by residual deformations left in the wake of the crack as it grows through previously deformed material. Hudson (1981) and Barsom (1976) models are based on root mean square approach for evaluating the average fatigue crack growth rate without accounting for the load interaction effects. If the influence of load interaction is small, these models predict reasonable result.

Since analytical modeling of crack closure is very difficult, models based on yield zone concept such as Wheeler (1972), Gallagher and Hughes (1974) and Gray and Gallagher (1976) are generally employed in the analytical investigation. The widely used Wheeler and Generalised Willenborg residual stress models are based on the assumption that crack growth retardation is caused by compressive residual stresses acting at the crack tip. Wheeler residual stress model is the simplest model among the various models available for VAL under yield zone concept. Wheeler model uses a retardation parameter (C_{pi}) to represent retardation due to overload. C_{pi} is the power function of the ratio of current plastic zone size and the distance from the crack tip to the border of the overload plastic zone size. The power coefficient used in the function for retardation parameter is generally called as shaping exponent (m_1). The shaping exponent is an adjustable calibration parameter that depends on the level of applied overload, crack length and plate width. There will be no retardation if m_1 is set to zero. Fig. 3 presents a graph of crack length against no. of flights for typical values of shaping exponent, given by Broek (1989). From Fig. 3, it can be observed that remaining life significantly increases as m_1 increases.

In general, shaping exponent can be obtained using one of the following procedures.

i. Calibrating ΔK and da/dN through number of experiments

ii. Analytically by trial and error process

Sheu, Song and Hwang (1995) conducted experiments on CT specimen to evaluate shaping exponent considering different OLRs and initial crack lengths. Their experimental result showed that m_1 values increase with OLR. It is also observed that m_1 is a function of OLR and ratio of crack length to width of the panel. Further, the value of m_1 increases when OL is applied at the larger initial crack length. After regression analysis, the following equations for m_1 in terms of OLR and a/W (where a= crack length and W = plate width) were proposed Sheu, Song and Hwang (1995).

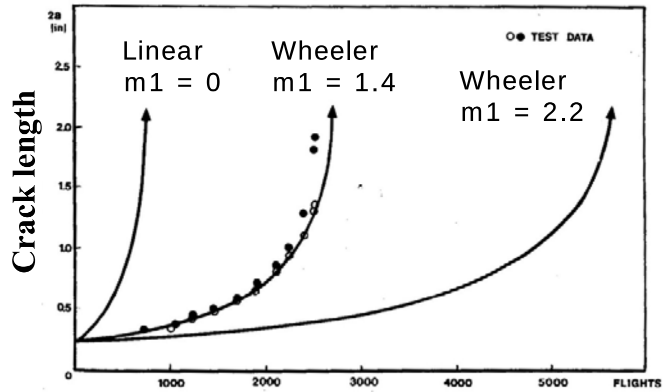

Figure 3 : Influence of shaping exponent on remaining life

$$m_1 = 2.522(\mathrm{OLR})^{0.733} - 3.092\left(1+\frac{a}{W}\right)^{-1.273}$$
for $5083-0$ Al alloy (1)

$$m_1 = 0.089(\mathrm{OLR})^{2.982} + 0.856\left(1+\frac{a}{W}\right)^{1.212}$$
for $6061-\mathrm{T651}$ Al alloy (2)

From close examination of eqns. 1 and 2, it can be seen that the indices and multiplication factors associated with OLR and $\frac{a}{W}$ are different for different grades of Al alloy. It is difficult to predict the physical significance of these. These eqns. are valid only for specific Al alloys. For any other material, the above equations will not be applicable straightaway to compute the retardation parameter and one would need to conduct number of experimental studies. Taheri, Trask and Pegg (2003) conducted number of experiments on 350 WT steel under VAL for remaining life prediction for center cracked plate panels and compared the experimental values with Wheeler residual stress model. They assumed the shaping exponent value by trial and error procedure, which provides the best fit of the experimental data. This resulted in predicting different shaping exponent for each OLR. This means that one need to conduct number of experiments for each specific loading scenario, requiring a huge database of test data. Finney (1989) studied the sensitivity of fatigue crack growth prediction using

Wheeler residual stress model. He expressed that the calibrated m_1 value is not a material constant but depends on the maximum stress in the spectrum and the crack shape. This contradicts the results reported by Sheu, Song and Hwang (1995). Gray and Gallagher (1976) and Arone (1990) concluded from their experimental studies that the shaping exponent is a function of OLR and ratio of crack length to width of plate panel.

From the foregoing discussions, it is observed that this limits the usefulness of the Wheeler residual stress model. Further, it is noted that the remaining life predictions are dependent upon the empirically determined shaping exponent. To the best of authors' knowledge, there are no general expressions available in the literature to evaluate the shaping exponent. As large amount of time and effort is required to calibrate the model through experiments to evaluate shaping exponent and also to overcome trial and error procedure, there is a need to develop simplified and general expression to evaluate the shaping exponent for any material with reasonable accuracy. In order to estimate the remaining life prediction of cracked plate panels under VAL, an improved Wheeler residual stress model consisting of simple, reliable and general expressions for shaping exponent for plate panels have been proposed in this paper.

In this paper an improved Wheeler residual stress model has been proposed for remaining life assessment of cracked plate panels under variable amplitude loading (VAL). Simple expressions for the computation of shaping exponent have been proposed for compact tension (CT) specimen and plate panels having a center crack or an edge crack. The remaining life assessment has been carried out by employing LEFM principles. In the present study, the degree of influence of overload ratio (OLR) and the shape factor (β) on shaping exponent have been investigated for remaining life assessment of cracked plate panels under tensile overload. Crack growth studies have been conducted on CT specimen and plate panels with a center crack or an edge crack subjected to tensile overloads for validating the proposed expressions. It is observed from the studies that the remaining life values computed using the improved Wheeler model for these plate panels are in close agreement with the experimental values reported in literature.

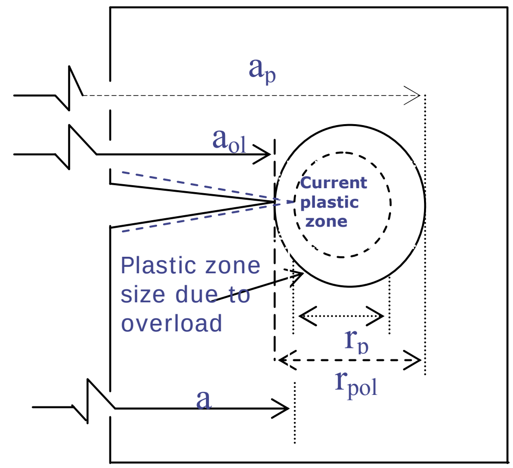

Figure 4 : Wheeler residual stress model

2 Wheeler Residual Stress Model

Wheeler (1972) employs the residual stress retardation model to account for crack growth retardation due to tensile overload (Fig. 4). The development of Wheeler model begins with the basic crack growth equation

$$\frac{da}{dN} = f(\Delta K) \tag{3}$$

Since the load is discontinuous variable, the crack growth can be computed using cycle-by- cycle approach

$$a_n = a_o + \sum_{i=1}^{N} f(\Delta K_i) \tag{4}$$

where, a_n = final crack length after N cycles a_0 = initial crack length.

ΔK_i= stress intensity factor range for cycle iTo account for crack growth retardation, Wheeler introduced a retardation parameter, C_{pi}, eqn. (4) then reduces to

$$a_n = a_o + \sum C_{pi} f(\Delta K_i) \tag{5}$$

The retardation parameter is calculated as shown below

$$\begin{aligned} C_{pi} &= \left(\frac{r_p}{(a_p - a)}\right)^{m_1} && \text{for } (a + r_p) < a_p \\ &= 1.0 && \text{for } (a + r_p) > a_p \end{aligned} \tag{6}$$

where, r_p = extent of current plastic zone $(a_p - a)$ = distance from crack tip to elastic – plastic interface (refer Fig. 4)

m_1= shaping exponent, which is generally obtained through experiments. The value of m_1 depends on applied overload, crack size and width of the plate.

It is observed that C_{pi} is minimum immediately after the application of overload, when $(a_p - a)$ has its maximum value. As 'a' approaches a_p, C_{pi} increases.

From Wheeler's model it can be observed that

- retardation decreases proportionately to the penetration of the crack into the overload plastic zone
- retardation occurs as long as the current plastic zone is within the plastic zone created by the overload
- retardation ceases as soon as the plastic zone touches the boundary of the plastic zone created by the overload

3 Improved Wheeler Residual Stress Model

As mentioned earlier the shaping exponent is primarily dependent on OLR and the ratio of crack length to width of plate panel. In the present study, the degree of influence of OLR and shape factor (β) on shaping exponent have been investigated for remaining life assessment of cracked plate panels under tensile overload. Extensive analytical investigations including regression analysis were conducted by Rama Chandra Murthy, Palani and Nagesh R. Iyer (2003) that contain different expressions for shaping exponent in terms of OLR and β to study the varying degree of influence on the remaining life prediction. Number of example problems were solved by Rama Chandra Murthy, Palani and Nagesh R. Iyer (2003) using these expressions for different materials such as steel and aluminium alloys. Extensive studies were carried out that considered

(i) variation of parameters β_e and β_c independently and concurrently

(ii) variation of parameter OLR independently and concurrently

(iii) various combinations of (i) and (ii)

Based on these studies among the various alternatives, the following shaping exponent expressions for a) CT specimen and plate panels with b) center crack and c) edge crack and under tensile overload are proposed in improved Wheeler model.

$$m_1 = \text{OLR} \quad (\text{for CT specimen}) \tag{7}$$

$$m_1 = \text{OLR} + \beta_c^2 \quad (\text{for center crack}) \tag{8}$$

$$m_1 = \frac{\text{OLR}}{2}\left(1+\sqrt{\beta_e}\right) \quad (\text{for edge crack}) \tag{9}$$

where OLR= $\frac{\sigma_{max,OL}}{\sigma_{max}}$, β_e = shape factor for edge cracked panels and β_c = shape factor for center cracked panels, which can be obtained using the following equations

For plate with edge crack

$$\beta_e = 1.12 - 0.231\alpha_e + 10.55\alpha_e^2 - 21.72\alpha_e^3 + 30.39\alpha_e^4 \tag{10}$$

$\alpha_e = \frac{a}{W}$ where, a = crack length, W = plate width

For plate with center crack

$$\beta_c = \left(1 - 0.025\alpha_c^2 + 0.06\alpha_c^4\right)\sqrt{\sec\left(\frac{\pi\alpha_c}{2}\right)} \tag{11}$$

where $\alpha_c = \frac{a}{W}$ where, a = Half crack length, W = Half plate width.

As stated earlier, it is observed that the parameters OLR and β have differing influences on the shaping exponent depending on plate configuration. Further, it may be noted that expressions (7) to (9) proposed are general in nature and seem to be independent of material used. This substantiates the observations made by Finney (1989). However, it may be noted that although the shaping exponent m_1 is shown to be independent of material used, the basic crack growth eqn (3) does depend on the material properties. From the above, it is observed that the shaping exponent is dependent on the geometry and OLR besides the location of crack. The retardation parameter in Wheeler residual stress model is computed by substituting appropriate shaping exponent in eqn (6). The remaining life is then estimated using the selected crack growth model.

4 Validation Studies

Studies have been conducted to validate the proposed the improved Wheeler residual stress model to quantify the influence of the proposed shaping exponent on CT specimen and plate panels having center crack and edge crack subjected to tensile overloads and for different initial crack lengths. For the studies, the experimental results reported by Sheu, Song and Hwang (1995), Stephens, Sheets and Njus (1977), Taheri, Trask and Pegg (2003), Dawicke (1997), Voorwald, Torres and Pinto (1991) in the literature for various materials such as steel and aluminium alloys have been used. Five example problems that consider (i) CT specimen (ii) a plate with a center crack and iii) a plate with edge crack have been presented here.

4.1 CT specimen - 6061-T651 Al Alloy

An example of a CT specimen made of 6061-T651 Al alloy has been chosen to validate the proposed Wheeler model by conducting crack growth analysis involving computation of delay cycles. This example has been studied by Sheu, Song and Hwang (1995). The data/information related to this problem is given in Tab.1.

Table 1 : CT specimen (6061-T651 Al alloy)

Thickness	:	9.59 mm
Material	:	6061-T651 Al alloy
Stress condition at crack tip	:	Plane strain
Max. load (P_{max})	:	4415 N
Min. load (P_{min})	:	22.75 N
Stress ratio	:	0.05
Yield stress	:	297.2 MPa
Crack growth equation	:	Paris
C	:	1.743e-11
m	:	2.82
OLRs	:	1.4,1.6,1.8,2.0
Initial crack length (a_o)	:	15, 18.5, 20 mm

Overload is applied at different initial crack lengths (15,18.5 and 20mm). Delay cycles for the respective load cases and initial crack lengths are shown in Tab. 2. Computation of delay cycles is carried out for all the OLRs and initial crack lengths. The delay cycles computed using the improved Wheeler model as well as reported by Sheu, Song and Hwang (1995) are shown in Tab. 2. From Tab. 2 it can be observed that the delay cycles computed using the improved Wheeler model are in good agreement with the experimental values. It can further be observed that the delay cycles are increasing with OLR. It is also observed that the delay cycles are decreasing when OL is applied at larger initial crack length. Thus, it is found that delay cycles for the initial crack lengths 18.5mm and 22.0mm are less compared to initial crack length 15mm for the respective OLRs.

4.2 CT Specimen (Man-Ten steel)

Another example of a CT Specimen made of Man-Ten steel has been chosen to validate the improved Wheeler model by conducting crack growth analysis and by computing remaining life. This example has been studied by Stephens, Sheets and Njus (1977). The data/information related to this problem is given Tab. 3.

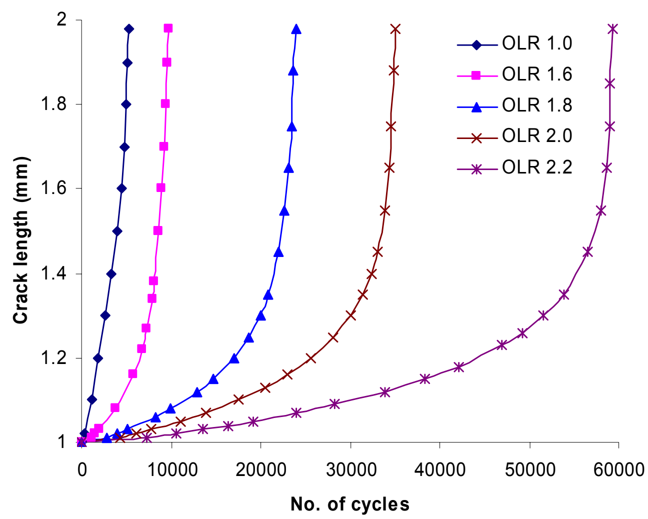

Figure 5 : Crack length vs No. of cycles for different OLRs

Overload is applied at initial crack length of 1.0in. The remaining life values for different overload ratios are shown in Tab. 4. From Tab. 4, it can be observed that the remaining life cycles computed using the improved Wheeler model are in good agreement with the experimental values given by Stephens, Sheets and Njus (1977). Fig. 5 shows the plot of crack length vs remaining life in cycles for different OLRs. From the plot it is observed that retardation is more for higher overload ratios and remaining life increases with the increase of OLR.

4.3 Plate (350WT Steel) with a center crack

An example of a plate made of 350WT steel with a center crack that has been studied by Taheri, Trask and Pegg

Table 2 : Delay cycles for CT specimen (6061-T651 Al alloy)

Overload ratio (OLR)	$P_{max,OL}$ KN	Initial crack length mm	Delay cycles computed using Wheeler model	Delay cycles computed using Improved Wheeler model	Exptl. Values by Sheu, Song and Hwang (1995)	% diff. in delay cycles using the improved Wheeler model w.r.t exptl. values
1.4	6.181	15	3566	3627	3633	0.16
1.6	7.064		7881	8110	8133	0.28
1.8	7.947		12402	12874	13233	2.71
2.0	8.83		22852	23462	24333	3.56
1.4	6.181	18.5	2445	2363	2500	5.48
1.6	7.064		5621	5478	5733	4.44
1.8	7.947		9134	9784	10133	3.44
2.0	8.83		18201	19023	19533	2.61
1.4	6.181	22.0	1986	1857	2033	8.66
1.6	7.064		4187	4124	4533	9.02
1.8	7.947		7419	7292	7633	4.46
2.0	8.83		15087	15413	15633	1.41

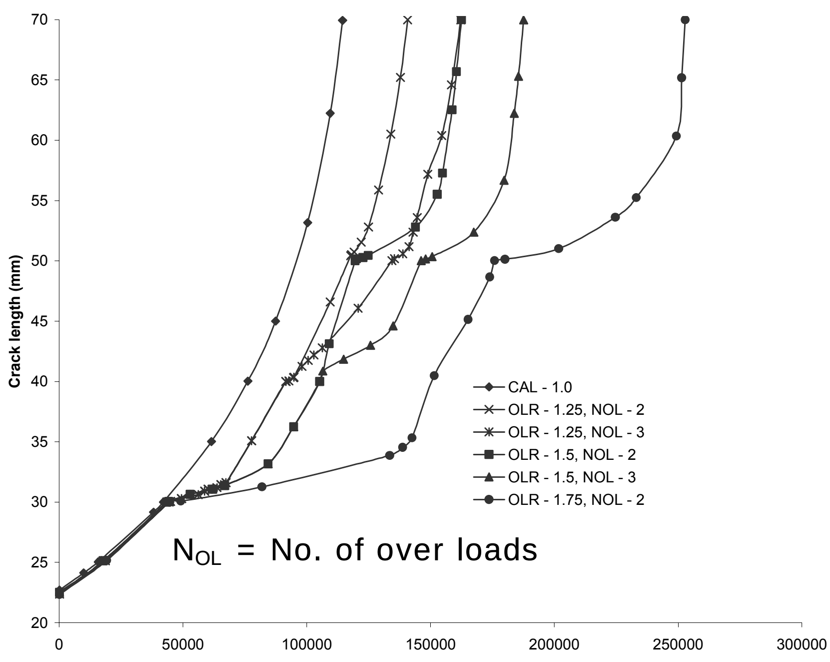

Figure 6 : Crack length vs remaining life

(2003) is selected. The data/information related to this problem is given in Tab. 5.

No. of overloads and occurrence of overload for each case is given in Tab. 5. The remaining life values computed using the proposed shaping exponent of improved Wheeler model and the corresponding experimental values reported by Taheri, Trask and Pegg (2003) are shown in Tab. 6. The remaining life values computed using the proposed shaping exponent are found to be within 4% of the experimental values. Fig. 6 shows the plot of crack

Table 3 : CT specimen – Man-Ten Steel

Thickness	:	5.715 mm
Material	:	Man-Ten Steel
P_{max}	:	23.17 kN
Stress conditions at crack tip	:	Plane stress
Stress ratio	:	0.0
Crack growth equation	:	Paris
C	:	2.73e-10
m	:	2.96
yield strength	:	372.31 MPa
Fracture toughness	:	164.82 MPa√m
Threshold SIF	:	73.62 MPa√m
OLRs	:	1.6,1.8,2.0,2.2
Initial crack length (a_o)	:	25.4 mm

Table 4 : Remaining life for CT specimen (Man-Ten steel)

OLR	$P_{max,OL}$ kN	Remaining life computed using improved Wheeler model	Exptl. Values by Stephens, Sheets and Njus (1977)	% diff.
1.0	23.17	5321	5526	3.71
1.6	37.1	9692	10000	3.08
1.8	41.77	23858	25526	6.53
2.0	46.39	35018	35657	1.79
2.2	51.04	59341	59868	0.88

length vs. remaining life values for different OLRs and no. of overloads. From Fig. 6, it can be observed that crack growth retardation increases with OLR. It can also be observed that the time of occurrence of overload influence the remaining life significantly. Early occurrence of overload causes significant increase in remaining life compared to later overloads. From Tab. 6, it can also be observed that the remaining life for OLR = 1.75 with two OLs is twice that of the life computed under CAL.

4.4 Plate (2024-T3 Al alloy) with a center crack

A plate made of 2024-T3 al alloy with a center crack has been chosen for study. Dawicke (1997) has conducted experimental studies on this problem. The

Table 5 : Centre crack – 350WT steel

Plate thickness	:	5mm
Material	:	350WT Steel
Stress condition at crack tip	:	Plane stress
Maximum stress (σ_{max})	:	114 MPa
Minimum stress (σ_{min})	:	11.4 MPa
Stress ratio	:	0.1
Crack growth equation	:	Paris
C	:	1.02 e-8
m	:	2.94
Fracture toughness	:	50 MPa √m
Yield Strength	:	350 MPa
OLRs	:	1.25,1.5,1.75
No. of overloads	:	2,3
Occurrence of overload (OL)	:	2 OL – 30, 50mm 3 OL– 30, 40 & 50 mm
Initial crack length (a_o)	:	22.4 mm

data/information related to the example problem is given in Tab. 7. Overload repeates at every 2500 constant amplitude load cycles.

The remaining life values computed using the improved Wheeler model and the corresponding experimental values available in Dawicke (1997) are shown in Tab. 8. The predicted remaining life values are found to be in good agreement with the experimental values. From Tab. 8, it can be observed that the remaining life value for OLR 1.5 is more than 12 times the value corresponding to CAL. This is due to the repetition of overload at every 2500 CAL cycles.

4.5 Plate (2024-T3 Al alloy) with an edge crack

The last example presented here is that of a plate made of 2024-T3 al alloy with an edge crack. The same has been studied experimentally by Voorwald, Torres and Pinto (1991). The data/information related to this problem is given in Tab. 9. Overload is applied at 6, 10,14,18 and 22 mm respectively.

The delay cycles have been computed using the proposed shaping exponent of improved Wheeler model for various OLRs and at corresponding crack lengths. The computed values are shown in Tab. 10. The delay cycles computed using the proposed shaping exponent are found to be in good agreement with the experimental values reported in Voorwald, Torres and Pinto (1991).

Table 6 : Remaining life values for plate with centre crack (350WT steel)

OLR	No. of over loads (N_{OL})	Remaining life values			% diff. in remaining life using the improved Wheeler model w.r.t exptl. Values	% increase in predicted life compared to CAL
		computed using		Exptl. Taheri, Trask and Pegg (2003)		
		Wheeler model	Improved Wheeler model			
1.25	2	135040	140764	146000	3.58	17.94
1.25	3	148692	162202	165000	1.69	29.86
1.5	2	158705	162503	167500	2.98	38.61
1.5	3	192526	187602	193000	2.79	68.15
1.75	2	235858	252883	255000	0.83	105.99

*No. of cycles computed under CAL 114499

Table 7 : Centre crack – 2024-T3 Al alloy

Thickness	:	2.286 mm
Material	:	2024-T3Al
Maximum stress (σ_{max})	:	68.94 MPa
Minimum stress (σ_{min})	:	1.38 MPa
Stress ratio	:	0.02
Stress condition at crack tip	:	Plane stress
Crack growth equation	:	Paris
C	:	0.829 e-8
m	:	3.284
Yield Strength	:	365.42 MPa
Fracture toughness	:	50.54 MPa √m
Overload ratio (OLR)	:	1.125, 1.5
Initial crack length($2a_o$)	:	25.4 mm

5 Discussion of Results

Studies on crack growth analysis of CT specimen, plate with a center crack or an edge crack have been conducted to validate the improved Wheeler residual stress model consisting of simple and generalized expressions for computation of shaping exponent under tensile overload. In general for all the example problems the values obtained by using the improved Wheeler residual stress model are in good agreement with the respective experimental values reported in the literature. In the case of CT specimen it is observed that the predicted delay cycles/remaining life values using improved Wheeler model are within 10% of the experimental values reported in the literature. Similarly in the case of center cracked plate, the predicted remaining life values are

Table 8 : Remaining life for plate with centre crack (2024-T3 Al alloy)

OLR	$\sigma_{max, OL}$, MPa	Remaining life computed using Improved Wheeler model	Exptl. Dawicke (1997)	% diff.
1.0	68.94	27339	30719	11.0
1.125	77.56	51624	52840	2.30
1.5	103.41	369712	385332	4.05

Table 9 : Edge crack – 2024-T3 Al alloy

Material	:	2024-T3 Al alloy
Thickness	:	1.27 mm
Width	:	90 mm
Max.load (P_{max})	:	7500 N
Min. load (P_{min})	:	2500 N
OLRs	:	1.2, 1.27, 1.33, 1.4
Crack Growth Equation	:	Modified Forman
C	:	3.6 e-08
m	:	3.019
Stress condition at crack tip	:	Plane stress
Initial crack length	:	5 mm
Yield stress	:	417 MPa

within 5% of the experimental values. In the case of edge cracked panels, the predicted life values are within ±12% of the experimental values reported in the literature. It is observed that the parameters OLR and β have differing influences on the shaping exponent depending on the location of the crack for remaining life prediction. In general, it is observed from the studies that the application of a overload cause a significant decrease in the crack growth rate for a large number of cycles subsequent to the overload. The amount of crack growth retardation increases as the OLR increases and is predominant for higher OLRs. Remaining life increases significantly as OLR increases. Remaining life is also influenced by the number of overloads and occurrence of overload. It is observed from the studies that occurrence of early overloads cause significant increase in remaining life compared to occurrence of later overloads.

6 Concluding Remarks

An improved Wheeler residual stress model for remaining life assessment of cracked plate panels under VAL (tensile overloads) has been proposed The improvement of the Wheeler residual stress model is in terms of the expressions for the shaping exponent (m_1), which is generally obtained through experiments. Simple expressions for the computation of shaping exponent have been proposed for CT specimen and plate panels having a center crack or an edge crack. The remaining life assessment has been carried out by employing LEFM principles. In the present study, the degree of OLR and the shape factor (β) on shaping exponent have been investigated for remaining life assessment of cracked plate panels under tensile overloads. It is observed that the parameters OLR and β have differing influences on the shaping exponent depending on the plate configuration. Further, it may be noted that proposed expressions for shaping exponent are general in nature and are independent of material used. However, it may be noted that although the shaping exponent is independent of material used, crack growth models account for material properties in terms of C and m. The shaping exponent is observed to be dependent on the geometry and load besides the location of crack. Extensive studies have been conducted to validate the proposed shaping exponent expressions by comparing with experimental results reported in the literature. It is observed from the studies that the remaining life values computed using the proposed shaping exponent expressions are in close agreement with the experimental values reported in literature for CT specimen and plate panels with a center crack or an edge crack. The procedures presented in the paper will be useful in the damage tolerant design of structural components in terms of crack growth analysis and remaining life prediction.

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Table 10 : Delay Cycles for a plate with an edge crack (2024-T3 Al alloy)

OLR	Delay cycles corresponding to crack length, mm	Delay cycles computed using Improved Wheeler model	Delay cycles Exptl. values by Voorwald, Torres and Pinto (1991)	% diff.
1.2	8	4595	5000	8.10
1.27		9767	11000	11.21
1.33		16765	18000	6.86
1.40		24120	25000	3.52
1.2	12	3726	4000	6.97
1.27		9941	10800	7.95
1.33		17102	18800	9.03
1.40		24064	25000	3.74
1.2	16	3514	3800	7.53
1.27		7842	8700	9.86
1.33		18761	20000	6.19
1.40		33102	35000	5.42

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