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Chance-Constrained Optimization of Pumping in Coastal Aquifers by Stochastic Boundary Element Method and Genetic Algorithm

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Abstract: In this paper the optimization of groundwater pumping in coastal aquifers under the threat of saltwater intrusion is investigated. The aquifer is inhomogeneous and contains several hydraulic conductivities zones. The aquifer data such as the hydraulic conductivities are uncertain, but with their expected mean and standard deviation values given. A stochastic boundary element method based on the perturbation technique is employed as the simulation tool. The stochastic optimization is handled by the chance-constrained programming. Genetic algorithm is selected as the optimization tool. Numerical examples of deterministic and stochastic problems are provided to demonstrate the feasibility of the proposed schemes.

keyword: Saltwater intrusion, optimization, uncertainty modeling, boundary element method, stochastic boundary element method, genetic algorithm, chance-constrained programming.

1 Introduction

Groundwater is the main source of freshwater supply in many parts of the world. As surface water is being depleted and/or contaminated, humanity's reliance on groundwater deepens with time. The over-exploitation of groundwater can cause short-terms problems such as the lowering of water table, the increase of pumping lift, wells running dry, and land subsidence. For the long-term, the depletion of water supply creates an unsustainable living environment and leads serious socioeconomic threats. The situation is even more critical in coastal aquifers, where the declining freshwater head causes seawater to invade inland and contaminate the freshwater sources. Once saltwater reaches the wells, the wells must be abandoned, which causes supply problem and lost of investment. These and many other issues of saltwater intrusion into coastal aquifers have been discussed in detail in the two books, Bear, et al. [1999] and Cheng and Ouazar [2003].

With these ongoing and potential threats, groundwater extraction in coastal aquifers needs to be carefully planned. One of the goals that water managers in coastal regions strives for is to maximize the yield of groundwater, yet without attracting saltwater into the wells. To achieve this goal, the mathematical tool of optimization can be utilized.

The optimization and management of water production in a groundwater basin has been widely studied; see, for example, Gorelick [1983] and Willis and Yeh [1987] for a review. Applications of these techniques to coastal aquifers, however, are relatively few. Only about a dozen such studies have addressed the issues of saltwater intrusion into coastal aquifers and their consequences in water supply. (See Qahman, et al. [2005] for a review.) In most of these studies, the saltwater intrusion into wells was indirectly addressed by using constraints such as monitoring salt concentration or piezometric head at certain control points, or minimizing the total saltwater volume in the aquifer. Only in the several recent studies the direct constraints of preventing the encroachment of saltwater front into the individual wells [Cheng, et al., 2000; Cheng, et al., 2003; Park and Aral, 2004], and the controlling of salt concentration in the wells [Qahman, et al., 2005], were directly addressed.

The current study is an extension of the previous work [Cheng, et al., 2000; Cheng, et al., 2003]. In Cheng, et al. [2000], analytical solutions were provided for maximum pumping involving one and two wells in the coastal

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zone. For multiple wells, genetic algorithm was employed to search for the near optimal solution. The input data, such as hydraulic conductivity, outflow rate, pumping rate, etc., were assumed to be deterministic. In Naji, et al. [1998a], analytical solutions for pumping induced saltwater intrusion under stochastic input data were considered for homogeneous aquifers, but without optimization. For inhomogeneous aquifers, numerical solution was needed, and it was handled using a stochastic boundary element method [Naji, et al., 1999]. In the next group of papers [Benhachmi, et al., 2003; Cheng, et al., 2003], pumping optimization was conducted under uncertain data input. The chance-constrained programming was used to convert the stochastic constraints into deterministic ones. The maximum allowable pumping rate was expressed in terms of the desirable reliability of prediction. The aquifer involved was homogeneous; hence analytical solution was used as the simulation tool, and the genetic algorithm was used for optimization. The present study extends the work of Benhachmi, et al. [2003] and Cheng, et al. [2003] by considering inhomogeneous aquifers that contain several hydraulic conductivity zones. This more realistic assumption prevents the use of analytical solution for simulation and a numerical solution tool is needed. The boundary element method, and particularly the stochastic boundary element method, is employed as the simulation tool in the present study.

In summary, the current study seeks to optimize the pumping pattern of a well group located in an inhomogeneous coastal aquifer. The input data, such as the hydraulic conductivities and outflow rates, are uncertain, but with their mean and the standard deviation values provided. The goal is to maximize the total pumping rate subject to the constraints of pumping capacity and no saltwater intrusion into the wells. A number of tools are needed for the present study. First, to solve a single realization of a deterministic problem of saltwater intrusion in an inhomogeneous aquifer with the sharp interface assumption, the boundary element method [Liu, et al., 1981; Taigbenu, et al., 1984] is utilized. In view of the uncertain input, the deterministic tool needs to be converted to a stochastic one using the perturbation technique [Cheng and Ouazar, 1995], which leads to the stochastic boundary element method [Naji, et al., 1999]. For the stochastic optimization problem, the chance-constrained programming of Charnes and Cooper [1959, 1963] is used to convert it to a deterministic optimization. Finally, due to the complexity of the problem, the evolutional optimization algorithm known as the genetic algorithm [Holland, 1975; Ouazar and Cheng, 1999] is used as the optimization tool. Numerical examples of deterministic and stochastic problems are provided to demonstrate the feasibility of the proposed schemes.

2 Sharp Interface Model

There are generally two approaches to model saltwater intrusion, the sharp interface and the miscible transport approach. (See Bear [1999] for the physical concepts behind these two approaches.) The sharp interface model assumes that the transition zone between the freshwater and saltwater is relatively narrow such that the two fluids can be considered immiscible with an interface separating them. The miscible model, on the other hand, considers the active solute transport of salt concentration by mechanisms of advection and dispersion, which further creates a density difference that drives the flow. In the present study, the sharp interface model is chosen for its simplicity in mathematical modeling.

Figure 1 gives the definition sketch of saltwater intrusion in confined and unconfined aquifers. We notice that h_f is the freshwater head, ξ is the interface location below the mean sea level, B is confined aquifer thickness, and d is the mean sea level above the aquifer datum. There exist two zones: in zone 2, which is in contact with the sea, saltwater and freshwater coexist, and in zone 1, only freshwater is found.

By utilizing the Dupuit approximation in a constant hydraulic conductivity zone, Strack [1976] has demonstrated that the single potential

$$\begin{aligned} \phi &= \frac{1}{2}\left[h_f^2 - s d^2\right] && \text{for zone 1} \\ &= \frac{s}{2(s-1)}\left(h_f - d\right)^2 && \text{for zone 2} \end{aligned} \tag{1}$$

for unconfined aquifer, and

$$\begin{aligned} \phi &= B h_f + \frac{(s-1)B^2}{2} - sBd && \text{for zone 1} \\ &= \frac{1}{2(s-1)}\left[h_f + (s-1)B - sd\right]^2 && \text{for zone 2} \end{aligned} \tag{2}$$

for confined aquifers, satisfies the Laplace equation

$$\nabla^2 \phi = 0 \tag{3}$$

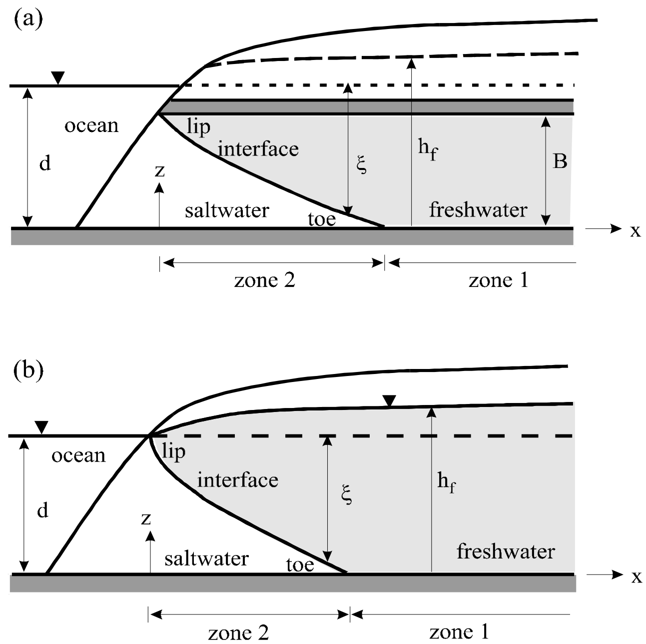

Figure 1 : Definition sketch of saltwater intrusion in (a) a confined aquifer, and (b) an unconfined aquifer.

continuously across the two zones, where the Laplacian is defined in the horizontal, or x–y plane. In the above $s = \rho_s/\rho_f$, and ρ_s and ρ_f are respectively the saltwater and the freshwater density. If there exist N pumping wells of discharge Q_i, $n = 1,\ldots,N$, they can be represented as Dirac delta functions δ, and the governing equation (3) becomes

$$\nabla^2\phi = \sum_{i=1}^{N} \frac{Q_i}{K}\delta(\mathbf{x}-\mathbf{x}_{\mathrm{w}}^i) \tag{4}$$

where K is the hydraulic conductivity, and $\mathbf{x}_{\mathrm{w}}^i$ is the pumping well location.

For boundary conditions, there exist the physical conditions of known piezometric head or flux on the external boundary. On the interfaces between the hydraulic conductivity zones, the following coupling conditions exist:

$$\begin{aligned} \phi_1 &= \phi_2 \\ K_1\frac{\partial\phi_1}{\partial n_1} &= -K_2\frac{\partial\phi_2}{\partial n_2} \end{aligned} \tag{5}$$

where the subscripts 1 and 2 denote the two adjacent zones. Equation (4) together with the boundary and interface conditions forms a well-posed boundary value problem that can be solved by an analytical or a numerical method. After (4) is solved, the toe of the saltwater wedge (Figure 1) is found at the location where the potential ϕ takes the value ϕ_{toe} given by

$$\begin{aligned} \phi_{\mathrm{toe}} &= \frac{s(s-1)}{2}d^2 \quad \text{for unconfined aquifer} \\ &= \frac{s-1}{2}B^2, \qquad \text{for confined aquifer} \end{aligned} \tag{6}$$

3 Deterministic Boundary Element Method

To solve the above boundary value problem in arbitrary aquifer geometry with multiple hydraulic conductivity zones, a numerical solution tool is needed. The boundary element method [Liu, et al., 1981; Taigbenu, et al., 1984] is selected for this purpose. The boundary integral equation solving the governing equation (4) can be represented as follows:

$$c\phi = \int_\Gamma \left(\phi\frac{\partial G}{\partial n} - G\frac{\partial\phi}{\partial n}\right) d\mathbf{x} + \frac{1}{2\pi K}\sum_{i=1}^{N} Q_i \ln r_i \tag{7}$$

where $r_i = \sqrt{(x-x_{\mathrm{w}}^i)^2+(y-y_{\mathrm{w}}^i)^2}$, $G = \ln r/2\pi$ is the free-space Green's function, Γ is the solution boundary, n is the outward normal of the boundary Γ, and c is the jump constant of the singularity which is equal to 1/2 on a smooth part of the boundary. Equation (7) is applied to each of the constant hydraulic conductivity zones. The numerical solution procedure for solving this type of boundary integral equations, which includes the discretization of boundary into elements, the interpolation of discrete potential values using polynomial shape functions, numerical integration of the kernel, the singularity removal, the formation of a linear system of equations, etc., is well known, hence will not be elaborated here.

4 Saltwater Front Tracking

Once the boundary integral equation (7) is solved to provide the full boundary and interface data, it can be utilized to find the potential at any interior point by placing the based point at that location. From the potential values found, it is necessary to trace the value ϕ_{toe} in the domain, which represents the trajectory of the saltwater front. This is accomplished by the following optimization procedure. Assume that the coastline is roughly parallel to the y-axis. A number of $y = y_i$ lines are drawn, where y_i are constants. On these lines, the x-coordinate x^i_{toe} where ϕ takes the value of ϕ_{toe} is to be found. These are the discrete locations of the saltwater front. The process is formulated as a nonlinear optimization by minimizing the objective function

$$F(x_1, x_2, \ldots, x_N) = \sum_{i=1}^{N} \left[\phi(x_i, y_i) - \phi_{\text{toe}}\right]^2 \tag{8}$$

In the above ϕ_{toe} is a known constant given by (6). Using the Gauss-Marquardt method [Naji, et al., 1998b], the objective function is minimized and the interface location can be found.

5 Stochastic Solution by Perturbation

In real-world groundwater problems, the hydrological and the hydrogeological input data, such as rainfall recharge, hydraulic conductivity, freshwater outflow rate, etc., are never certain. In the deterministic approach, the estimated mean values are used as input data. The deterministic result produced is interpreted as the mean of the output prediction. However, as the stochastic analysis shows, the mean input does not necessarily produce the mean output; also, the reliability of the prediction, hence the risk, is not known. So it is more desirable to face the random nature of the practical world and perform analysis in the statistical space. In the stochastic analysis, the input data are provided as the mean, the variance, and the covariance; and the output prediction is likewise given. Particularly, the question of reliability of prediction can be addressed and risk assessment can be conducted.

There are a number of ways to solve stochastic problems. The most widely used technique is the perturbation method. This technique assumes that the variation of a random variable ζ about its mean $\overline{\zeta}$ is relatively small. Hence, given a function g that is dependent on the random variables $(\zeta_1, \zeta_2, \ldots, \zeta_m)$, we can expand it into a Taylor series as

$$\begin{aligned} g(\zeta_1, \zeta_2, \ldots, \zeta_m) &= g(\overline{\zeta}_1 + \zeta'_1, \overline{\zeta}_2 + \zeta'_2, \ldots, \overline{\zeta}_m + \zeta'_m) \\ &= g(\overline{\zeta}_1, \overline{\zeta}_2, \ldots, \overline{\zeta}_m) + \sum_{i=1}^{m} \frac{\partial g(\overline{\zeta}_1, \overline{\zeta}_2, \ldots, \overline{\zeta}_m)}{\partial \overline{\zeta}_i} \zeta'_i + \\ &\quad \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\partial^2 g(\overline{\zeta}_1, \overline{\zeta}_2, \ldots, \overline{\zeta}_m)}{\partial \overline{\zeta}_i \partial \overline{\zeta}_j} \zeta'_i \zeta'_j + \cdots \end{aligned} \tag{9}$$

where $\zeta'_i = \zeta_i - \overline{\zeta}_i$ is the fluctuation from the mean. Taking mean value of the above equation, we obtain

$$\begin{aligned} \overline{g}(\zeta_1, \zeta_2, \ldots, \zeta_m) &\approx g(\overline{\zeta}_1, \overline{\zeta}_2, \ldots, \overline{\zeta}_m) \\ &\quad + \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\partial^2 g(\overline{\zeta}_1, \overline{\zeta}_2, \ldots, \overline{\zeta}_m)}{\partial \overline{\zeta}_i \partial \overline{\zeta}_j} \sigma_{\zeta_i \zeta_j} \end{aligned} \tag{10}$$

where $\sigma_{\zeta_i \zeta_j}$ is the covariance between the variables ζ_i and ζ_j, and when $i = j$ the covariance becomes the variance $\sigma^2_{\zeta_i}$. Similarly, we can find the variance of the random function g as [Cheng and Ouazar, 1995]

$$\begin{aligned} &\sigma^2_g(\zeta_1, \zeta_2, \ldots, \zeta_m) \\ &\approx \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\partial g(\overline{\zeta}_1, \overline{\zeta}_2, \ldots, \overline{\zeta}_m)}{\partial \overline{\zeta}_i} \frac{\partial g(\overline{\zeta}_1, \overline{\zeta}_2, \ldots, \overline{\zeta}_m)}{\partial \overline{\zeta}_j} \sigma_{\zeta_i \zeta_j} \end{aligned} \tag{11}$$

Hence given the statistical information of the input parameters $\overline{\zeta}_i$ and $\sigma_{\zeta_i \zeta_j}$, (10) and (11) allows the mean and variance of the output, $\overline{g}$ and σ^2_g, to be evaluated.

For the current problem, the random function g represents the n discrete saltwater front locations x_i along a set of rays $y = y_i$. The random variables ζ_i are the input parameters such as hydraulic conductivity K, freshwater outflow rate q, pumping rate Q, etc. Hence we can express the random functions as $x_i(\{K_1, K_2, \ldots\}, \{q_1, q_2, \ldots\}, \{Q_1, Q_2, \ldots\})$. For each of the input data, we are given its statistical moments such as mean ($\overline{K}$, $\overline{q}$, $\overline{Q}$), standard deviation (σ^2_K, σ^2_q, σ^2_Q), and covariance (σ_{Kq}, σ_{KQ}, …). Our goal is to find the solution, i.e., the saltwater front, in terms of mean $\overline{x}_i$ and standard deviation $\sigma^2_{x_i}$. Equations (10) and (11) are the approximation formulas that allow the evaluations of these statistical results via the perturbation of deterministic solution.

6 Stochastic Boundary Element Method

The boundary element method introduced above is for deterministic problems and needs to be modified to solve stochastic problems. There are two ways to accomplish this goal. One is to use the exact stochastic boundary element method formulation which directly models and solves for the mean, standard deviation, and covariances of the prediction [Cheng and Lafe, 1991; Cheng, et al., 1993]. However, the size of the solution system, which is the square of the discrete unknowns for the covariances involved, can be too large to be suitable for the present applications. We hence adopt the perturbation based methodology as described in the preceding section, which provides an efficient, but approximate solution.

Equation (10) shows that the mean of saltwater front location consists of two parts. The first part is obtained from the conventional deterministic solution by using the mean input parameters, $\overline{K}$, $\overline{q}$, $\overline{Q}$, etc., in the deterministic solution tool, such as the boundary element method defined by (7). The second part needs the second derivatives of the saltwater front location subjected to the variation of input parameters K, q, Q, etc. This is accomplished by small perturbations of input parameters and the finite difference approximation. In other words, we solve again the boundary value problem using the deterministic BEM, but with perturbed input data $K \pm \Delta K$, $q \pm \Delta q$, etc., to find the new saltwater front locations. The second derivatives can be approximated by the central difference formula such as the following

$$\frac{\partial^2 x_i(\overline{K},\overline{q},\overline{Q})}{\partial \overline{K}^2} \approx \frac{x_i(\overline{K}+\Delta K,\overline{q},\overline{Q}) - 2x_i(\overline{K},\overline{q},\overline{Q}) + x_i(\overline{K}-\Delta K,\overline{q},\overline{Q})}{\Delta K^2} \tag{12}$$

Since the standard deviations and covariances, σ_K^2, σ_q^2, σ_{Kq}, etc., are provided data, (10) allows the evaluation of the predicted mean location of saltwater front. Similarly by utilizing (11) and the central difference for the first derivative,

$$\frac{\partial x_i(\overline{K},\overline{q},\overline{Q})}{\partial \overline{K}} \approx \frac{x_i(\overline{K}+\Delta K,\overline{q},\overline{Q}) - x_i(\overline{K}-\Delta K,\overline{q},\overline{Q})}{2\Delta K} \tag{13}$$

the variances and covariances of the toe location can be found [Naji, et al., 1999].

7 Pumping Optimization in Coastal Aquifer

The objectives and constraints of water management in coastal regions can be quite complex. The goals can include maximizing the pumped volume of water, minimizing the utility cost as well as the capital investment, keeping the salt concentration in water to a tolerable level, spreading the risk of water shortage by the conjunctive use of surface water and groundwater, and other quality management, resources allocation, economic development, and policy decision objectives. To accomplish these goals, there may exist constraints such as maximum pumping rate due to equipment capacity, minimum pumping rate for cost effective operation, drawdown limits due to land subsidence and pumping lift concerns, prevention of intrusion of saltwater front into the wells, avoiding irreversible damage to aquifers, and other environmental, ecological, and social-economical constraints. In the current study, we focus on only a few of the above objectives and constraints. Particularly, our objective is to maximize the pumping rate from all wells combined. This goal is constrained by the conditions that each well has a minimum and maximum pumping limit, and that no pumping wells can be invaded by the saltwater front.

The mathematical statement of optimization is given as follows [Cheng et al., 2000]:

$$\max_{Q_i} Z = \sum_{i=1}^{N} Q_i \tag{14}$$

where Z is the objective function to be maximized with respect to the design variables Q_i, the pumping rate. The constraints of the optimization are

$$x_{\mathrm{toe}}^i < x_{\mathrm{w}}^i; \quad i = 1,\ldots,N \tag{15}$$

where x_{w}^i is the distance of the well to the coast, and x_{toe}^i is the saltwater toe location in front of the well, and

$$Q_{\min} \le Q_i \le Q_{\max} \quad \text{or} \quad Q_i = 0 \tag{16}$$

in which we notice that the last condition allows the wells to be shut down.

Since most search methods work only in the unconstrained search space, the constrained optimization needs to be transformed into an unconstrained one. This can be accomplished by the penalty method. The constraint (15) is incorporated into the objective function as a penalty

$$\max_{Q_i} Z = \sum_{i=1}^{N} Q_i - r\,\mathrm{H}_i(x_{\mathrm{toe}}^i - x_{\mathrm{w}}^i)\left(\frac{x_{\mathrm{toe}}^i}{x_{\mathrm{w}}^i} - 1\right)^2 \tag{17}$$

where r is a penalty factor, H_i is the Heaviside unit step function, and $\mathrm{H}_i = 1$ for $x^i_{\mathrm{toe}} \geq x^i_{\mathrm{w}}$ and $\mathrm{H}_i = 0$ for $x^i_{\mathrm{toe}} < x^i_{\mathrm{w}}$. The constraint (16) is not incorporated because it will be automatically satisfied in the genetic algorithm optimization through the selection of the population space as described below.

8 Chance-Constrained Optimization

As the current problem is of random nature, the optimization needs to be conducted in the probability space. For example, the deterministic constraint of no saltwater intrusion into the wells as described in (15) needs to be modified to be a probabilistic constraint:

$$\mathrm{Prob}(x^i_{\mathrm{toe}} < x^i_{\mathrm{w}}) > R \tag{18}$$

which states that the probability of a well not being intruded is greater than a reliability level R.

Optimization in the probability space is a difficult task. Often an approximation is sought using the deterministic method. One popular method that allows this conversion is the chance-constrained programming pioneered by Charnes and Cooper [1959, 1963]. The chance-constrained programming transforms a stochastic optimization into a deterministic equivalent with reliability measure incorporated as a decision variable. The deterministic equivalent of (18) is then:

$$\overline{x}^i_{\mathrm{toe}} + F^{-1}(R)\,\sigma_{x^i_{\mathrm{toe}}} < x^i_{\mathrm{w}} \tag{19}$$

where $\overline{x}^i_{\mathrm{toe}}$ is the expectation of toe location and $\sigma_{x^i_{\mathrm{toe}}}$ is its standard deviation, and F^{-1} is the value of standard normal cumulative probability distribution corresponding to the reliability level R. The mean and standard deviation of toe location can be computed using the stochastic boundary element method as described above.

Similar to the deterministic optimization, it is necessary to convert the constrained optimization into an unconstrained one. This is again accomplished by the penalty method. Hence the stochastic optimization problem becomes:

$$\max_{Q_i} Z = \sum_{i=1}^{n} Q_i - r\,\mathrm{H}_i(\overline{x}^i_{\mathrm{toe}} + F^{-1}(R)\sigma_{x^i_{\mathrm{toe}}} - x^i_{\mathrm{w}}) \left(\frac{\overline{x}^i_{\mathrm{toe}} + F^{-1}(R)\,\sigma_{x^i_{\mathrm{toe}}}}{x^i_{\mathrm{w}}} - 1 \right)^2 \tag{20}$$

9 Simple Genetic Algorithm

Genetic algorithm, first introduced by Holland [1975], is an optimization technique based on natural evolution phenomena. It has been extensively used to solve engineering design problems such as pipe network optimization, dynamic groundwater remediation management, free surface flow, etc. [Ouazar and Cheng, 1999]. It has also been used to optimize saltwater intrusion problems [Cheng, et al., 2000; Cheng, et al., 2003, Park and Aral, 2004].

The simple genetic algorithm works with a design family consisting of a population of individuals. Each individual of the population represents one trial case of the design variables. In the present case, this is given by a selection of $\{Q_1, Q_2, \ldots, Q_n\}$ taken from the population space $Q_{\min} \leq Q_i \leq Q_{\max}$. Each individual is regarded as a chromosome and is coded into a binary string $(0110\ldots01)$. The length of the string is defined by the precision required to evaluate the corresponding design variables. Within an evolutional iteration, three basic genetic operators, known as selection, crossover, and mutation, are applied to the family of individuals to produce stronger offsprings. Starting with a random selection of an initial population, the fitness (objective function value) of each individual (trial case) is computed. The solutions of higher fitness values are considered as better parents who receive higher priority to reproduce (selection) by a variety of mating procedures (crossover). To allow for the diversity of offsprings, and not to converge into an evolutional dead end, the bits can be randomly flipped based on certain probability for mutation. This procedure produces a generation of offsprings, which in turn becomes the parents of another generation. The process continues until a convergence is detected or a specified maximum number of generation is reached. The reader is referred to a textbook of genetic algorithm [Michalewicz, 1992] for details of this optimization technique.

In the present application, 5 individuals are used to form each generation, and each string of the chromosome is represented using 15 binary bits. The selection algorithm is the tournament scheme with a shuffling technique for choosing random pairs for mating. The routines used also include jump mutation, creep mutation, the options for single-point or uniform crossover, niching, and an option for variable number of children per pair of parents.

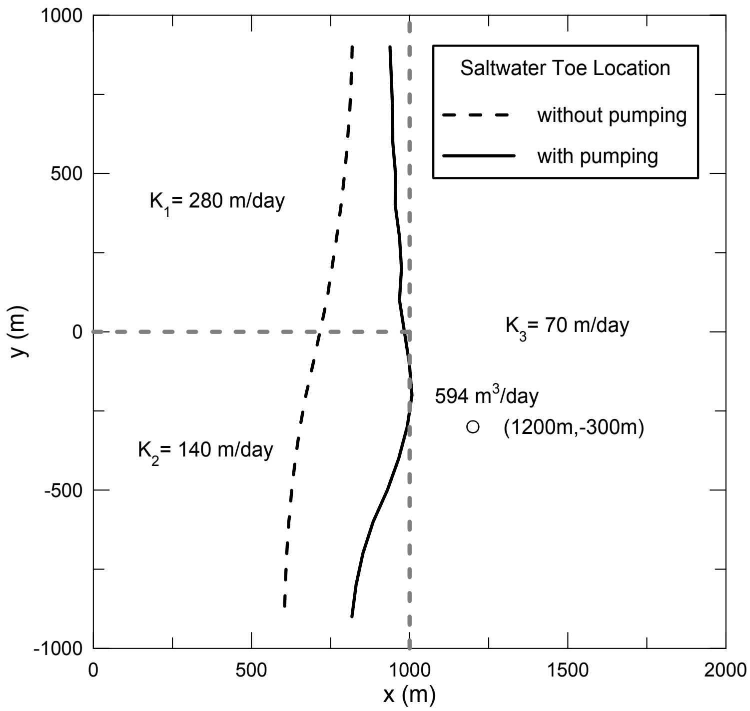

Figure 2 : Deterministic saltwater toe location for optimal pumping from one well at (1200m, -300m).

10 Numerical Examples

To demonstrate the feasibility of the above presented algorithms, the procedure is tested on a fictitious aquifer. We assume an unconfined aquifer of the horizontal dimension $2\,\text{km} \times 2\,\text{km}$, as shown in Figure 2. The aquifer has three hydraulic conductivity zones of $K_1 = 280$ m/day, $K_2 = 140$ m/day, and $K_3 = 70$ m/day, as indicated in the figure. Other input parameters include the freshwater and saltwater densities $\rho_f = 1.0$ g/cm^3 and $\rho_s = 1.025$ g/m^3, and the mean sea level above the aquifer datum $d = 20$ m (Figure 1b). The problem is defined by the following boundary conditions: the left side of the boundary is the seashore with freshwater head $h_f = 20$ m; the right side boundary is maintained at a constant head of $h_f = 21.6$ m; and the top and bottom sides are assumed to have no flow across the boundary, hence $\partial h_f / \partial n = 0$. These conditions are converted to the Strack potential as defined in (1) and expressed as

$$\begin{aligned} &\phi(0,y) = 0 \\ &\phi(2000\,\text{m}, y) = 28.28\,\text{m}^2 \\ &\frac{\partial \phi}{\partial n}(x, 1000\,\text{m}) = \frac{\partial \phi}{\partial n}(x, -1000\,\text{m}) = 0 \end{aligned} \tag{21}$$

Together with the Laplace equation (4), these define a well-posed boundary value problem. The toe location is represented by the equipotential line $\phi = \phi_{\text{toe}} = 5.125$ m^2, as defined in (6). The above aquifer, with a number of pumping wells deployed, is first solved as a deterministic optimization problem, and then as a stochastic optimization problem.

10.1 Deterministic Optimization Problem

In the first set of the examples, we assume that all input data are given with certainty; hence the solution is deterministic. The first problem involves the optimization of one pumping well located at $(1200\,\text{m}, -300\,\text{m})$. (See Figure 2.) The solution procedure for this one well and later multiple well problems is described as follows:

1. First, we start with the genetic algorithm by creating a family of 5 trial cases. For each case, pumping rate for each well is randomly selected within the minimum and maximum pumping rate constraints.

2. By the desirable decimal precision, these pumping rates are converted into binary codes for genetic operations.

3. For each individual (trial case), the boundary value problem defined by (4) and (21) is solved using the boundary integral equation (7).

4. Along rays perpendicular to the coastline and intersecting the wells, Gauss-Marquardt method is used to find the toe location by searching the potential value that equals to ϕ_{toe}.

5. The "fitness" of each individual is assessed using the objective function (17), which contains a penalty if wells are invaded.

6. Once the fitness of the whole generation is evaluated, the best solution so far is identified and stored.

7. Genetic operators, such as selection, crossover, and mutation are applied to these "chromosomes" to generate new offsprings.

8. Once the new family is formed, it loops back to the solution procedure as described in step 2.

For the current one-well problem, the above procedure produces the optimal discharge of $Q = 594$ m^3/day. The saltwater front location from this pumping is plotted in Figure 2. For comparison, we also show the original saltwater front due to the natural head difference without the presence of pumping well. The effect of pumping in attracting saltwater front to invade inland is clearly shown.

In the next case, we position the well to a new location at $(1200\,\text{m}, 300\,\text{m})$. (See Figure 3.) The well is now located in front of the more permeable zone with $K_1 = 280$ m/day, instead of $K_2 = 140$ m/day as the previous case. In this case, we obtain the optimal pumping rate of $Q = 443$ m^3/day, which is a significant decrease from the previous case. This reduction is anticipated as it is easier for saltwater to move through the higher hydraulic conductivity zone. The well in this case is less protected than the previous case; hence a smaller pumping rate is found. The toe location is plotted in Figure 3.

Finally, we test the case with three pumping wells, respectively located at $(1100\,\text{m}, 700\,\text{m})$ for well 1, $(1300\,\text{m}, -200\,\text{m})$ for well 2, and $(1200\,\text{m}, -500\,\text{m})$ for well 3. (See Figure 4.) All wells are subject to the pumping limits of $Q_{\min} = 100$ m^3/day and $Q_{\max} = 700$ m^3/day. The optimal pumping pattern is found to be $Q_1 = 108$ m^3/day, $Q_2 = 512$ m^3/day, and $Q_3 = 250$ m^3/day. Figure 4 shows the resultant saltwater front. We observe that for the two lower wells, well 2 is able to pump at a higher rate than well 3 because it is farther away from the sea. These two wells combine to give a pumping of 762 m^3/day, which is more than the single well case represented in Figure 2. The upper well is able to add another $Q_1 = 108$ m^3/day, which is much less than the single well case shown as Figure 3. The total pumping rate is 870 m^3/day. So the three well system is more effective in producing freshwater from this aquifer than the one well cases.

10.2 Stochastic Optimization Problem

In this section we consider cases with uncertain input data. In reality, the hydraulic conductivity values provided cannot be certain, and should only be interpreted as the estimated mean values; hence we assign the hydraulic conductivity for the three zones as $\overline{K}_1 = 280$ m/day, $\overline{K}_2 = 140$ m/day, and $\overline{K}_3 = 70$ m/day. The degree of uncertainty needs to be quantified in order to produce results useful to managers. This can be described by the coefficient of variation, which is set to 10% for the current case. This value is translated into the standard deviations of $\sigma_{K_1} = 28$ m/day, $\sigma_{K_2} = 14$ m/day, and $\sigma_{K_3} = 7$ m/day. We assume that there is no correlation between the hydraulic conductivity zones; hence $\sigma_{K_1K_2}, \sigma_{K_1K_3}, \ldots = 0$. All other input parameters are assumed to be deterministic. For the output, the manager can set the reliability of prediction, which is chosen to be 90%, meaning that the chance of failure of prediction is 10%.

In the first example, we treat the same three well case as reported in the preceding section, except that the hydraulic conductivities are uncertain with the data given above. The solution procedure follows the same steps of the deterministic case, except for these additions and modifications:

4a. Step 4 is repeated by changing the hydraulic conductivity data. The hydraulic conductivity is increased, and then decreased by a small amount ΔK, one at a time for each zone. For each change, the new boundary value problem is solved and the saltwater toe location is found.

4b. Equations (12) and (13) are utilized to find the first and second derivatives of toe location subject to hydraulic conductivity changes.

4c. Equations (10) and (11) are used to find the mean and the standard deviation of toe location, $\overline{x}^i_{\text{toe}}$ and $\sigma_{x^i_{\text{toe}}}$.

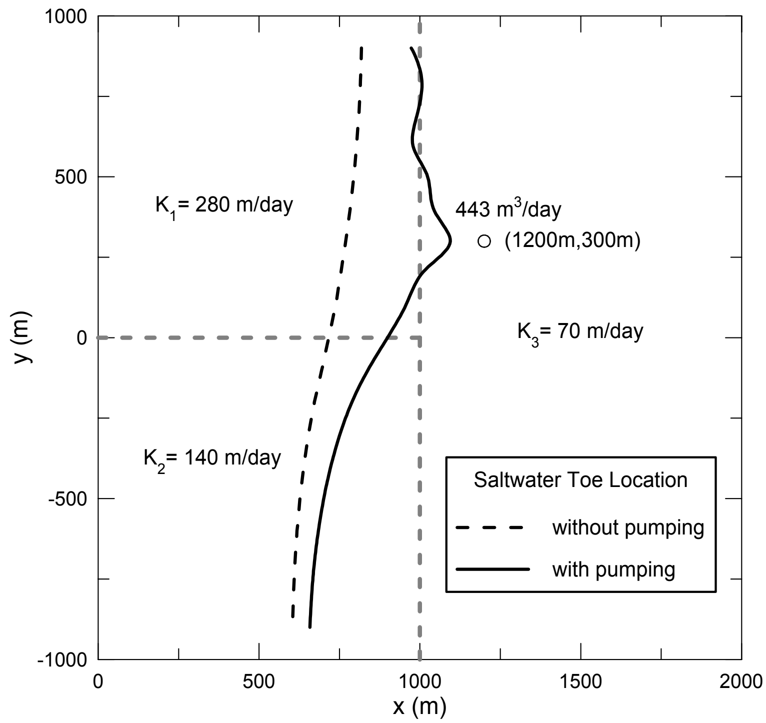

Figure 3 : Deterministic saltwater toe location for optimal pumping from one well at (1200m, 300m).

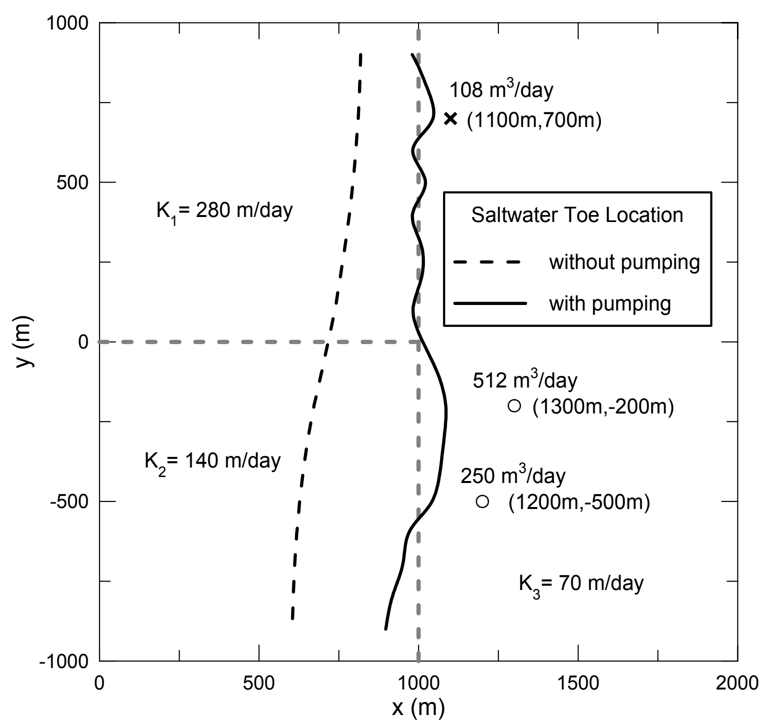

Figure 4 : Deterministic saltwater toe location for optimal pumping from three wells.

5. The "fitness" of each individual is assessed using the objective function (20).

In the genetic optimization procedure, we allow one more level of sophistication: in addition to pumping between the prescribed minimum and maximum rates, wells are allowed to shut down, and the saltwater front is permitted to invade the inactive wells. Shutting down wells nearer to the coast can allow other wells to pump more, if their maximum capacities have not been reached. The total pumped water can be more than the case that all wells are pumping. To allow for this option, the genetic algorithm is modified by adding a random bit (0 or 1) at the end of each substrings in the chromosome. The value of this added bit, with certain probability, decides whether the well is turned on or off. For the objective function, if an inactive well is invaded, no penalty is imposed.

For the stochastic optimization cases, the pumping rate constraints are set to be $Q_{\min} = 150$ m^3/day and $Q_{\max} = 700$ m^3/day. The optimal solution from genetic algorithm indicates that well 1 is shut down and only two wells are pumping. (See Figure 5.) The pumping rates are $Q_1 = 0$, $Q_2 = 571$ m^3/day, and $Q_3 = 250$ m^3/day. We notice that well 2 is allowed to pump more than that in the deterministic case, despite the added reliability constraint. This is likely to be the consequence of the shutting down of well 1. The total pumping is 821 m^3/day, which is less than the deterministic case, as expected. (The deterministic case can be roughly viewed as the case of 50% reliability. The higher the required reliability, the lower the pumping rate.) Figure 5 shows the mean saltwater front location from the suggested pumping rates. Also shown in the figure are the 90% reliability envelopes from the uncertainty analysis.

The second example treated has 5 pumping wells, located at: well 1 $(1100\,\mathrm{m}, 700\,\mathrm{m})$, well 2 $(1300\,\mathrm{m}, -200\,\mathrm{m})$, well 3 $(1200\,\mathrm{m}, -500\,\mathrm{m})$, well 4 $(1800\,\mathrm{m}, 100\,\mathrm{m})$, and well 5 $(1300\,\mathrm{m}, 300\,\mathrm{m})$. (See Figure 6.) The coefficient of variation for hydraulic conductivities is increased to 20%. Other parameters are the same as the above three well case. The result of the optimization again shows that well 1 is shut down, and the optimal pumping rates are: $Q_1 = 0$, $Q_2 = 313$ m^3/day, $Q_3 = 184$ m^3/day, $Q_4 = 696$ m^3/day, and $Q_5 = 269$ m^3/day. As compared to the three well case, we observe that wells 2 and 3 are pumped much less. But the two new wells 4 and 5 provide extra pumping. Particularly well 4, being farthest from the sea and placed in a gap between other wells, is able to pump at a rather large rate. The total pumping rate is 1462 m^3/day, which is significantly larger than the three well case. Figure 6 gives the mean and the 90% reliability saltwater front locations based on the optimal pumping rate.

11 Conclusion

In this paper we used a number of advanced techniques, including the stochastic boundary element method, chance-constrained programming, and the genetic algorithm to solve problems of pumping optimization in coastal aquifers containing multiple hydraulic conductivity zones with uncertain data input. The output is presented as a reliability analysis, which is most useful to the water managers and decision makers. The stochastic analysis shows that if the input data has high degree of uncertainty, the allowable pumping rate must be reduced to ensure the same level of reliability in the prediction. If a high degree of reliability is demanded by the manager, the pumping rate must also be lowered. These trends can be interpreted the other way around. If money can be invested in hydrological and hydrogeological investigations to reduce the uncertainty, then higher pumping rate is allowed. Or, if there exist alternative water resources in case of prediction failure, then the reliability demand can be lessened, and more water can be produced from the aquifers. As the water shortage becomes more and more severe in many parts of the world, this type of simulation capability as developed in this paper can assist the society to achieve the most effective use of the valuable water resources and to have a sustainable living environment.

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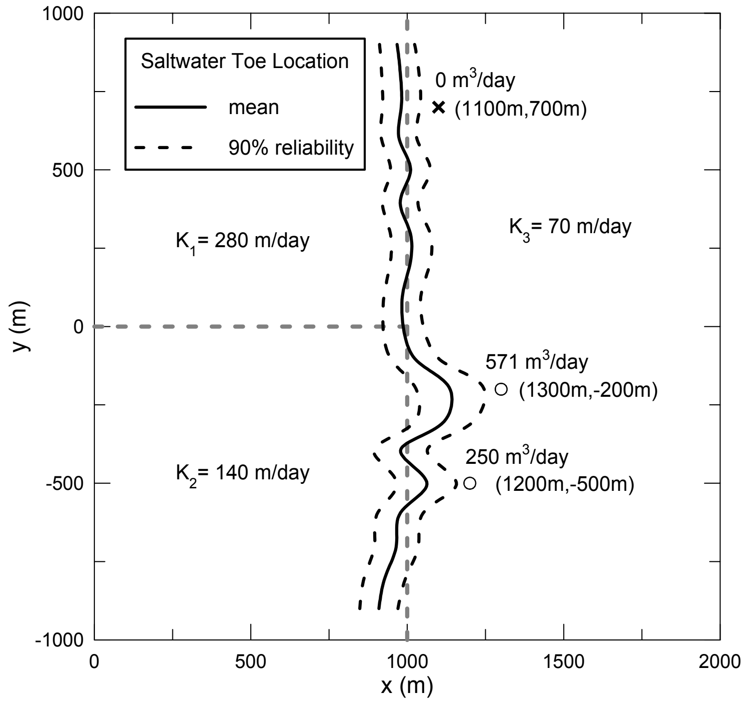

Figure 5 : Stochastic saltwater toe location for optimal pumping from three wells. (◯: active well; ×: inactive well)

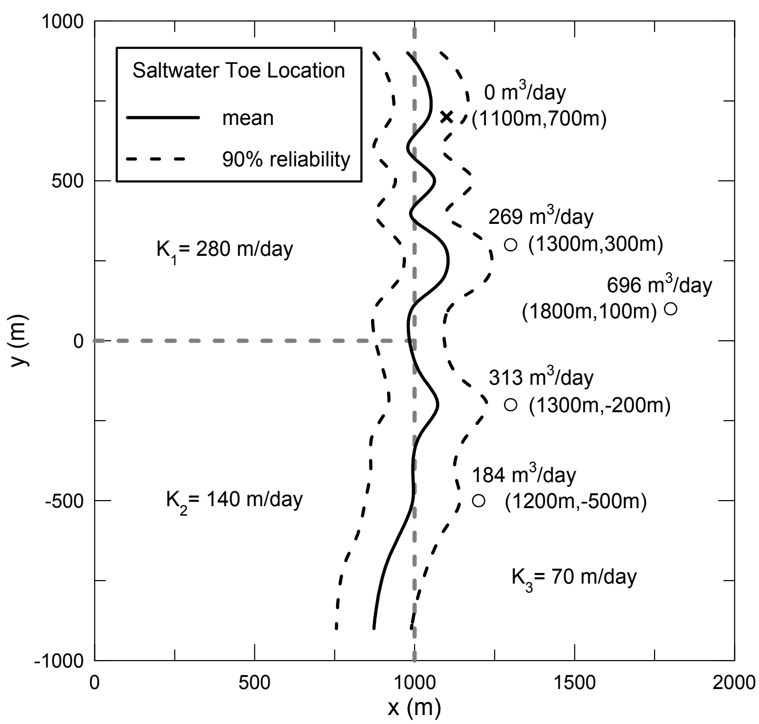

Figure 6 : Stochastic saltwater toe location for optimal pumping from five wells. (◯: active well; ×: inactive well)

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