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A Meshless Approach Based upon Radial Basis Function Hermite Collocation Method for Predicting the Cooling and the Freezing Times of Foods

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Abstract: This work presents a meshless numerical scheme for the solution of time dependent non linear heat transfer problems in terms of a radial basis function Hermite collocation approach. The proposed scheme is applied to foodstuff's samples during freezing process; evaluation of the time evolution of the temperature profile along the sample, as well as at the core, is carried out. The moving phase-change zone is identified in the domain and plotted at several timesteps. The robustness of the proposed scheme is tested by a comparison of the obtained numerical results with those found using a Finite Volume Method and with experimental results.

1 Introduction

One of the main applications of refrigeration can be identified in the preservation of food. As matter of fact, activity of enzymes and micro-organisms, which are responsible for the deterioration of organic products, is effectively reduced by cooling and freezing food items. Beside, crystallisation of water reduces the amount of liquid water available for microbial growth.

One of the key issues in the frozen food industry is the production of high quality items; the growing demand of such products in the market has stimulated researches and industries to develop new food processing practices and technologies. Refrigeration equipments need to respect quality standards as well as to be cost effective and, therefore, an appropriate design need to be carried out, for this purpose a good estimation of the cooling and freezing times is necessary.

In literature a wide collection of methods and approaches, which can provide accurate estimation of temperature and times are available. For instance, semi analytical methods have been developed by Becker and Fricke (2002). Numerical methods based on finite volumes or finite elements technique have been proposed to predict freezing times (see Hu and Sun (2000) and Puri and Anantheswaran (1993)). Beside that, the finite volume method has been the main approach used in commercial computational fluid dynamics (CFD) software packages which are already exist to simulate thermal processes of food for analysing complex flux behaviour.

In recent years, meshless approaches have become a popular numerical scheme to solve partial differential equations; La Rocca, Hernandez and Power (2004) show the versatility of the radial basis functions (RBFs) Hermite collocation approach to solve time dependent linear convective diffusion problems with constant coefficients. As it is point out in their work, the numerical solution of such transient problem using meshless approaches present several advantages in comparison with the use of traditional scheme as the finite difference, elements and volume.

The use of a mesh is a basic characteristic of traditional numerical approaches for the solution of partial differential equations. In those approaches, assumptions are made for the local approximation of the primitive variables, which require mesh to support them. During recent years, considerable effort has been given to the development of so-called mesh free methods (meshless approach). The aim of this type of approach is to eliminate at least the structure of the mesh and approximate the solution entirely using nodes values inside and/or in the boundary quasi-random distributed in the domain. For instance, the meshlees local Petrov-Galerkin and local boundary integral methods were given by Atluri and Zhu (1998) and (2000), respectively. These two methods basically transform the original problem into a local weak formulation over each subdomain, and the shape functions were constructed from using the moving least-squares approximation to interpolate the solution.

In this work we used a truly mesh free numerical scheme based upon radial basis functions Hermite interpolation

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techniques. In recent years the theory of radial basis functions (RBFs) has undergone intensive research and enjoyed considerable success as a technique for interpolating multivariable data and functions. Although most work to date on RBFs relates to scattered data approximation and in general to interpolation theory there has recently been an increased interest in their use for solving PDEs. This approach, which approximates the whole solution of the PDE by the direct use of RBFs, is very attractive due to the fact that this is truly meshfree technique. Kansa (1990a) (1990b) introduced the concept of solving PDEs using RBFs direct interpolation approach. He focused upon the multiquadric function and argued that PDEs are intrinsically related to the interpolation scheme from which PDE solvers are derived.

The above approach has been applied successfully in several cases (see for example Dubal (1994), Hon and Mao (1998), Zerroukat, Power and Chen (1998) and Amaziane, Naji and Ouazar (2004)), including initial and boundary value problems. However, no existence of solution and convergence analysis is available in the literature and for some cases, it has been reported that the resulting matrix was extremely ill-conditioned and even singular for some distribution of the nodal points (see Dubal, Olivera and Matzner (1993)). Several techniques have been proposed to improve the conditioning of the coefficient matrix and the solution accuracy, as are: replacement of global solvers by block partitioning, LU decomposition schemes, matrix preconditioners, multizone methods etc (see Kansa and Hon (2000)). Besides some variations of the original Kansa's idea, which improve the performance of the scheme, have been also suggested. Among them it is worthy to mention the works of Shu and co-workers and Tran-Cond and co-workers.

Wu and Shu (2002) suggested using differential quadrature approximation to represent the differential operators of a given problem, instead of the direct differentiation used in the Kansa's method. The proposed differential scheme is the same one used in the dual reciprocity method (DRM) of the boundary element technique to approximate the derivatives of the variables in the volume integrals to be transformed to surface integrals (for more details see Partridge, Brebbia and Wrobel (1992)). In this type of differentiation the basic unknowns are the nodal values of the function instead of the expansion coefficients. Shu, Ding and Yeo (2005) implemented this type of approach to find the numerical solution of complex incompressible Newtonian fluid flow problems at moderate Reynold's number. Wu and Hon (2003) recently solved transient heat diffusion problems using a scheme similar to the one proposed by Wu and Shu. Besides showing the implementation of the approach, they also gave error estimate on the convergence of the scheme.

On the other hand, Mai-Duy and Trang-Cong (2001) proposed to approximate directly the higher derivative of the differential operator by the radial basis function instead of the function itself and find the lower derivatives and the function via symbolic integrations, instead of finding the derivatives by differentiation of the radial basis function. By this idea, they were able to improve the accuracy of the scheme without increasing the computational cost, in particular the approximation of the higher derivatives. This approach has been also used to solve both transient and steady state problems (see Mai-Cao and Tran-Cong (2005), Mai-Duy (2004) and May-Duy and Tran-Cong (2003)). However as is the case of the original Kansa's approach, none of the above modifications are completely mathematically robust and rigorous proof of their existence of solution is not available.

Fasshauer (1996) suggested an alternative approach based on Hermite interpolation technique using radial basis functions, which allows not only the interpolation a given function but also its derivatives. The convergence proof for RBF Hermite-Brikhoff interpolation was given by Wu (1992) who also recently proved the convergence of this approach when solving PDEs (see Wu (1998) and Schaback and Franke (1998)). From a series of simple steady state numerical examples Fasshauer (1996) concluded that those methods base on Hermite interpolation performs slightly better than those based on direct interpolation. Jumarhon, Amini and Chen (2000) and Leitao (2001) observed similar improvement when using Hermite algorithms instead of direct approaches.

More recently Power and Barraco (2002) found that direct interpolation methods have some difficulties solving convection-diffusion problems at high Péclet number, which do not come across using the Hermite approach. Li and Chen(2003) pointed out that these inconveniences that the direct approach has to predict high Péclet can be improve by using higher-order radial basis functions and overlapping domain decomposition technique.

Furthermore it is important to mention that most of the works reported in the literature based on the Hermite approach are used to solve steady state non-homogeneous

problems with constant coefficients. In the present work, we report how this approach (Hermit collocation) can be implemented to find the numerical solution of time dependent non linear diffusion problems with variable coefficients with application to cooling and freezing foods. The numerical results obtained with the proposed meshless approach are compared with experimental values and with the results of a finite volume scheme, previously reported in literature.

2 Mathematical model

In this article we present an implementation of the radial basis function Hermite collocation method for the solution of time dependent non linear heat transfer problems. To model heat transfer through solid foods Fourier's Equation of heat conduction is normally used, which can be written as:

$$\rho \cdot C(T)\frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x_j}\left(K_j(T)\frac{\partial T(x,t)}{\partial x_j}\right) \qquad x \in \Omega \subset R^d,\ t > 0 \tag{1}$$

where $K(T)$ and $C(T)$ are the thermal properties of the substance, which are function of the temperature field and therefore function of the position x at time t, ρ is the density, which is considered constant in the present work, Ω is a bounded domain in R^d, d is the dimension of the problem.

In general the above differential equation is required to satisfy the following boundary and initial conditions:

$$A\,T(x,t) + B\frac{\partial T(x,t)}{\partial n} = f(x,t), \qquad x \in \Gamma,\ t > 0 \tag{2a}$$

$$T(x,t) = T_0(x), \quad t = o \tag{2b}$$

where $A$and $B$are known constants and $f(x,t)$ and $T_0(x)$ are known functions. When $B = 0$ and $A \neq 0$, we refer to this type of boundary condition as of the Dirichlet type, when $A = 0$ and $B \neq 0$as the Neumann type and when $B \neq 0$ and $A \neq 0$as the mixed (radiation) or Robin type.

Thermal proprieties of food items strongly depend on temperature and might be determined using temperature dependent models. Under this conditions the above problem becomes strongly non linear. Therefore, cases such as freezing processes, in which large ranges of temperature are involved, are challenging to solve due to of the rapid variation of the coefficients with temperature. However, the problem can be reduced to a more simple form by introducing a change of variable in terms of the Kirchhoff's transformation (see Carslaw and Jeager (1959)), i.e.

$$\varphi = \int_{T(x,t)}^{T_0} K(\xi)\ d\xi \tag{3}$$

which implies

$$\begin{aligned}
\frac{\partial \varphi}{\partial t} &= -K\frac{\partial T}{\partial t} \\
\frac{\partial \varphi}{\partial x_j} &= -K\frac{\partial T}{\partial x_j} \\
\frac{\partial^2 \varphi}{\partial x_j^2} &= \frac{\partial}{\partial x_j}\left(-K\frac{\partial T}{\partial x_j}\right)
\end{aligned} \tag{4}$$

Using this well known technique follows that the form of the heat equation is preserved with a diffusivity function of T; in this way equation (1) becomes:

$$\frac{\partial \varphi}{\partial t} = k(T)\frac{\partial^2}{\partial x_j^2}\varphi \tag{5}$$

where $k(T) = K(T)/[\rho \cdot c(T)]$, which can be solved by a direct iterative scheme.

In the above transformation $T_0$is a referent temperature, which in the case of initial constant temperature can be chosen to be equal to the initial value.

The boundary and initial conditions of the problem need to be adapted according to the transformation as follows:

The initial condition $T(x,0) = T(0,t)$ implies $\varphi(t) = \int_{T(x,0)}^{T_0} K(\xi)d\xi$, $x \in \Omega$, which reduces to φ=*0* in the case of constant initial temperature, i.e., $T(x,0) = T(0)$.

A boundary condition of the Dirichlet type $T(x,t) = D(x,t)$,$x \in \Gamma$ implies $\varphi(t) = \int_{D(x,t)}^{T_0} K(\xi)d\xi$ with $x \in \Gamma$ and a boundary condition of the Neumann type $\frac{\partial T(x,t)}{\partial n} = N(x,t)$,$x \in \Gamma$, implies $\frac{\partial \varphi(x,t)}{\partial n} = -K(T)N(x,t)$, $x \in \Gamma$, while a boundary condition of the Robin type is a mixture of the Dirichlet and Neumann conditions.

As can be observed a boundary condition of the Neumann or Robin type, needs to be included in the iterative scheme. Moreover a simple homogeneous Neumann condition reduces to$\frac{\partial \varphi(x,t)}{\partial n} = 0$

In equation (5), it is possible to approximate the time derivative of the partial differential operator by a simple

forward difference using Crank-Nicholson (θ weighted) method, i.e.

$$\frac{\varphi(x,t+\Delta t)-\varphi(x,t)}{\Delta t} = \theta\, k(T)\frac{\partial^2\varphi(x,t+\Delta t)}{\partial x_j^2} + (1-\theta)\, k(T)\frac{\partial^2\varphi(x,t)}{\partial x_j^2} \quad (6)$$

In this way at each time step the transient problem can be seen as a steady state non-homogeneous problem, with the non-homogenous term as function of the solution at the previous time step, i.e.

$$\theta\left(\Delta t\, k(T)\frac{\partial^2}{\partial x_j^2} - \frac{1}{\theta}\right)\varphi(x,t+\Delta t) = (\theta-1)\left(\Delta t\, k(T)\frac{\partial^2}{\partial x_j^2} - \frac{1}{(\theta-1)}\right)\varphi(x,t) \quad (7)$$

After obtaining at each time step the function φ, the temperature field is found by inverting of the integral equation (3).

3 Radial basis function meshless Hermite approach

In recent years the theory of radial basis functions (RBFs) has undergone intensive research and enjoyed considerable success as a technique for interpolating multivariable data and functions. A radial basis function, $\Psi(x-x_j)=\psi\left(\left\|x-x_j\right\|\right)$ depends upon the separation distances of a sub set of data centres, $X\subset\Re^n, \left\{x_j\in X, j=1,2,...,N\right\}$. Due to RBFs spherical symmetry about the centres x_j (nodal or collocation points), they are called radial. The distances, $\left\|x-x_j\right\|$, are usually taken to be the Euclidean metric, although other metrics are possible.

The most popular RBFs are:

$r^{2m-2}\log r$ (generalized thin plate spine),
$\left(\mathrm{r}^2+\mathrm{c}^2\right)^{m/2}$ (generalized multiquadric),
exp(- r / c) (Gaussian)

where m is an integer number and $r=\left\|x-x_j\right\|$.

The Gaussian and the inverse multiquadric i.e. $m<0$ in the generalized multiquadric function, are positive definite functions, while the thin-plate splines and the multiquadric i.e. $m>0$ in the generalized multiquadric function, are conditionally positive definite functions of order m, which require the addition of a polynomial term P of order m-1 together with a given homogeneous constraint conditions (see equation (10)) in order to obtain an invertible interpolation matrix. The multiquadric functions with values of m=1 and c=0 are often referred to as conical functions whilst with m=3 and c=0 as Duchon cubics.

Duchon (1977) derived the thin plate splines (TPS) as an optimum solution to the interpolation problem in a certain Hilbert space via the construction of a reproducing kernel. Therefore, they are the natural generalisation of cubic splines in n>1 dimension. Even though TPS have been considered optimal in interpolating multivariate functions they do, however, only converge linearly (see Powell (1994)). On the other hand, the Hardy multiquadratics (MQ) functions converge exponentially and always produce a minimal semi-norm error as proved by Madych and Nelson (1990). However, despite MQ's excellent performance, it contains a free parameter, c^2, often referred to as the shape parameter. When c is small the resulting interpolating surface is pulled tightly to the data points, forming a cone-like basis functions. As c increases, the peak of the cone gradually flatters.

In a typical interpolation problem we have N pairs of data points $\left\{(x_j,F(x_j))_{j=1}^N\right\}$, which are assumed to be samples of some unknown function F that is to be interpolated by the function f, i.e.

$$f(x)=\sum_{j=1}^{N}\lambda_j\Psi\left(\left\|x-x_j\right\|\right)+P_m(x) \quad x\in\Re^2 \quad (8)$$

in the sense that

$$F(x_i)=\sum_{j=1}^{N}\lambda_j\Psi\left(\left\|x_i-x_j\right\|\right)+P_m(x) \quad (9)$$

along with the constrains condition

$$\sum_{j=1}^{N}\lambda_j P_k(x_j)=0 \;\; 1\le k\le m \quad (10)$$

Here the numbers λ_j, j=1,2,...,N, are real coefficients and Ψ is a radial basis function.

The matrix formulation of the above interpolation problem can be written as Ax = b with

$$A=\begin{pmatrix}\Psi & P_m\\ P_m^T & 0\end{pmatrix} \quad (11)$$

x^T (λ, β) and b^T = (F, 0), where β are the coefficients of the polynomial.

Micchelli (1986) proved that for a case when the nodal points are all distinct, the matrix resulting from the above radial basis function interpolation is always non singular. Although theoretically the resulting matrix from the above interpolation technique is always invertible, numerical experiments show that the condition number of the matrix obtained with the use of smooth RBFs like Gaussian or multiquadrics are extremely large when compared with those resulting from non-smooth RBFs like the thin-plate splines for low values of m(see Schaback (1995)). Similar difficulties to those encountered with smooth functions are found when using non-smooth functions with large values of m.

Let us now consider a boundary value problem

$$L[u](x) = f(x) \tag{12a}$$

$$B[u](x) = g(x) \tag{12b}$$

where the operators L and B are linear partial differential operators on the domain Ω and at the contour Γ respectively. For the above problem, a Hermite RBF collocation method represents the solution u by an interpolation function of the following type:

$$u(x) = \sum_{k=1}^{n} \lambda_k B_\xi \Psi(\|x-\xi_k\|) + \sum_{k=n+1}^{N} \lambda_k L_\xi \Psi(\|x-\xi_k\|) + P_m(x) \tag{13}$$

with n as the number of nodes on the boundary of Ω, $N-n$ the number of internal nodes, L_ξ and B_ξ are the differential operators in (12-a,b), but acting on Ψ viewed as a function of the second argument ξ.

This expansion for u leads to a collocation matrix A, which is of the form

$$A = \begin{pmatrix} B_x B_\xi[\Psi] & B_x L_\xi[\Psi] & B_x P_m \\ L_x B_\xi[\Psi] & L_x L_\xi[\Psi] & L_x P_m \\ B_x P_m^T & L_x P_m^T & 0 \end{pmatrix} \tag{14}$$

The matrix (14) is of the same type as the scattered Hermite interpolation matrices and thus non-singular as long as Ψ is chosen appropriately. The convergence of the above approach has been proven by Schaback and Franke (1998) in terms of a generalized Fourier transform analysis (see also Wu (1998)).

Due to the uncertainty regarding which RBF is the best to use in a collocation method for the solution of boundary value problems for partial differential equations in this work we will use the generalised TPS.

In the matrix representation (14) of the Hermite collocation numerical solution of equation (7), in the cases when boundary conditions of the first, second and mixed kind (Dirichlet, Neumann and Robin) are prescribed, the partial differential operators are given by the following expressions:

$$L_x = L_\xi = \theta \left[\Delta t \left(k(T) \frac{\partial^2}{\partial x_j^2} \right) - \frac{1}{\theta} \right] \tag{15}$$

where the partial differential operator L_x is self adjoin, and

$$\begin{aligned} B_x^D &= 1 \quad B_x^N = \frac{\partial}{\partial x_j} n_j(x) \\ B_x^R &= A(x) + C(x) \frac{\partial}{\partial x_j} n_j(x) \\ B_\xi^D &= 1 \quad B_\xi^N = \frac{\partial}{\partial \xi_j} n_j(\xi) \\ B_\xi^R &= A(\xi) + C(\xi) \frac{\partial}{\partial \xi_j} n_j(\xi) \end{aligned} \tag{16}$$

In the above relations the super index D,N,R in the operator B represent the type of boundary conditions implemented, i.e. Dirichlet, Neumann and Robin. Furthermore to avoid singularity at $r = 0$ on the resulting differential operators of the matrix A, we use in the representation formula (13) the generalized TPS

$$\psi = r^6 \log r \tag{17}$$

together with the corresponding cubic polynomial.

$$\begin{aligned} P(x) = {} & \lambda_{N+1} x_1^3 + \lambda_{N+2} x_2^3 + \lambda_{N+3} x_1^2 x_2 \\ & + \lambda_{N+4} x_1 x_2^2 + \lambda_{N+5} x_1^2 + \lambda_{N+6} x_2^2 \\ & + \lambda_{N+7} x_1 x_2 + \lambda_{N+8} x_1 + \lambda_{N+9} x_2 + \lambda_{N+10} \end{aligned} \tag{18}$$

The non-homogeneous term is obtained by the multiplication of the rectangular matrix $\tilde{A} = \left[\tilde{L}_x B_\xi[\Psi] \quad \tilde{L}_x L_\xi[\Psi] \quad \tilde{L}_x P_m \right]$, defined at the

internal points, by the λ coefficient found at the previous time step, where

$$\tilde{L}_x = (\theta - 1)\left[\Delta t\left(k(T)\frac{\partial^2}{\partial x_j^2}\right) - \frac{1}{(\theta - 1)}\right] \tag{19}$$

As can be observed the above system of equations need to be solved iteratively at each time step of the transient analysis, due to the presence of a variable k(T). In the proposed iterative algorithm, at each time step the initial guess value of the diffusivity coefficient is given in term of the temperature field of the previous time step, which is then update with the new obtained value of the temperature field. In order to progress to a new time step it is necessary to achieve convergence of the iterative scheme at the previous time step with a given tolerance of the relative variation of the temperature field $(T_{i+1}(x) - T_i(x))/T_i(x)$, with i as the number of iteration.

4 Numerical examples

To perform heat transfer calculation thermal properties of foods must be known. One of the main constituent commonly found in food is water. Below the initial freezing point which is different for each foodstuff due to the different concentration of solid dissolved, such water begin to crystallize. Therefore water content and is change of state, influences the thermophysical proprieties of foods; It is nearly impossible to experimentally determine such proprieties, and mathematical models need to be used. As pointed out by Comina, Bonacina and Toffan (1973) and confirmed recently by La Rocca and Morale (1998), the phase change can be assumed to take place between two sharply defined temperatures, initial phase change temperature T_l and final phase change temperature T_s. Therefore, in such interval and considering an homogeneous food material, the values of the thermal conductivity K can be predicted using the following expressions:

$$\begin{aligned} &K(T) = K_l, \quad T \le T_l \\ &K(T) = K_s + \frac{(K_l - K_s)(T - T_s)}{T_l - T_s}, \quad T_s \le T \le T_l \\ &K(T) = K_s, \quad T \le T_s \end{aligned} \tag{20}$$

which is defines a linear relation to interpolate thermal conductivity in the phase change area; while a triangle is used to approximate heat capacity, as follows:

$$\begin{aligned} &C(T) = C_l, \quad T \le T_l \\ &C(T) = C_l + \frac{(C_{\max} - C_l)(T_l - T)}{T_l - T_{\max}}, \quad T_{\max} \le T \le T \\ &C(T) = C_s + \frac{(C_{\max} - C_s)(T - T_s)}{T_{\max} - T_s}, \quad T_s \le T \le T_{\max} \\ &C(T) = C_s, \quad T \le T_s \end{aligned} \tag{21}$$

with

$$C_{\max} = \frac{(2\Delta u - C_S(T_{\max} - T_S) - C_L(T_L - T_{\max}))}{(T_L - T_S)}$$

been Δu the latent heat of fusion.

It is important to point out that for simplicity constant values of thermal properties are employed below and above the final and initial crystallization temperature. Moreover, it has to be admitted that better approximation function are available in literature, however for the purpose of this paper the above estimation can be considered satisfactory.

A plot of the resulting thermal diffusivity $k(T) = K(T)/[\rho \cdot C(T)]$ versus temperature is given in figure1; as it can be noted it changes dramatically with temperature. Moreover, values above and below the phase changing zone are drastically different. Due to the dependency of thermal properties on temperature and the difference in magnitude of the physical parameters involved, the numerical analysis of the present freezing process is not a trivial task.

Knowing the above variation of the thermal diffusivity with the temperature field, equation (3) can be integrated to yield the following relation between the variable φ and T :

$$\begin{aligned} &\varphi = K_l(T_0 - T), \quad T_l \le T \le T_0 \\ &\varphi = (K_l)(T_0 - T_l) + K_s(T_l - T) - \frac{K_l - K_s}{T_l - T_s}T_s(T_l - T) \\ &\quad + \frac{K_l - K_s}{T_l - T_s}\frac{T_l^2 - T^2}{2}, \quad T_s \le T \le T_l \\ &\varphi = (K_l)(T_0 - T_l) + K_s(T_l - T_s) - \frac{K_l - K_s}{T_l - T_s}T_s(T_l - T_s) \\ &\quad + \frac{K_l - K_s}{T_l - T_s}\frac{T_l^2 - T_s^2}{2} + K_s(T_l - T), \quad T \le T_s \end{aligned} \tag{22}$$

which can be directly inverted by solving the corresponding quadratic equation to find the value of T knowing φ, that will be used at each step of the iterative algorithm.

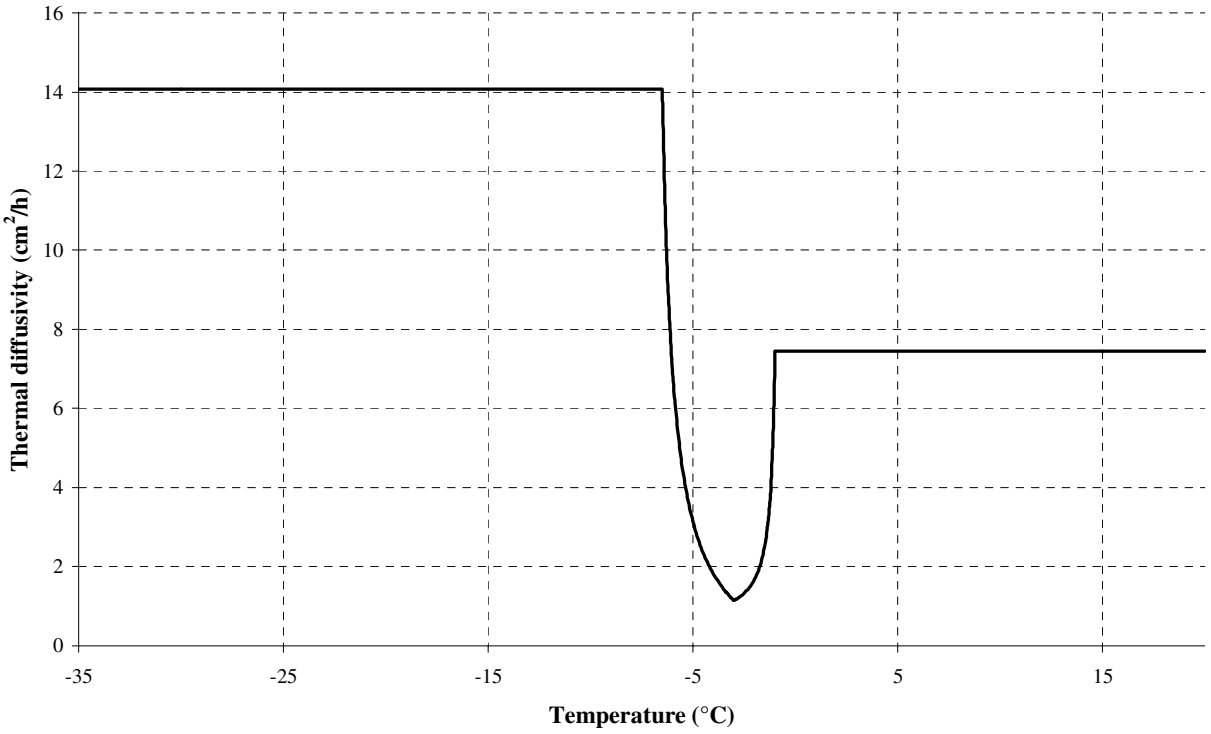

Figure 1 : Thermal diffusivity profile in the range of temperature explored here, for the hamburger sample

Table 1 : Thermo-physical parameters for the hamburger sample.

heat capacity c		Thermal conductivity K		Density	Latent heat Δu	T_l	T_m	T_s	2l
J/(kg K)		W/(m K)		Kg/m^3	J/kg	° C	° C	° C	M
Cl	Cs	Kl	Ks						
3100	2100	0.43	1.2	990	199000	-1	-3	-6.5	0.0425

For the purpose of validation of the proposed numerical approach based upon the Hermite radial function collocation technique, two test problems are considered. In each cases a comparison between the obtained numerical results of the time variation of the temperature at the core of the sample with a finite volume results as well as experiments value reported by La Rocca and Morale (1998), are presented.

4.1 First numerical example

In the first example a hamburger sample is considered. The sample has circular cross section, of diameter *D* and a thickness s, which respects the condition s/D$<<$1, bounded by a pair of parallel planes. The lateral surface of the cylinder was kept insulated, i.e. non-flux of heat through surface.

For this example, the thermo-physical parameters in the governing equation can be evaluated using the data given table 1.

Our main object at this point is to validate the proposed meshless scheme, by comparing the present results with those previously obtained with a finite volume method scheme and with experimental values. In figure 2 we present a comparison between the temperature distribution obtained using the Hermite collocation approach at the core of the sample with those found with a finite volume method.

A uniform temperature field, function of time, is applied at each end surfaces of the cylinder (dash line in figure 2). Due to the symmetry, the problem was solved only in

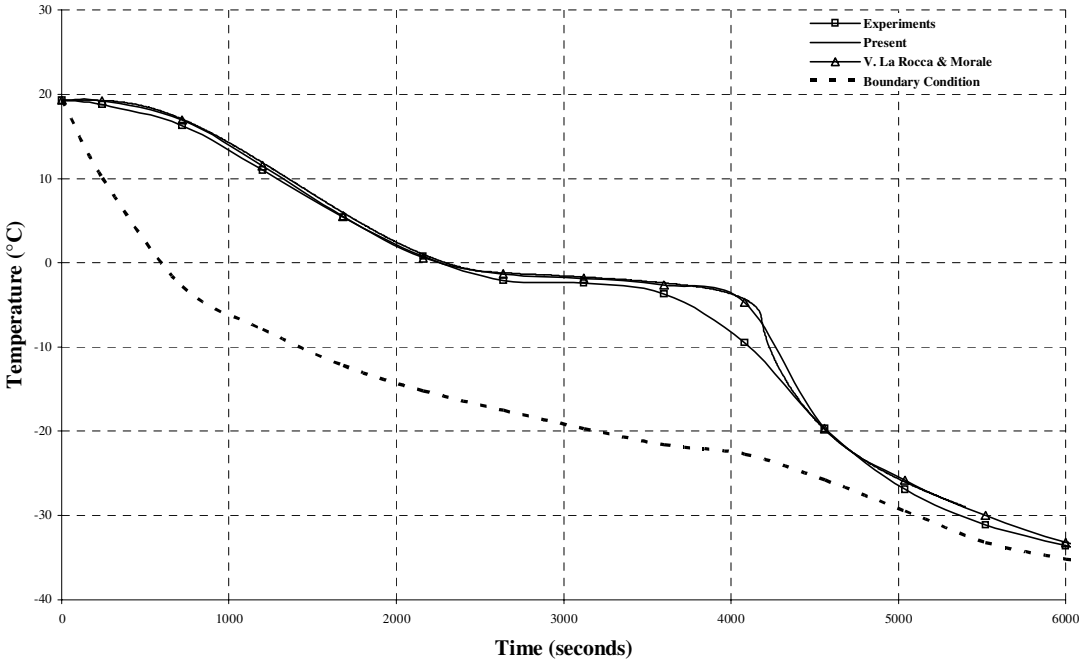

Figure 2 : Comparison between experimental (), numerical results of the temperature obtained using a finite volume method (Δ) and the results found using the meshless approach (continuous line)

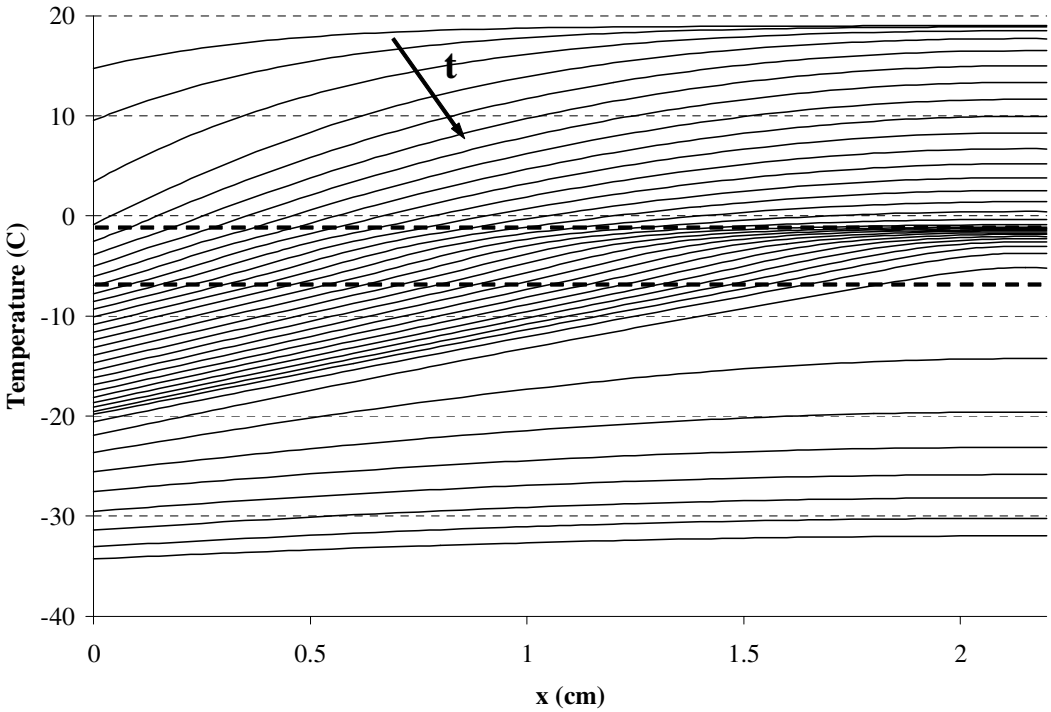

Figure 3 : Time evolution of the temperature profile along the centre line of the sample.

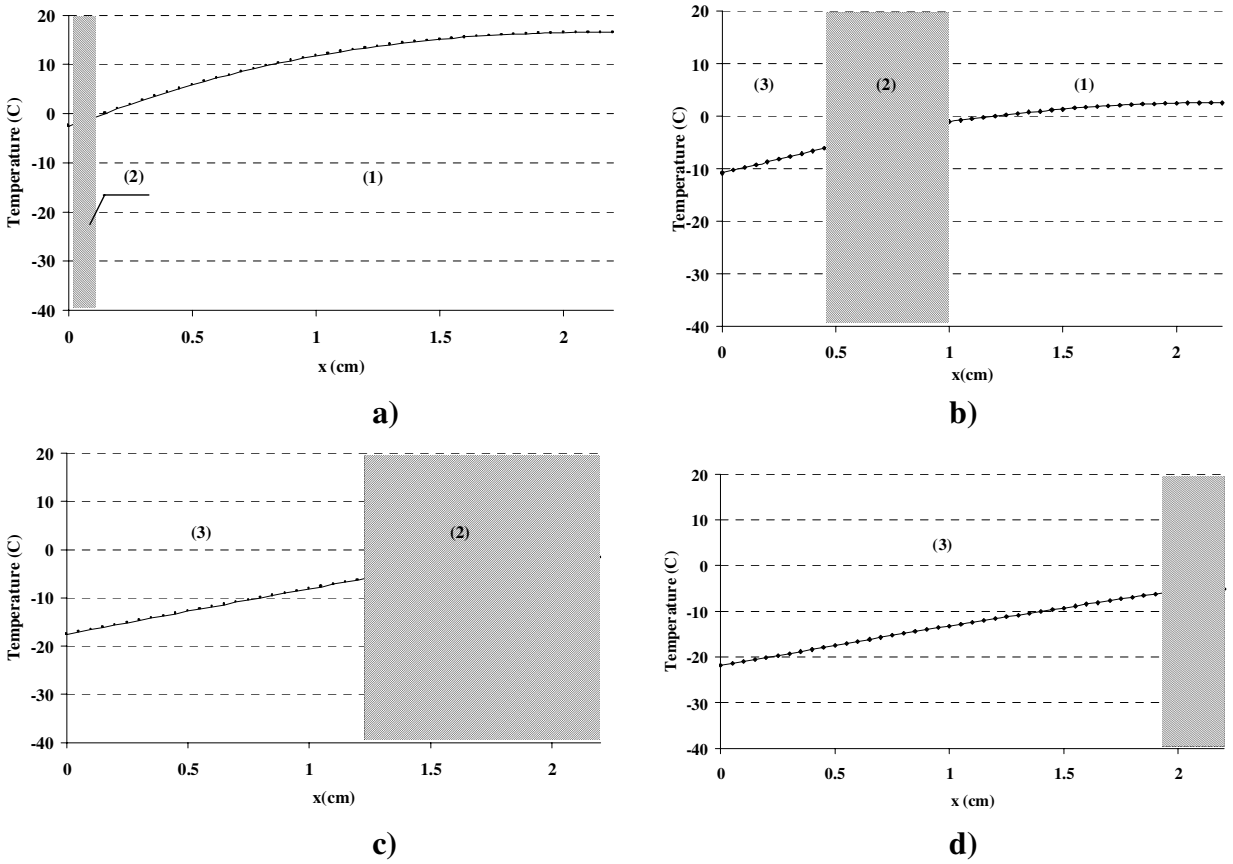

Figure 4 : Phase change zone plotted at different time steps; (1) Unfrozen zone, (2) Phase changing zone, (3) Frozen zone.

half of the domain with a zero flux condition at the middle cross section of the cylinder. As it can be seen both numerical models are in agreement with the experimental data. A small discrepancy between the numerical and experimental data at the final part of the simulation is due to the simple model use to interpolate the thermal properties of the foodstuff. However, for the purpose of this work the error introduced by such simple approximating function, is considered acceptable.

The obtained results of the time evolution of the temperature profile along the centre line of the cylinder is plotted in figure 3. The two dash line helps to identify the range of temperatures were the phase change takes place, i.e., T_l and T_s.

As point out before, water is the predominant constituent in most foods. Therefore as the temperature reaches the freezing point of water, crystallization begins. Actually, ice formation starts when the temperature in the sample reaches the value T_l , usually between -1 ° C and -3 ° C rather then the freezing point of pure water, due to the dissolved chemical in solution. Moreover, below such temperature, some of the liquid water crystallizes and therefore the liquid solution becomes more concentrate.

The versatility of our scheme allows us to follow the phase changing zone inside the cylinder. It is interesting to notice that along the sample three different zones can be identified: 1) Unfrozen zone, in which the water contained in the food is not frozen; 2) Phase changing zone, where crystals of ice coexist with the liquid solution; 3) Frozen zone characterised by the complete crystallization of the water present in the sample. However, even though it is assumed here that the crystallization process ends at the temperature T_s, it is well know that as solute in solution depress further the freezing point, foods may never be completely frozen. In Figure 4, four different instants of the freezing process are shown; it can be seen that the phase change zone moves from the boundary surface (left side) to the core of the sample (right side).

Table 2 : Thermo-physical parameters for the spinach sample.

heat capacity c		Thermal conductivity K		Density	Latent heat Δu	T_l	T_m	T_s	s
J/(kg K)		W/(m K)		Kg/m^3	J/kg	$^\circ$ C	$^\circ$ C	$^\circ$ C	m
Cl	Cs	Kl	Ks						
4000	2010	0.55	2.04	954	199000	-1	-3	-7	0.043

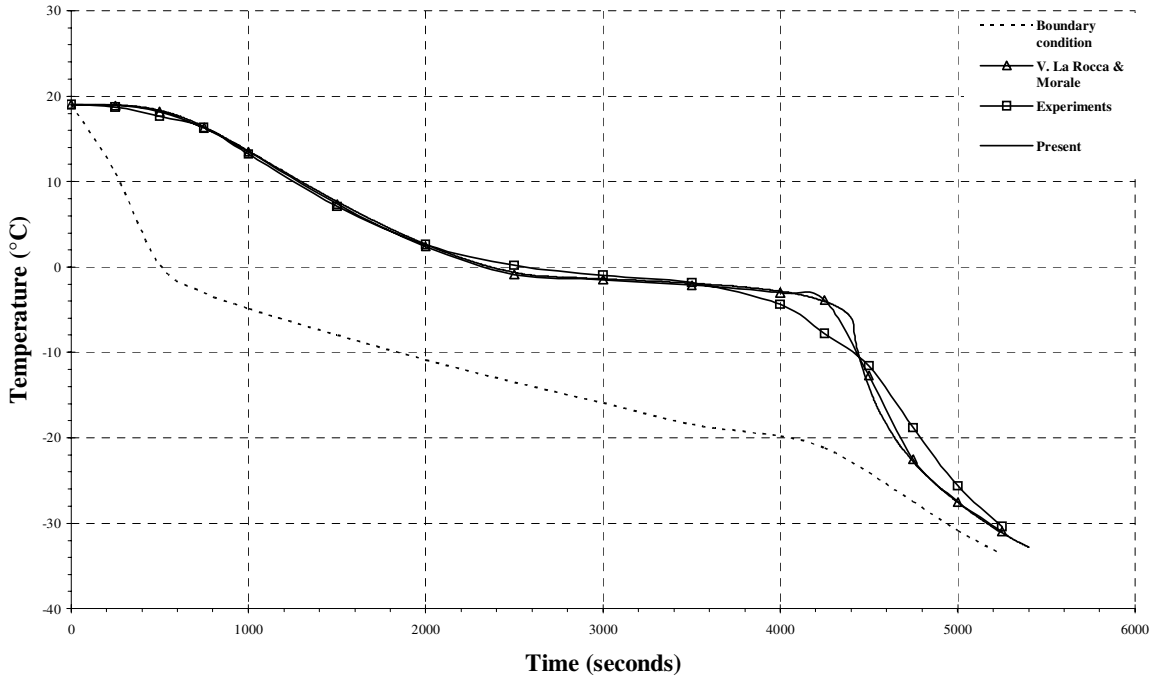

Figure 5 : Comparison between experimental (), numerical results of the temperature obtained using a finite volume method (Δ) and the results found using the meshless approach (continuous line)

4.2 Second numerical example

Basically in this example the analysis carried out in the previous example is repeated but in this case a sample of spinach is considered. Similar geometry characterize the problem, while the new thermo-physical parameters in the governing equation can be evaluated using table 2

In figure 5 a comparison between the two numerical schemes and the experimental data is given

As in the previous case good agreement between the two numerical results and the experimental data is observed, except at the final part of the simulation, which is due to the simplified interpolating function used to define the variation of $k(T)$.

5 Conclusions

The results present in the previous section show the versatility of the radial basis function Hermite collocation approach to solve time dependent freezing problems including cases of variable coefficient. Using the standard Crank-Nicholson weighed method to represent the time derivative in the partial differential equation the original problem reduces to the solution of a steady state non-homogeneous diffusion problem at each time step, with the non-homogeneous term proportional to the solution of the previous time step. When the thermal properties depend on the temperature, the numerical solution of the problem is more complicated due to the introduced non linearity. However, incorporating a Kirchhoff transfor-

mation the difficulties introduced by the non linearity can be significantly reduced, preserving the structure of the heat equation. One of the major contributions of this article is to show the accuracy of the scheme even in cases of strong variation of thermal properties. Moreover the versatility of our scheme allows us to follow the phase change zone, in which crystallization takes place, along the domain.

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