Numerical Investigation of the Multiple Dynamic Crack Branching Phenomena

T.Nishioka[1] , S.Tchouikov[1] and T.Fujimoto[1]

Abstract: In this study, phenomena of multiple branching of dynamically propagating crack are investigated numerically. The complicated paths of cracks propagating in a material are simulated by moving finite element method based on Delaunay automatic triangulation (**MFEM BODAT**), which was extended for such problems. For evaluation of fracture parameters for propagating and branching cracks switching method of the path independent dynamic J integral was used. Using these techniques the generation phase simulation of multiple dynamic crack branching was performed. Various dynamic fracture parameters, which are almost impossible to obtain by experimental technique alone, were accurately evaluated.

keyword: Dynamic crack bifurcation, Dynamic fracture, Crack propagation and arrest, Moving finite element method, Dynamic J integral, Fracture prediction criteria, Multiple crack branching

1 Introduction

The phenomena of multiple crack branching are very often observed in dynamic fracture of brittle materials (See Fig.1). However, the detailed mechanism of multiple crack branching, (some cracks branch, and arrest, some continue propagating and bifurcate again), has not been fully elucidated yet. Recently the problem of governing condition of dynamic crack branching was investigated in our experimental studies [Nishioka, Kishimoto, Ono and Sakakura(1999a, 1999b), Nishioka, Matsumoto, Fujimoto and Sakakura(2003)]. The experiments on dynamic branching phenomena revealed that the total energy flux per unit time into a propagating crack tip or into a fracture process zone governs the dynamic two crack branching [**total energy flux criterion**].

Over the past few decades, many numerical methods have been proposed to model crack and other prob-
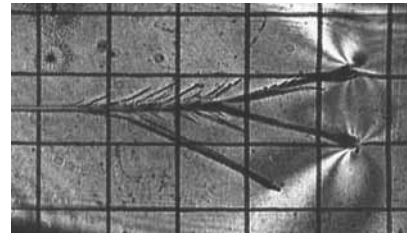

Figure 1 : An ultra-high speed photograph of multiple crack branching

lems. For example, some MLPG method [Han and Atluri (2004)] and Material point method [Bardenhagen and Kober (2004), Guo and Nairn (2004)] have been reported. In our previous studies [Nishioka, Furutuka, Tchouikov and Fujimoto(2002), Nishioka, Tchouikov and Fujimoto(2002)], the authors developed the moving finite element method based on Delaunay automatic triangulation (**MFEM BODAT**) [Nishioka, Tokudome and Kinoshita (2001)], which allows the simulation of the phenomenon of dynamic two cracks bifurcation. In the present work, this method is extended for simulation of the multiple crack branching phenomena. The cracks are modeled by the Delaunay Automatic triangulation for each time step. Using this crack modeling technique the moving singularities at the tips of dynamically propagating cracks are treated accurately and even fairly complicated fracture paths are precisely generated. The applicability of the moving finite element method were demonstrated for various cases [Fujimoto and Nishioka (2001), Nishioka, Hashimoto and Fujimoto (2001), Nishioka, Hu and Fujimoto (2001), Nishioka, Ichikawa and Maeda (1995), Tchouikov, Nishioka and Fujimoto (2005), Nihsioka, Maruoka and Fujimoto (2004), Nishioka and Stan (2003), Nishioka, Stan and Fujimoto (2002, 2001), Nishioka, Syano and Fujimoto (2000), Nishioka, Tchouikov and Fujimoto (2004, 2001), Nishioka, Yoshimura, Nishi and Sakakura (1995), Tchouikov, Nishioka and Fujimoto (2004)].

In this work, we carried out the generation phase simu-

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lation of multiple dynamic crack branching based on experimental fracture history data. Various dynamic fracture parameters, such as the dynamic J integral [Nishioka and Atluri (1983)], dynamic stress intensity factors, energy flux are accurately evaluated even immediately after the crack branching. The same simulations were also performed for two crack bifurcation without preceding multiple crack branching. Comparing the calculation results for both cases the influence of multiple crack branching on crack propagation was investigated.

Theoretical and computational aspect of dynamic fracture simulations were reviewed in [Fujimoto and Nishioka (2001), Nishioka (2002, 2001, 2000, 1999a, 1999b)].

2 Moving Finite Element Method for Dynamic Crack Branching Phenomena

In this study for mesh generation we used the modified Delaunay automatic triangulation [Taniguchi(1992)], which requires only exterior, interior boundary points and specified interior points (if they are necessary). In consideration of the stress singularity at each propagating crack tip, the crack tips are always surrounded by the specified interior points.
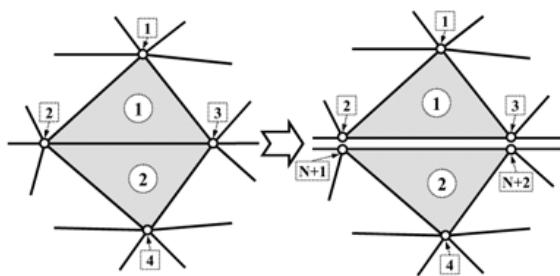

Figure 2 : Crack modeling by distinguishing the crack surfaces after mesh generation (*N* is the total number of nodal points after mesh generation)

At the Delaunay automatic mesh generation stage the two surfaces of crack path have common nodal points, and the crack surfaces are described by element boundaries. In order to distinguish both surfaces of crack after Delaunay automatic mesh generation, dual nodes setting on crack path are used, so that, the nodal points with the same coordinates have different nodal numbers if there are lying on opposite crack surfaces (See Fig.2). Therefore, the total number of nodal points increases and the

element-nodes relations are changed accordingly. During crack propagation, when crack length is increased more than a certain value, new nodal points are placed on crack path behind the group of surrounding interior points around the crack tip. Furthermore, only an area of the group of specified interior points with its neighborhood is actually re-meshed during crack propagation, the rest of the mesh pattern is remaining fixed for more accuracy of analysis. For the time integration of the finite element equation of motion the Newmark method is used. The details of time integration procedures are given in [Nishioka, Furutuka, Tchouikov and Fujimoto(2002)].

3 Evaluation of Dynamic Fracture Mechanics Parameters

In this study, to evaluate various fracture mechanics parameters for a dynamically propagating and branching cracks the path independent dynamic J integral derived by Nishioka and Atluri (1983) is used.

For most numerical analyses, considering dynamically propagating crack in an elastic solid, the global-axis components of the dynamic J integral (J') can be evaluated by the following expression [Nishioka and Atluri(1983)]:

$$J'_k = \int_{\Gamma+\Gamma_c} [(W+K)n_k - t_i u_{i,k}]dS$$
$$+ \int_{V_\Gamma - V\varepsilon} [(\rho\ddot{u}_i - \overline{f}_i)u_{i,k} - \rho\dot{u}_i\dot{u}_{i,k}]dV \quad (1)$$

where $u_i, t_i, \overline{f}_i, n_k$ and ρ denote the displacement, traction, body force, outward direction cosine, and mass density, respectively. W and K are the strain and kinetic energy densities, respectively, and $()_{,k} = \partial()/\partial X_k$. The integral paths $\Gamma_\varepsilon, \Gamma$ and Γ_c are shown in Fig.3 denote a near-field path, far-field path and crack surface path, respectively. V_Γ is the region surrounded by Γ, while V_ε is the region surrounded by Γ_ε.

The crack-axis components of the dynamic J integral can be evaluated by the following coordinate transformation:

$$J'^{(0)}_l = \alpha_{lk}(\theta_0)J'_k, \quad (2)$$

where α_{lk} is the coordinate transformation tensor and θ_0 is the angle between the global X_1 and the crack axis x_1^0. The tangential component of the dynamic J integral J'^0_1 corresponds to the rate of change in the potential energy
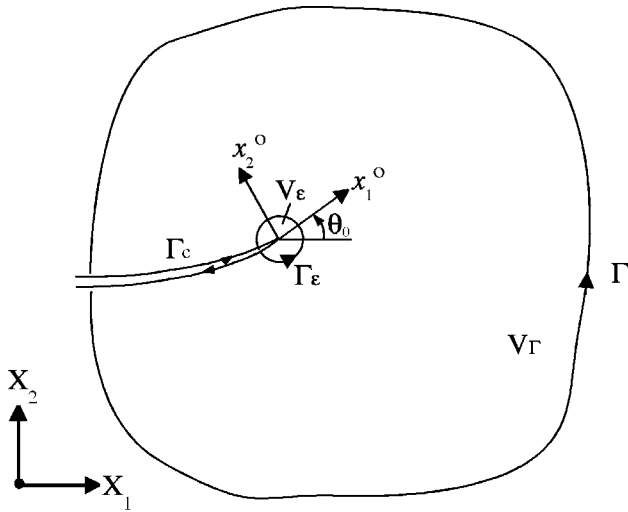
Figure 3 : Definition of integral paths

per unit crack extension, namely, the dynamic energy release rate G.

To accurately evaluate the inplane mixed-mode stress intensity factors from the dynamic J integral values, we used the component separation method [Nishioka (1994)] which can be expressed as:

$$K_I = \delta_I \left\{ \frac{2\mu J_1'^{\,0}\beta_2}{A_I(\delta_I^2\beta_2 + \delta_{II}^2\beta_1)} \right\}^{1/2}$$

$$= \delta_I \left\{ \frac{2\mu G\beta_2}{A_I(\delta_I^2\beta_2 + \delta_{II}^2\beta_1)} \right\}^{1/2} \tag{3}$$

$$K_{II} = \delta_{II} \left\{ \frac{2\mu J_1'^{\,0}\beta_2}{A_{II}(\delta_I^2\beta_2 + \delta_{II}^2\beta_1)} \right\}^{1/2}$$

$$= \delta_{II} \left\{ \frac{2\mu G\beta_2}{A_{II}(\delta_I^2\beta_2 + \delta_{II}^2\beta_1)} \right\}^{1/2} \tag{4}$$

Where, δ_I and δ_{II} are the mode I and mode II crack opening displacements, μ is the shear modulus, β_1, β_2 are the crack velocity parameters normalized by the dilatational and shear wave velocities, and $A_I(C), A_{II}(C)$ are functions of crack velocity [Nishioka and Atruli(1983), Nishioka, Furutuka, Tchouikov and Fujimoto(2002)].

Some of the features of the component separation method are: (i) mixed - mode stress intensity factors can be evaluated by ordinary non - singular elements, and (ii) the

signs of K_I and K_{II} are automatically determined by the signs of δ_I and δ_{II}, respectively.

Because of difficulty in setting far - field integral path separately for each just bifurcated crack tip, a switching method of the path independent dynamic J integral [Nishioka, Furutuka, Tchouikov and Fujimoto (2002)] was proposed:

For the time integration of the finite element equation of motion the Newmark method is used. To fulfill the unconditionally stable condition the Newmark's parameters are chosen to be $\beta = 1/4$ and $\delta = 1/2$ [Bathe and Wilson (1976)]. For further details about time integration procedures readers may confer [Nishioka, Furutuka, Tchouikov and Fujimoto (2002)].

3.1 Evaluation of Fracture Mechanics Parameters

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For most numerical analyses, considering dynamically propagating crack in an elastic solid, the global-axis components of the dynamic J integral (J') can be evaluated by the following expression:

$$\begin{aligned}
J_k' = & \int_{\Gamma+\Gamma_C} [(W+K)n_k - t_i u_{i,k}]s\,dS \\
& + \int_{V_\Gamma} [\{(\rho\ddot{u}_i - f)u_{i,k} - \rho\dot{u}_i\dot{u}_{i,k}\}s\,dV \\
& + \sigma_{ij}u_{i,k}s_{,j} - (W+K)s_{,k}]dV
\end{aligned} \tag{5}$$

where Γ is a far-field integral path, that encloses all branched crack tips and s is a continuous function defined in V_Γ.

For calculation of the dynamic J integral for certain crack tip the s function is set as s=1 for the point at that crack tip and for the points in whole domain and s=0 for the points at the others crack tips. Equation (5) made it possible to accurately by evaluate the dynamic J integral components for interacting branched crack tips.

4 Simulation Results

Basing on the experimental data and the history of dynamic crack branching, the generation phase simulation was carried out using the moving finite element method. Considering the stress singularity each propagating crack
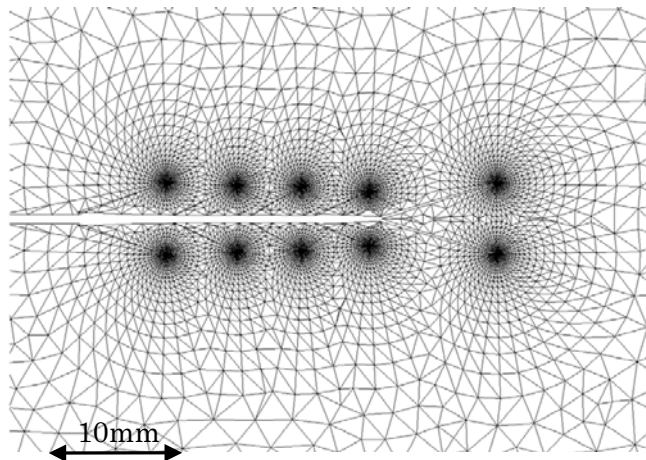
Figure 4 : Deformed mesh pattern at the crack branching area (150μs)

(a) 127 μ s

(b) 128 μ s

(c) 150 μ s

Figure 5 : Equivalent stress distribution at branching area

tip is always surrounded by the specified interior points, placed regularly around the crack tip by 28 points in the radial direction and by 6° increment in the circumferential direction. Therefore, although the initial number of elements and nodes were 6756 and 3500 respectively, due to large number of crack tips propagating, the total numbers of elements and nodes increased exceedingly and was 22516 and 11668 at the stage shown in Fig.4. The time increment of $\Delta t = 1\mu s$ was used. Furthermore, the generation phase simulation of two cracks bifurcation without multiple branching of central crack based on the same experimental data was performed and simulation results for two cases were compared.

The distributions of the equivalent stress at different time steps are shown in Fig.5. Due to the crack tip singularity, a large stress concentration can be seen around the crack tips. The stress in the vicinity of central crack tip is much larger than those around others cracks before branching (a), but become almost equal right after the bifurcation (b). After the last bifurcation, the stress concentrations around the two main propagating cracks become much larger, than stress around arrested cracks (c).

The dynamic stress intensity factors for central main crack tip and one of the cracks after last bifurcation in multiple crack branching case (a) and simple two crack branching case (b) are plotted in Fig.6. The K_I factor for single straight crack (b) is much larger than K_I for straight multiply branching cracks (a) and has a maximum around 110μs. However it can be noted, that values of stress intensity factors for last two branches are very
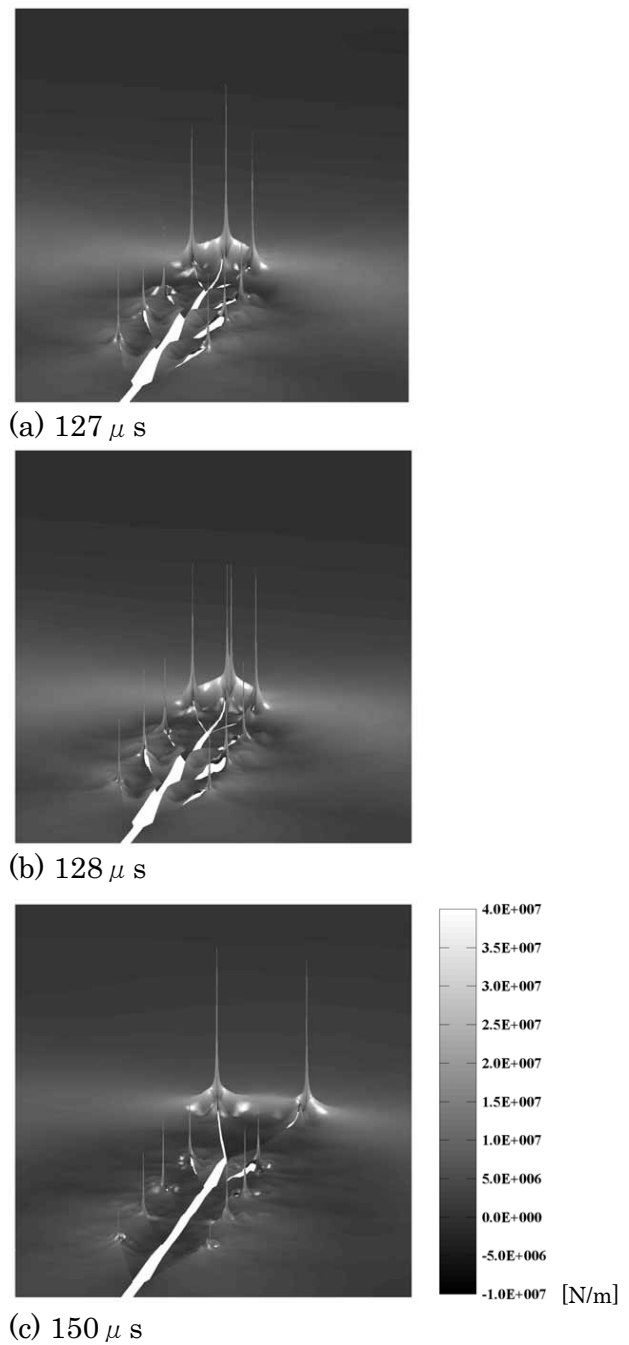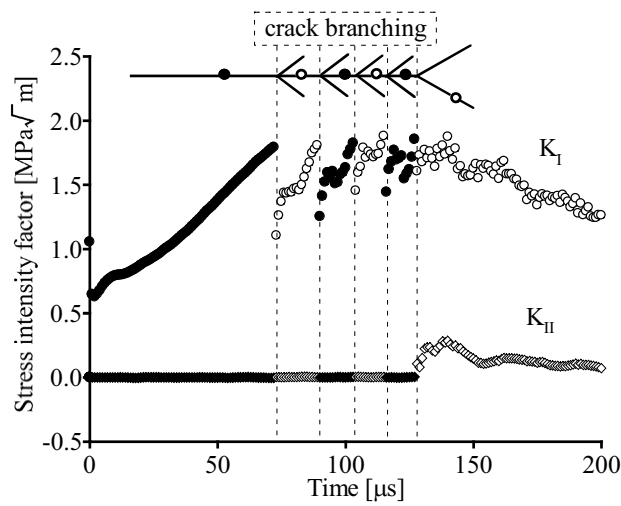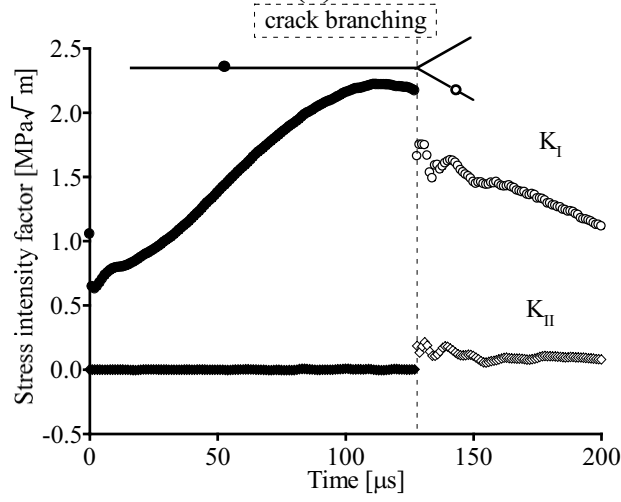
similar in both cases.

The energy flux to the propagating crack tip per unit time was calculated as
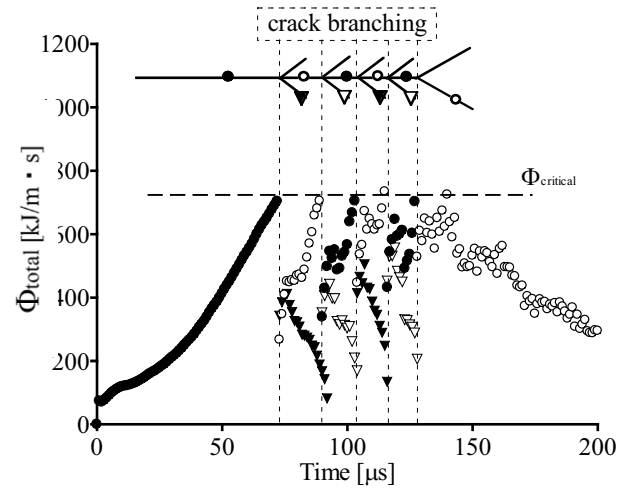
$$\Phi_{total} = J' \cdot C \tag{6}$$

(a)

(a)

(b)

(b)

Figure 6 : Variations of K factors for main crack tip (a) multiple crack branching (b) two crack branching

Figure 7 : Variations of energy flux (a) multiple crack branching (b) two crack branching
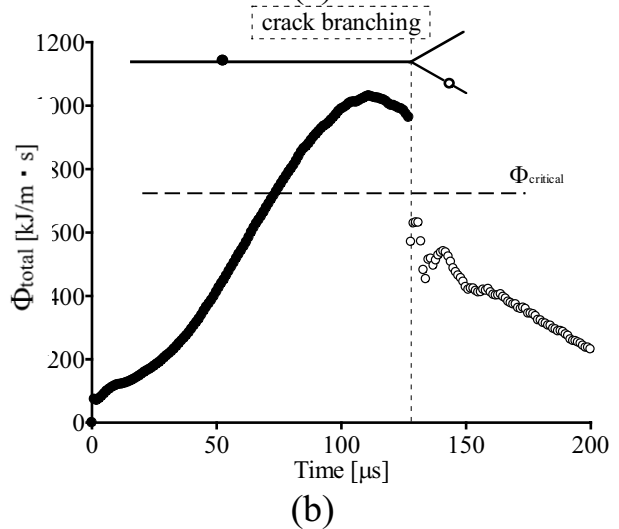
and plotted in Fig.7. In case of multiple crack branching, observed in the experiment, the bifurcation of central crack occurs when the energy flux to the crack tip reaches some critical value, represented by dashed line in Fig.7. It can be seen that energy flux for side crack much smaller than for energy flux for central crack in the case of multiple crack branching (Fig.7a).

5 Conclusions

In this study, the moving finite element method based on Delaunay automatic triangulation extended for simulation of complicated crack branching problems, such as

multiple cracks branching phenomenon was developed. Experimentally observed phenomenon of multiple cracks branching was successfully reproduced by the generation phase simulation. The mechanism of multiple crack branching was modeled and various fracture mechanics parameters were accurately obtained. The simulation results confirmed the idea, that the total energy flux per unit time into a propagating crack tip governs the dynamic crack branching, thus, for dynamic crack branching, the total energy flux criterion was repeatedly verified.

In this study, the moving finite element method based on Delaunay automatic triangulation was further developed for the numerical simulation of complicated dy-

namic crack bifurcation phenomena, such as branching into two or three branches and multiple branching. Various dynamic fracture parameters were accurately evaluated by the switching method of the path independent dynamic J integral even immediately after the crack bifurcation.

The dynamic crack bifurcation phenomena observed experimentally was successfully predicted by the numerical simulations with conjunction of the local symmetry criterion, the dynamic fracture toughness criterion and the critical total energy flux criterion.

References

Bathe, K.J.; Wilson, E.L. (1962): Numerical methods in finite element analysis, Prentice-Hall, Englewood Cliffs, NJ.

Bardenhagen, S. G.; Kober, E. M. (2004): The Generalized Interpolation Material Point Method, CMES: *CMES: Computer Modeling in Engineering & Sciences*, Vol. 5, No. 6, pp. 477-496.

Fujimoto, T.; Nishioka, T. (2001): Numerical analysis of fracture under dynamic large deformation using a moving finite element method, in *Advances in Computational Engineering and Sciences*, eds. S.N. Atluri, T. Nishioka and M. Kikuchi, Tech Science Press, Chapter 7, Paper 21, pp.1-6.

Guo, Y.; and Nairn J. A. (2004):Calculation of J-Integral and Stress Intensity Factors using the Material Point Method, CMES: *CMES: Computer Modeling in Engineering & Sciences*, Vol. 6, No. 3, pp. 295-308.

Han, Z. D.; Atluri S. N. (2004):A Meshless Local Petrov-Galerkin (MLPG) Approach for 3-Dimensional Elasto-dynamics ,CMC: *CMC: Computerts, Materials, and Continua*, Vol. 1, No. 2, pp. 129-140.

Nishioka, T. (2002): Computational aspect of dynamic fracture, in *Comprehensive Structural Integrity*, (R.de Borst), chapter4, Elsevier Science, pp.211-254.

Nishioka, T. (2001):Recent theoretical and computational studies on dynamic fracture phenomena, in *Advances in Computational Engineering and Sciences*, eds. S.N. Atluri, T. Nishioka and M. Kikuchi, *Tech Science Press*, Chapter 7, Paper 27, pp.1-6.

Nishioka, T. (2000): Hybrid numerical methods in static and dynamic fracture mechanics, *Optics and Laser Eng.*, Vol.32, pp.205-255.

Nishioka, T. (1999a): Theoretical and computational aspects of the dynamic J integral in dynamic fracture mechanics for homogeneous and nonhomogeneous materials, *CMES: Computer Modeling in Engineering & Sciences*, Vol.4, pp.29-36.

Nishioka, T. (1999b): Theoretical and Computational Aspects of the Dynamic J Integral in Dynamic Fracture Mechanics for Homogeneous and Nonhomogeneous Materials", *CMES: Computer Modeling in Engineering & Sciences*, Vol.4, No.1, pp.29-36.

Nishioka, T. (1998): On the dynamic J integral in dynamic fracture mechanics, in Fracture: *A Topical Encyclopedia of Current Knowledge*, ed. G.P. Cherepanov, Krieger Publishing Company, Melbourne, USA, pp.575-617.

Nishioka, T. (1997): Computational dynamic fracture mechanics, *International Journal of fracture*, 1997, Vol. 86, pp.127-159.

Nishioka, T. (1995):Recent developments in computational dynamic fracture mechanics, in *dynamic fracture*, edited by M.H. Aiabadi, computational mechanics publicaton,chapter 1, pp1-60.

Nishioka, T. (1994): The state of the art in computational dynamic fracture mechanics, *JSME International Journal*, Ser. A, Vol.37, pp.313-333.

Nishioka, T.; Atluri, S.N. (1983): Path independent integrals, energy release rates and general solutions of near-tip fields in mixed-mode dynamic fracture mechanics, *Engineering Fracture Mechanics*, vol.18, No.1, pp.1-22.

Nishioka, T.; Furutuka, J.; Tchouikov, S.; Fujimoto, T. (2002): Generation-phase simulation of dynamic crack bifurcation phenomenon using moving finite element method based on Delaunay automatic triangulation, *CMES: Computer Modeling in Engineering & Sciences*, Vol.3, pp.129-145.

Nishioka, T.; Hashimoto, M.; Fujimoto, T. (2001): Numerical simulation of fracture paths in static and dynamic fracture phenomena using a moving finite element

method, in *Advances in Computational Engineering and Sciences*, eds. S.N. Atluri, T. Nishioka and M. Kikuchi, *Tech Science Press*, Chapter 7, Paper 22, pp.1-6.

Nishioka, T.; Hu, Q.H.; Fujimoto, T. (2001): Generation-phase simulation of dynamic interfacial fracture under impact loading", in *Impact Engineering and Application*, eds. A. Chiba, S. Tanimura and K. Hokamoto, *Elsevier Science*, I, pp.381-388.

Nishioka, T.; Ichikawa, Y.; Maeda, N. (1995): Numerical Study on Three-Dimensional Dynamic Fracture, *Dynamic Fracture, Failure and Deformation*, (Nishioka, T. and J.S. Epstein, Editors), ASME Publication, PVP-Vol.300, pp.73-85.

Nishioka, T.; Kishimoto, T.; Ono, Y.; Sakakura, K. (1999-a): Basic studies on the governing criterion for dynamic crack branching phenomena, *Transactions of the Japan Society of Mechanical Engineers*, vol.65, no.633, Ser. A, pp.1123-1131 (in Japanese).

Nishioka, T.; Kishimoto, T.; Ono, Y.; Sakakura, K. (1999-b): Governing criterion of dynamic crack bifurcation, In: F. Ellyin and J.W. Provan (eds.) *Progress in mechanical behaviour of materials*, Vol. I, pp.255-260.

Nishioka, T,; Maruoka, A.; Fujimoto, T. (2004): Numerical simulation to predict dynamic fracture path in material containing circular holes, *JSME 17th computational mechanics conference*, No.04-40, pp.91-92.

Nishioka, T.; Matsumoto, K.; Fujimoto, T.; Sakakura, K. (2003): Evaluation of dynamic crack bifurcation by method of caustic, *Journal of the Japanese Society for Experimental Mechanics*, Vol.3, No.2, pp.21-28 (in Japanese).

Nishioka, T.; Stan, F. (2003): A Hybrid Experimental-Numerical Study on the Mechanism of Three-Dimensional Dynamic Fracture,*CMES: Computer Modeling in Engineering and Sciences*, Vol.4, No.1, pp.119-139.

Nishioka, T,; Stan, F.; Fujimoto, T. (2002): Dynamic J integral and dynamic stress intensity factor distributions along naturally and dynamically propagating three-dimensional fracture fronts, *JSME International Journal*, Special Issue on Fracture Mechanics, Series A, Vol. 45, No. 4, pp.523-537.

Nishioka, T,; Stan, F.; Fujimoto, T. (2001): Dynamic J integral distributions along dynamically propagating three-dimensional fracture fronts, in *Proc. Asian Pacific Conf. on Fracture and Strength and Int. Conf. Adv. Tech.*

in Experimental Mechanics, Sendai, Japan, pp.724-729.

Nishioka, T.; Syano, S.; Fujiimoto, T. (2000): A Study on crack acceleration and deceleration effects in dynamic fracture phenomena, *Advances in Computational Engineering and Sciences*, (S.N. Atluri and F.W. Brust, editors),,Vol.1, pp. 942-947.

Nishioka, T,; Tchouikov, S.; Fujimoto, T. (2004): Numerical Investigation of the Multiple Dynamic Crack Branching Phenomena, International Conference on Computational Engineering and Science, Funchal, Portugal, July 26-29, *The Proceedings of ICCES 2004*, pp.1-6.

Nishioka, T.; Tchouikov, S; Fujimoto, T. (2002): Study of dynamic branching fracture by numerical simulation, *Zairyo/Journal of the Society of Materials Science*, Vol. 51, No.12, December, p 1359-1366 (in Japanese).

Nishioka, T,; Tchouikov, S.; Fujimoto, T. (2001): Fracture path prediction simulations of dynamic bifurcation fracture phenomena, in *Advances in Computational Engineering and Sciences*, eds. S.N. Atluri, T. Nishioka and M. Kikuchi, *Tech Science Press*, Chapter 7, Paper 17,pp.1-6.

Nishioka, T.; Tokudome, T.; Kinoshita, M. (2001): Mixed-phase simulation with fracture-path prediction mode using moving finite element method based on Delaunay automatic mesh generation, *International Journal of solid and Structures*, Vol.38, pp.5237-5301.

Nishioka, T.; Yoshimura, S.; Nishi, M.; Sakakura, K. (1995): Experimental Study on Three-Dimensional Dynamic Fracture, *Dynamic Fracture, Failure and Deformation,* (Nishioka, T. and J.S. Epstein, Editors), ASME Publication, PVP-Vol.300, pp.87-97.

Taniguchi, T. (1992): Automatic mesh generation for FEM: use of Delaunay automatic triangulation. Morikita Publishing (in Japanese).

Tchouikov, S.; Nishioka, T.; Fujimoto, T. (2004): Fracture Numerical prediction of dynamically propagating and branching cracks using moving finite element method, *CMC: Computers, Materials and Cotinua*, Tech science press, Vol.1,No.2,pp.191-204.

Tchouikov, S.; Nishioka, T.; Fujimoto, T. (2005): Numerical Prediction of Dynamically Propagating and Branching Cracks Using Moving Finite Element Method, *CMC: Computers, Materials and Continua*, Tech Science Press, (To be submitted)