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A Consistent Computation of Magnetization Reversal under a Circularly Polarized Field and an Anisotropy Field

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Abstract: In this paper the Landau-Lifshitz equation is subjected to a circularly polarized field in the plane, as well as both a dc field and an anisotropy field along the vertical easy axis perpendicular to the plane. The representation of Landau-Lifshitz equation in the Minkowski space is a Lie-type system. By performing a computation through the Lie-group solvers we can develop a consistent numerical method, which satisfies the consistency condition exactly, and thus can retain the invariant behavior. Then, we use the consistent numerical method to investigate the magnetization reversal, whose switching criterion is displayed through the minimum curve of the vertical magnetization component as a function of exciting frequency. When the anisotropy field is considered, the minimum curve may exhibit a discontinuity between reversal magnetization range and non-reversal magnetization range. Without exception, when the exciting frequency of the circularly polarized field is high, the magnetization reversal will not occur.

Keyword: Landau-Lifshitz equation, Magnetization reversal, Consistency condition, Consistent numerical method, Lie-type system, Lie-group solver

1 Introduction

Among the many physical important issues about magnetization memory devices, the most subtle process of magnetization motion perhaps is its reversal of direction. Thirion, Wernsdorfer and Mailly (2003) were experimentally identified that in the presence of magnetic anisotropy, reversing the magnetic field while simultaneously applying a polarized field at a right angle to the applied dc field can significiantly lower the threshold value for switching. Some theoretical analyses of a similar case were given by Bertotti, Serpico and Mayergoyz (2001). Those studies indicated that the most effective frequency for small amplitude oscillations corresponded to the uniform-mode ferromagnetic resonance frequency.

Motivated by its great application potential in the magnetic data storage and random access memories, the magnetization reversal in magnetic particles and thin films has been a continuously interesting topic in the past several decades. In recent years, it is possible to produce nano magnetic sample with well-controlled shape and structure. However, the magnetic anisotropy of these samples makes the dynamic magnetization processes highly nonlinear, and a thorough understanding of the micromagnetic dynamics is desirable, as many efforts have already been made in this issue. In order to simulate the magnetization reversal phenomenon of ferromagnetic materials, we use the following model as our investigating tool:

$$\dot{\mathbf{M}} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} - \frac{\gamma\alpha}{M_s} \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}}), \tag{1}$$

which was proposed by Landau and Lifschitz (1935).

The effective field $\mathbf{H}_{\text{eff}}$ is the sum of applied field, demagnetizing field, anisotropy field and exchange field. However, in this paper we only consider the case of a uniformly magnetized sample. Therefore, by discarding the demagnetizing field and the exchange field, the possible interaction of nonuniform modes is not taken into account here [Liu and Ku (2005)].

Throughout this paper, a dot between two vectors stands for their scalar product, and $\| \bullet \|$ denotes the magnitude of vector. The two material

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parameters of $\gamma > 0$ and $\alpha \geq 0$ are, respectively, the absolute value of gyromagnetic ratio and the damping constant. From Eq. (1) it is apparent that $\mathbf{M} \cdot \dot{\mathbf{M}} = 0$; hence, the magnitude of magnetization vector $\mathbf{M}(t)$ is conserved, i.e., $\|\mathbf{M}(t)\| = M_s$, where M_s is a constant saturation magnetization. The above result shows that the magnetization described by the Landau-Lishitz equation possesses an invariant behavior, restricting the magnetizing vector on a sphere with a radius M_s. When we calculate the magnetization behavior by numerical method, this point to keep the invariance is very important. A numerical method that can exactly preserve $\|\mathbf{M}(t)\| = M_s$ for all time will be called a *consistent* numerical method, and the equality $\|\mathbf{M}(t)\| = M_s$ is called a consistency condition.

In this paper we study the Landau-Lifshitz equation (1) under a circularly polarized field, a constant dc field as well as an anisotropy field:

$$\mathbf{H}_{\text{eff}} = M_s(H_0 \cos \omega t, H_0 \sin \omega t, H_z + k_{\text{eff}} M_3/M_s)^{\text{T}}, \tag{2}$$

where H_0 is a constant amplitude, and ω is the excitation frequency. While the polarized field $M_s(H_0 \cos \omega t, H_0 \sin \omega t, 0)$ is rotated counterclockwisely in the (x, y) plane at an angular frequency ω, H_z is a dc field in the vertical z-direction. The term $k_{\text{eff}} M_3$ presents an effective anisotropy field along the vertical direction. We shall consider the magnetization reversal of a uniformly magnetized ferromagnetic material with uniaxial anisotropy. The easy axis is oriented along the z-axis. The reversal of magnetization means that under the effective field the magnetization vector can rotate its direction with the value of the vertical component changing from positive to negative, or vice versa. Otherwise, it is a non-reversal magnetization.

About the magnetization under the above field, some analytical results were obtained by Bertotti, Serpico and Mayergoyz (2001), Bertotti, Magni, Mayergoyz and Serpico (2001) and Bertotti, Mayergoyz and Serpico (2001, 2004) by assuming the magnetic body exhibiting rotational symmetry about the z-axis. Besides that very few analytic solutions are known for the nonlinear large magnetization motions. Usually, the majority of nonlinear studies are carried out by the numerical methods [Serpico, Mayergoyz and Bertotti (2001); Krishnaprasad and Tan (2001); Frank (2004); Liu and Ku (2005); d'Aquino, Serpico and Miano (2005)]. More recently, Rivkin and Ketterson (2006) have studied the magnetization reversal using various rf magnetic pulses, numerically showing that the magnetic switching is possible with simple sinusoidal pulses. Lee and Yuan (2007) have used an oscillating field to study the magnetization reversal, showing that the oscillating field reduces the coercivity significiantly.

The issue of developing a suitable time-stepping technique for the Landau-Lifshitz equation that preserves relevant properties has received much attentions [Slodicka and Cimrak (2003); Cimrak and Slodicka (2004); Banas and Slodicka (2005); Frank (2004); Krishnaprasad and Tan (2001); Lewis and Nigam (2003); and Sun, Ma and Qin (2004)]. The general point of view of these contributions is that the use of suitable geometrical integrators designed to preserve the geometrical properties can help us to further understand the magnetization behavior. Some researchers have based on the Lie group $SO(3)$ to develop the numerical integrators for the Landau-Lifshitz equation, for example, Frank (2004) and Lewis and Nigam (2003). However, for the Landau-Lifshitz equation the Lie-group $SO(3)$ is a coadjoint action, exhibiting a nonlinear structure. Although there are many existing numerical integration techniques for the time stepping of the Landau-Lifshitz equation, some of them may corrupt the conserved properties of magnetization dynamics. For these reasons, it is important to develop a numerical method based on the Lie group $SO_o(3,1)$ that can retain the conserving properties automatically.

In Section 2 of this paper we first give an outline of the Lie-type representation of the Landau-Lifshitz equation in the Minkowski space. The Lie-type representation is important for getting the closed-form solution as shown by Liu (2007) for the magnetization under an AC field, of which the Lie-type representation is a linear system. Usually, it is a formidable task to find the analytical solution of the Landau-Lifshitz magnetization equation including damping and/or other nonlinear effects. However, with the aid of the Lie-type representation in a four-dimensional Minkowski space and a new formula derived in Section 3, we will derive an approximate solution of the magnetization motion in Section 4, which can satisfy the consistency condition $\|\mathbf{M}(t)\| = M_s$ exactly, and thus it is a consistent numerical method as just defined in the above. In Section 5 we apply the consistent numerical method to study the magnetization reversal under the effective field in Eq. (2). Finally, we draw conclusions in Section 6.

2 A Lie-type representation in the Minkowski space

Let us define a unit vector

$$\mathbf{m} := \frac{\mathbf{M}}{\|\mathbf{M}\|} = \frac{\mathbf{M}}{M_s}, \tag{3}$$

as well as use a new time scale $\tau := \gamma M_s t$ and a new field

$$\mathbf{H} := \frac{\mathbf{H}_{\text{eff}}}{M_s} = (H_0 \cos \omega t, H_0 \sin \omega t, H_z + k_{\text{eff}} m_3)^{\text{T}}, \tag{4}$$

such that Eq. (1) can be rearranged to

$$\mathbf{m}' = \hat{\mathbf{H}}\mathbf{m} + \alpha\mathbf{H} - \alpha\mathbf{H}\cdot\mathbf{m}\mathbf{m}, \tag{5}$$

where the prime denotes the differential with respect to τ,

$$\hat{\mathbf{H}} := \begin{bmatrix} 0 & -H_3 & H_2 \\ H_3 & 0 & -H_1 \\ -H_2 & H_1 & 0 \end{bmatrix} \tag{6}$$

is a skew-symmetric matrix, and H_i, $i = 1,2,3$, are three independent components of $\mathbf{H}$.

Liu (2004) has proved that the Landau-Lifshitz equation (5) can be written as

$$\mathbf{X}' = \mathbf{A}\mathbf{X} \tag{7}$$

in the four-dimensional Minkowski space with $\mathbf{X} \in \mathbb{M}^4$ satisfying the cone condition of $\mathbf{X}^{\text{T}}\mathbf{g}\mathbf{X} = 0$, where T denotes the transpose and $\mathbf{g}$ is a Minkowski metric given by

$$\mathbf{g} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{0}_{3\times1} \\ \mathbf{0}_{1\times3} & -1 \end{bmatrix}, \tag{8}$$

where $\mathbf{I}_3$ is the third order identity matrix. In above, we have defined

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}^s \\ X^0 \end{bmatrix} = \begin{bmatrix} X^1 \\ X^2 \\ X^3 \\ X^0 \end{bmatrix} := X^0 \begin{bmatrix} \mathbf{m} \\ 1 \end{bmatrix} \tag{9}$$

as an augmented state vector, and

$$\mathbf{A} := \begin{bmatrix} \hat{\mathbf{H}} & \alpha\mathbf{H} \\ \alpha\mathbf{H}^{\text{T}} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -H_3 & H_2 & \alpha H_1 \\ H_3 & 0 & -H_1 & \alpha H_2 \\ -H_2 & H_1 & 0 & \alpha H_3 \\ \alpha H_1 & \alpha H_2 & \alpha H_3 & 0 \end{bmatrix} \tag{10}$$

as the system matrix, satisfying the Lie algebraic property of $\mathbf{A}^{\text{T}}\mathbf{g} + \mathbf{g}\mathbf{A} = \mathbf{0}$, which is known as the Lie algebra for the Lorentz group $SO_o(3,1)$ [Liu (2001)]. Eq. (7) is a Lie-type system, because $\mathbf{A} \in so(3,1)$ is a Lie algebra element.

For this formulation the cone condition is very crucial, because by Eqs. (9), (8) and (3) we can derive

$$\mathbf{X}^{\text{T}}\mathbf{g}\mathbf{X} = 0 \Leftrightarrow \|\mathbf{m}\| = 1 \Leftrightarrow \|\mathbf{M}\| = M_s. \tag{11}$$

Therefore, the preservation of the cone condition is equivalent to the preservation of the consistency condition.

The solution of Eq. (7) may be expressed by the following state transition formula:

$$\mathbf{X}(\tau) = [\mathbf{G}(\tau)\mathbf{G}^{-1}(\tau_i)]\mathbf{X}(\tau_i), \tag{12}$$

where $\mathbf{G}(\tau)$ is a transformation matrix satisfying

$$\mathbf{G}'(\tau) = \mathbf{A}(\tau)\mathbf{G}(\tau), \;\; \mathbf{G}(0) = \mathbf{I}_4, \tag{13}$$

and τ_i is an initial time. A Lie-group solver will be developed in Section 4, which can enforce the $\mathbf{X}$ calculated by Eq. (12) to satisfy Eq. (11) automatically.

3 A useful formula

In this section we consider a special case of Eq. (7) allowing the dimensions to be n and with

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{n\times n} & \mathbf{Q} \\ \pm\mathbf{Q}^{\text{T}} & 0 \end{bmatrix}, \tag{14}$$

where

$$\mathbf{Q}'(\tau) = \mathbf{\Omega}\mathbf{Q}(\tau), \tag{15}$$

and $\mathbf{\Omega}$ is a constant skew-symmetric matrix, namely, $\mathbf{\Omega}^{\mathrm{T}} = -\mathbf{\Omega}$.

Let

$$\mathbf{A}_1 = \begin{bmatrix} \mathbf{\Omega} & \mathbf{0}_{n\times 1} \\ \mathbf{0}_{1\times n} & 0 \end{bmatrix},$$
$$\mathbf{A}_2 = \mathbf{A}(0) - \mathbf{A}_1 = \begin{bmatrix} -\mathbf{\Omega} & \mathbf{Q}(0) \\ \pm\mathbf{Q}^{\mathrm{T}}(0) & 0 \end{bmatrix}. \tag{16}$$

Notice that both $\mathbf{A}_1$ and $\mathbf{A}_2$ are constant matrices, but $\mathbf{A}$ is a time-dependent matrix.

It is not difficult to show that

$$\mathbf{A}' = \mathbf{A}_1\mathbf{A} - \mathbf{A}\mathbf{A}_1, \quad \mathbf{A}(\tau) = e^{\mathbf{A}_1\tau}\mathbf{A}(0)e^{-\mathbf{A}_1\tau}, \tag{17}$$

and that the solution of $\mathbf{G}'(\tau) = \mathbf{A}(\tau)\mathbf{G}(\tau)$, $\mathbf{G}(0) = \mathbf{I}_{n+1}$ is given by

$$\mathbf{G}(\tau) = e^{\mathbf{A}_1\tau}e^{\mathbf{A}_2\tau}. \tag{18}$$

It means that corresponding to the $\mathbf{A}$ given by Eqs. (14) and (15), we have a closed-form solution of $\mathbf{G}(\tau)$ as expressed by Eq. (18).

4 Approximate solutions

4.1 A new system

In this section we consider an approximate solution of Eq. (7) under the field (2) with $k_{\mathrm{eff}} = 0$. Under this condition, Eq. (7) is a linear Lie-type system.

For the purpose of deriving a new system let us define a new excitation frequency

$$\Omega = \frac{\omega}{\gamma M_s}. \tag{19}$$

Then, Eq. (7) can be written as

$$\mathbf{X}' = \mathbf{A}\mathbf{X} = (\mathbf{A}_3 + \mathbf{A}_4)\mathbf{X}, \tag{20}$$

where

$$\mathbf{A}_3 := \begin{bmatrix} 0 & -H_3 & 0 & 0 \\ H_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha H_3 \\ 0 & 0 & \alpha H_3 & 0 \end{bmatrix}, \tag{21}$$

$$\mathbf{A}_4 := \begin{bmatrix} 0 & 0 & M1 & M2 \\ 0 & 0 & -M3 & M4 \\ -M1 & M3 & 0 & 0 \\ M2 & M4 & 0 & 0 \end{bmatrix}. \tag{22}$$

where

$$M1 = H_0 \sin\Omega\tau \quad M2 = \alpha H_0 \cos\Omega\tau$$
$$M3 = H_0 \cos\Omega\tau \quad M4 = \alpha H_0 \sin\Omega\tau.$$

Due to $k_{\mathrm{eff}} = 0$, $\mathbf{A}_3$ is a constant matrix. Let

$$\mathbf{Y} = \exp(-\mathbf{A}_3\tau)\mathbf{X}, \tag{23}$$

and then from Eq. (20) it follows that

$$\mathbf{Y}' = \exp(-\mathbf{A}_3\tau)\mathbf{A}_4\exp(\mathbf{A}_3\tau)\mathbf{Y}. \tag{24}$$

In terms of

$$\exp(\mathbf{A}_3\tau) = \begin{bmatrix} \cos H_3\tau & -\sin H_3\tau & 0 & 0 \\ \sin H_3\tau & \cos H_3\tau & 0 & 0 \\ 0 & 0 & \cosh\alpha H_3\tau & \sinh\alpha H_3\tau \\ 0 & 0 & \sinh\alpha H_3\tau & \cosh\alpha H_3\tau \end{bmatrix}, \tag{25}$$

and through a lengthy calculation we can obtain

$$\mathbf{Y}' = \begin{bmatrix} 0 & 0 & a & c \\ 0 & 0 & b & d \\ -a & -b & 0 & 0 \\ c & d & 0 & 0 \end{bmatrix}\mathbf{Y}, \tag{26}$$

where

$$a := H_0\sin(\Omega - H_3)\tau\cosh\alpha H_3\tau + \alpha H_0\cos(\Omega - H_3)\tau\sinh\alpha H_3\tau, \tag{27}$$

$$b := -H_0\cos(\Omega - H_3)\tau\cosh\alpha H_3\tau + \alpha H_0\sin(\Omega - H_3)\tau\sinh\alpha H_3\tau, \tag{28}$$

$$c := H_0\sin(\Omega - H_3)\tau\sinh\alpha H_3\tau + \alpha H_0\cos(\Omega - H_3)\tau\cosh\alpha H_3\tau, \tag{29}$$

$$d := -H_0\cos(\Omega - H_3)\tau\sinh\alpha H_3\tau + \alpha H_0\sin(\Omega - H_3)\tau\cosh\alpha H_3\tau. \tag{30}$$

The linear system (26) can be transformed to a new system as follows:

$$\mathbf{Z}'(\tau) = \mathbf{B}(\tau)\mathbf{Z}(\tau), \tag{31}$$

where

$$\mathbf{B} := \begin{bmatrix} \mathbf{0}_3 & \mathbf{U} \\ \mathbf{U}^{\mathrm{T}} & 0 \end{bmatrix}, \tag{32}$$

$$\mathbf{Z} := \begin{bmatrix} \mathbf{G}_3^{-1} & \mathbf{0}_{3\times 1} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix} \mathbf{Y}, \tag{33}$$

in which $\mathbf{G}_3$ satisfies

$$\mathbf{G}_3' = \begin{bmatrix} 0 & 0 & a \\ 0 & 0 & b \\ -a & -b & 0 \end{bmatrix} \mathbf{G}_3, \quad \mathbf{G}_3(0) = \mathbf{I}_3, \tag{34}$$

and $\mathbf{U}$ is given by

$$\mathbf{U} := \mathbf{G}_3^{-1} \begin{bmatrix} c \\ d \\ 0 \end{bmatrix}. \tag{35}$$

From Eq. (34), $\mathbf{G}_3$ is known to be a three-dimensional rotation matrix, because the system matrix is a 3×3 skew-symmetric matrix. And thus, we can replace $\mathbf{G}_3^{-1}$ by $\mathbf{G}_3^{\mathrm{T}}$.

4.2 Consistent numerical method

Regretably it is difficult to give an exact solution of Eq. (31) even it is a linear differential equation, because $\mathbf{B}$ is a rather complex matrix function of τ. However, for the calculational purpose we can adopt the following numerical method:

$$\mathbf{Z}(\ell+1) = \exp[\Delta\tau \mathbf{B}(\bar{\ell})]\mathbf{Z}(\ell) = \begin{bmatrix} M5 & M6 \\ M7 & M8 \end{bmatrix} \mathbf{Z}(\ell). \tag{36}$$

where

$$M5 = \mathbf{I}_3 + \frac{\cosh\left(\Delta\tau\sqrt{\mathbf{U}(\bar{\ell})\cdot\mathbf{U}(\bar{\ell})}\right)-1}{\mathbf{U}(\bar{\ell})\cdot\mathbf{U}(\bar{\ell})}\mathbf{U}(\bar{\ell})\mathbf{U}^{\mathrm{T}}(\bar{\ell})$$

$$M6 = \frac{\sinh\left(\Delta\tau\sqrt{\mathbf{U}(\bar{\ell})\cdot\mathbf{U}(\bar{\ell})}\right)}{\sqrt{\mathbf{U}(\bar{\ell})\cdot\mathbf{U}(\bar{\ell})}}\mathbf{U}(\bar{\ell})$$

$$M7 = \frac{\sinh\left(\Delta\tau\sqrt{\mathbf{U}(\bar{\ell})\cdot\mathbf{U}(\bar{\ell})}\right)}{\sqrt{\mathbf{U}(\bar{\ell})\cdot\mathbf{U}(\bar{\ell})}}\mathbf{U}^{\mathrm{T}}(\bar{\ell})$$

$$M8 = \cosh\left(\Delta\tau\sqrt{\mathbf{U}(\bar{\ell})\cdot\mathbf{U}(\bar{\ell})}\right).$$

Here, $\Delta\tau$ is a small increment of τ, $\mathbf{Z}(\ell)$ denotes the numerical value of $\mathbf{Z}$ at the ℓ-th step, i.e., $\mathbf{Z}(\tau_\ell)$, and the value of $\mathbf{U}(\bar{\ell})$ means that $\mathbf{U}(\bar{\ell}) = \mathbf{U}((\ell+1/2)\Delta\tau)$. Upon $\mathbf{Z}$ is available, we can use Eq. (33) to calculate $\mathbf{Y}$, and then Eq. (23) to calculate $\mathbf{X}$.

Therefore, we come to a new integration method for $\mathbf{X}$:

$$\begin{aligned} \mathbf{X}(\ell+1) &= \mathbf{G}(\ell+1,\ell)\mathbf{X}(\ell) \\ &:= \exp[\mathbf{A}_3\tau_{\ell+1}] \begin{bmatrix} \mathbf{G}_3(\ell+1) & \mathbf{0}_{3\times 1} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix} \\ &\cdot \exp[\Delta\tau\mathbf{B}(\bar{\ell})] \begin{bmatrix} \mathbf{G}_3^{\mathrm{T}}(\ell) & \mathbf{0}_{3\times 1} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix} \\ &\cdot \exp[-\mathbf{A}_3\tau_\ell]\mathbf{X}(\ell). \end{aligned} \tag{37}$$

Notice that each matrix on the right-hand side is an element of the Lorentz group $SO_o(3,1)$. Therefore, by using the closure property of the Lie group we can conclude that the state transition matrix $\mathbf{G}(\ell+1,\ell)$ as a product of these matrices is also an element of the Lorentz group $SO_o(3,1)$, which has an important property [Liu (2001)]:

$$\mathbf{G}^{\mathrm{T}}\mathbf{g}\mathbf{G} = \mathbf{g}, \tag{38}$$

where $\mathbf{g}$ was defined by Eq. (8).

Now, by using Eqs. (37) and (38) we can prove that the cone condition can be preserved exactly by

$$\mathbf{X}^{\mathrm{T}}(\ell)\mathbf{g}\mathbf{X}(\ell) = 0 \Longrightarrow \mathbf{X}^{\mathrm{T}}(\ell+1)\mathbf{g}\mathbf{X}(\ell+1) = 0, \tag{39}$$

which by means of Eq. (11) can be written as

$$\|\mathbf{M}(\ell)\| = M_s \Longrightarrow \|\mathbf{M}(\ell+1)\| = M_s. \tag{40}$$

The above equation indicates that the consistency condition can be preserved for all time if $\|\mathbf{M}(0)\| = M_s$ holds initially. Therefore, we can claim that the present numerical method is a consistent numerical method as defined in Section 1. The success for such a development of consistent numerical method is that we can derive the Lie-group solutions as these revealed by five matrices given in Eq. (37).

4.3 A particular case of $H_3 = 0$

In this section we consider a special case of $H_3 = 0$, i.e., $H_z = k_{\text{eff}} = 0$, such that from Eqs. (27) and (28) we have

$$a := H_0 \sin \Omega\tau, \tag{41}$$

$$b := -H_0 \cos \Omega\tau. \tag{42}$$

It is obvious that

$$\frac{d}{d\tau}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 & -\Omega \\ \Omega & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}, \tag{43}$$

and in view of Eqs. (15) and (16) it follows that

$$\mathbf{A}_1 = \begin{bmatrix} 0 & -\Omega & 0 \\ \Omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 0 & \Omega & 0 \\ -\Omega & 0 & -H_0 \\ 0 & H_0 & 0 \end{bmatrix}. \tag{44}$$

By using the method in Section 3 we can derive

$$\mathbf{G}_3 = \begin{bmatrix} \cos\Omega\tau & -\sin\Omega\tau & 0 \\ \sin\Omega\tau & \cos\Omega\tau & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} M9 & M10 & M11 \\ M12 & M13 & M14 \\ M15 & M16 & M17 \end{bmatrix} = \begin{bmatrix} M18 & M19 & M20 \\ M21 & M22 & M23 \\ M15 & M16 & M17 \end{bmatrix}, \tag{45}$$

where $m = \sqrt{\Omega^2 + H_0^2}$,

$$M9 = \frac{\Omega^2}{m^2}\cos m\tau + \frac{H_0^2}{m^2} \qquad M10 = \frac{\Omega}{m}\sin m\tau$$

$$M11 = \frac{H_0\Omega}{m^2}\cos m\tau - \frac{H_0\Omega}{m^2} \qquad M12 = -\frac{\Omega}{m}\sin m\tau$$

$$M13 = \cos m\tau \qquad M14 = -\frac{H_0}{m}\sin m\tau$$

$$M15 = \frac{H_0\Omega}{m^2}\cos m\tau - \frac{H_0\Omega}{m^2} \qquad M16 = \frac{H_0}{m}\sin m\tau$$

$$M17 = \frac{H_0^2}{m^2}\cos m\tau + \frac{\Omega^2}{m^2}$$

and

$$M18 = \frac{\Omega^2}{m^2}\cos m\tau\cos\Omega\tau + \frac{H_0^2}{m^2}\cos\Omega\tau + \frac{\Omega}{m}\sin m\tau\sin\Omega\tau$$

$$M19 = \frac{\Omega}{m}\sin m\tau\cos\Omega\tau - \cos m\tau\sin\Omega\tau$$

$$M20 = \frac{H_0\Omega}{m^2}\cos m\tau\cos\Omega\tau - \frac{H_0\Omega}{m^2}\cos\Omega\tau + \frac{H_0}{m}\sin m\tau\sin\Omega\tau$$

$$M21 = \frac{\Omega^2}{m^2}\cos m\tau\sin\Omega\tau + \frac{H_0^2}{m^2}\sin\Omega\tau - \frac{\Omega}{m}\sin m\tau\cos\Omega\tau$$

$$M22 = \frac{\Omega}{m}\sin m\tau\sin\Omega\tau + \cos m\tau\cos\Omega\tau$$

$$M23 = \frac{H_0\Omega}{m^2}\cos m\tau\sin\Omega\tau - \frac{H_0\Omega}{m^2}\sin\Omega\tau - \frac{H_0}{m}\sin m\tau\cos\Omega\tau.$$

Inserting the above $\mathbf{G}_3$ and c and d defined in Eqs. (29) and (30) into Eq. (35), and then inserting that $\mathbf{U}$ into Eq. (36) we obtain a numerical solution for this problem. When $\mathbf{Z}$ is available, we can use Eq. (33) to calculate $\mathbf{Y}$, and then Eq. (23) to calculate $\mathbf{X}$. As mentioned, this numerical method is a consistent one.

4.4 Small damping case

Usually, the damping constant α is smaller than one. Here, we further suppose that the damping coefficient α is much smaller than one. Under this condition we can neglect the time variations of these two terms $\cosh\alpha H_3\tau$ and $\sinh\alpha H_3\tau$ in Eqs. (27) and (28). Such that we still have a similar equation for a and b as that given by Eq. (43), but merely replacing the Ω by $\Omega - H_3$ for the present case. Therefore, when we replace all the Ω in $\mathbf{G}_3$ by $\Omega - H_3$, and follow a similar procedure as that for the above case, we can obtain a consistent numerical method for this problem.

When the anisotropy field was taken into account, Eq. (7) becomes nonlinear because of the dependence of $\mathbf{A}$ on m_3. However, the numerical method can still be applied for this case but with the value of m_3 being taken as a constant at each time step in the numerical computation. Simultaneously, for a stable reason the time stepsize needs to be shortened, for example, $\Delta\tau$=0.005.

5 Magnetization reversal

In order to test the performance of our numerical method, we first calculate a simple numerical example under the following parameters $H_3 = 0$, $H_0 = 0.4$, $\Omega = 1.5$ and $\alpha = 0.001$. The initial values of $\mathbf{m}$ are taken to be $m_1 = m_2 = m_3 = 1/\sqrt{3}$. The time stepsize used in this calculation was fixed to be $\Delta\tau = 0.01$. For the magnetization problem, it is utmost important that the numerical method can preserve the magnitude with $\|\mathbf{m}\| = 1$. In Fig. 1(a) we plot the numerical error of magnitude defined by $|\|\mathbf{m}\| - 1|$, i.e., the absolute value of $\|\mathbf{m}\| - 1$, from which we can see that the present numerical method can preserve the quantity $\|\mathbf{m}\| = 1$ very well. Even, for a very long history of the magnetization motion the magnitude error is still much smaller than 10^{-12}. It is also interesting that the time history of the vertical component of the magnetization m_3 as shown in Fig. 1(b) does not change its direction, keeping in the positive value for all time, i.e., no magnetization reversal.

Then, we calculate a more complex case under the following parameters $H_0 = 0.9, H_z = 0.5, \Omega = 0.4, \alpha = 0.001$ and $k_{\text{eff}} = -2$. The initial values of $\mathbf{m}$ are taken to be $m_1 = m_2 = -m_3 = 1/\sqrt{3}$. The time stepsize used in this calculation was fixed to be $\Delta\tau = 0.005$. We plot the numerical error of magnitude in Fig. 1(a) as shown by the dashed line, from which we can see that the present numerical method can preserve the quantity $\|\mathbf{m}\| = 1$ very well. Even, for a very long history of the magnetization motion the magnitude error is still much smaller than 10^{-14}. For this nonlinear example our consistent numerical method performs better than the above linear case in the preservation of $\|\mathbf{m}\| = 1$. It is also interesting that the time history of the vertical component of the magnetization m_3 as shown in Fig. 1(c) changes its direction rapidly, which is very different from the above example as shown in Fig. 1(b), and the magnetization direction switches with a highly oscillating motion in the range of $-1 < m_3 < 1$. In Figs. 2 and 3 we plot the magnetization motions for the above two numerical examples. It is obvious that the first example is of the non-reversal magnetization, while the second example is of the reversal magnetization.

Therefore, we attempt to know what parameters values will cause the magnetization direction reversal. To investigate the magnetization reversal we assume that the magnetization initially along the positive z-direction with $m_1 = m_2 = 0, m_3 = 1$. Then, within one-period of the input, i.e., $\tau \leq 2\pi/\Omega$, we search the minimal value of m_3. Fig. 4 shows the minimum of m_3 as a function of Ω for three different values of $(H_0, \alpha) = (0.6, 0.001)$, $(H_0, \alpha) = (0.8, 0.001)$ and $(H_0, \alpha) = (0.8, 0.02)$. It can be seen that under the same damping constant value of $\alpha = 0.001$, the case with $H_0 = 0.8$ has a larger range of exciting frequencies which allow the magnetization reversal than the case with $H_0 = 0.6$. For the last two cases under the same $H_0 = 0.8$, a larger damping constant with $\alpha = 0.02$ will produce a larger reversal than the smaller one with $\alpha = 0.001$; however, the damping constant α makes little influence of the minimum curve on the non-reversal portion.

In Fig. 5 we compare five different curves of the minimum of m_3 with respect to Ω in the range of $\Omega \in (0, 2)$ under five different values of $(H_0, H_z, \alpha, k_{\text{eff}}) = (0.6, 0.5, 0.001, -2)$, $(H_0, H_z, \alpha, k_{\text{eff}}) = (0.9, 0.5, 0.001, -2)$, $(H_0, H_z, \alpha, k_{\text{eff}}) = (0.6, 0, 0.001, -2)$, $(H_0, H_z, \alpha, k_{\text{eff}}) = (0.6, 0, 0.001, -0.5)$ and $(H_0, H_z, \alpha, k_{\text{eff}}) = (0.6, 0, 0.001, 0)$. First, we can observe that the present curves with k_{eff} nonzero are very different from those curves in Fig. 4 and the curve in Fig. 5 under $k_{\text{eff}} = 0$, where the major features are that these curves in Fig. 5 with nonzero k_{eff} all have a big jump (discontinuity) at a certain exciting frequency. Then, under the same values of $(H_z, \alpha, k_{\text{eff}}) = (0.5, 0.001, -2)$, the larger case with $H_0 = 0.9$ than the case with $H_0 = 0.6$ produces a larger range of exciting frequencies which allow the magnetization reversal and also has a larger reversal value. Without considering the dc field in the vertical direction, i.e., $H_z = 0$, the range of exciting frequencies to allow the magnetization reversal becomes

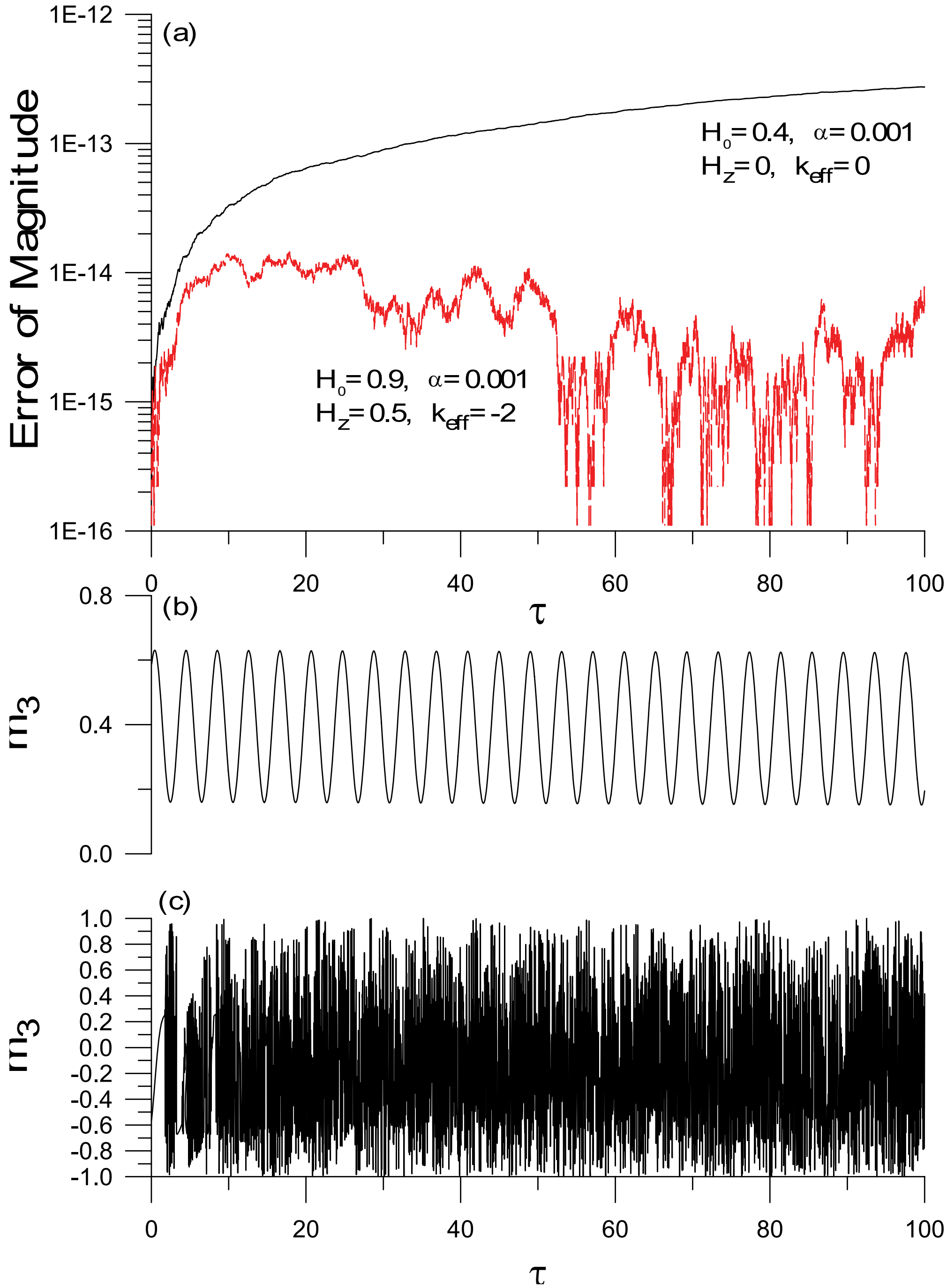

Figure 1: For these two numerical examples considered here: (a) showing the numerical errors of magnetization magnitude, (b) the non-reversal magnetization time history of the first example, (c) the reversal magnetization time history of the second example.

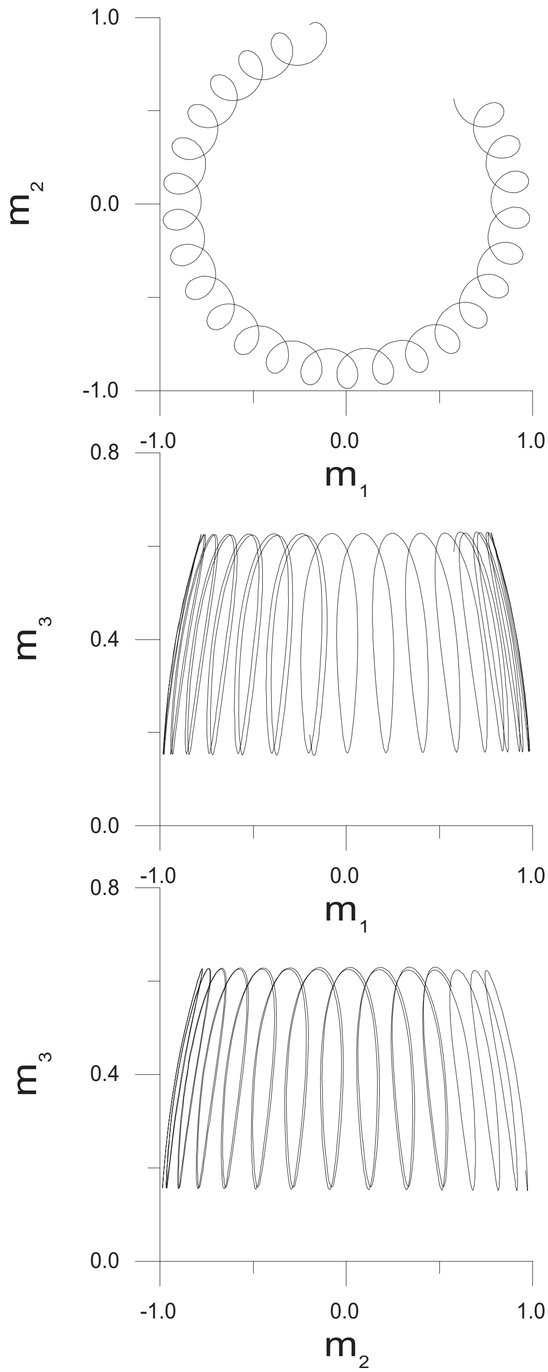

Figure 2: The magnetization motion of the first example.

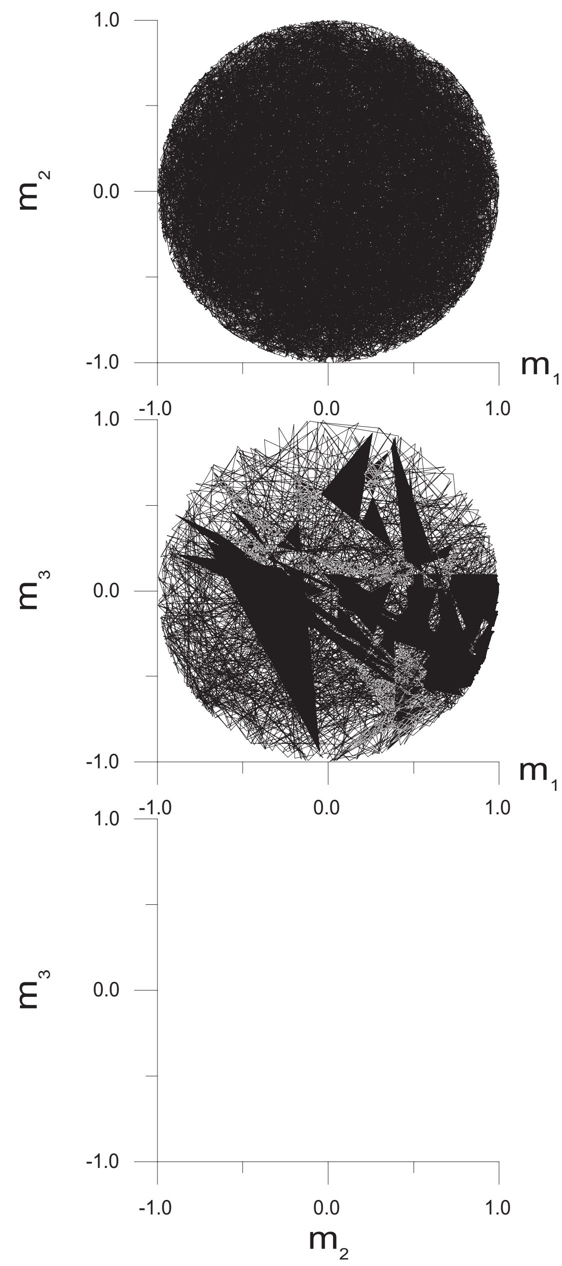

Figure 3: The magnetization motion of the second example.

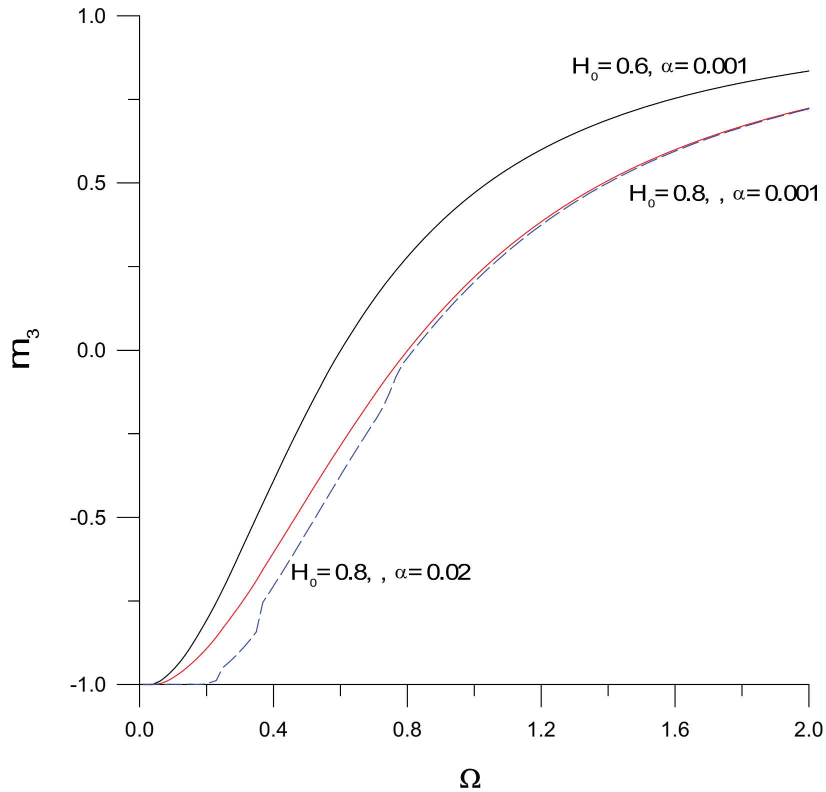

Figure 4: The minimum curves of the vertical component vs. exciting frequency for the case of $H_3 = 0$.

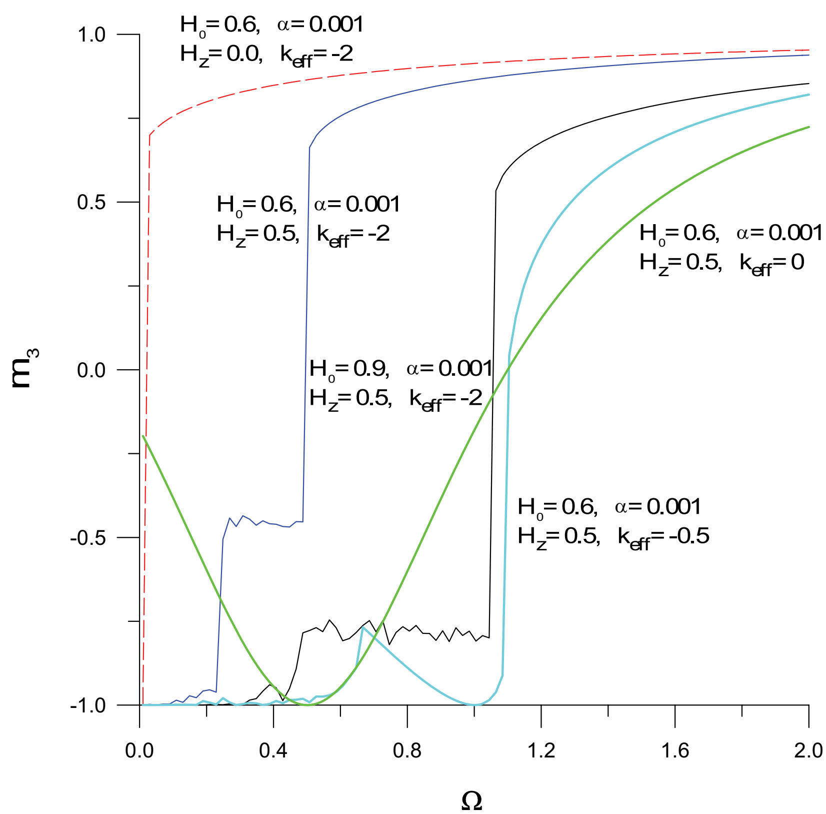

Figure 5: The minimum curves of the vertical component vs. exciting frequency for the cases of $H_3 \neq 0$.

very narrow near to the left end with $\Omega = 0$ as shown by the dashed line in Fig. 5. The last three minimum curves compare the effect of different k_{eff}; when the absolute value of k_{eff} is smaller the range of exciting frequencies which allow the magnetization reversal is larger. For k_{eff} zero, the minimum curve is a smooth convex function of Ω. Even we do not calculate the minimum curves to a higher value of excitation frequency, from Figs. 4 and 5 it can be seen that when the exciting frequency of the circularly polarized field is high, the magnetization reversal is impossible.

6 Conclusions

According to the Lie-type representation of the Landau-Lifshitz equation, we have derived a consistent numerical method for computing the magnetization by subjecting to a circularly polarized field, a dc field and an anisotropy field along the vertical direction of easy axis. The new consistent method can be used to correctly simulate the switching of magnetization direction and the magnetization reversal of a magnetic thin film. We proposed a minimum curve of the vertical component to detect the magnetization reversal as varying the circularly polarized exciting frequency. Then, the influences of parameters values on the magnetization reversal were studied. To assist the magnetization reversal, a dc field along the easy axis is necessary. Then, a larger amplitude of the circularly polarized exciting field will give rise a more large range of exciting frequency for magnetization reversal. When the anisotropy field is included, the minimum curves exhibit discontinuities between the reversal magnetization and non-reversal magnetization. Without exception, for higher exciting frequency of the circularly polarized field, the magnetization reversal does not happen.

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