Acoustoelastic Effects on Borehole Flexural Waves in Anisotropic Formations under Horizontal Terrestrial Stress Field

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Abstract: Applying the Stroh theory and based on the works of Hwu and Ting (1989), the complex function solution of stress and displacement fields around an open borehole in intrinsic anisotropic formation under horizontal terrestrial stress field is obtained. For cross-dipole flexural wave propagation along borehole axis, using the perturbation method, the acoustoelastic equation describing the relation between the alteration in phase velocity and terrestrial stress as well as formation intrinsic anisotropy is derived. At last, the numerical examples are provided for both the cases of fast and slow formation where the symmetry axis of a transversely isotropic (TI) formation makes an angle with the borehole axis. The phase velocity dispersion curves of borehole flexural wave and the corresponding velocity-stress coefficient are investigated. Computational results indicate that different from the stressed intrinsic isotropic formation situation, the variation in the phase velocity of flexural wave in stressed intrinsic anisotropic formation is dominated by two factors, one is the intrinsic formation anisotropy itself and the other is the stress-induced anisotropy. The former factor merely causes the borehole flexural wave split while the latter factor induces the dispersion curves intersection for two flexural waves polarized orthogonally. The combined effect of the two factors could strengthen or weaken the phenomenon of crossover for flexural wave dispersion curves. Thus, the dispersion curves of flexural waves may not intersect even under the unequal horizontal terrestrial stress field, whereas it is still possible to observe the crossover of the flexural wave dispersion curves under the equal horizontal terrestrial stress field. The polarized direction of the low-frequency fast flexural wave is no longer consistent with the direction of the maximum horizontal terrestrial stress all the time. Therefore, the crossover of the borehole flexural wave dispersion curves means that the terrestrial stress must exist. On the other hand, we can't exclude the possibility of the existence of terrestrial stress even if the flexural wave dispersion curves do not intersect. Based on the above researches, the method for terrestrial stress inversion from borehole flexural wave dispersion curves obtained by cross-dipole sonic logging in stressed intrinsic anisotropic formation is simply discussed.

Keyword: Acoustoelastic effect, borehole flexural wave, stressed intrinsic anisotropic formation, dispersion curve, terrestrial stress inversion method

1 Introduction

The quantitative information of terrestrial stress has an important impact on borehole stability and the oilfield production. In the initial stage of oilfield development, it will be great significance for making oil and gas field development scheme, stability design of borehole wall and well pattern optimization if we understand the situation of stress distribution in exploitation area and reservoir local stress. In oilfield production period, the variation of abnormal formation stresses may be caused by geological condition alteration, oilfield long-term exploitation and other human factors. Making this clear will be beneficial to take timely

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measures to prevent or reduce the phenomenon of oil well damage which often appears in oilfield. At present, the main methods for local stress measurement in oilfield include the borehole breakout and hydraulic fracturing technique, their costly and devastating borehole restrict their application in certain extent.

In propagation medium, elastic wave velocities are affected by prestresses (terrestrial stress). The theory of acoustoelasticity [Pao, Sachse and Fukuoda (1984)] was proposed in 1960s. And recently it has been applied to borehole problems, which is the theoretical basis of terrestrial stress nondestructive examination. Semi-analytical perturbation method and pure numerical method are the two main methods for investigation of the borehole modes in complex formation. In 1994, Norris, Sinha and Kostek (1994) first applied the theory of acoustoelasticity to a prestressed heterogeneous medium comprising of inviscid fluid and solid parts. Considering the influence of fluid-solid interfacial slip, the boundary conditions, constitutive relations and motion equations in the reference, intermediate and current coordinates were established, respectively. Then, adopting the perturbation procedure, the first-order perturbation in the eigenfrequency of Stonely wave in a borehole with pressurized fluid and prestrained solid in the intermediate coordinate was deduced. Based on it, Sinha, Kostek and Norris (1995), Sinha and Kostek (1996) further studied the dispersion alteration of Stonely and flexural waves induced by fluid pressure and uniaxial horizontal terrestrial stress, respectively. Their results indicate that under an uniaxial horizontal terrestrial stress field, the borehole flexural wave depends on the polarization direction, which results in the dispersion curves intersection for two flexural waves polarized orthogonally. Because the intrinsic formation anisotropy merely causes the flexural dispersion curves split, rather than intersection, according to the crossover of flexural wave dispersion curves, we can distinguish the stress-induced anisotropy from the intrinsic anisotropy of the formation. Subsequently, Winkler, Sinha and Plona (1998) proved both by theory and experiments that the intersection of the flexural wave dispersion curves can be used as an indicator of existence of terrestrial stress. Applying nonlinear acoustoelastic model and perturbation integral procedure, considering the uniaxial horizontal terrestrial stress and borehole fluid pressure jointly, Cao, Wang, Li, Xie, Liu and Lu (2003), Cao, Wang and Ma (2004) investigated the acoustoelastic effects of Stonely and flexural wave as well as the sensitive coefficient and velocity-stress coefficient. Furthermore, Li, Yin and Su (2006) analyzed the influence of triaxial terrestrial stresses on borehole modes. The results indicate that for stressed intrinsic isotropic formation, it is only the horizontal deviatoric terrestrial stress will cause the crossover of flexural wave dispersion curves, and which is independent with the horizontal mean terrestrial stress, superimposed stress and fluid pressure. For cased hole, without considering the terrestrial stress and applying perturbation method, Li, Su and Yin (2007) investigated the flexural wave in anisotropic formation. The results indicate that because of the influence of the casing, the flexural wave dispersion curves in cased hole in both fast and slow anisotropic formations all almost tend toward an identical Stoneley wave velocity at higher frequency.

Generally, the perturbation method is only suitable to analyze the dispersion curves of borehole mode. In order to obtain the full-wave field in time domain, a purely numerical method has to be adopted, mainly the finite element method (FEM) and finite difference method (FDM). For general homogeneous anisotropic medium, an effective meshless method based on the local Petrov-Galerkin approach is proposed by Sladek J, Sladek V and Atluri (2004) for solution elastodynamic problem. Subsequently, Atluri, Liu and Han (2006) developed a mixed finite difference method within the framework of the meshless local Petrov-Galerkin approach for solving solid mechanics problems. Later, a new numerical algorithm based on meshless local Petrov-Galerkin approach and modified moving least square method was proposed by Gao, Liu K, Liu Y (2006) for analyzing the wave propagation and dynamic fracture problems in elastic media. For borehole problem, Sinha, Liu and Kostek (1997), Liu and Sinha (2000, 2003) discretized the dynamic equation of stress-velocity form directly by applying staggered-grid high-order finite difference method. They investigated the dispersion curves as well as the waveforms in time domain of Stonely and flexural wave in fluid-filled borehole in biaxial and triaxial terrestrial stressed formation, using absorbing boundary condition to eliminate the reflection at an artificial finite boundary. But for purely numerical method in borehole modes problem, it is extremely high demands on the computer resources and its result is hard to be further analysis, which limits its development and application range. In generally, comparing to the purely numerical method, the analytic and semi-analytic method is more suitable for theoretical study.

The above researches are all based the assumption that the formation is an intrinsic isotropic elastic solid, and merely the influence of stress-induced anisotropy on borehole mode is analyzed. However, actually, the formation is not isotropic, but anisotropic in various degrees. The stresses and displacements field around a borehole in anisotropic formation is completely different from that in isotropic formation under terrestrial stresses, which directly influences the wave field and dispersion of borehole mode. Therefore, the study of the acoustoelastic effect on borehole flexural wave in intrinsic anisotropic formation under terrestrial stress field has important theoretical significance for realization of terrestrial stress nondestructive examination in stressed intrinsic anisotropic formation through cross-dipole sonic logging.

In this paper, considering an intrinsic anisotropic formation under horizontal terrestrial stress field, using the Stroh theory and based on the work of Hwu and Ting (1989), the complex function solutions of stresses and displacements around an open borehole are obtained firstly. Then, applying the nonlinear theory of acoustoelasticity and perturbation integral procedure, the expression of first-order correction of the phase velocity for borehole modes is derived. Finally, the numerical examples are provided for both fast and slow azimuthal anisotropic formation. The results indicate that the variation of flexural wave phase velocity is controlled by two factors, one is the intrinsic formation anisotropy and the other is the stress-induced anisotropy. The former factor merely causes flexural wave split while the latter factor results in flexural wave dispersion curves intersection. The combined effect of the two factors could strengthen or weaken the phenomenon of crossover for flexural wave dispersion curves. Thus even under an unequal horizontal terrestrial stress field, flexural wave dispersion curves may not intersect. Whereas it is still possible to observe the crossover of flexural wave dispersion curves under an equal horizontal terrestrial field because of the non-axisymmetry of the stresses around the borehole in anisotropic formation, besides which, the polarized direction of the low-frequency fast flexural wave is no longer keep consistent with the direction of the maximum horizontal terrestrial stress all the time.

Therefore, in actual cross-dipole sonic logging, if the formation is intrinsic anisotropic, a crossover of the flexural wave dispersion curves means it must exist terrestrial stress. On the other hand, we can't exclude the existent possibility of terrestrial stress if the flexural wave dispersion curves do not intersect. Based on the above researches, we simply discuss the acoustoelastic inversion model and the method for terrestrial stress inversion from borehole flexural wave dispersion curves.

2 Stresses and displacements field around an open borehole in anisotropic formation under horizontal terrestrial stress field

First of all, we give solutions of the statics problem that an open borehole is in anisotropic formation under horizontal terrestrial stress field. As shown in Figure 1, the formation is anisotropic elastic solid and C_{ijks} is its elastic constant. The horizontal terrestrial stress in x and y direction are S_x and S_y respectively. The radius of the borehole is a. Without considering the variation of terrestrial stress along the depths, the problem can be simplified as a generalized anisotropic plane strain problem [Ting (1996)], which can be solved by using Stroh theory. Hwu and Ting (1989) first studied the two-dimension problem of the anisotropic elastic solid with an elliptic hole or rigid inclusion subjected to a uniform loading at infinity applying Stroh theory. Based on their works, the complex function solution for the problem here can be obtained easily.

Assuming in infinite anisotropic elastic medium, in z plane, the circular hole boundary L is

$$z = x_1 + ix_2 = a\cos\psi + \mathrm{i}a\sin\psi \tag{1}$$

Where a is the radius of the circular hole, ψ is a real parameter, x_1 and x_2 are the components of the two Cartesian coordinate axis.

In z_α plane, the corresponding circular hole boundary L_α is

$$z_\alpha = x_1 + p_\alpha x_2 \tag{2}$$

Considering the conformal mapping [Ting (1996)]

$$z_\alpha = c_\alpha \zeta_\alpha + d_a \zeta_\alpha^{-1}, \quad (\alpha = 1,2,3) \tag{3}$$

where

$$c_\alpha = \frac{a}{2}(1 - \mathrm{i}p_\alpha), \quad d_\alpha = \frac{a}{2}(1 + \mathrm{i}p_\alpha) \tag{4}$$

$$\zeta_\alpha = \frac{z_\alpha + \sqrt{z_\alpha^2 - a^2(1 + p_\alpha^2)}}{a(1 - \mathrm{i}p_\alpha)} \tag{5}$$

$$\zeta_\alpha^{-1} = \frac{z_\alpha - \sqrt{z_\alpha^2 - a^2(1 + p_\alpha^2)}}{a(1 + \mathrm{i}p_\alpha)} \tag{6}$$

in equations (4)$\sim$(6), $p_\alpha(\alpha = 1,2,3)$ is the three complex eigenvalues of the eigenvalue problem

$$\left\{Q + p(R + R^{\mathrm{T}}) + p^2 T\right\} a = 0 \tag{7}$$

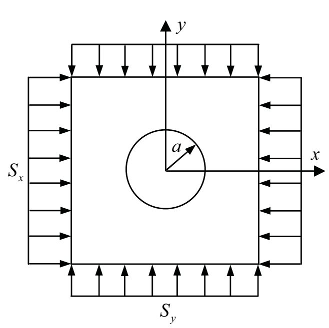

Figure 1: An open borehole in anisotropic formation under horizontal terrestrial stress field

and let $Imp_\alpha > 0(\alpha = 1,2,3)$, the corresponding eigenvectors is $a_\alpha(\alpha = 1,2,3)$. In equation (7), the superscript "T" denotes transpose. $a = (a_1, a_2, a_3)^{\mathrm{T}}$, Q, R and T are 3×3 matrix, their component respectively is

$$Q_{ik} = C_{i1k1}, \quad R_{ik} = C_{i1k2}, \quad T_{ik} = C_{i2k2} \tag{8}$$

and let

$$\begin{aligned} &A = (a_1, a_2, a_3), \quad b = (R^{\mathrm{T}} + pT)a, \\ &B = (b_1, b_2, b_3) \end{aligned} \tag{9}$$

the mapping (3) transforms the region outside the circular hole in z_α plane to the region outside the unit circle in ζ_α plane.

Let σ_{ij}^∞ and ε_{ij}^∞ be the stress and strain at infinity. They are related by the stress-strain laws

$$\sigma_{ij}^\infty = C_{ijks}\varepsilon_{ij}^\infty \tag{10}$$

For generalized plane strain problem, we have the condition $\varepsilon_{33}^\infty = 0$. If strain is known, stressed can be obtained through stress-strain relation and condition $\varepsilon_{33}^\infty = 0$, and vice versa. Assuming that σ_{ij}^∞ and ε_{ij}^∞ are known, writing

$$u^\infty = x_1\varepsilon_1^\infty + x_2\varepsilon_2^\infty, \quad \varphi^\infty = x_1 t_2^\infty - x_2 t_1^\infty \tag{11}$$

in which

$$\begin{aligned} &\varepsilon_1^\infty = \begin{pmatrix} \varepsilon_{11}^\infty \\ 0 \\ 2\varepsilon_{13}^\infty \end{pmatrix} = u_{,1}^\infty, \quad \varepsilon_2^\infty = \begin{pmatrix} 2\varepsilon_{12}^\infty \\ \varepsilon_{22}^\infty \\ 2\varepsilon_{23}^\infty \end{pmatrix} = u_{,2}^\infty \\ &t_1^\infty = \begin{pmatrix} \sigma_{11}^\infty \\ \sigma_{12}^\infty \\ \sigma_{13}^\infty \end{pmatrix} = -\varphi_{,2}^\infty, \quad t_2^\infty = \begin{pmatrix} \sigma_{12}^\infty \\ \sigma_{22}^\infty \\ \sigma_{23}^\infty \end{pmatrix} = \varphi_{,1}^\infty \end{aligned} \tag{12}$$

then the solution of displacement u and stress function φ can be chosen as the from

$$\begin{aligned} &u = x_1\varepsilon_1^\infty + x_2\varepsilon_2^\infty + 2Re\left\{A\left\langle\zeta_*^{-1}\right\rangle q\right\} \\ &\varphi = x_1 t_2^\infty - x_2 t_1^\infty + 2Re\left\{B\left\langle\zeta_*^{-1}\right\rangle q\right\} \end{aligned} \tag{13}$$

where q is a constant vector to be determined by boundary condition. $\langle f(z_*)\rangle$ is the diagonal matrix

$$\langle f(z_*)\rangle = \mathrm{diag}[f(z_1), f(z_2), f(z_3)] \tag{14}$$

Then, we determine the constant vector q by boundary condition.

At the circular hole boundary L, $x_1 = a\cos\psi$ and $x_2 = a\sin\psi$, thus we have

$$\begin{aligned} x_1\varepsilon_1^\infty + x_2\varepsilon_2^\infty &= Re[\mathrm{e}^{-\mathrm{i}\psi}a(\varepsilon_1^\infty + \mathrm{i}\varepsilon_{21}^\infty)] \\ x_1t_2^\infty - x_2t_1^\infty &= Re[\mathrm{e}^{-\mathrm{i}\psi}a(t_2^\infty - \mathrm{i}t_1^\infty)] \end{aligned} \tag{15}$$

and in z_α plane, we have $\zeta_\alpha = \mathrm{e}^{\mathrm{i}\psi}$ at the circular hole boundary L_α, so $\zeta_\alpha^{-1} = \mathrm{e}^{-\mathrm{i}\psi}$. From equations (13) and (15), the displacement u_L and the stress function φ_L at the circular hole boundary L are

$$\begin{aligned} u_L &= Re[\mathrm{e}^{-\mathrm{i}\psi}(a\varepsilon_1^\infty + \mathrm{i}a\varepsilon_2^\infty + 2Aq)] \\ \varphi_L &= Re[\mathrm{e}^{-\mathrm{i}\psi}(at_2^\infty - \mathrm{i}at_2^\infty + 2Bq)] \end{aligned} \tag{16}$$

The traction on the surface of a circular hole vanishes. Setting $\varphi_L = 0$ in equation (16), we obtain

$$q = -\frac{1}{2}B^{-1}a(t_2^\infty - \mathrm{i}t_1^\infty) \tag{17}$$

Substituting equation (17) into equation (13), we obtain the displacement u and stress function φ respectively as

$$\begin{aligned} u &= x_1\varepsilon_1^\infty + x_2\varepsilon_2^\infty + Re\left\{A\left\langle\zeta_*^{-1}\right\rangle B^{-1}a(-t_2^\infty + \mathrm{i}t_1^\infty)\right\} \\ \varphi &= x_1t_2^\infty - x_2t_1^\infty + Re\left\{B\left\langle\zeta_*^{-1}\right\rangle B^{-1}a(-t_2^\infty + \mathrm{i}t_1^\infty)\right\} \end{aligned} \tag{18}$$

For the problem in this paper, the anisotropic formation with a circular borehole is subjected to horizontal stresses S_x and S_y at infinity, thus we have

$$\sigma_{11}^\infty = S_x, \quad \sigma_{22}^\infty = S_y, \quad \sigma_{12}^\infty = \sigma_{13}^\infty = \sigma_{23}^\infty = 0 \tag{19}$$

According to the stress-strain relation (10) and condition $\varepsilon_{33}^\infty = 0$, the stress σ_{33}^∞ and strain ε_{11}^∞, ε_{22}^∞, ε_{12}^∞, ε_{13}^∞ and ε_{23}^∞ can be determined. Therefore, the stress σ_{ij}^∞ and strain ε_{ij}^∞ are all known. After gaining the displacement and stress function from equation (18), the stress and strain can be further obtained. From the relation

$$\begin{aligned} \varepsilon_1 &= \begin{pmatrix}\varepsilon_{11}\\0\\2\varepsilon_{13}\end{pmatrix} = u_{,1}, \quad \varepsilon_2 = \begin{pmatrix}2\varepsilon_{12}\\\varepsilon_{22}\\2\varepsilon_{23}\end{pmatrix} = u_{,2} \\ t_1 &= \begin{pmatrix}\sigma_{11}\\\sigma_{12}\\\sigma_{13}\end{pmatrix} = -\varphi_{,2}, \quad t_2 = \begin{pmatrix}\sigma_{12}\\\sigma_{22}\\\sigma_{23}\end{pmatrix} = \varphi_{,1} \end{aligned} \tag{20}$$

we have

$$\begin{aligned} t_1 &= -\varphi_{,2} = t_1^\infty - Re\left\{B\left\langle\zeta_*^{-1}\right\rangle_{,2}B^{-1}a(-t_2^\infty + \mathrm{i}t_1^\infty)\right\} \\ t_2 &= \varphi_{,1} = t_2^\infty + Re\left\{B\left\langle\zeta_*^{-1}\right\rangle_{,1}B^{-1}a(-t_2^\infty + \mathrm{i}t_1^\infty)\right\} \\ \varepsilon_1 &= u_{,1} = \varepsilon_1^\infty + Re\left\{A\left\langle\zeta_*^{-1}\right\rangle_{,1}B^{-1}a(-t_2^\infty + \mathrm{i}t_1^\infty)\right\} \\ \varepsilon_2 &= u_{,2} = \varepsilon_2^\infty + Re\left\{B\left\langle\zeta_*^{-1}\right\rangle_{,2}B^{-1}a(-t_2^\infty + \mathrm{i}t_1^\infty)\right\} \end{aligned} \tag{21}$$

where

$$\begin{aligned} \left\langle\zeta_*^{-1}\right\rangle_{,1} &= \left\langle\frac{\partial\zeta_*^{-1}}{\partial x_1}\right\rangle \\ &= \left\langle\frac{1}{a(1+\mathrm{i}p_*)}\left(1 - \frac{z_*}{\sqrt{z_*^2 - a^2(1+p_*^2)}}\right)\right\rangle \\ \left\langle\zeta_*^{-1}\right\rangle_{,2} &= \left\langle\frac{\partial\zeta_*^{-1}}{\partial x_2}\right\rangle \\ &= \left\langle\frac{p_*}{a(1+\mathrm{i}p_*)}\left(1 - \frac{z_*}{\sqrt{z_*^2 - a^2(1+p_*^2)}}\right)\right\rangle \end{aligned} \tag{22}$$

Acquired the stress and strain from equation (21), using stress-strain relation and condition $\varepsilon_{33} = 0$, stress σ_{33} is also gained. Hereto, all the stress, strain and displacement are obtained. For borehole problem, the cylindrical coordinate is always adopted for convenience. Therefore, the displacement, stress and strain should be transformed into cylindrical coordinate. The specific coordinate transformation relation can refer to literature [Auld (1973)].

3 Acoustoelastic equation of borehole mode in anisotropic formation

For a fluid-filled open borehole in anisotropic formation under terrestrial stress field, applying the theory of acoustroelasticity and perturbation integral procedure, the first-order perturbation in the phase velocity of borehole mode can be expressed as [Norris, Sinha and Kostek (1994)]

$$\frac{\Delta v}{v_\mathrm{R}^m} = \frac{v^m - v_\mathrm{R}^m}{v_\mathrm{R}^m} = \frac{\Delta\omega}{\omega_m} = \frac{\int_V \hat{c}_{L\gamma M\nu}u_{\nu,M}^m(u_{\gamma,L}^m)^*\mathrm{d}V}{2\omega_m^2\int_V \rho u_\gamma^m(u_\gamma^m)^*\mathrm{d}V} \tag{23}$$

where superscript m refers to the family of guided wave modes, $m=0$ and $m=1$ denote Stonely and flexural wave, respectively. v_{R}^m and v^m are the phase velocity of m family guided wave modes in the unperturbed reference and biasing stress-induced current state. $\Delta\omega$ is the variation of eigenfrequency ω_m between the reference and current state. u_γ^m is the displacement of m family guided wave modes in the reference state, superscript "$*$" denotes the complex conjugate. Comma "," represents the partial differential with respect to its corresponding geometric coordinate, the summation convention on repeated subscripts is implied. V is the integral volume, including the fluid in the borehole and the formation around it. $\hat{c}_{L\gamma M\nu}$ is the incremental part of equivalent elastic module of anisotropic formation in terrestrial stress-induced state compared to the second-order elastic module $c^{\circ}_{L\gamma M\nu}$ of formation in unperturbed reference state, namely

$$\hat{c}_{L\gamma M\nu} = c_{L\gamma M\nu} - c^{\circ}_{L\gamma M\nu} + T_{LM}\delta_{\gamma\nu} + c_{L\gamma KM}w_{\nu,K} + c_{LKM\nu}w_{\gamma,K} + c_{L\gamma M\nu AB}E_{AB} \quad (24)$$

where $c_{L\gamma M\nu}$ and $c_{L\gamma M\nu AB}$ are the second-order and the third-order elastic module tensor of formation in current state, respectively, $\delta_{\gamma\nu}$ is Kronecker symbol. T_{LM}, E_{AB} and $w_{\gamma,K}$ are static biasing stresses, strains and static displacement gradients of the borehole problem, respectively. According to the acquired complex function solutions previously, the static displacement gradient can be obtained through derivation to static displacement. In cylindrical coordinate, we have

$$\begin{aligned}
&w_{r,r} = \frac{\partial w_r}{\partial r} = E_{rr},\\
&w_{\theta,\theta} = \frac{1}{r}\left(\frac{\partial w_\theta}{\partial\theta} + w_r\right) = E_{\theta\theta},\\
&w_{z,z} = \frac{\partial w_z}{\partial z} = E_{zz} = 0, \quad w_{r,\theta} = \frac{1}{r}\left(\frac{\partial w_r}{\partial\theta} - w_\theta\right),\\
&w_{\theta,r} = \frac{\partial w_\theta}{\partial r}, \quad w_{z,\theta} = \frac{1}{r}\frac{\partial w_z}{\partial\theta}, \quad w_{\theta,z} = \frac{\partial w_\theta}{\partial z},\\
&w_{z,r} = \frac{\partial w_z}{\partial r}, \quad w_{r,z} = \frac{\partial w_r}{\partial z},
\end{aligned} \quad (25)$$

Generally, when using perturbation integral procedure to compute the alteration in phase velocity of borehole modes caused by terrestrial stress, we should select the state of anisotropic formation without terrestrial stress as the unperturbed reference state, and the state of formation deformation under terrestrial stress as the perturbed state. In this way, the second-order elastic module of formation in unperturbed reference state is equal to that in perturbed state, namely, $c^{\circ}_{L\gamma M\nu} = c_{L\gamma M\nu}$, and the equation (24) can be simplified. However, before using the equation (23), the solutions of displacement field for borehole mode in unperturbed state need to be known first. Unfortunately, for general anisotropic formation, we haven't the complete analytical solution of displacement field till present, which brings us great difficulties to calculate the phase velocity of borehole mode using perturbation integral equation (23). In order to solve this problem, we choose an appropriate isotropic formation without terrestrial stress as the reference unperturbed state, and the state of the intrinsic anisotropic formation under terrestrial stress is still selected as the current perturbed state. In this way, we can easily obtain the analytical solution of displacement for borehole mode in unperturbed state. The next, the key is how to choose the reference isotropic formation in unperturbed state. We know that for two transverse waves propagation along the borehole axis and polarized orthogonally in general anisotropic formation, their velocities are different. Because the velocity of the transverse wave in isotropic elastic solid is determined by elastic constant λ and μ, the reference isotropic formation in unperturbed reference state can be chosen from the velocities of plane waves propagation along borehole axis in anisotropic formation [Sinha, Norris and Chang (1994)], namely

$$\begin{aligned}
&\mu_{\mathrm{qSV}} = \rho V_{\mathrm{qSV}}^2, \quad \lambda_{\mathrm{qSV}} = \rho(V_{\mathrm{qP}}^2 - 2V_{\mathrm{qSV}}^2)\\
&\mu_{\mathrm{qSH}} = \rho V_{\mathrm{qSH}}^2, \quad \lambda_{\mathrm{qSH}} = \rho(V_{\mathrm{qP}}^2 - 2V_{\mathrm{qSH}}^2)
\end{aligned} \quad (26)$$

where V_{qP}, V_{qSV} and V_{qSH} are the velocities of the three plane wave propagation along the borehole axis in anisotropic formation, respectively. λ_{qSH}, μ_{qSH} and λ_{qSV},μ_{qSV} are the elastic constants of the reference isotropic formation in unperturbed state when study the qSH and qSV polarized flexural wave, respectively, which means that two different isotropic unperturbed reference models are established to gain the displacement solutions of cross-dipole flexural waves polarized in different direction in unperturbed state. This choice of the unperturbed reference isotropic formation model has been fully considered the difference in phase velocity for borehole flexural waves polarized orthogonally in anisotropic formation and makes the perturbative correction minimal, which results in high accuracy of perturbed solution [Sinha, Norris and Chang (1994)].

Because it still hasn't been found in relevant literatures about the measured values of the third-order elastic module for anisotropic medium until recently, using the known values of the third-order elastic modules for isotropic medium instead, and assuming they are identical in different direction. In this way, according to convention, the three independent third-order elastic modules are written as c_{111}, c_{112} and c_{123}, respectively, and have the relation

$$\begin{aligned} c_{144} &= \frac{1}{2}(c_{112}-c_{123}), \\ c_{155} &= \frac{1}{4}(c_{111}-c_{112}), \\ c_{456} &= \frac{1}{8}(c_{111}-3c_{112}+2c_{123}) \end{aligned} \tag{27}$$

Expanding the equation (23), it can be written as

$$\begin{aligned} \frac{\Delta v}{v_{\mathrm{R}}^m} &= \frac{\Delta v_{\mathrm{ani}}+\Delta v_{\mathrm{str}}}{v_{\mathrm{R}}^m} = \frac{v^m - v_{\mathrm{R}}^m}{v_{\mathrm{R}}^m} \\ \frac{\Delta v_{\mathrm{ani}}}{v_{\mathrm{R}}^m} &= \frac{\int_V \left(c_{L\gamma M\nu} - c^{\circ}_{L\gamma M\nu}\right) u_{\nu,M}(u_{\gamma,L})^* \mathrm{d}V}{2\omega^2 \int_V \rho u_\gamma (u_\gamma)^* \mathrm{d}V} \\ \frac{\Delta v_{\mathrm{str}}}{v_{\mathrm{R}}^m} &= \frac{1}{2\omega^2 \int_V \rho u_\gamma (u_\gamma)^* \mathrm{d}V} \\ &\quad \left[\int_V \chi_0 \mathrm{d}V + c_{111}\int_V \chi_1 \mathrm{d}V + c_{112}\int_V \chi_2 \mathrm{d}V \right.\\ &\quad + c_{123}\int_V \chi_3 \mathrm{d}V + c_{144}\int_V \chi_4 \mathrm{d}V \\ &\quad \left. + c_{155}\int_V \chi_5 \mathrm{d}V + c_{456}\int_V \chi_6 \mathrm{d}V \right] \end{aligned} \tag{28}$$

where Δv_{ani} and Δv_{str} are the variations of the phase velocity of borehole guided mode caused by the formation anisotropy and the terrestrial stress, respectively. In equation (28), we have

$$\begin{aligned} \chi_0 &= \left(T_{LM}\delta_{\gamma\nu} + c_{L\gamma KM} w_{\nu,K} + c_{LKM\nu} w_{\gamma,K}\right) u_{\nu,M} u^*_{\gamma,L} \\ \chi_1 &= E_{11} u_{1,1} u^*_{1,1} + E_{22} u_{2,2} u^*_{2,2} \\ \chi_2 &= E_{11}\left[u_{2,2}(2u^*_{1,1}+u^*_{2,2}) + u_{3,3}(2u^*_{1,1}+u^*_{3,3})\right] \\ &\quad + E_{22}\left[(u_{1,1}+2u_{2,2})u^*_{1,1} + u_{3,3}(2u^*_{2,2}+u^*_{3,3})\right] \\ \chi_3 &= 2\left[E_{11}(u_{3,3}u^*_{2,2}) + E_{22}(u_{3,3}u^*_{1,1})\right] \\ \chi_4 &= E_{11}\left[u_{3,2}u^*_{3,2} + u_{2,3}(u^*_{2,3}+2u^*_{3,2})\right] \\ &\quad + E_{22}\left[u_{3,1}u^*_{3,1} + u_{1,3}(u^*_{1,3}+2u^*_{3,1})\right] \\ &\quad + 4E_{12}u_{3,3}(u^*_{1,2}+u^*_{2,1}) \\ &\quad + 4E_{13}(u_{1,3}u^*_{2,2}+u_{2,2}u^*_{3,1}) \\ &\quad + 4E_{23}\left[(u_{2,3}+u_{3,2})u^*_{1,1}\right] \\ \chi_5 &= (E_{11}+E_{22})\left[u_{2,1}u^*_{2,1} + u_{1,2}(u^*_{1,2}+2u^*_{2,1})\right] \\ &\quad + E_{11}\left[u_{3,1}u^*_{3,1} + u_{1,3}(u^*_{1,3}+2u^*_{3,1})\right] \\ &\quad + E_{22}\left[u_{2,3}(u^*_{2,3}+2u^*_{3,2}) + u_{3,2}u^*_{3,2}\right] \\ &\quad + 4E_{12}\left[(u_{1,2}+u_{2,1})u^*_{1,1} + u_{2,2}(u^*_{1,2}+u^*_{2,1})\right] \\ &\quad + 4E_{13}\left[(u_{1,3}+u_{3,1})u^*_{1,1} + u_{3,3}(u^*_{1,3}+u^*_{3,1})\right] \\ &\quad + 4E_{23}\left[(u_{2,3}+u_{3,2})u^*_{2,2} + u_{3,3}(u^*_{2,3}+u^*_{3,2})\right] \\ \chi_6 &= 4E_{12}\left[u_{3,2}u^*_{3,1} + u_{2,3}(u^*_{1,3}+u^*_{3,1}) + u_{1,3}u^*_{3,2}\right] \\ &\quad + 4E_{13}(u_{2,3}+u_{3,2})(u^*_{1,2}+u^*_{2,1}) \\ &\quad + 4E_{23}\left[u_{3,1}u^*_{2,1} + u_{1,3}(u^*_{1,2}+u^*_{2,1}) + u_{1,2}u^*_{3,1}\right] \end{aligned} \tag{29}$$

In equation (29), the superscript m in displacement components is omitted for convenience, and the subscript $1,2,3$ corresponds to the r,θ,z in cylindrical coordinate system, respectively.

Using relation (27), integration after substituting the analytical solutions of the static stress, strain and displacement into equation (28), we have the acoustoelatic equation of borehole modes as following

$$\begin{aligned} \frac{\Delta v}{v_{\mathrm{R}}^m} &= \frac{\Delta v_{\mathrm{ani}}+\Delta v_{\mathrm{str}}}{v_{\mathrm{R}}^m} = \frac{v^m - v_{\mathrm{R}}^m}{v_{\mathrm{R}}^m} \\ &= \frac{\Delta v_{\mathrm{ani}}}{v_{\mathrm{R}}^m} + Q^{S_x}S_x + Q^{S_y}S_y \end{aligned} \tag{30}$$

where Q^{S_x} and Q^{S_y} are called as velocity-stress coefficients corresponding to horizontal terrestrial stress S_x and S_y, respectively. They are related to eigenfrequency. From equation (30), noticing that the last term of equation (24) is related to the third-order elastic module, so Q^{S_x} and Q^{S_y} can be expressed as the summation of the following four terms

$$Q^{S_x} = \sum_{i=1}^{4} Q_i^{S_x} = C_1^{S_x} + \frac{C_2^{S_x} c_{111}}{c_{66}} + \frac{C_3^{S_x} c_{112}}{c_{66}} + \frac{C_4^{S_x} c_{123}}{c_{66}}$$
$$Q^{S_y} = \sum_{i=1}^{4} Q_i^{S_y} = C_1^{S_y} + \frac{C_2^{S_y} c_{111}}{c_{66}} + \frac{C_3^{S_y} c_{112}}{c_{66}} + \frac{C_4^{S_y} c_{123}}{c_{66}} \tag{31}$$

where $C_i^{S_x}$ and $C_i^{S_y}(i=1,2,3,4)$ are called as sensitivity coefficients corresponding to S_x and S_y, respectively. They depend on frequency and merely can be obtained by the numerical method. According to the equations (30) and (31), the sensitivity coefficients do not influence the phase velocity of borehole mode directly, but through the third-order elastic module, they affect the first-order correction of phase velocity by velocity-stress coefficient. Therefore, we can investigate the influence of terrestrial stress on phase velocity quantitatively by the velocity-stress coefficient.

4 Numerical computation and results

The influence of stress concentration around a borehole in anisotropic formation on phase velocity of borehole modes are all included in the velocity-stress coefficient. The next, we provide a numerical example for quantitative analysis of the flexural wave dispersion in anisotropic formation under terrestrial stress. Assuming the formation is TI elastic solid. A schematic of a borehole and the global coordinate system xyz are shown in Figure 2. The symmetry axis of the formation is in xz plane and makes an angle φ with the borehole axis z. The local Cartesian coordinate $x'y'z'$ is established in formation, z' axis coincides with the TI symmetry axis and y' axis is coincident with y axis in global coordinate system. Fluid is full filled in the borehole and the horizontal terrestrial stresses in x and y direction are S_x and S_y, respectively.

In computation, we set the fluid compression wave velocity in borehole $v_{\mathrm{f}} = 1500\mathrm{m/s}$, fluid density $\rho_{\mathrm{f}} = 1000\mathrm{kg/m^3}$, and the borehole radius $a = 0.1016\mathrm{m}$. The material parameters of the TI formation are listed in Table 1 and Table 2, respectively.

For the TI formation as shown in Figure 2, the polarized directions of the plane qSV and SH waves propagation along borehole axis parallel with x and y axis, respectively, which correspond to 0° and 90° azimuth in cylindrical coordinate. Therefore, it refers to the 0° and 90° azimuthally polarized flexural waves respectively while using the equations (26) and (28) to calculate.

The validation calculation is given first. When elastic modules $C_{11} = C_{33} = \lambda + 2\mu$, $C_{12} = C_{13} = \lambda$, $C_{44} = (C_{11} - C_{12})/2 = \mu$, the TI formation is degenerated to isotropic one. Setting the parameters of the formation and the fluid be identical with that in reference [Li, Yin and Su (2006)], and uniaxial horizontal terrestrial stress $S_x = -5\mathrm{MPa}$, the flexural wave dispersion curves obtained by using method in this paper is shown in Figure 3, in which curves 2 and 3 are almost consistent with those in reference [Li, Yin and Su (2006)]. This means the method in this paper is reliable.

4.1 Fast formation

For fast TI formation, the Figure 4 shows the flexural wave velocity-stress coefficients corresponding to S_x and the dispersion curves when the symmetry axis of TI formation makes different angle φ with the borehole axis under the terrestrial stress field $S_x = -50\mathrm{MPa}$, $S_y = 0$.

Because of the anisotropy induced by stress concentration around a borehole and the formation intrinsic itself, the formation is not material symmetry about the borehole axis. In addition the asymmetry of dipole sources, thus the two flexural waves polarized in $\phi = 0°$ and $\phi = 90°$ azimuth are different. From Figure 4, the velocity-stress coefficients in $\phi = 0°$ azimuth entirely differ from that in $\phi = 90°$. Since the third-order elastic modules are much larger than the second-order elastic constants for several magnitudes for the solid formation, the last three velocity-stress coefficients related to third-order modules c_{111}, c_{112} and c_{123} are dominated and the first one almost can be ignored. The result coincides to equation (31). With the φ change from 0° to 90°, the curve shape of velocity-stress coefficient $Q_i^{S_x}(i =$

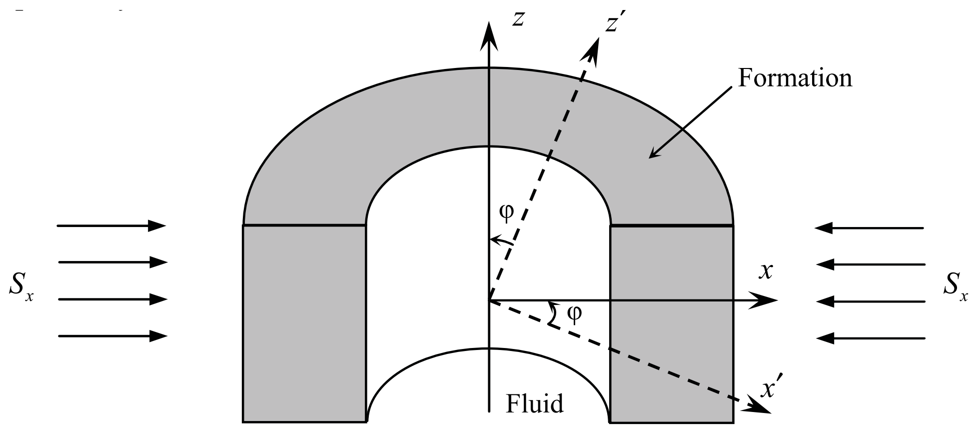

Figure 2: A fluid-filled borehole in TI formation under terrestrial stress

Table 1: The second-order elastic module and density of TI formation

Parameter	C_{11}/(GPa)	C_{12}/(GPa)	C_{13}/(GPa)	C_{33}/(GPa)	C_{44}/(GPa)	ρ(Kg/m^3)
Fast stratum	37.03	12.91	8.91	27.03	9.06	2600
Slow stratum	21.58	10.18	7.5	16.6	5.3	2560

Table 2: The third-order elastic module of TI formation

Parameter	C_{111}/(GPa)	C_{112}/(GPa)	C_{123}/(GPa)
Fast stratum	-21217	-3044	2361
Slow stratum	-21217	-3044	2361

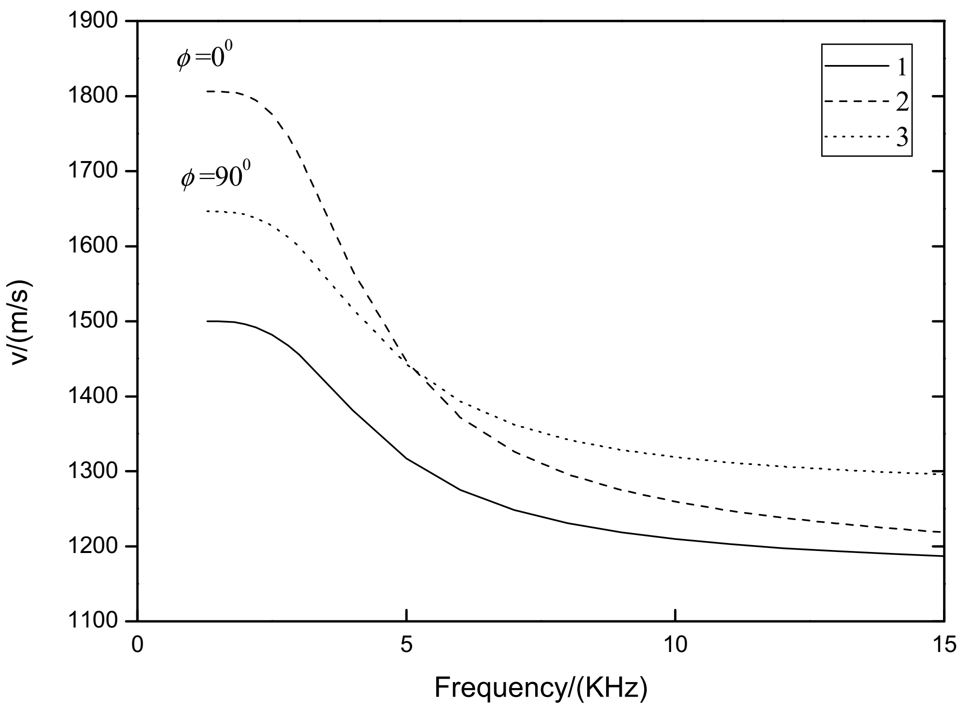

Figure 3: Flexural wave dispersion when degenerated to isotropic formation under uniaxial terrestrial stress 1 reference state; 2 0° azimuth; 3 90° azimuth

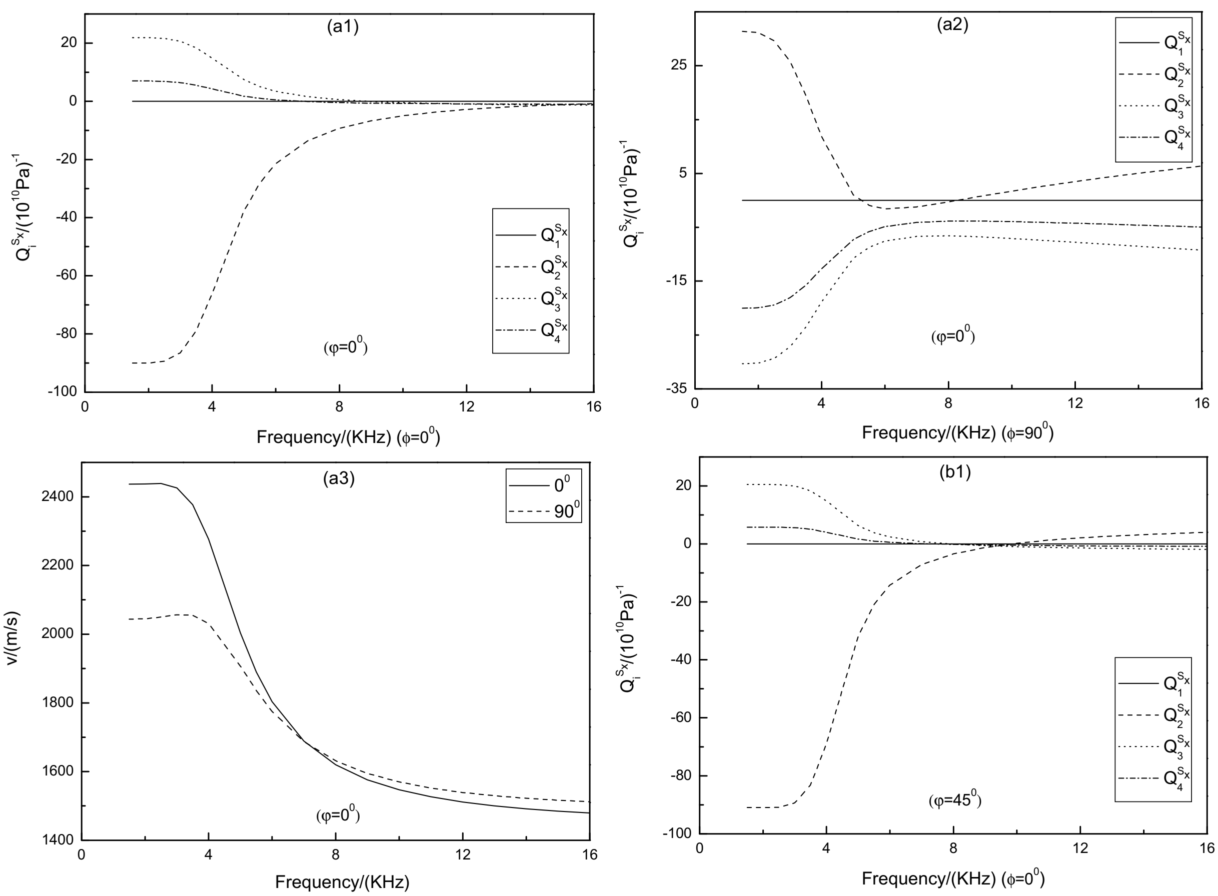

Figure 4

$2,3,4)$ is approximately similar, in which the variations of $Q_2^{S_x}$ is the most obvious, especially at low frequency. From Figures 4a3, 4b3 and 4c3, a crossover of dispersion curves for two flexural waves polarized orthogonally is clear. And with φ increase from 0° to 90°, the frequencies corresponding to the intersection point are of little difference and all around 7KHz. It is same with the conclusion of the acoustoelastic effect on borehole flexural wave in intrinsic isotropic formation [Sinha and Kostek (1996)], the crossover of flexural wave dispersion curves in intrinsic anisotropic formation is also the unique feature of stress-induced anisotropy, which can be used as an indicator of terrestrial stress existence. Although the phenomenon of dispersion curves crossover for cross-dipole flexural wave has been already observed in laboratory and acoustic logging, the theoretical research is still limited to the case of intrinsic isotropic formation under terrestrial stress. Here, we proved that it still has the characteristic of dispersion curves intersection for flexural wave in stressed intrinsic anisotropic formation, which lays a theoretical foundation for expanding the terrestrial stress nondestructive examination method through cross-dipole sonic logging to the condition of intrinsic anisotropic formation.

Under the uniaxial terrestrial stress in y axis direction of $S_x = 0$, $S_y = -50$MPa, the Figure 5 shows the flexural wave velocity-stress coefficient related to S_y and the dispersion curves with different φ. From Figure 5, the curve shape of velocity-stress coefficient $Q_i^{S_x} (i = 2,3,4)$ is of slight change when φ increase from 0° to 90°. In $\phi = 90^\circ$ azimuth, the absolute values of $Q_2^{S_y}$, $Q_3^{S_y}$ and $Q_4^{S_y}$ at an identical frequency decrease gradually respectively in low frequency with the φ increase, in which the decreasing range for the absolute value of $Q_2^{S_y}$ is the most obvious. In

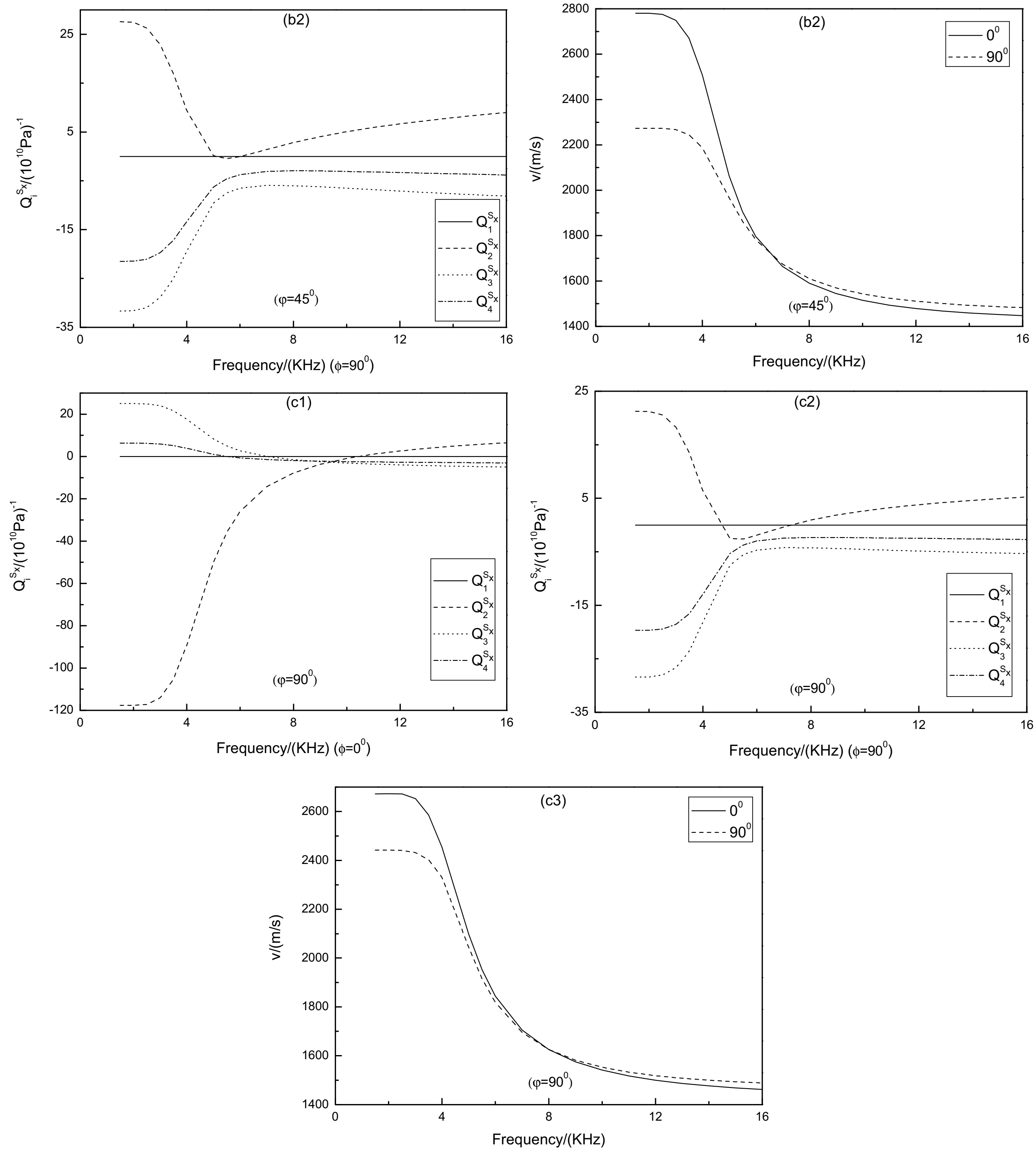

Figure 4: Flexural wave velocity-stress coefficients corresponding to S and dispersion curves in fast TI formation when $S_x = -50\text{MPa}$, $S_y = 0$. (a) $\phi = 0$; (b) $\phi = 45^\circ$; (c) $\phi = 90^\circ$

$\phi = 0^\circ$ azimuth, the alteration in absolute values of $Q_2^{S_y}$, $Q_3^{S_y}$ and $Q_4^{S_y}$ is not very great with φ increase. Comparing the Figure 5a1, 5a2 with the Figure 4a1, 4a2, respectively, it can be found that the velocity-stress coefficients related to S_x in $\phi = 0^\circ$ and $\phi = 90^\circ$ azimuth are exactly identical with coefficients for S_y in $\phi = 90^\circ$ and $\phi = 0^\circ$ azimuth. The reason is that the TI formation is material symmetry about borehole axis when $\varphi = 0^\circ$. For two cases that the formation is subjected to an equal stress in x and y axis direction respectively, the stress distribution around a borehole is completely identical except for the x and y coordinate interchange. Thus the velocity-stress coefficients related to S_x and S_y just exchange each other in $\phi = 0^\circ$ and $\phi = 90^\circ$ azimuth. If $\varphi \neq 0^\circ$, the TI formation is not material symmetry about the borehole axis, the stress distribution around a borehole under the two kinds of uniaxial stress condition is completely different. Therefore, the velocity-stress coefficients related to S_x and S_y no longer equal each other, which can be seen by comparing Figure 5b, 5c with Figure 4b, 4c. We can also find from the Figure 5 that the degree of crossover of flexural wave dispersion curves is quite different with the variation of φ. When $\varphi = 0^\circ$, comparing Figure 5a3 from Figure 4a3, the two group dispersion curves are completely identical except exchange in 0° and 90° azimuth. That is because the formation is intrinsic symmetry about the borehole axis when $\varphi = 0^\circ$, the phase velocity alteration caused by formation intrinsic anisotropy Δv_{ani} is completely same in 0° and 90° azimuth, while the phase velocity alteration induced by terrestrial stress Δv_{str} exchange in 0° and 90° azimuth. When $\varphi = 45^\circ$, from Figure 5b3, the difference in phase velocity of two flexural wave is much small at high frequency, which makes the crossover of dispersion curves indistinct. Through local amplification, the crossover is still can be seen. When $\varphi = 90^\circ$, from Figure 5c3, the difference in two flexural waves is the most, and also a crossover of dispersion curves is the most significant. From above discussion, we find that for intrinsic anisotropic formation, even under the unequal terrestrial stress state, an obvious intersection of flexural wave dispersion curves may not be observed. The reason is that the stress-induced anisotropy and the intrinsic anisotropy of formation maybe partially counteract each other in this case, which makes the difference in phase velocity of two flexural waves polarized orthogonally reduction, especially in high frequency.

4.2 Slow formation

For slow TI formation in Table 1, the velocities of plane qSV and SH wave are all less than the velocity of fluid compression wave in borehole when φ increase from 0° to 90°. The flexural wave dispersion curves and velocity-stress coefficient are investigated under two different terrestrial stress fields.

Figure 6 denotes the dispersion curves and velocity-stress coefficient related to S_x in slow TI formation when $S_x = -10$MPa, $S_y = 0$. Similar to the case in fast formation, the three velocity-stress coefficients related to the third-order modules c_{111}, c_{112} and c_{123} are major. The first one can be almost neglected. In computation, the third-order elastic modules of slow formation is chosen as same as that of fast formation, from equation (28)$\sim$(31), we know that the absolute value of velocity-stress coefficient in slow formation is greater than that in fast formation. Under the uniaxial terrestrial stress $S_x = -10$MPa, the intersection of dispersion curves comes to more obvious gradually with the φ increase form 0° and 90°. Moreover, the intersection point moves to low frequency and the corresponding phase velocity come to high speed with φ increase.

For slow TI formation, under uniaxial terrestrial stress field $S_x = 0$, $S_y = -10$MPa, velocity-stress coefficient related to S_y and dispersion curves of flexural wave are shown in Figure 7. With φ increase from 0° to 90°, the variation of three velocity-stress coefficient $Q_2^{S_y}$, $Q_3^{S_y}$ and $Q_4^{S_y}$ is obvious, especially for $Q_2^{S_y}$. Simultaneously, the crossover of dispersion curves of two flexural waves becomes more distinct gradually. The intersection points are all around 8KHz, and the corresponding phase velocities have no significant difference.

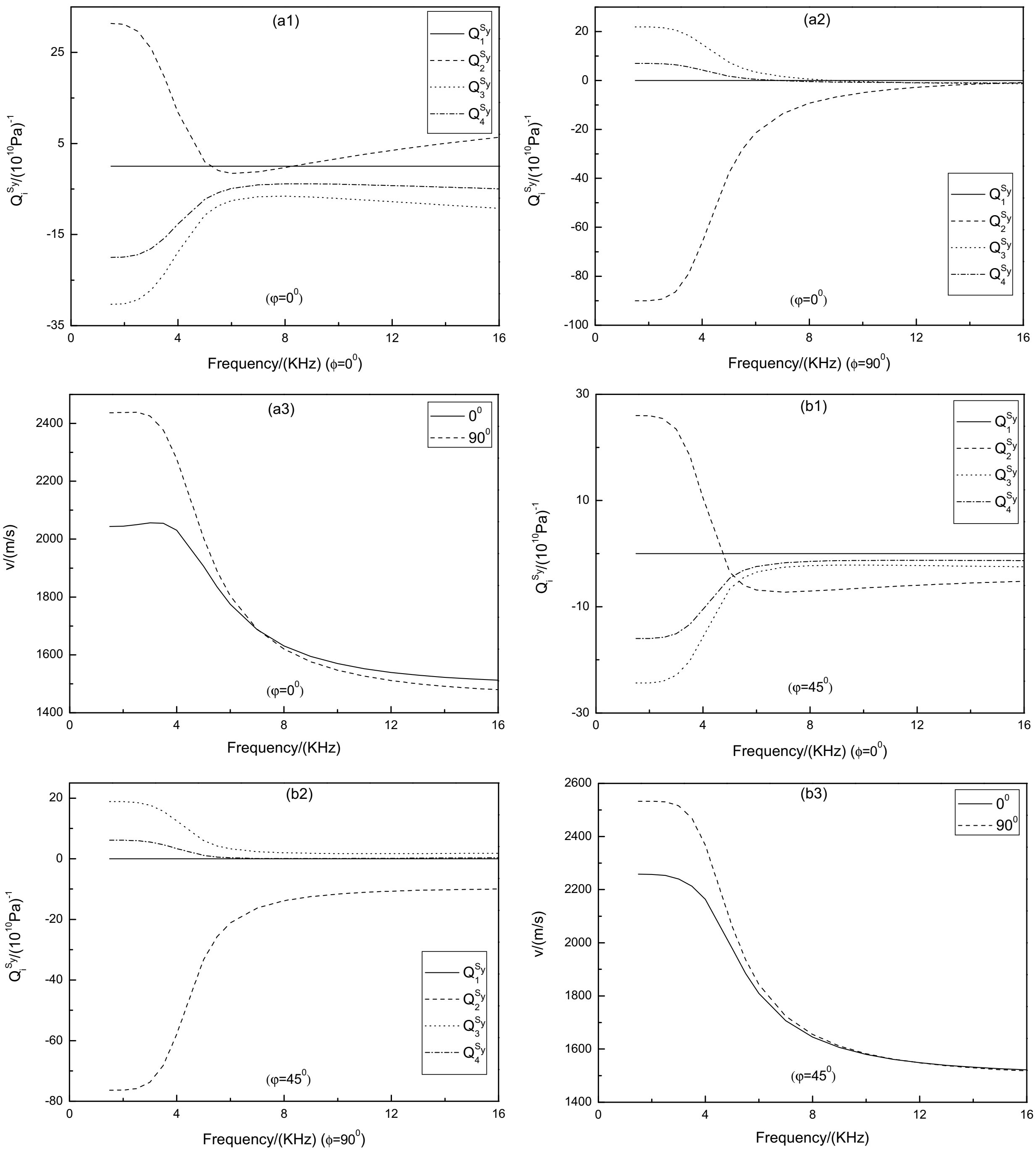
(a1)
(a2)
(a3)
(b1)
(b2)
(b3)
$Q_i^{S_y}/(10^{10}Pa)^{-1}$
v/(m/s)
$Q_1^{S_y}$
$Q_2^{S_y}$
$Q_3^{S_y}$
$Q_4^{S_y}$
0^0
90^0
($\varphi=0^0$)
($\varphi=45^0$)
Frequency/(KHz) ($\phi=0^0$)
Frequency/(KHz) ($\phi=90^0$)
Frequency/(KHz)

Figure 5

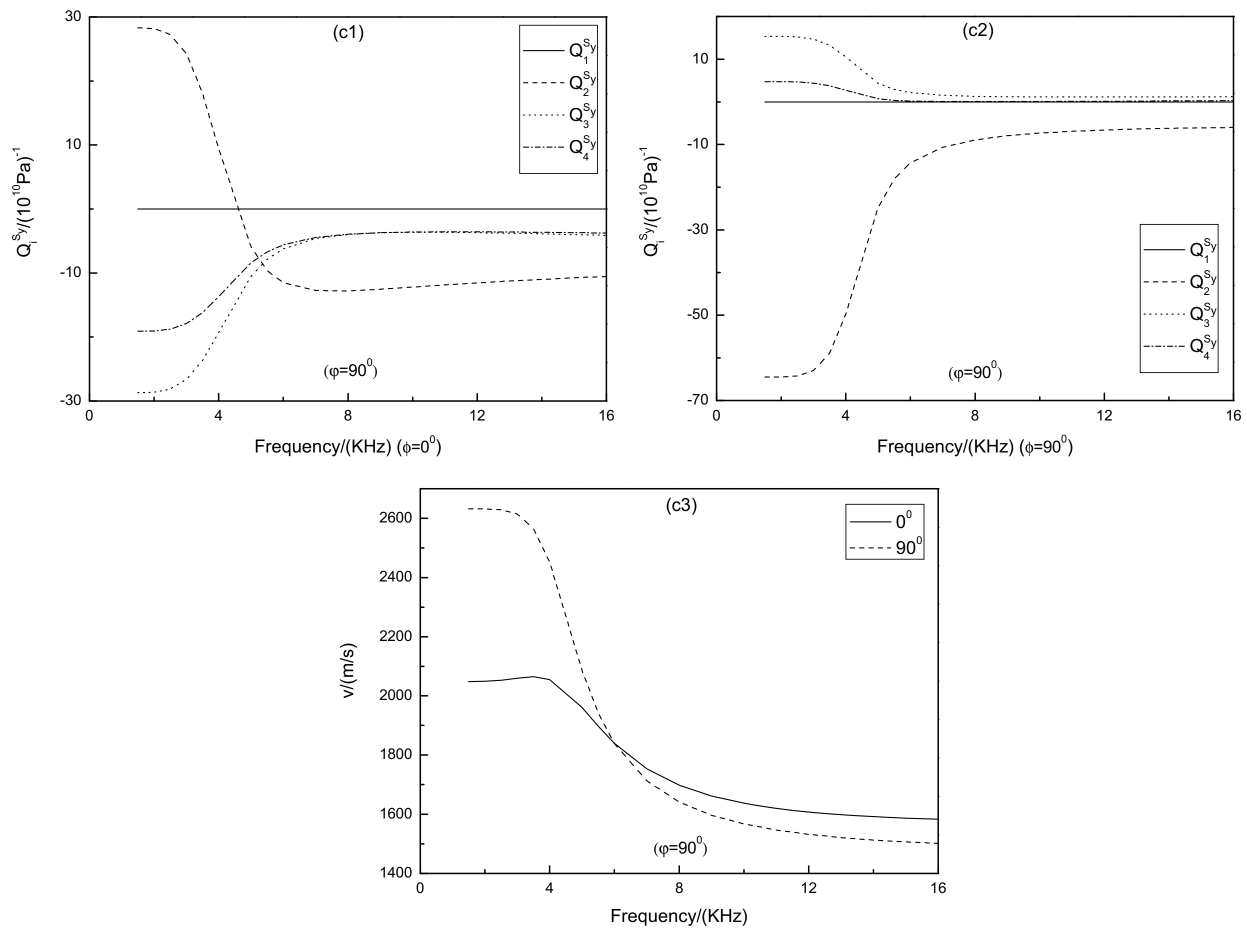

Figure 5: Flexural wave velocity-stress coefficients corresponding to S and dispersion curves in fast TI formation when $S_x = 0$, $S_y = -50$MPa. (a) $\phi = 0$; (b) $\phi = 45°$; (c) $\phi = 90°$

4.3 *Influence of ratio of horizontal terrestrial stress on crossover of dispersion curves*

If it exist horizontal terrestrial stress in x, y axis direction simultaneously, we let $S_y = \eta S_x$, in which η is the ratio of horizontal terrestrial stress in y axis direction to that in x axis direction, ranging from 0 to 1. The TI formation with $\varphi = 90°$ is chosen as example to study the effect of the horizontal terrestrial stress ration on the dispersion curves of flexural waves polarized orthogonally. The material parameters are listed in Table 1 and Table 2.

For fast TI formation, letting $S_x = -50$MPa, the flexural wave dispersion curves with η ranging from 0 to 1 are plotted in Figure 8. Comparing with the case of isotropic formation, in intrinsic anisotropic formation, the relation between the crossover of dispersion curves and the terrestrial stress is more complicated, some new phenomenon appears. From Figure 8a, when $\eta = 0$, the flexural wave dispersion curves intersect. In low frequency, the phase velocity of flexural wave at $\phi = 0°$ is greater. The polarized direction of low-frequency fast flexural wave coincides with the direction of maximum horizontal terrestrial stress, namely x axis direction. However, with η increase to 0.4, from Figure 8b, although the flexural wave split, their dispersion curves do not intersect. The polarized direction of low-frequency fast flexural wave is still consistent with the direction of maximum horizontal terrestrial stress. From Figure 8c, when $\eta = 0.8$, the crossover of dispersion curves can be observed again although it is not obvious. But the polarized direc-

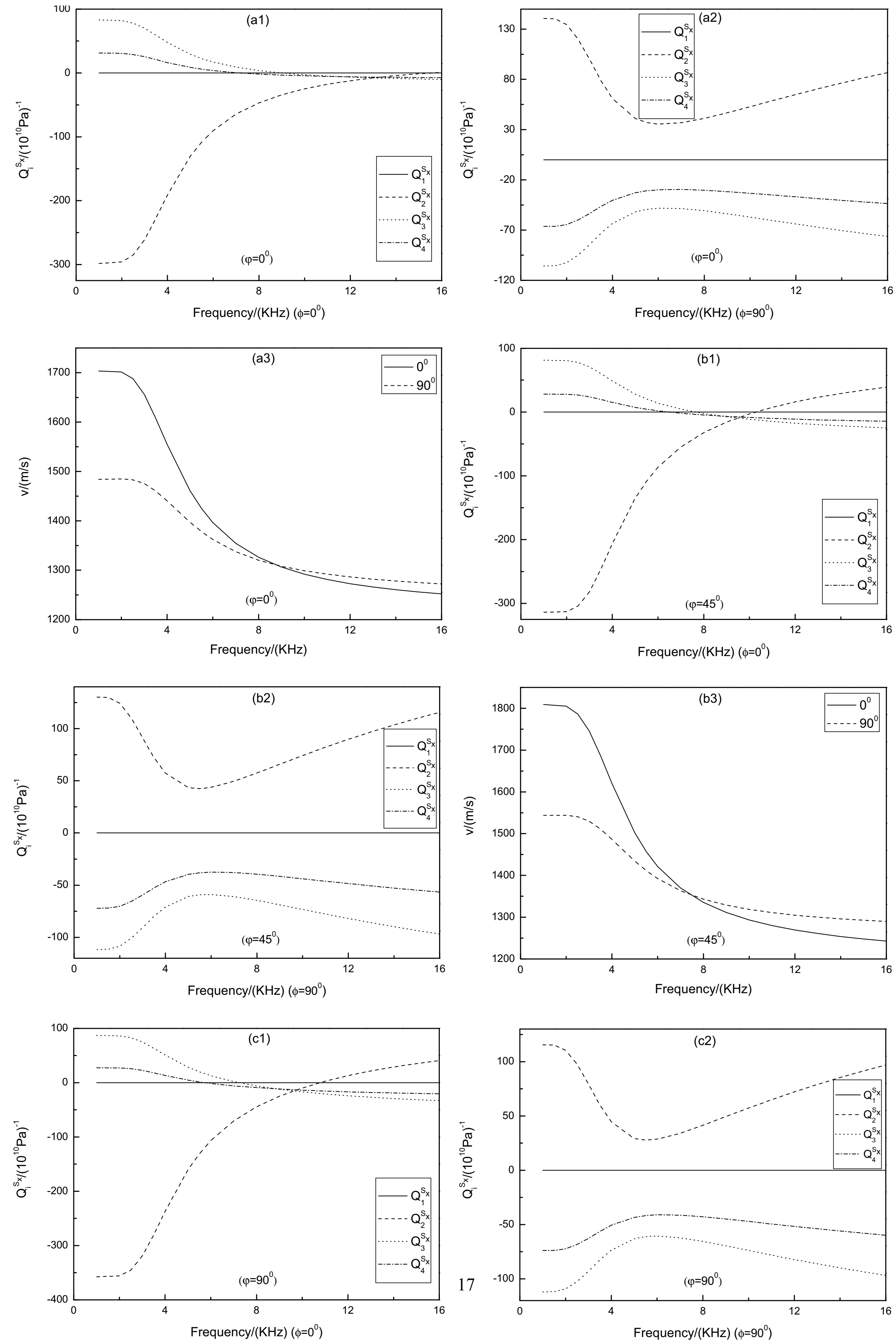
(a1)
$Q_i^{S_x}/(10^{10}Pa)^{-1}$
Frequency/(KHz) ($\phi=0^0$)
$Q_1^{S_x}$
$Q_2^{S_x}$
$Q_3^{S_x}$
$Q_4^{S_x}$
($\varphi=0^0$)
(a2)
$Q_i^{S_x}/(10^{10}Pa)^{-1}$
Frequency/(KHz) ($\phi=90^0$)
$Q_1^{S_x}$
$Q_2^{S_x}$
$Q_3^{S_x}$
$Q_4^{S_x}$
($\varphi=0^0$)
(a3)
v/(m/s)
Frequency/(KHz)
0^0
90^0
($\varphi=0^0$)
(b1)
$Q_i^{S_x}/(10^{10}Pa)^{-1}$
Frequency/(KHz) ($\phi=0^0$)
$Q_1^{S_x}$
$Q_2^{S_x}$
$Q_3^{S_x}$
$Q_4^{S_x}$
($\varphi=45^0$)
(b2)
$Q_i^{S_x}/(10^{10}Pa)^{-1}$
Frequency/(KHz) ($\phi=90^0$)
$Q_1^{S_x}$
$Q_2^{S_x}$
$Q_3^{S_x}$
$Q_4^{S_x}$
($\varphi=45^0$)
(b3)
v/(m/s)
Frequency/(KHz)
0^0
90^0
($\varphi=45^0$)
(c1)
$Q_i^{S_x}/(10^{10}Pa)^{-1}$
Frequency/(KHz) ($\phi=0^0$)
$Q_1^{S_x}$
$Q_2^{S_x}$
$Q_3^{S_x}$
$Q_4^{S_x}$
($\varphi=90^0$)
(c2)
$Q_i^{S_x}/(10^{10}Pa)^{-1}$
Frequency/(KHz) ($\phi=90^0$)
$Q_1^{S_x}$
$Q_2^{S_x}$
$Q_3^{S_x}$
$Q_4^{S_x}$
($\varphi=90^0$)

Figure 6

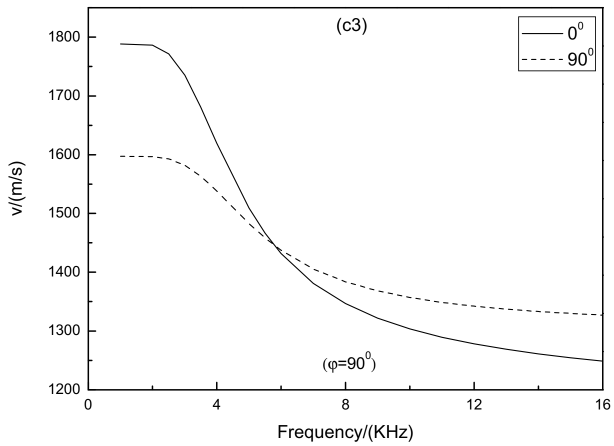

Figure 6: Flexural wave velocity-stress coefficients corresponding to and dispersion curves in fast TI formation when $S_x = -10$MPa, $S_y = 0$. (a) $\phi = 0$; (b) $\phi = 45°$; (c) $\phi = 90°$

tion of low-frequency fast flexural wave becomes to y axis direction, which is not the direction of maximum horizontal terrestrial stress. It is the sphere stress state when $\eta = 1.0$. In this condition, for isotropic formation, the flexural wave dispersion curves do not intersect [Li, Yin and Su (2006)]. Whereas for anisotropic formation, from Figure 8d, the crossover of dispersion curves becomes more obvious. Furthermore, with the frequency increase, the azimuth of fast flexural wave changes from 90° to 0°, which is exactly contrary to the case of $\eta = 0$. From above discussion, we find that for intrinsic anisotropic formation, the dispersion curves of flexural waves may not intersect even under the unequal horizontal terrestrial stress field, whereas it is still possible to observe the crossover of flexural wave dispersion curves under the equal horizontal terrestrial stress field. Moreover, the polarized direction of the low-frequency fast flexural wave is no longer consistent with the direction of the maximum horizontal terrestrial stress all the time. The above phenomenon is the unique feature of the stressed intrinsic anisotropic formation.

For slow TI formation, setting $S_x = -10$MPa, the influence of η on crossover of flexural wave dispersion curves is shown in Figure 9. According to Figure 9, we can find that with η increase from 0 to 0.8, the degree of intersection of dispersion curves decrease gradually, and the polarized direction of the low-frequency fast flexural wave always coincides with the direction of the maximum horizontal terrestrial stress. While when $\eta = 1.0$, the flexural wave merely splits and their dispersion curves do not coincide or intersect. That is the result of combined effect of the formation intrinsic anisotropy and the stress induced anisotropy, which is also completely different with the case of isotropic formation [Li, Yin and Su (2006)].

Base on the previous analysis, the conclusion can be made that in stressed intrinsic anisotropic formation, the alteration in phase velocity of flexural wave is controlled by two factors, one is the intrinsic formation anisotropy and the other is the stress-induced anisotropy, which also can be seen from equation (30). The two factors all can cause the flexural wave split, but only the second factor result in the intersection of dispersion curves. The combined effect of the two factors could strengthen or weaken the phenomenon of intersection of flexural wave dispersion curves. Therefore, the dispersion curves of flexural waves may not intersect even under the unequal horizontal terrestrial stress field, whereas it is still possible to observe the crossover of the flexural wave dispersion curves under the equal horizontal terrestrial stress field. The polarized direction of the low-frequency fast flexural wave is no longer consistent with the direction of the maximum horizontal terrestrial stress all the time.

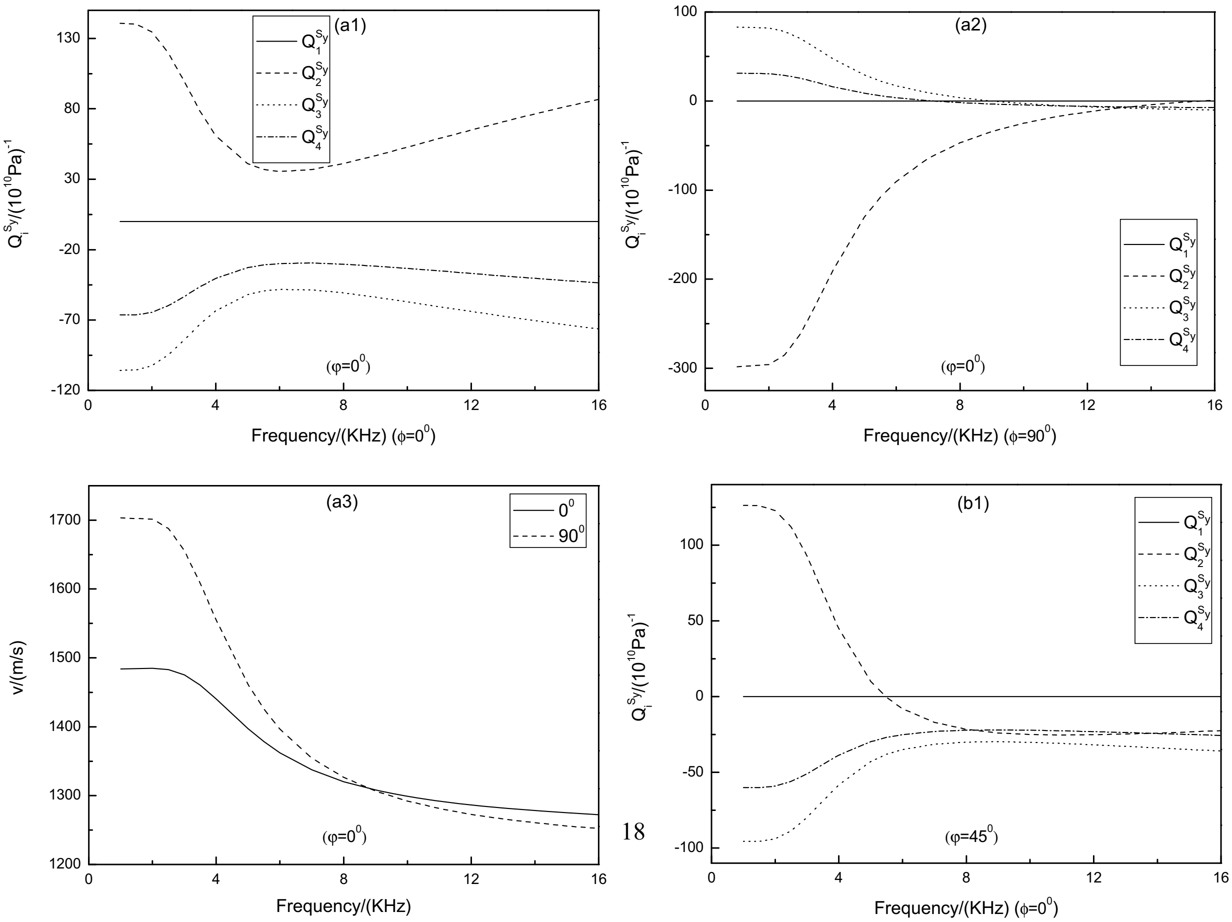

Figure 7

5 The acoustoelastic model and method to inversion terrestrial stress

The magnitude and the direction of the terrestrial stress have an important significance for the oilfield development and production. The research of the acoustoelastic effect on borehole mode induced by terrestrial stress is the theoretical foundation of the terrestrial stress nondestructive examination using cross-dipole sonic logging. The waveform of fast and slow flexural wave in time domain can be obtained from the 4-component (4-c) data in cross-dipole array acoustic logging. Based on it, the dispersion curves of the flexural waves polarized in two main directions can be also calculated. On the basis of the previous researches, the acoustoelastic inversion model and method for terrestrial stress inversion from borehole flexural wave dispersion curves obtained by cross-dipole sonic logging in stressed intrinsic anisotropic formation is simply discussed.

To determine terrestrial stress from flexural wave dispersion curves, we assume that the second-order elastic constant of the formation is known, and the third-order elastic module as well as the terrestrial stress is unknown quantities to be determined. The third-order elastic module is also determined while inversion terrestrial stress. A general anisotropic solid has 56 independent third-order elastic modules. While the actual formation is usually TI elastic solid, the number of the independent third-order elastic modules is much less than that of general anisotropic media. Thus, we can assume the number of the independent third-order elastic modules according to practical requirements. While discussing the method for terrestrial stress inversion, without losing generality, we assume it has 6 independent third-order elas-

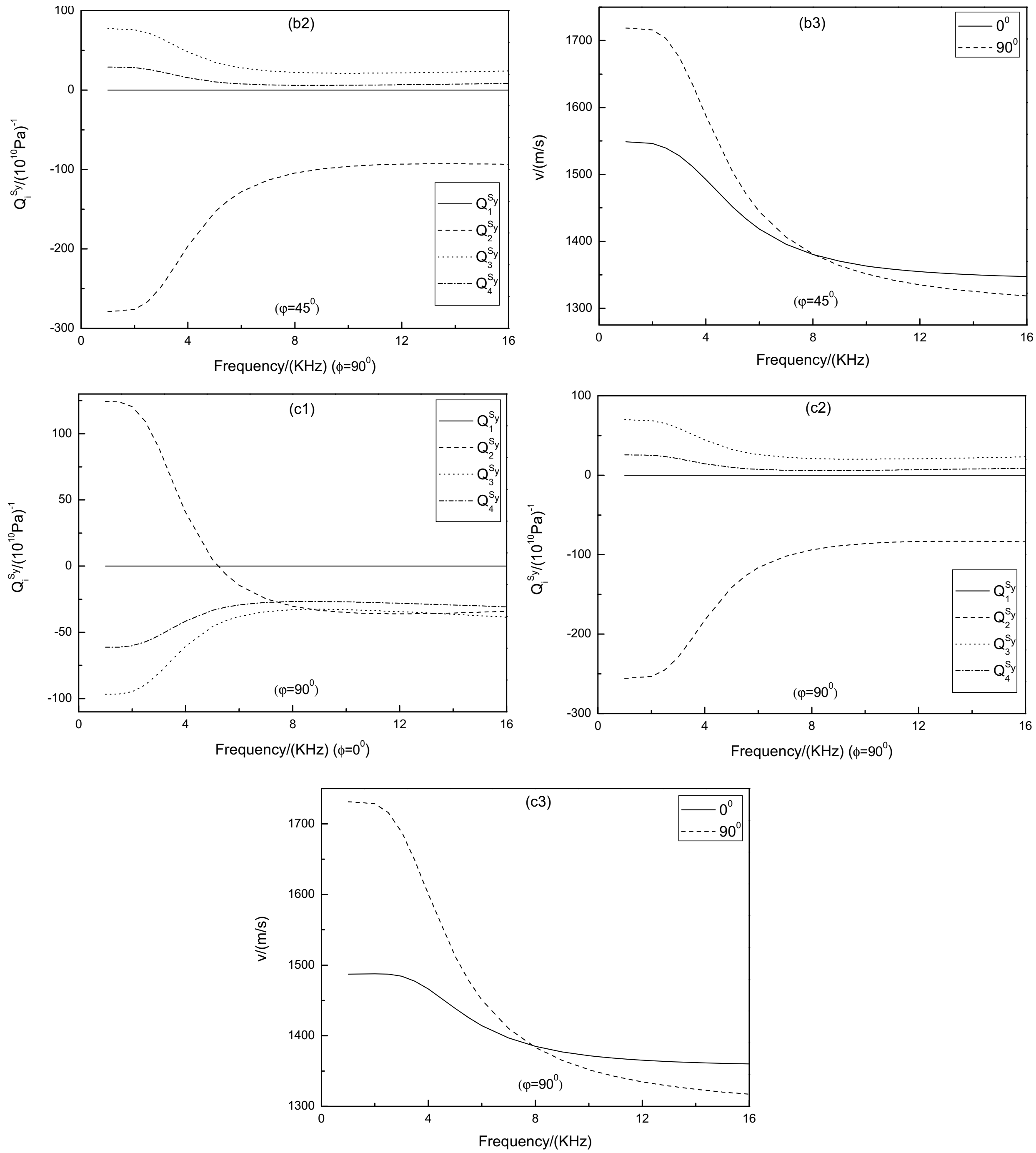

Figure 7: Flexural wave velocity-stress coefficients corresponding to and dispersion curves in fast TI formation when $S_x = 0$, $S_y = -0$MPa. (a) $\phi = 0$; (b) $\phi = 45°$; (c) $\phi = 90°$

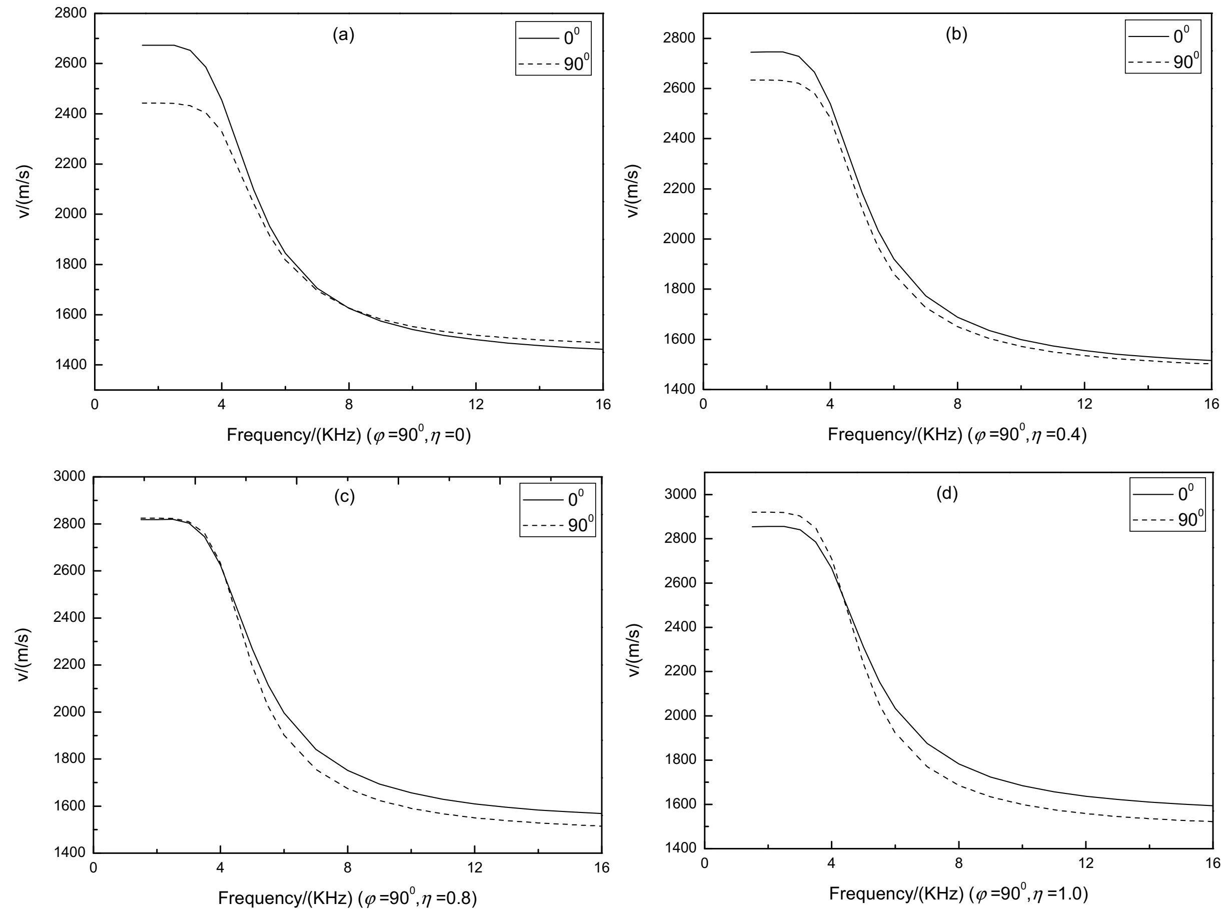

Figure 8: The influence of ratio of horizontal terrestrial stress on crossover of flexural wave dispersion curves in fast TI formation when $\phi = 90°$ (a) $\eta = 0$; (b) $\eta = 0.4$; (c) $\eta = 0.8$; (d) $\eta = 1.0$

tic modules of the formation here, namely c_{111}, c_{112}, c_{123}, c_{144}, c_{155} and c_{456}. There are 8 unknown quantities to be determined including two horizontal stresses S_x and S_y. Integration equation (28), we have the acoustoelastic equation as following form

$$\begin{aligned}\frac{\Delta v}{v_R^m} &= \frac{v^m - v_R^m}{v_R^m} = \frac{\Delta v_{\text{ani}} + \Delta v_{\text{str}}}{v_R^m} = \frac{\Delta v_{\text{ani}}}{v_R^m} \\ &+ \left(C_1^{S_x} + \frac{C_2^{S_x} c_{111}}{c_{66}} + \frac{C_3^{S_x} c_{112}}{c_{66}} + \frac{C_4^{S_x} c_{123}}{c_{66}} \right. \\ &\left. + \frac{C_5^{S_x} c_{144}}{c_{66}} + \frac{C_6^{S_x} c_{155}}{c_{66}} + \frac{C_7^{S_x} c_{456}}{c_{66}} \right) S_x \\ &+ \left(C_1^{S_y} + \frac{C_2^{S_y} c_{111}}{c_{66}} + \frac{C_3^{S_y} c_{112}}{c_{66}} + \frac{C_4^{S_y} c_{123}}{c_{66}} \right. \\ &\left. + \frac{C_5^{S_x} c_{144}}{c_{66}} + \frac{C_6^{S_x} c_{155}}{c_{66}} + \frac{C_7^{S_x} c_{456}}{c_{66}} \right) S_y \end{aligned} \quad (32)$$

Based on the equation (32), the specific computation procedures of the method to inverse terrestrial stress from flexural wave dispersion curves is:

(1) Acquiring the waveform of the fast and slow flexural main wave from the 4-component data in cross-dipole array acoustic logging, and then, the dispersion curves of the flexural waves polarized in two main directions are calculated. Selecting 8 points in dispersion curves as the computational points, get the frequency and flexural wave phase velocity v^m for each computational point;

(2) Computing the flexural wave phase velocity v_R^m in unperturbed reference state for each computational point, obtain the alteration in phase velocity caused by terrestrial stress and formation intrinsic anisotropy $\Delta v = \Delta v_{\text{str}} +$

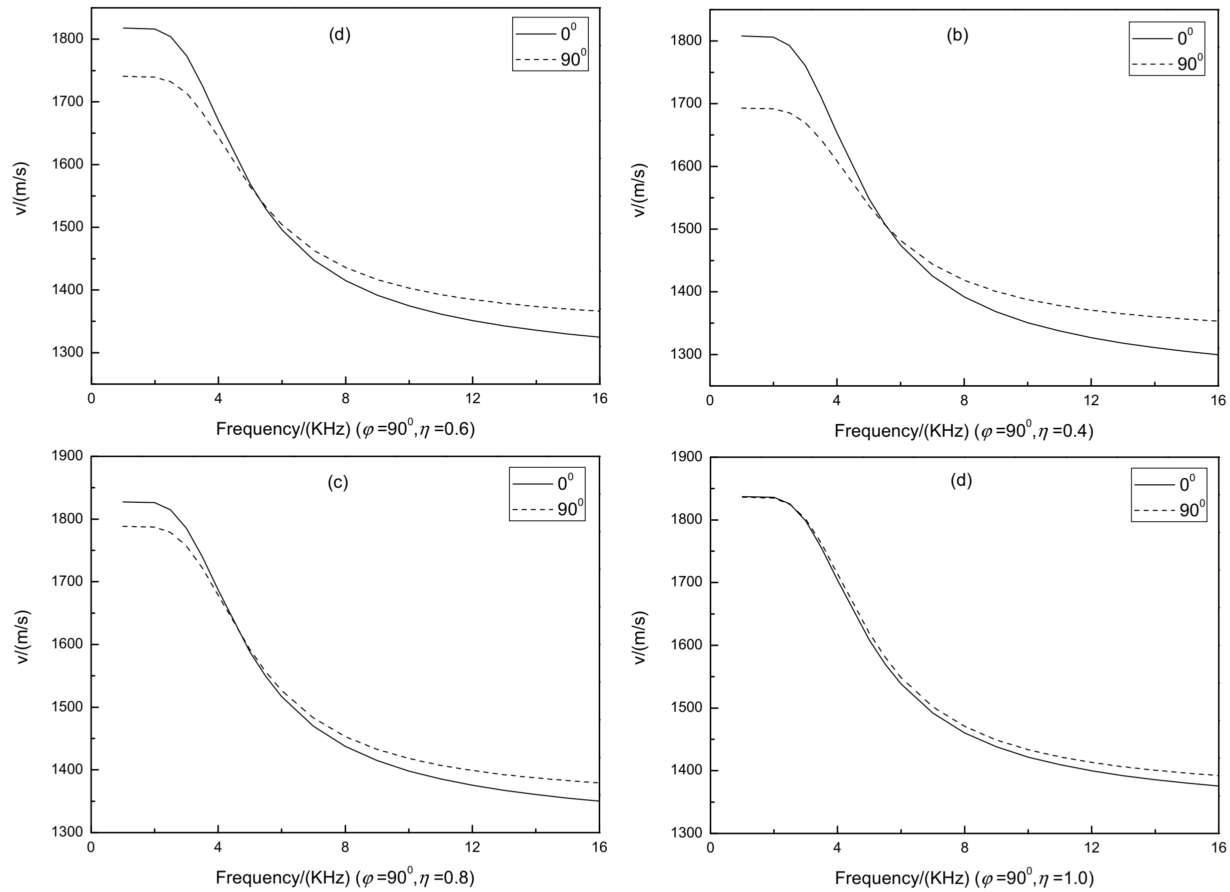

Figure 9: The influence of ratio of horizontal terrestrial stress on crossover of flexural wave dispersion curves in slow TI formation when $\phi = 90^\circ$ (a) $\eta = 0$; (b) $\eta = 0.4$; (c) $\eta = 0.8$; (d) $\eta = 1.0$

$\Delta v_{\text{ani}} = v^m - v_R^m$;

(3) Calculating the sensitive coefficients $C_i^{S_x}, C_i^{S_y}, (i = 1,...,7)$ and the alteration in phase velocity caused by formation anisotropy itself Δv_{ani} for each computational point using equation (28);

(4) According to equation (32), 8 independent equations describing the variation of flexural wave phase velocity with 8 undetermined quantities can be established. Solving the equation, the 2 horizontal terrestrial stresses and the 6 third-order elastic modules can be obtained.

In intrinsic anisotropic formation, because the direction of the maximum horizontal terrestrial stress may be inconsistent with the polarized direction of low-frequency fast flexural wave, in order to get the proper direction of the maximum horizontal terrestrial stress, the forward calculation is carried out assuming the polarized direction of low-frequency fast flexural wave is the direction of the maximum horizontal terrestrial stress after obtaining the magnitudes of the 2 horizontal terrestrial stresses and the 6 independent third-order elastic modules. If the computed flexural wave dispersion curves coincide with those of actual acoustic logging, which means the assumed direction is correct, otherwise, the direction of the maximum horizontal terrestrial stress is the polarized direction of low-frequency slow flexural wave.

6 Conclusion

Some foundational problems in cross-dipole sonic logging in stressed intrinsic anisotropic formation are studied in this paper. Adopting the Stroh theory, nonlinear theory of acoustoelasticity and perturbation method, the influence of the anisotropy of formation itself and induced by terrestrial stress on dispersion curves of borehole flexural waves is discussed. The results indicate that for stressed intrinsic anisotropic formation, the variation of flexural wave phase velocity is dominated by the formation intrinsic anisotropy and the terrestrial stress-induced anisotropy jointly, which makes the relation between the intersection of flexural wave dispersion curves and the terrestrial stress completely different with the case of intrinsic isotropic formation. Even under the unequal horizontal terrestrial stress field, the dispersion curves of flexural waves may not intersect, whereas it is still possible to observe the intersection of flexural wave dispersion curves under the equal horizontal terrestrial stress field. Moreover, the polarized direction of the low-frequency fast flexural wave is no longer consistent with the direction of the maximum horizontal terrestrial stress all the time. Based on the above researches, the terrestrial stress inversion method from flexural wave dispersion curves obtained by cross-dipole sonic logging in intrinsic anisotropic formation is simply discussed.

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