 CMC, vol.10, no.2, pp.139-161, 2009

Influence of Temperature and High Electric Field on Power Consumption by Piezoelectric Actuated Integrated Structure

Deepak A Apte[1] **and Ranjan Ganguli**[1,2]

Abstract: The influence of electric field and temperature on power consumption of piezoelectric actuated integrated structure is studied by using a single degree of freedom mass-spring-damper system model coupled with a piezoactuator. The material lead zirconate titanate, is considered as it is capable of producing relatively high strains (e.g., $3000\mu\varepsilon$). Actuators are often subject to high electric fields to increase the induced strain produced, resulting in field dependant piezoelectric coefficient d_{31}, dielectric coefficient ε_{33} and dissipation factor δ. Piezostructures are also likely to be used across a wide range of temperatures in aerospace and undersea operations. Again, the piezoelectric properties can vary with temperature. Recent experimental studies by physics researchers have looked at the effect of high electric field and temperature on piezoelectric properties. These properties are used together with an impedance based power consumption model. Results show that including the nonlinear variation of dielectric permittivity and dissipation factor with electric field is important. Temperature dependence of the dielectric constant also should be considered.

1 Introduction

Smart structures typically involve the actuation of operating structures using smart materials and their control. Piezoelectric materials [Cheng and Chen (2004), Wu, Lo and Chao (2005), Wu and Syu(2006), Singh, Rokne and Dhaliwal(2008)], shape memory alloys [Auricchio, Petrini, Pietrabissa and Sacco (2003), and magnetostrictives [Zhou, Zhou and Zheng (2007)] are used as sensors and actuators in smart structures. Piezoelectric materials such as lead zirconate titanate (PZT) have the capability of undergoing strain on the application of an electric field [Shi and Atluri

[1] Department of Aerospace Engineering, Indian Institute of Science, Bangalore.
[2] Corresponding author. Email: ganguli@aero.iisc.ernet.in, Tel: 91-80-22933017, Fax: 91-80-23600134

(1990), Im and Atluri (1989), Crawley and Deluis (1987), Crawley and Lazarus (1991)].

The piezoelectric actuated integrated structures are among the most important 'smart' and 'adaptive' structures and are being studied for applications in diverse systems ranging from smart skins for submarines, controlling large space structures, active control of helicopter and aircraft vibrations and other fields [Chopra (2002), Umesh and Ganguli (2009), Thakkar and Ganguli (2004), Beom and Atluri (2003)]. A recent review of analysis of piezoelectric plates and shells is given by [Wu, Chiu and Wang (2008)].

Considerable work has been done in the computational modeling of piezoelectric materials in recent years. [Arockiarajan and Menzel (2007)] used a micromechanics model to study rate dependant properties of piezoelectric materials. A coupled three dimensional finite element was used and rate dependant properties under cyclic electrical and mechanical loading were studied. The model proposed in their study provides insight into rate dependant behavior of piezoelectric materials observed in experiments. In another micromechanical approach, [Jayabal et al (2008)] captured nonlinear dissipative effects in polycrystal ferroelectric. Taking into consideration of all the domain switching possibilities, they were able to improve the modeling and response of polycrystal ferroelectric under electromechanical loading condition.

Several researches have investigated piezoelectric structures. [Wu and Chen (2007)] developed a two dimensional, self-similar formulation for piezoelectric materials. The method was used to derive explicit dynamic Green's functions and analytical results were obtained for hexagonal 6mm materials including quartz. [Dziatkiewicz and Fedelinski (2007)] used the dual reciprocating boundary element method to compute frequencies and mode shapes of two-dimensional piezoelectric structures. They considered the piezoelectric material to be homogenous, linear-elastic, transversely isotropic and dielectric. Numerical results were compared with analytical solutions from the literature. [Han et al (2005)] used an analytical-numerical method to study the effect of anisotropy and piezoelectricity on wave propagation. A multilayered piezoelectric plate was divided into a number of layered elements and analyzed. Coupling between elastic field and electric field was considered in each element. [Nguyen et al (2008)] developed a novel smoothed four-node piezoelectric element based on linear elastic analysis. A strain smoothing method of mesh-free conforming nodal integration was used. Several examples and comparisons with the published literature showed the the proposed element was computationally efficient and easy to implement.

The papers discussed till now typically ignored the dependance of piezoelectric coefficients on temperature and voltage and used linear analysis. [Sladek et al

(2007)] developed a meshless method based on the local Petrov-Galerkin approach to solve boundary value problems. Meshless methods have become a popular idea in computational mechanics [Sladek, Sladek and Atluri (2004), Han and Atluri (2004), Liu, Han, Rajendran and Atluri (2006), Li, Shu and Atluri (2008), Atluri, Liu and Han (2006)]. Transient dynamics was considered and time dependance was removed by Laplace transforms. This paper considered the fact that material properties of piezoelectric materials are influenced by thermal fields. [Saldek et al (2006)] also developed the meshless local Petrov-Galerkin method for plane piezoelectricity. [Wu et al (2008)] developed a meshless differential reproducing kernel particle (DRKP) method for static analysis of simply supported multilayered piezoelectric plates under electro-mechanical loads. [Wu and Liu (2007)] found exact solutions of simply supported, doubly curved functionally graded (FG) elastic and piezoelectric shells using a state space approach. The direct and converse effects on the static behavior of doubly curved, multilayered and FG piezoelectric shells was studied and the accuracy and convergence of the proposed state space approach was evaluated. [Wu and Syu (2006) found asymptotic solutions for multilayered piezoelectric hollow cylinder using the method of perturbations. The twenty-two basic equations of piezoelectricity were reduced to eight different equations in terms of eight primary variables of elastic and electric fields. It was shown that the asymptotic solutions approach 3D piezoelectric solutions. [Chen et al (2009)] used the regularized meshless method to solve antiplane piezoelectricity problems with multiple inclusions. They addressed a singularity problem by using the subtract and adding techniques.

We see that most studies on the computational modeling of piezoelectric structures have focussed on calculations of the displacements, stress, natural frequencies and mode shapes. The research effort is directed at improving these predictions, however, the important parameter of power consumed by the effects of variations is piezoelectric materials properties due to temperature and electric field which is important for applications is typically not addressed. This paper hopes to bring this important problem to the attention of computational modeling researchers.

Piezoceramic actuator is a transformer that converts electrical energy into mechanical energy. Apparent power is supplied to the piezoelectric actuator. Apparent power consists of two parts: real power or dissipative power or active power and reactive power. Dissipative power is converted into some other form of energy such as heat which can not be reused. The reactive energy however flows and remains within the system. When piezoelectric actuator are bonded to a structure, it allows the alteration of system characteristics as well as the system response. The amount of power needed to drive the piezostructure is a key design parameter. Thus actuator power consumption is a very important issue in the application and design of intelligent material systems and structures and other devices . However, far less research is focused on studying the power consumption of piezo actuated integrated structure compared to studies on the structural analysis of such structures [Ling et al (1997); Zhou and Rogers (1995); Ling et al (1996); Ling et al (1993)].

Piezoelectric ceramics such as Lead Zirconate Titanate, $\text{PbZ}r_{0.53}\text{T}i_{0.47}O_3$ i.e. PZT-5H are typically used for actuator applications. Unlike, sensor applications, actuator applications need a high electric field to maximize strain obtained. Also, PZT works under varying temperature conditions and is used in aircraft applications where temperature variation ranging from -150^o C to 100^o C are possible. Under these circumstances, material properties provided by manufacturers are no longer applicable to describe actuator performance since they were measured at low electric fields and neglect the effects of variation in temperature. Piezoelectric material exhibits nonlinearity with parameters such as applied voltage [Ling et al (1993); Li et al (1991); Wang et al (1999); Sirohi and Chopra (2000); Kugel and Cross (1998); Masys et al (2003)]. Classical literature assumes it as linear while calculating power consumption in electro-mechanical systems. Hence, at higher applied voltages electrical power requirements can be different from that given by a constant model. The piezoelectric strain coefficients also depends on temperature [Wang et al (2003)]. This fact is not included in most constitutive models of piezoelectric materials. The power supply of the smart structure must be able to cope with the demands of structure when it is in operation. If the power supplied is less than required, the active control algorithms will not be able to actuate properly and there can be a significant deterioration in the performance of the structure.

Some work [Ling et al (1997); Zhou et al (1995); Ling et al (1996); Ling et al (1993)] has been done for analyzing power consumption for piezoceramics. Here, power is calculated assuming constant strain coefficient with respect to electric field and temperature. In this paper, we carry out the analysis for power losses in bending and study the effect of high electric field and variations in temperature on power consumption. The analysis is validated for bending mode with constant coefficients, as results are available for that mode from previous studies [Ling et al (1997); Zhou et al (1995); Ling et al (1996); Ling et al (1993)].

2 Electro-mechanical admittance method

The governing constitutive equation [Sirohi and Chopra (2000)] for the PZT actuator can be given as

$$S_1 = d_{31}E_3 + \bar{s}_{11}\sigma_1 \tag{1}$$

$$D_3 = \bar{\varepsilon}_{33}E_3 + d_{31}\sigma_1 \tag{2}$$

Here, E_3 is the applied electric field, σ_1 is the stress, $\bar{s}_{11}$ is the complex compliance at zero electric field and is equal to $1/\overline{Y}_{11}$. $\overline{Y}_{11}$ is the complex modulus of PZT at zero electric field and is equal to $(Y_{11}(1+j\eta))$, where η is the mechanical loss factor of PZT, d_{31} is the piezoelectric coefficient, D_3 is the electrical displacement, S_1 is the strain, $\bar{\varepsilon}_{33}$ is the dielectric permittivity and is equal to $\varepsilon_{33}\ (1-tan\ \delta)$, where δ is the dielectric loss coefficient.

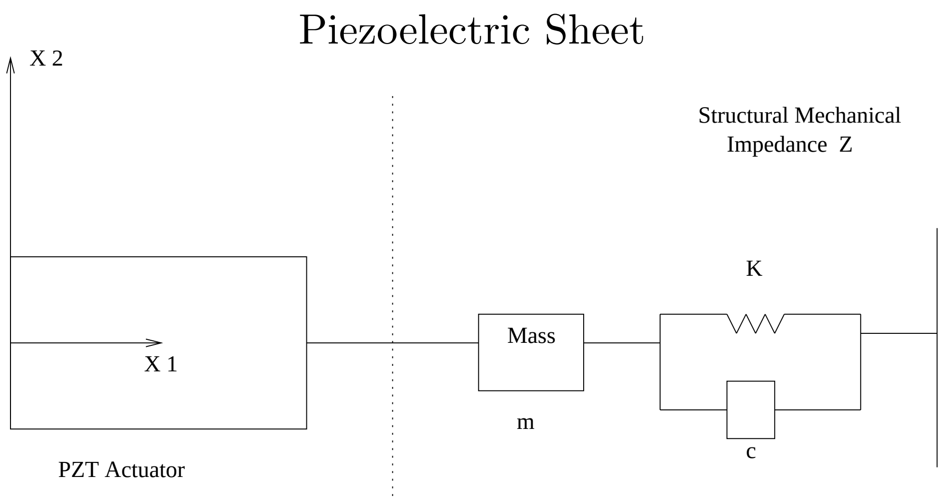

Figure 1: A Schematic of a PZT actuator-driven structure

Power requirement of the integrated structure (piezoelectric material and structure) as shown in Fig.1 can be calculated using the electro-mechanical impedance method derived using the constitutive Eq. (1) and Eq. (2) above. This model represents the structural dynamics using a single degree of freedom mass-spring-damper system and provides a simple model for the preliminary design of the power supply system. As is well known, the resonant natural frequency is a critical feature of a structural dynamics model [Young, Tsai, Lin and Chen (2006), Reutskiy (2005), Reddy and Ganguli (2007), Altenbach and Eremeyev (2009)]. The piezoelectric material is poled in the 3 direction and the electric field is applied in the 3 direction. The coupled electromechanical admittance for piezoelectric actuator bonded to structure in bending mode can be given [Ling et al (1997)] as,

$$Y = j\omega\frac{w_A l_A}{h_A}(\bar{\varepsilon}_{33} - \frac{Z}{Z_A+Z}d_{31}^2\overline{Y}_{11}) \tag{3}$$

Here, Z is impedance of the structure,

$$Z = c + m\frac{\omega^2 - \omega_n^2}{j\omega} \tag{4}$$

and m is the mass of an equivalent single degree of freedom system, c is the damping, ω_n is the natural resonant frequency of the system and ω is the excitation frequency. The actuator impedance Z_A is given as,

$$Z_A = \frac{K_A(1+\eta j)}{\omega}\frac{kl_A}{tan(kl_A)j} \tag{5}$$

Here, K_A is equal to $w_A h_A / l_A \bar{s}_{11}$, w_A, l_A, h_A are the width, length and thickness of the actuator, k is $\sqrt{\omega^2\rho/\overline{Y}_{11}}$ and ρ is the density of the PZT. Putting Eq. (4) and Eq. (5) in Eq. (3) and simplifying, we can write
$Y = Real(Y)$+ j $Imag(Y)$. Where,

$$Real(Y) = \frac{A}{B} \quad ; \quad Imag(Y) = \frac{C}{B}$$

where,

$$A = -(\omega w_A l_A^2 d_{31}^2 \overline{Y}_{11} tan(kl_A) K_A k(\omega^2 m\eta - m\omega_n^2\eta - c\omega)) \tag{6}$$

$$\begin{aligned} B = {} & (h_A(K_A^2\eta^2k^2l_A^2 + 2K_A\eta kl_A c\omega tan(kl_A) + c^2\omega^2 tan(kl_A)^2 + K_A^2k^2l_A^2 \\ & + 2K_A kl_A m\omega^2 tan(kl_A) - 2K_A kl_A m\omega_n^2 tan(kl_A) + m^2\omega^4 tan(kl_A)^2 \\ & - 2m^2\omega_n^2\omega^2 tan(kl_A)^2 + m^2\omega_n^4 tan(kl_A)^2)) \end{aligned} \tag{7}$$

$$\begin{aligned} C = {} & (\omega w_A l_A(\bar{\varepsilon}_{33}K_A^2\eta^2k^2l_A^2 + 2\bar{\varepsilon}_{33}K_A\eta kl_A c\omega tan(kl_A) + \bar{\varepsilon}_{33}c^2\omega^2 tan(kl_A)^2 \\ & + \bar{\varepsilon}_{33}K_A^2k^2l_A^2 + 2\bar{\varepsilon}_{33}K_A kl_A m\omega^2 tan(kl_A) - 2\bar{\varepsilon}_{33}K_A kl_A m\omega_n^2 tan(kl_A) \\ & + \bar{\varepsilon}_{33}m^2\omega^4 tan(kl_A)^2 - 2\bar{\varepsilon}_{33}m^2\omega_n^2\omega^2 tan(kl_A)^2 + \bar{\varepsilon}_{33}m^2\omega_n^4 tan(kl_A)^2 \\ & - l_A d_{31}^2\overline{Y}_{11}c\omega tan(kl_A)K_A\eta k - \omega^2 d_{31}^2\overline{Y}_{11}c^2 tan(kl_A)^2 \\ & - \omega^2 l_A d_{31}^2\overline{Y}_{11} tan(kl_A)K_A km - \omega^4 d_{31}^2\overline{Y}_{11} tan(kl_A)^2 m^2 \\ & + 2\omega^2 d_{31}^2\overline{Y}_{11} tan(kl_A)^2 m^2\omega_n^2 + l_A d_{31}^2\overline{Y}_{11} tan(kl_A)K_A km\omega_n^2 \\ & - d_{31}^2\overline{Y}_{11} tan(kl_A)^2 m^2\omega_n^4)) \end{aligned} \tag{8}$$

Eq. (3) shows that admittance of the integrated structure depends upon impedance of the structure, impedance of the actuator, dimension of the piezoelectric actuator, piezoelectric coefficient, dielectric constant and dissipation factor. For a given piezostructure, the impedance of the structure and impedance of actuator are fixed.

3 Electrical power requirement of actuator

When a voltage V = V_o sin (ωt) is applied on the piezoelectric actuator, current I = I_o sin(ω t + ϕ) flows through the circuit, where ϕ is the phase between the current and voltage. V_0 and I_0 are the peak values of voltage and current respectively. Current depends upon the admittance of integrated structures when applied voltage V is constant. Positive sign of phase ϕ indicates that current leads the voltage. As PZT is capacitive in nature, ideally current should lead the voltage by 90 degrees. Admittance of the integrated structure as seen in Eq. (3) indicates a purely reactive load, but due to the losses taking place in dielectric material as well as in the structure, current will lead the voltage by an angle of less than 90 degree, but close to 90 degrees. Admittance will have two parts, real part and reactive part. Electrical power depends upon applied voltage and current flowing in the circuit. In other words, it depends upon voltage and admittance of the integrated structure. Hence, apparent power, real power and reactive power, can be calculated as follows.
Apparent Power (A):

$$A = VI = \frac{V_0^2}{2} * Y \tag{9}$$

Real Power (P)

$$P = Real(A) = \frac{V_o^2}{2} * Real(Y) \tag{10}$$

Reactive Power (Q)

$$Q = Imag(A) = \frac{V_o^2}{2} * Imag(Y) \tag{11}$$

The total power or apparent power can also be expressed in terms of real power and reactive power as

$$A = P + jQ \tag{12}$$

Thus, the magnitude of complex power or apparent power can be written as

$$|A| = \sqrt{P^2 + Q^2} \tag{13}$$

$$PowerFactor = PF = cos\phi = \frac{P}{|A|} \tag{14}$$

Here, V and I are the RMS values of voltage and current; Real power loss is caused because of three things, due to dielectric losses in the PZT (δ), mechanical losses in the PZT (η), and due to structural damping (ζ) related power loss in the structure. The real or dissipative power is converted into heat, sound and mechanical energy. More energy dissipation can increase the temperature considerably and can cause degradation in the piezoelectric properties [Chen et al (2003)]. The reactive power remains and flows within the system. The amount of reactive power required increases the size and weight of the amplifier [Chopra (2002)]. The apparent power is the power supplied to the piezoceramic actuators. As an example of an integrated structure, consider a PZT of length l_A = 5.08 cm, width w_A = 2.54 cm, and thickness h_A = 0.2 cm driving a one degree of freedom mass-spring-damper system. The natural frequency of the system is 500 Hz. Mass m is 2 kg, with a damping ζ of 0.01. The damping constant c is calculated using $c = 2\zeta m\omega_n$. The material properties of the PZT G-1195 are given in Table 1. These dimensions of the piezostructure and properties are taken from [Ling et al (1997)]. Fig. 2 shows the two components of admittance along with total admittance, and is identical to that given in [Ling et al (1997)]. Thus, the analysis for calculation of power by the piezostructure in [Ling et al (1997)] has been correctly implemented.

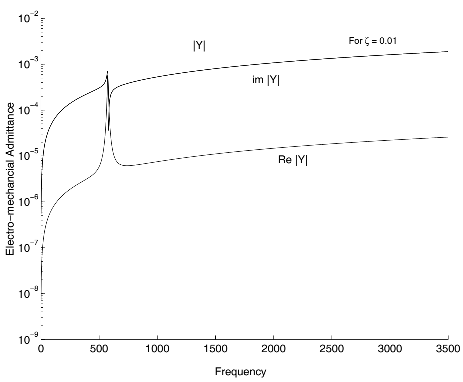

Figure 2: The coupled electromechanical admittance of the PZT driven mass-spring-damper actuator system with ζ= 0.01

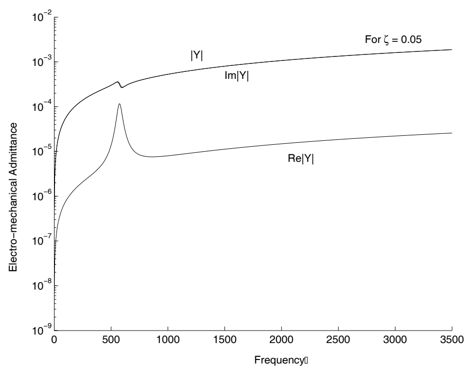

Figure 3: The coupled electromechanical admittance of the PZT driven mass-spring-damper actuator system with ζ= 0.05

From Fig.2 it can be observed that the real part of admittance is much lower than the imaginary part, except at resonance ($\omega = 500$ Hz), where the real part becomes

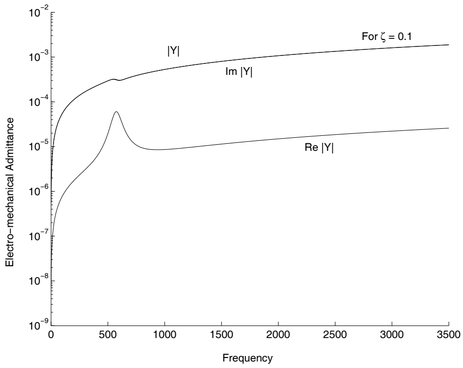

Figure 4: The coupled electromechanical admittance of the PZT driven mass-spring-damper actuator system with ζ= 0.1

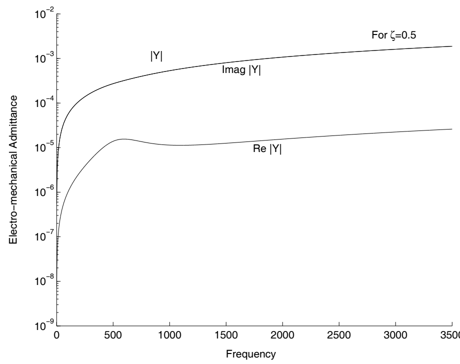

Figure 5: The coupled electromechanical admittance of the PZT driven mass-spring-damper actuator system with ζ= 0.5

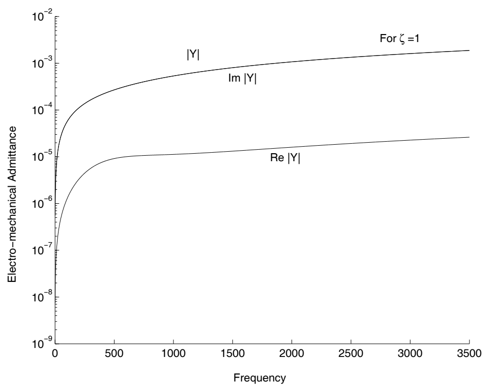

Figure 6: The coupled electromechanical admittance of the PZT driven mass-spring-damper actuator system with ζ= 1

comparable in magnitude to the imaginary part. Fig.3 to Fig.6 show the electromechanical admittance variation with frequency as the damping ratio increases from $\zeta = 0.05$ to $\zeta = 1$. With the increase in ζ the peaks in the admittance are smoothed and at $\zeta = 1$ the peaks disappear and the real admittance is much lesser than the imaginary admittance at all frequencies. It can be concluded that at resonance there is a fall in reactive power for low damped cases which means that compact and less bulky amplifiers can be used.

Table 1: Constant Baseline Material Properties of G1195 PZT

d_{32} $(m/volt)$	$-166*10^{-12}$
Y_{22} (N/m^2)	$6.3*10^{10}$
ρ (kg/m^3)	7560
ε $(farads/mr)$	$1.5*10^{-8}$
δ	0.012
η	0.001

4 Power consumption studies

In recent years, PZT-5H material has been extensively studied for actuator applications. This material has relatively large strain coefficient (e.g. $3000\mu\varepsilon$) compared to other piezoceramics. It is therefore a good candidate for applications requiring maximum displacements such as linear drive motors [Wang et al (1998)]. Future results in the paper are carried out using PZT-5H properties. Baseline material properties supplied by manufacturers are shown in Table 2. The structural properties remains same in either case, as given before (m = 2 Kg, ω_n = 500 Hz, ζ = 0.01). This analysis is carried out to observe the effect of temperature and electric field dependant coefficients of PZT-5H type material on power requirements of the integrated structure. The power consumption of the integrated structure depends upon admittance of the integrated structure, and applied voltage as can be observed from Eqs.(6)-(8). Electromechanical admittance depends upon piezoelectric coefficient (d_{31}), dielectric permittivity (ε_{33}) and dissipation constant (δ). It is seen from experiments that these parameters vary with applied electric field and temperature. Hence, the applied field and temperature conditions can also change the load on the power supply. The following sections show the effect of variation in electric field and temperature on power requirements of the integrated structure. At first, electrical power required to drive the system is calculated assuming constant coefficients which are supplied by manufacturers. The electrical power for various voltage levels is predicted. Later, constant parameters are replaced by polynomial fits to experimental data. These data have been obtained in past few years by physics researchers working on characterization of the properties of piezoceramics. Again, required driving electric power is predicted for various voltage levels and variation in electrical power requirement of load with respect to electric field is observed. Similar procedure is followed to see the effect of variation of temperature.

Table 2: Constant Baseline Material Properties of PZT-5H

$d_{32}\ (m/volt)$	$-288 * 10^{-12}$
$Y_{22}\ (N/m^2)$	$6.3 * 10^{10}$
$\rho\ (kg/m^3)$	7600
$\varepsilon\ (farads/mr)$	$301 * 10^{-10}$
δ	0.021
η	0.001

4.1 Effect of high electric field :

The effects of high electric fields on the piezoelectric coefficient d_{31}, dielectric permittivity ε_{33} and dissipation factor δ on power consumption by piezostructure is studied. These results assume constant temperature.

4.1.1 Piezoelectric coefficient (d_{31}) :

There exists a nonlinear relation between piezoelectric coefficient d_{31} and electric field which is experimentally proved but often ignored in analytical work. (Kugel and Cross (1998)), have highlighted nonlinearity in d_{31} for soft piezoelectric material PZT-5H. They investigated experimentally, the dependence of coefficient of d_{31} on electric field. Variation of d_{31} is plotted with respect to RMS value of amplitude of electric field as per the available data by experiment. This relation was observed to be nonlinear. Similarly, most recently in [Masys et al (2003)] have investigated electromechanical response of piezoceramic as the function of amplitude and frequency of applied voltage and studied the effect of dc bias field. Piezoceramic coefficient is calculated as a function of frequency and applied electric field. Experiments were done for both soft and hard PZT's. For both type of PZT it was observed that dependence of d coefficient over frequency (0.01-1000 HZ) is very less. But these coefficient shows large dependence on applied electric field. The following polynomial fit is obtained using experimental data for PZT-5H given in [Kugel and Cross (1998)].

$$k1 = 1 + 0.1013E_m + 0.4125E_m^2 - 0.3928E_m^3 + 0.1313E_m^4 \tag{15}$$

Here, E_m is KV_{rms}/cm and $k1$ indicates relative change in d_{31}^o. Also, d_{31}^o is the low field value as given in Table 2. The electrical power for various voltage level is predicted for constant d_{31} as well as nonlinear d_{31}, using the procedure described earlier. The d_{31} coefficient increases with electric field as shown in Fig. 7 where an increase of almost 40 percent takes place at an electric field of 1.4 KV/cm. As seen from Fig. 8 and Fig. 9, the real power or active power consumption remains almost same for both constant as well as d_{31} dependant model, but reactive power required for driving the electro-mechanical system as indicated by d_{31} voltage dependant model is less than that predicted by constant d_{31} model. At lower electric field ($<$ 600 V/cm), reactive power required for the load is same in either case, but as higher electric field ($>$ 600V/cm) is applied, reactive power required by the system as predicted by the models differs. Required reactive power at 1.38KV/cm electric field is less by 13 percent when field dependant model is considered. However, the important point to note here is that the effect of nonlinear d_{31} variation with the electric field is a fall in the power requirement with the electric field. This happens because the value of d_{31} increases at higher electric fields leading to greater induced strain ($d_{31}E$) compared to the constant d_{31} case. Thus, designing the power supply based on constant d_{31} is a conservative practice.

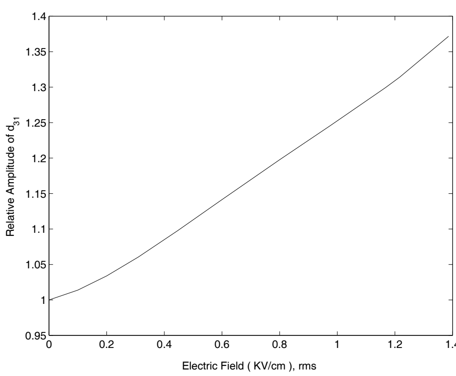

Figure 7: Electric field dependence of piezoelectric strain coefficient of PZT-5H

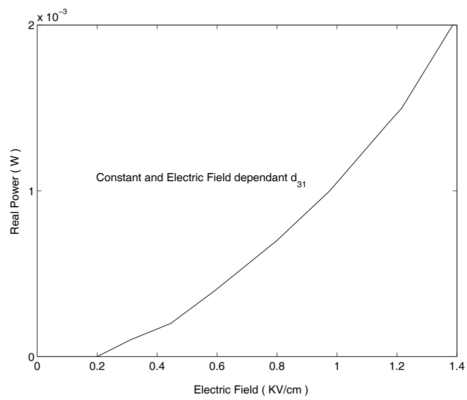

Figure 8: Variation of real power with respect to electric field with constant and field dependant piezoelectric coefficient

4.1.2 Dielectric permittivity (ε_{33})

Dielectric permittivity is the degree to which a medium resists the flow of electric charge, and is defined as the ratio of the electric displacement to the electric field strength. It is equal to the product of the relative dielctric permittivity of a substance (dielectric constant of material) and the permittivity of free space. Dielectric constant is an expression of the extent to which a material concentrates electric flux, and is the electrical equivalent of relative magnetic permeability. Materials with

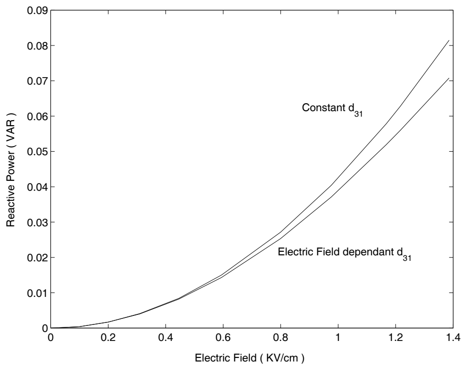

Figure 9: Variation of reactive power with respect to electric field with constant and field dependant piezoelectric coefficient

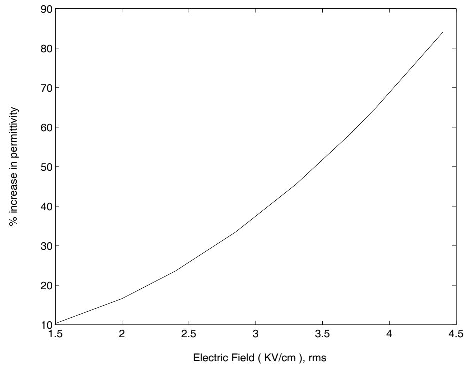

Figure 10: Electric field dependence of dielectric permittivity of PZT-5H

high dielectric constants result in the high-value capacitors. Electro-mechanical impedance of the integrated structure depends upon dielectric permittivity. It is found that dielectric constant is not constant as often assumed and it also varies as a function of electric field as shown in Fig. 10. Dielectric constant increases upto 80 percent on application of large electric field as stated by [Sirohi and Chopra (2000)]. The following polynomial fit is given in [Shirohi and Chopra (2000)] from experimental data.

$$k_e = 7.32 - 5.9754E_m + 5.3187E_m^2 \tag{16}$$

where, k_e is percent increase in the dielectric constant, E_m is the electric field is KV_{rms}/cm. Large value of dielectric constant results in increased value of capacitance. This increases the load on power supply as shown by Eq. (3). Required power is predicted by varying applied electric field. It is found that, the nonlinear nature of dielectric permittivity results in increased power requirements at higher electric field. It causes increase in reactive power as shown by Fig. 11. Therefore it can be expected that much greater power will be needed to drive the piezostructure at high electric fields than suggested by constant dielectric permittivity. Hence, power supply design should consider nonlinear variation in dielectric permittivity.

4.1.3 Dissipation factor (δ) :

An ideal capacitor is loss free. But in a real capacitor some energy losses always takes place and as a result current will lead the voltage by an angle less than 90 degrees but close to 90 degrees. These losses are due to conduction currents in the

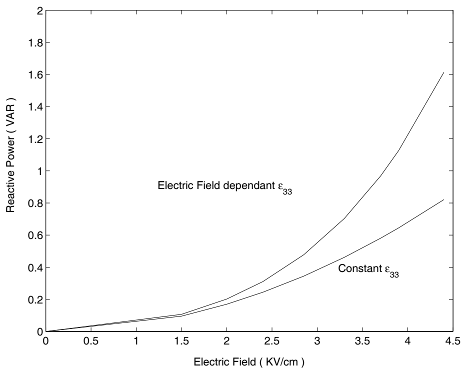

Figure 11: Variation of reactive power with respect to electric Field with constant and field dependant dielectric constant

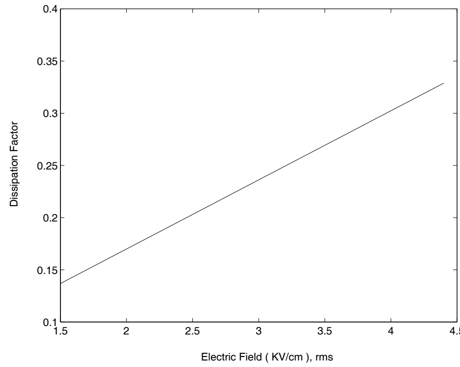

Figure 12: Electric field dependence of dissipation factor of PZT-5H

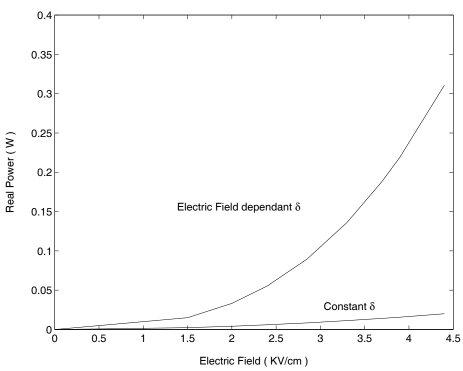

Figure 13: Variation of real power with respect to electric field with constant and field dependant dissipation constant

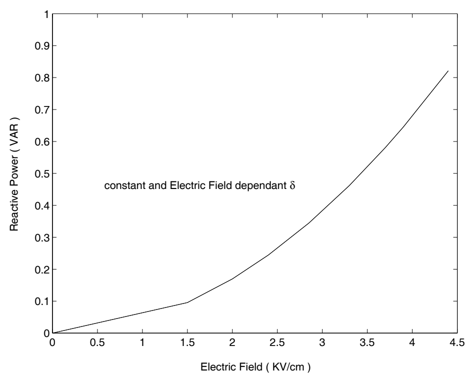

Figure 14: Variation of reactive power with respect to electric field with constant and field dependant dissipation constant

dielectric as well as molecular friction opposing the rotation of dielectric dipoles. This causes the current to lead the voltage by a phase angle δ of less than 90 degrees. The energy losses appear as ohmic heating in the shunt resistance. The dissipation factor tanδ is therefore a measure of the energy loss in the capacitor and consequently, the power consumed by the actuator. It is found from the experiments that the dissipation factor is not a constant as often assumed but varies linearly as a function of electric field as shown in Fig. 12. The dissipation factor

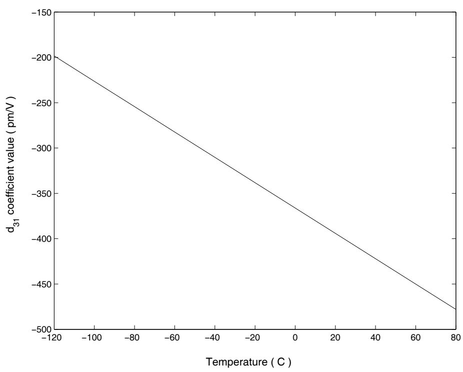

Figure 15: Temperature dependence of piezoelectric strain coefficients of PZT-5H

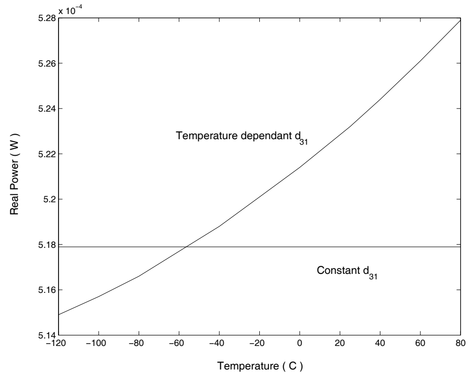

Figure 16: Variation of real power with respect to temperature with constant and field dependant piezoelectric constant

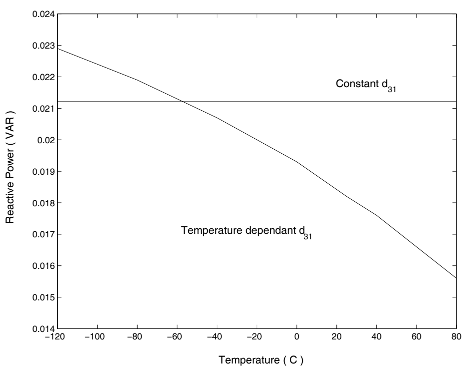

Figure 17: Variation of reactive power with respect to temperature with constant and piezoelectric constant

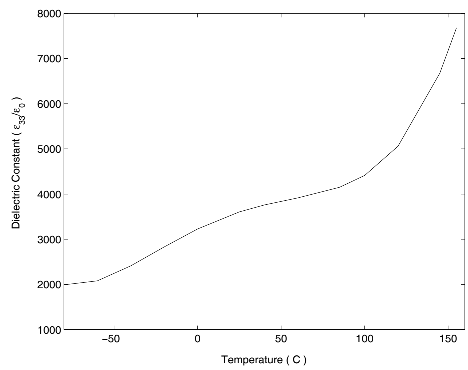

Figure 18: Temperature dependence of dielectric constant of PZT-5H

more than doubles as the electric field is increased from 1.5 KV/cm to 4.4 KV/cm. The following polynomial fit is proposed in [Shirohi and Chopra (2000)] using experimental data.

$$tan\delta = 0.0376 + 0.0662E_m \tag{17}$$

It can be observed from Fig 13 and Fig 14 that field dependant nature of dissipation factor affects active power demand on power supply, while reactive power remains unaffected. Variation in dissipation factor causes large change in power consumption at high electric field. At an electric field of 4.4 KV/cm, real power as predicted

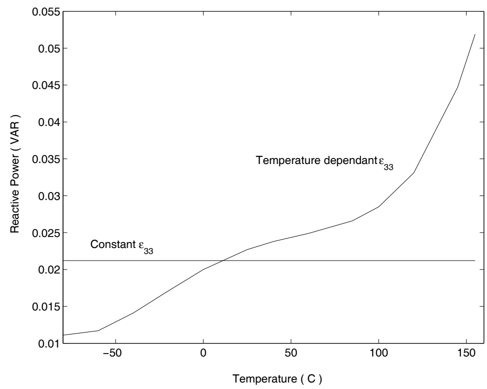

Figure 19: Variation of reactive power with respect to temperature with constant and field dependant dielectric constant

by constant model is 0.02 W but as per field dependant model its value is 0.31 W, indicating large difference in prediction of real power. Therefore, it is important to consider the field dependant nature of the dissipation factor when designing the power supply.

4.2 Effect of temperature :

PZT 5H shows temperature dependant behavior [Wang et al (1998)]. It is seen in recent experimental studies that piezoelectric strain coefficients depend upon temperature. Hence, temperature plays an important role in indicating the electromechanical response of piezoelectric materials. Many applications, involving smart material like PZT 5H have to work in diverse temperature conditions. Accepted constitutive relations for these materials do not include the influence of temperature on the electromechanical properties. Therefore, most analytical models are inappropriate when temperature is introduced as a design variable. In addition, the effect of temperature on power supply requirements of piezostructure needs to be investigated. The results below are obtained at high electric field and at low electric field for piezoelectric coefficient (d_{31}) and dielectric constant respectively (ε_{33}), respectively, based on the data available from [Wang et al (1998)].

4.2.1 Piezoelectric effect (d_{31}) :

The effect of temperature can be included by using a curve fit model. For these, using data given in (Wang et al (1998)), the following curve fit is obtained.

$$d_{31} = (-366.1869 - 1.3979 * T) \tag{18}$$

The temperature is varied between -120 degrees to 80 degrees centigrade as shown in Fig. 15. Note that the magnitude of d_{31} and therefore the induced strain obtained increases with temperature in a linear manner. The constant d_{31} model is not appropriate as electrical power predicted by such a model is constant at all temperature levels. Fig. 16 and Fig. 17 show that for temperature dependant d_{31} model, active power required for the system increases with an increase in temperature, while reactive power requirements of the system decreases with increase in temperature. At lower temperature, the active power predicted by the constant model is more, while reactive power is less than that predicted by temperature-dependant model. At higher temperature active power predicted by the constant model is less while reactive power predicted by constant model is more than that predicted by temperature dependant model. For real power, the difference between the temperature dependant model and the constant model is -0.45, 1.05 and 1.87 percent at -120 degrees, 25 degrees and 80 degrees, respectively. And for reactive power it is 6.14, -14.36 and -25.30 percent, respectively, at -120 degrees, 25 degrees and 80 degrees. Negative sign indicates that power predicted by temperature dependant model is less than that predicted by the constant model. It indicates that real power requirement of the system remains almost constant in the temperature range of -120 degrees to 80 degrees primarily because real power in piezostructures is much less than reactive power. At very low temperature reactive power requirement of the system is more than that predicted by constant model while in the temperature range of -70 to 80 degrees, temperature variation results in fall in reactive power requirement of system.

4.2.2 Dielectric permittivity (ε_{33}) :

Dielectric permittivity is also a temperature dependant parameter. Experimental data shows that relative permittivity (dielectric constant of material) also varies linearly with temperature up to 120 degrees. Above that temperature, the relative permittivity increases at a very rapid rate due to the impending phase transformation at the Curie temperature of 190 degrees. It is necessary to include the effect of temperature on permittivity. A curve fit showing variation of relative permittivity with temperature is obtained using available data [Wang et al (1998)].

$$\begin{aligned} \varepsilon_{33} &= 3227.49 + 18.11 * T - 0.1086035 * T^2 \\ &- 0.78142667 * 10^{-3} * T^3 + 0.1241328 * 10^{-4} * T^4 \end{aligned} \tag{19}$$

The temperature is varied between -100 degrees to 150 degrees as shown in Fig. 18. Constant ε_{33} model does not take into account effect of temperature. The power predicted by such a model is constant at all temperature levels. Temperature dependant model shows that, increase in temperature causes increase in power requirement of actuator. Fig. 19 shows that the reactive power requirement of the integrated system increases with increase in temperature. The demand for power increases rapidly above 120 degrees temperature. At lower temperature, the power required is less then that predicted by constant model, and at higher temperature power required is more than that predicted by constant model. For real power, the difference between temperature dependant model and constant model is -52, -14 and 81.98 percent at temperatures of -80 degrees, 25 degrees and 150 degrees respectively. For reactive power, the difference is -47.67, 7 and 144.65 percent respectively. This indicates that variation in dielectric constant of material with respect to temperature results in large change in real and reactive power requirements of the system. Hence, while designing power supply for such a system, it is important to consider nonlinear variation of dielectric constant with respect to temperature.

5 Conclusion

The influence of temperature and high electric field on power consumption of a piezoelectric actuated structure is considered in this study. Experimental data from recent work on the physics of piezoceramics is used in conjunction with a power consumption model of a piezoceramic actuating a single degree of freedom spring-mass-damper system. The following conclusions are drawn from these studies:

1. Nonlinear variation of piezoelectric coefficient with electric field has negligible effect on real power but reduces the reactive power required because of higher induced strain ($d_{31}E$) produced by large value of d_{31} at higher electric field. Use of the constant d_{31} coefficients therefore overpredicts the power required and is a conservative practice.

2. Nonlinear variation of dielectric permittivity with electric field results in considerable increase in reactive power.

3. Field dependant variation in dissipation factor substantially effects the real power while reactive power does not change.

4. Temperature variation of piezoelectric coefficient effects both real and reactive power but in opposite directions. But, variation in real power is very small. Constant d_{31} model overpredicts the reactive power at high temperatures and its use is a conservative practice.

5. Temperature variation of dielectric constant results in decrease in real and

reactive power requirements at very low temperatures while at higher temperatures, both real and reactive power increases substantially.

It is therefore possible to conclude that the effect of electric field and temperature on power consumption of piezostructures is important and should be considered for design of power supply. In addition, the temperature and electric field dependence of piezoelectric constants shown in this paper can be easily integrated into computational studies on piezostructures which can lead to more accurate analysis and insights into this design.

Reference

Altenbach, H.; Eremeyev, V, A. (2009): Eigen-vibrations of Plates made of Functionally Graded Material. *CMES: Computer Modeling In Engineering and Sciences*, vol. 9, no. 2, pp. 153-178.

Arockiarajan, A.; Menzel, A.(2007): On the modelling of rate-dependent domain switching in piezoelectric materials under superimposed stresses. *CMES: Computer Modeling in Engineering and Sciences*, vol. 20, no. 1, pp. 55-72.

Atluri, S, N.; Liu, H, T.; Han, Z, D. (2006): Meshless Local Petrov-Galerkin (MLPG) mixed collocation method for elasticity problems. *CMES: Computer Modeling In Engineering and Sciences*, vol. 14, no. 2, pp. 141-152.

Auricchio, F.; Petrini, L.; Pietrabissa, R.; Sacco, E.(2003): Numerical Modeling o Shape-memory Alloys in Orthodontics. *CMES: Computer Modeling In Engineering and Sciences*, vol. 4, no. 3-4, pp. 365-380.

Beom, H, G.; Atluri, S, N.(2003): Effect of electric fields on fracture behavior of ferroelectric ceramics. *Computers and Structures*, vol. 51, no. 6, pp. 1107-1125.

Chen, W, P.; Chong, C, P.; Chan, H, L, W.(2003): $Materials\ Science\ and\ Engineering\ B-solid\ Materials\ for\ Advanced\ Technology$, vol. 99, no. 1-3, pp. 203-206.

Chen, K, H.; Kao, J, H.; Chen, J, T.(2009): Regularized meshless method for antiplane piezoelectricity problems with multiple inclusions. *CMC: Computers Materials and Continua*, vol. 9, no. 3, pp. 253-280.

Cheng, C, H.; Chen, S, C.(2004): The simulation of diaphragm deflection actuated by shear mode piezoelectric actuator in microdroplet ejector. *CMC: Computers Materials and Continua*, vol. 1, no. 3, pp. 205-212.

Crawley, E, F.; DeLuis, J.(1987): Use of Piezoelectric actuators as elements of intelligent structures. *American Institute of Aeronautics and Astronautics Journal*, vol. 25, no. 10, pp. 1373-1385.

Crawley, E, F.; Lazarus, K, B.(1991): Induced strain actuation of isotropic and anisotropic plates. *American Institute of Aeronautics and Astronautics Journal*, vol. 29, no. 6, pp. 944-951.

Chih, W, P.; Chiu, K, H.; Wang, Y, M.(2008): A Differential Reproducing Kernel Particle Method for the Analysis of Multilayered Elastic and Piezoelectric Plates. *CMES: Computer Modeling in Engineering and Sciences*, vol. 27, no. 3, pp. 163-186.

Chopra, I. (2002): Review of state of art of smart structures and integrated systems. *AIAA Journal*, vol. 17, no. 1, pp. 35-46.

Dziatkiewicz, G.; Fedelinski, P.(2007): Dynamic analysis of piezoelectric structure by the dual reciprocity boundry element method. *CMES: Computer Modeling in Engineering and Sciences*, vol. 17, no. 1, pp. 35-46.

Han, X.; Ding, H.; Liu, G, R.(2005): Elastic waves in a hybrid multilayered piezoelectric plate. Source: *CMES: Computer Modeling in Engineering and Sciences*, vol. 15, no. 1, pp. 49-55.

Han, Z, D.; Atluri, S, N.(2004): A Meshless Local Petrov-Galerkin (MLPG) Approach for 3-Dimensional Elasto-dynamics. *CMC: Computers Materials and Continua*, vol. 2, no.2, pp. 129-140.

Han, F.; Pan, E.; Roy, A. K.; Yue, Z, Q.(2006): Response of piezoelectric, transversely isotropic, functionally graded, and multilayered half spaces to uniform circular surface loadings. *CMC: Computers Materials and Continua*, vol. 14, no. 1, pp. 15-29.

Im, S.; Atluri, S, N.(1989): Effects of a Piezo-Actuator on a Finitely Deformed Beam Subjected to General Loading. *American Institute of Aeronautics and Astronautics Journal*, vol. 27, no. 12, pp. 1801-1807.

Jayabal, K.; Arockiarajan, A.; Sivakumar, S, P.(2008): A Micromechanical Model for Polycrystal Ferroelectrics with Grain Boundary Effects. *CMES: Computer Modeling in Engineering and Sciences*, vol. 27, no. 2, pp. 111-124.

Kugel, V, D.; Cross, L, E.(1998): Behavior of soft piezoelectric ceramics under high sinusoidal electric fields. *Journal of Applied Physics*, vol. 84, no. 5, pp. 2815-2830.

Li, S, P.; Cao, W, W.; Cross, L, E.(1991): The Extrinsic Nature of Nonlinear Behavior Observed in Lead Zirconate Titanate Ferroelectric Cermnic. *Journal of Applied Physics*, vol. 69, no. 10, pp. 7219-7224.

Li, S.; Atluri, S, N.(2008): The MLPG Mixed Collocation Method for Material Orientation and Topology Optimization of Anisotropic Solids and Structures. *CMES: Computer Modeling in Engineering and Sciences*, vol. 30, no. 1, pp. 37-56.

Liu, H, T.; Han, Z, D.; Ranjendran, A, M.; Atluri, S, N.(2006): Computational modeling of impact response with the RG damage model and the Meshless Loacal Petrov-Galerkin (MLPG) approaches. *CMC: Computers Materials and Continua*, vol. 4, pp. 43-54.

Ling, C.; Sun, F, P.; Rogers, C, A.(1993): Impedence method for the dynamic analysis of active material systems. *Proc.34th SDM Conf.LaJolla, CA*, pp. 3587-3599.

Ling, C.; Sun, F, P.; Rogers, C, A.(1996): Electro-mechanical impedence modelling of active systems. *Smart Materials and Structure*, vol. 5, no. 2, pp. 171-186.

Ling, C.; Sun, F, P.; Rogers, C, A.(1997): An impedence method for dynamic analysis of active materials systems. *Journal of Intellingent Material Systems and Structures*, vol. 8 no. 4, pp. 323-334.

Masys, A, J.; Ren, W.; Yang, G.; et al.(2003): piezoelectric strain in lead zirconate titanate ceramics as a function of electric field. *Journal of Applied Physics*, vol. 94, no. 2, pp. 1155-1162.

Nguyen, H, V.; Mai, N, D.; Tran, T, C.(2008): A Smoothed Four-Node Piezoelectric Element for Analysis of Two-Dimensional Smart Structures. *CMES: Computer Modeling in Engineering and Sciences*, vol. 23, no. 2, pp. 209-222.

Reddy, K, V.; Ganguli, R.(2007): Fourier Analysis of Mode Shapes of Damaged Beams. *CMC: Computers Materials and Continua*, vol. 5, no. 2, pp. 79-98.

Reutskiy, S, Y.(2005): The Method of Fundamental Solutions for Eigenproblems with Laplace and Biharmonic Operators. *CMC: Computers Materials and Continua*, vol. 2, no. 3, pp. 177-188.

Sanz, J, A.; Solis, M.; Dominguez, J.(2007): Hypersingular BEM for piezoelectric solids:Formulation and applications for fracture mechanics. *CMES: Computer Modeling in Engineering and Sciences*, vol. 17, no. 3, pp. 215-229.

Shi, G.; Atluri, S, N.;(1990): Active Control of Nonlinear Dynamic-Response of Space-Frames Using Piezoelectric Actuators. *Computers and Structures*, vol. 34, no. 4, pp. 549-564.

Shing, B, M.; Rokne, J.; Dhaliwal.(2008): Electroelastic Problem of Two Antiplane Collinear Cracks at the Interface of Two Bonded Dissimilar Piezoelectric Layers. *SDHM: Structural Durability and Health Monitoring*, vol. 4, no. 2. pp. 95-106.

Sirohi, J.; Chopra(2000): Fundamental behavior of piezoelectric sheet actuators. *Journal of Intellingent Material Systems and Structures*. vol. 11 no. 1, pp. 47-61.

Sladek, J.; Sladek, V.; Atluri, S, N.(2004): Meshless Local Petrov-Galerkin Method in Anisotropic Elasticity. *CMC: Computer, Materials and Continua*, vol. 6, no. 3, pp. 477-490.

Sladek, J.; Sladek, V.; Zhang, Ch.; Garcia-Sanche, F.; Wunsche, M.(2006): Meshless Local Petrov-Galerkin Method for Plane Piezoelectricity. *CMC: Computer, Materials and Continua*, vol. 4, no. 2, pp. 109-118.

Sladek, J.; Saldek, V.; Zhang, C.; Solek, P.(2007) : Application of the MLPG to Thermo-Piezoelectricity. *CMES: Computer Modeling in Engineering and Sciences*, vol. 22, no. 3, pp. 217-234.

Thakkar, D.; Ganguli, R.(2004): Dynamic Response of Rotating Beams with Piexoceramic Actuation. *Journal of Sound and Vibration*, vol. 270, no. 4-5, pp. 729-753.

Umesh, K.; Ganguli, R.(2009): Shape and Vibration Control of a Smart Composite Plate with Matix Cracks. *Smart Materials and Structures*, vol. 18, no. 2, pp. 025002.

Wang, D.; Fotinich, Y.; Carman, G, P.(1998): Influnce of tempetrature on the electromechanical and fatigue behavior of piezoelectric ceramics. *Journal of Applied Physics*, vol. 83, no. 10, pp. 5342-5350.

Wang, Q, M.; Zhang, Q, M.; Xu, B, M.; et al.(1999): Nonlinear piezoelectric behavior of ceramic bending mode actuators under strong electrie fields. *Journal of Applied Physics*, vol. 86, no. 6, pp. 3352-3360.

Wu, K, C.; Chen, S, H.(2007): Two dimensional dynamic Green's functions for piezoelectric materials. *CMES: Computer Modeling in Engineering and Sciences*, vol. 20, no. 3, pp. 147-156.

Wu, C, P.; Chiu, K, H.; Wang, Y, M.(2008): A Differential Reproducing Kernel Particle Method for the Analysis of Multilayered Elastic and Piezoelectric Plates. *CMES: Computer Modeling in Engineering and Sciences*, vol. 27, no. 3, pp. 163-186.

WU, C, P.; Chiu, K, H.; Wang, Y, M.(2008): A Review on the Three-Dimensional Analytical Approaches of Multilayered and Functionally Graded Piezoelectric Plates and Shells. *CMC: Computer, Materials and Continua*, vol. 8, no. 2, pp. 93-132.

Wu, C, P.; Lo, J, Y.; Chao, J, K.(2005): A Three-Dimensional Asymptotic Theory of Laminated Piezoelectric Shells. *CMC: Computer, Materials and Continua*, vol. 2, no. 2, pp. 119-137.

Wu, C, P.; Liu, K, Y.(2007): A State Space Approach for the Analysis of Doubly Curved Functionally Graded Elastic and Piezoelectric Shells. *CMC: Computer, Materials and Continua*, vol. 6, no. 3, pp. 177-200.

Wu, C, P.; Syu, Y, S.(2006): Asymptotic Solutions for Multilayered Piezoelectric Cylinders Under Electromechanical Loads. *CMC: Computers Materials and Continua*, vol. 4, no. 2, pp. 87-108.

Yang, B.; Tewary, V, K. (2006): Efficent Green's function modeling of line and surface defects in multilayered anisotropic elastec and piezoelectric materials. *CMES: Computer Modeling in Engineering and Sciences*, vol. 15, no. 3, pp. 165-177.

Young, D, L.; Tsai, C, C.; Lin, Y, C.; Chen, C, S.(2006): The Method of Fundamental Solutions for Eigenfrequencies of Plate Vibrationn. *CMC: Computers Materials and Continua*, vol. 4, pp. 1-10.

Zhou, S, W.; Rogers, C, A.(1995): Power-Flow and Consumption in Piezoelectrically Actuated Structures. *American Institute of Aeronautics and Astronautics Journal*, vol. 33, no. 7, pp.1305-1311.

Zhou, H.; Zhou, Y.; Zheng, X.(2007): Numerical Simulation of Nonlinear Dynamic Responses of Beams Laminated with Giant Magnetostrictive Actuators. *CMC: Computers Materials and Continua*, vol. 6, no. 3, pp. 1-10.