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A General Magnetoelastic Coupling Theory of Deformable Magnetized Medium Including Magnetic Forces and Magnetostriction Effects

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Abstract: From the viewpoint of energy, a general magnetoelastic coupling theory including magnetic forces and magnetostriction effects is proposed for deformable magnetized medium. Firstly, a Taylor series expansion of independent variables of stress and magnetization in the elastic Gibbs free energy function is applied to obtain a polynomial expression; and then based on the magnetoelastic coupling mechanism, appropriate transcendental functions are substituted for some terms in a polynomial constitutive relationship derived by way of substituting the polynomial Gibbs free energy function in thermodynamic equations to achieve a more compact magnetostrictive constitutive relationship. The numerical simulation exhibits that the predicted magnetostrictive strain and magnetization curves under various pre-stresses are in good agreement with the experimental data given by Kuruzar et al (1971) and Jiles et al (1984). Secondly, based on the above magnetization constitutive relationship, a general magnetic forces expression is presented according to the variational principle for the total energy functional of the coupling system of the 3-d deformable magnetized materials. It is found that for the case of linear isotropic ferromagnetic materials, the magnetic forces expression can be degenerated into the Zhou-Zheng Model (1999). Combining the above nonlinear magnetostrictive constitutive relationship and magnetic forces expression, a general nonlinear magnetoelastic coupling theory is presented in this paper for deformable magnetized medium.

Keywords: Magnetoelastic coupling, Magnetic forces, Magnetostriction effects

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1 Introduction

No matter for ferromagnetic materials or giant magnetostrictive materials (GMM), an important functional material in smart material and structure field [Liu and Zheng (2005); Deepak and Ranjan (2009); Erdogan and Murat (2009); Chen, Kao and Chen (2009); Wu and Liu (2007); Wu, Chiu and Wang (2008); Arockiarajan and Menzel (2007)], there simultaneously exist the magnetic forces and magnetostriction effects when an external magnetic field is exerted on them. Most researches on GMM usually consider the magnetostriction effects only [Ghosh and Gopalakrishnan (2004)], while those on ferromagnetic materials mainly focus on the magnetic forces. In fact, Delaere et al (2002) indicated that the magnetic forces are the major cause of noises and vibrations in rotating electric machinery and magnetostriction effects are the principal cause of noises in non-rotating machinery. Although there were many researches about the magnetic forces [Zhou and Zheng (1996, 1997); Zhou, Zheng and Miya (1997)] or magnetostriction effects [Zhou and Zhou (2007); Zhou, Zheng and Zhou (2006); Zhou, Zhou and Zheng (2007); Jiang and Ding (2006)] in external magnetic filed, a general nonlinear magnetoelastic coupling theory simultaneously including magnetic forces and magnetostrctive interaction is necessary both for noise and vibration control of electric machinery and development of GMM device with high precision.

In earlier theoretical studies on magnetic forces and magnetostriction effects, the magnetostriction effects were reduced to an equivalent magnetostrictive force. For example, Pao (1978) and Reyne (1987) considered the magnetic susceptibility as a function of stress and presented an expression of magnetostrictive forces for deformable magnetized medium. Since the magnetostrictive forces contain the terms of derivative of magnetic susceptibility with respect to density, this expression cannot be applied in a numerical simulation. On the basis of the principle of virtual work, Besbes et al (1996) gave an expression of magnetic forces in which the term containing magnetic susceptibility that changes with stress represents the magnetostrictive force. In the study of Delaere et al (2000), the magnetostrictive force was simply treated as a thermal stress, and the magnetic forces and mechanic forces were added to calculate the magnetoelastic deformation. By means of applying a magnetostrictive stress tensor directly derived from the experiment result and the expression of magnetic forces deduced from Chu expression or Ampere expression, Vandevelde et al [Vandevelde and Melkebeek (2002); Vandevelde, Hilgert and Melkebeek (2004); Vandevelde and Melkebeek (2003); Vandevelde and Gyselinck et al (2004)] calculated the deformation of magneto-elasticity. Considering the limitation of the expression on the magnetostrictive forces, Hilgert et al (2005) proposed a calculative model for magnetic forces (i.e. Maxwell stress tensor) and magnetostriction effects (i.e. magnetostrictive strain directly derived from experiment result) in silicon steel sheets of electric machines. Since the complexity of magnetoelastic coupling problem and the effect of stress on both magnetization and strain, it is still not precise to use the magnetostrictive force as the equivalent of magnetostriction effects. Thus, Hirsinger and Billardon (1995) presented global (structural) couplings and local (material) couplings theory, but the magnetostrictive strain and the magnetization in their local couplings do not change when the stress changes. Vandevelde and Melkebeek (2001, 2002) suggested that the magnetostrictive constitutive relationship could be deduced from the free energy. However, they did not give any explicit expression of the constitutive relationship; instead, the magnetostrictive stress tensor was still used in their finite element method (FEM) code.

In fact, the experiment results [Kuruzar and Cullity (1971); Jiles and Atherton (1984)] have indicated that both the magnetostrictive strains and the magnetization are strongly coupled with the bias magnetic field and the pre-stress. And as known to all, the expression of magnetic forces is related with magnetization. Therefore the research simultaneously considering the magnetostriction effects and magnetic forces is difficult. In this paper, a more general nonlinear magnetostrictive constitutive relationship, in which the magnetostrictive strain and the magnetization change in accordance with the stress, will be established for deformable magnetized medium on the basis of thermodynamic theory. At the same time, based on the magnetization constitutive relationship, a general expression of magnetic forces is deduced from the variation of total magnetic energy to displacement. So, a general nonlinear magnetoelastic coupling theory simultaneously considering magnetic forces and magnetostriction is presented.

2 The Magnetostrictive Constitutive Relationship

Generally, the total differential of the internal energy density function $U(\varepsilon_{ij}, M_k, S)$ can be expressed as

$$dU = \sigma_{ij} d\varepsilon_{ij} + \mu_0 H_k dM_k + T dS \tag{1}$$

where σ_{ij} is stress, ε_{ij} is strain, M_k is magnetization, T is the temperature, S is the entropy density, and $\mu_0 = 4\pi \times 10^{-7}\mathrm{H/m}$ is the vacuum permeability. The elastic Gibbs free energy density function $G(\sigma_{ij}, M_k, T)$ is defined as

$$G(\sigma_{ij}, M_k, T) = U - TS - \sigma_{ij}\varepsilon_{ij} \tag{2}$$

Thus, its total differential can be written as

$$dG = -\varepsilon_{ij} d\sigma_{ij} + \mu_0 H_k dM_k - SdT \tag{3}$$

Ignoring the temperature change (i.e.$dT = 0$), one can get the following thermodynamic relations

$$\varepsilon_{ij} = -\frac{\partial G}{\partial \sigma_{ij}}, \quad \mu_0 H_k = \frac{\partial G}{\partial M_k} \tag{4}$$

To obtain polynomial constitutive relationships, the elastic Gibbs free energy function $G(\sigma_{ij}, M_k)$ is expressed as the Taylor series expansion of independent variables of stress σ_{ij} and magnetization M_k at the reference point $(\sigma_{ij}, M_k) = (0,0)$. Here, the constant term has been ignored since it does not make any contribution to the partial derivatives. And the linear terms will also disappear because $\varepsilon_{ij} = 0$, $H_k = 0$ when $\sigma_{ij} = 0$, $M_k = 0$. Furthermore, all the odd order terms of M_k will be zero because the magnetic field is always an odd function of the magnetization. Reserving only the coupling terms containing $M_k M_l$ and $M_k M_l M_i M_j$, such as $\sigma_{ij} M_k M_l$, $\sigma_{ij}\sigma_{mn} M_k M_l$, $\sigma_{ij} M_k M_l M_i M_j$, $\sigma_{ij}\sigma_{mn} M_k M_l M_i M_j$ and so on, we can obtain the polynomial expressions of strain ε_{ij} and magnetic field H_k from Eq. 4, as follows

$$\begin{aligned}
\varepsilon_{ij} = &-\frac{\partial^2 G}{\partial \sigma_{ij}\partial \sigma_{kl}}\sigma_{kl} - \frac{1}{2}\frac{\partial^3 G}{\partial \sigma_{ij}\partial \sigma_{kl}\partial \sigma_{mn}}\sigma_{kl}\sigma_{mn} + \cdots \\
&- \frac{1}{2}\left(\frac{\partial^3 G}{\partial \sigma_{ij}\partial M_k \partial M_l} + \frac{\partial^4 G}{\partial \sigma_{ij}\partial \sigma_{mn}\partial M_k \partial M_l}\sigma_{mn} + \cdots\right) M_k M_l \\
&- \frac{1}{4!}\left(\frac{\partial^5 G}{\partial \sigma_{ij}\partial M_k \partial M_l \partial M_i \partial M_j} + \frac{\partial^6 G}{\partial \sigma_{ij}\partial \sigma_{mn}\partial M_k \partial M_l \partial M_i \partial M_j}\sigma_{mn} + \cdots\right) M_k M_l M_i M_j
\end{aligned} \tag{5}$$

$$\begin{aligned}
\mu_0 H_k = &\frac{\partial^2 G}{\partial M_k \partial M_l} M_l + \frac{1}{3!}\frac{\partial^4 G}{\partial M_k \partial M_l \partial M_i \partial M_j} M_l M_i M_j + \cdots \\
&+ \left(\frac{\partial^3 G}{\partial \sigma_{ij}\partial M_k \partial M_l}\sigma_{ij} + \frac{1}{2}\frac{\partial^4 G}{\partial \sigma_{ij}\partial \sigma_{mn}\partial M_k \partial M_l}\sigma_{ij}\sigma_{mn} + \cdots\right) M_l \\
&+ \left(\frac{1}{6}\frac{\partial^5 G}{\partial \sigma_{ij}\partial M_k \partial M_l \partial M_i \partial M_j}\sigma_{ij} + \frac{1}{12}\frac{\partial^6 G}{\partial \sigma_{ij}\partial \sigma_{mn}\partial M_k \partial M_l \partial M_i \partial M_j}\sigma_{ij}\sigma_{mn} + \cdots\right) \\
&M_l M_i M_j
\end{aligned} \tag{6}$$

Based on some magnetoelastic coupling mechanisms, for example, on micro scale, magnetostriction and magnetization will originate from magnetic domain wall motion and rotation, and the macro effect is that magnetostrictive strain first increases to maximum value, then decreases until it becomes saturated, some transcendental functions are employed to substitute some polynomial terms in Eqs. 5 and 6 in order to obtain a compact expression with fewer constants as follows. The detailed deduction process can be found in the published paper of the authors [Zhou, Zhou and Zheng (2009)].

$$\varepsilon_{ij} = S^{(s)}_{ijkl}\sigma_{kl} + \lambda_{0ij}(\sigma_{mn}) + [m_{ijkl} - \frac{\lambda_{0ij}(\sigma_{mn})}{M^2_{\mathrm{ws}}}\delta_{kl}]M_k M_l - \frac{\theta_{ij} m_{ijkl}}{M^2_{ws}} M_k M_l M_i M_j \quad (7)$$

$$H_k = f_k^{-1}(M_l) - 2\mu_0^{-1}[m_{ijkl}\sigma_{ij} - \frac{\Lambda_0(\sigma_{mn})}{M^2_{\mathrm{ws}}}\delta_{kl}]M_l + 4\mu_0^{-1}\frac{\theta_{ij} m_{ijkl}\sigma_{ij}}{M^2_{ws}} M_l M_i M_j \quad (8)$$

where, $S^{(s)}_{ijkl}$ is the intrinsic compliance tensor; $\lambda_{0ij}(\sigma_{mn})$ is a nonlinear tensor function; the constant tensor m_{ijkl} is introduced to describe the free magnetostriction without a pre-stress; M_{ws} is the saturation domain wall motion magnetization when the pre-stress is zero; δ_{kl} is the Kronecker delta; θ_{ij} is a jump tensor function; $f_k(M_l)$ is a nonlinear vector function and $\Lambda_0(\sigma_{mn})$ is the primary function of $\lambda_{0ij}(\sigma_{mn})$.

For the convenience of engineering applications, the constant tensor and nonlinear vector/tensor functions can be further simplified in Eqs. 7 and 8, and the three dimensional model for an isotropic material will be derived from the general model, expressed as follows.

$$\varepsilon_{ij} = \frac{1}{E}[(1+\nu)\sigma_{ij} - \nu\sigma_{kk}\delta_{ij}] + \frac{\lambda_s}{M^2_{\mathrm{ws}}}[\frac{3}{2}M_i M_j - M_k M_k(\frac{1}{2}\delta_{ij} + \tilde{\sigma}_{ij}/\sigma_s)] - \frac{\lambda_s}{M^4_{\mathrm{ws}}}(\frac{3}{2}M_i^2 M_j^2 - \frac{1}{2}M_k^2 M_k^2 \delta_{ij}) \quad (9)$$

$$H_k = \{\frac{1}{kM} f^{-1}\left(\frac{M}{M_s}\right)\delta_{kl} - \frac{\lambda_s}{\mu_0 M^2_{\mathrm{ws}}}[2\tilde{\sigma}_{kl} - (\mathrm{I}^2_\sigma - 3\mathrm{II}_\sigma)\delta_{kl}/\sigma_s]\}M_l + \frac{\lambda_s(6\sigma_{kl} - 2\sigma_{mm}\delta_{kl})}{\mu_0 M^4_{ws}} M_l^3 \quad (10)$$

where, λ_s is the maximum magnetostrictive strain, E is elastic modulus, M_{ws} is the saturation magnetization when $\sigma = 0$, k is a relaxation factor, and $\tilde{\sigma}_{mn} = 3\sigma_{mn}/2 - \sigma_{kk}\delta_{mn}/2$ is $3/2$ times as much as the deviatoric part of the stress tensor σ_{mn}.

One dimensional rod is commonly used in engineering applications, and the nonlinear functions in Eqs. 7 and 8 are easy to select for one dimensional situation. So it's not difficult to get the following one dimensional nonlinear constitutive relation. The detailed deduction process is to found in the published paper by the authors [Zhou, Zhou and Zheng (2008)].

$$\varepsilon = \frac{\sigma}{E_s} + \begin{cases} \lambda_s \tanh(\frac{\sigma}{\sigma_s}) + [1 - \tanh(\frac{\sigma}{\sigma_s})]\frac{\lambda_s M^2}{M^2_{\mathrm{ws}}} - \frac{\theta\lambda_s(M^4 - M_0^4(\sigma))}{M^4_{\mathrm{ws}}} & \frac{\sigma}{\sigma_s} \geq 0 \\ \frac{\lambda_s}{2}\tanh(\frac{2\sigma}{\sigma_s}) + [1 - \frac{1}{2}\tanh(\frac{2\sigma}{\sigma_s})]\frac{\lambda_s M^2}{M^2_{\mathrm{ws}}} - \frac{\theta\lambda_s(M^4 - M_0^4(\sigma))}{M^4_{\mathrm{ws}}} & \frac{\sigma}{\sigma_s} < 0 \end{cases} \quad (11)$$

$$H=\frac{1}{k}f^{-1}\left(\frac{M}{M_s}\right)+\begin{cases}-\{\sigma-\sigma_s\ln[\cosh(\frac{\sigma}{\sigma_s})]\}\frac{2\lambda_s M}{\mu_0 M_{ws}^2}+\frac{4\theta\lambda_s\sigma(M^3-M_0^3(\sigma))}{\mu_0 M_{ws}^4} & \frac{\sigma}{\sigma_s}\geq 0\\ -\{\sigma-\frac{\sigma_s}{4}\ln[\cosh(\frac{2\sigma}{\sigma_s})]\}\frac{2\lambda_s M}{\mu_0 M_{ws}^2}+\frac{4\theta\lambda_s\sigma(M^3-M_0^3(\sigma))}{\mu_0 M_{ws}^4} & \frac{\sigma}{\sigma_s}<0\end{cases} \tag{12}$$

3 Generalized Magnetic Forces Expression

As shown in the general magnetostrictive constitutive relationships of deformable magnetized materials (see Eqs. 7 and 8), the magnetization and magnetostrictive strain are complex functions of stress and magnetic field. For the convenience of deducing the expression of magnetic forces, we can rewrite the Eq. 8 as $\mathbf{H}=\mathbf{M}/\chi_m$, and then the magnetic induction can be noted as $\mathbf{B}=\mu_m\mathbf{H}$. Obviously, susceptibility χ_m and permeability μ_m in this paper are functions of stress and magnetic field. i.e.$\chi_m=\chi_m(H,\sigma)$, $\mu_m=\mu_m(H,\sigma)$.

Introducing magnetic scalar potential ϕ, and ensuring $\mathbf{H}=-\nabla\phi$, the functional of the total energy of the magnetoelastic interaction for the deformable magnetic medium can be written as

$$\mathbf{W}_{mag}\{\phi,\mathbf{u}\}=\int_{\Omega^+(\mathbf{u})}\left(\int_0^{H^+}B^+dH^+\right)dv+\frac{1}{2}\int_{\Omega^-(\mathbf{u})}\mu_0(\nabla\phi^-)^2dv + \int_{\Gamma_0}\mathbf{n}_0\bullet\mathbf{B}_0\phi^-ds \tag{13}$$

Here, the superscripts "+" and "-" are used respectively to represent those variables in and out of the deformable magnetic medium bodies; $\Omega^+(\mathbf{u})$ and $\Omega^-(\mathbf{u})$ represent the regions in and out of the deformable magnetic medium bodies respectively; Γ_0 is the surface which encloses and is far away from the deformable magnetic medium; $\mathbf{n}_0$ is the unit vector for exterior normal.

The total magnetic force $\mathbf{F}$ is the change rate between magnetic energy and the displacement when the magnetic medium is undergoing an incremental displacement with the magnetic excitation held fixed, and its expression is as follows [Belahcen (2004)],

$$\mathbf{F}^T=-\frac{\delta W_{mag}}{\delta\mathbf{u}} \tag{14}$$

For the magneto-mechanics coupling system of magnetic medium, we first calculate the variation of total magnetic energy to displacement as follows,

$$
\begin{aligned}
\delta_{\mathbf{u}}\mathbf{W}_{mag}\{\phi,\mathbf{u}\} &= \left(\int_{\Omega^+(\mathbf{u}+\delta\mathbf{u})} - \int_{\Omega^+(\mathbf{u})}\right)(\int_0^{H^+} B^+ dH^+)dv \\
&\quad + \frac{1}{2}\left(\int_{\Omega^-(\mathbf{u}+\delta\mathbf{u})} - \int_{\Omega^-(\mathbf{u})}\right)\mu_0(\nabla\varphi^-)^2 dv \\
&= \int_{\Omega^+(\mathbf{u}+\delta\mathbf{u})\cap\Omega^-(\mathbf{u})}[\int_0^{H^+} B^+ dH^+ - \frac{1}{2}\mu_0(\nabla\varphi^-)^2]dv \\
&\quad + \int_{\Omega^-(\mathbf{u}+\delta\mathbf{u})\cap\Omega^+(\mathbf{u})}[\int_0^{H^+} B^+ dH^+ - \frac{1}{2}\mu_0(\nabla\varphi^-)^2]dv \\
&= \oint_S[\int_0^{H^+} B^+ dH^+ - \frac{1}{2}\mu_0(\nabla\varphi^-)^2]\mathbf{n}^+ \bullet \delta\mathbf{u}ds
\end{aligned}
\tag{15}
$$

The following Equation is used in the derivation process of Eq. 15.

$$
\{\Omega^+(\mathbf{u}+\delta\mathbf{u})\cap\Omega^-(\mathbf{u})\}\cup\{\Omega^-(\mathbf{u}+\delta\mathbf{u})\cap\Omega^+(\mathbf{u})\} = S\bullet(\mathbf{n}^+\bullet\delta\mathbf{u}) \tag{16}
$$

Considering the junction condition of magnetic field in boundary S

$$
\begin{cases}
B_n^+ = B_n^-, \quad H_\tau^+ = H_\tau^- \\
(H^+)^2 = (H_n^+)^2 + (H_\tau^+)^2, \quad (H^-)^2 = (H_n^-)^2 + (H_\tau^-)^2
\end{cases}
\tag{17}
$$

using Gauss integral formula and ignoring the variation of volume strain (i.e. $\nabla\cdot\delta\mathbf{u}\approx 0$), the Eq. 15 can be further simplified as follows,

$$
\begin{aligned}
\delta_{\mathbf{u}}\Pi_{em}\{\phi,\mathbf{u}\} &= \oint_S[\int_0^{H^+} B^+ dH^+ - \frac{1}{2}\mu_0(\mathbf{H}^-)^2]\mathbf{n}^+\delta\mathbf{u}ds \\
&= \oint_S\{\int_0^{H^+} B^+ dH^+ - \frac{1}{2}\mu_0[\frac{\mu_m^2}{\mu_0^2}(H_n^+)^2 + (H_\tau^+)^2]\}\mathbf{n}^+\delta\mathbf{u}ds \\
&= -\int_{\Omega^+}\nabla[\frac{\mu_m^2}{2\mu_0}(\mathbf{H}^+)^2 - \int_0^{H^+} B^+ dH^+]\delta\mathbf{u}dv \\
&\quad - \oint_S[-\frac{1}{2\mu_0}(\mu_m^2-\mu_0^2)(H_\tau^+)^2]\mathbf{n}^+\delta\mathbf{u}ds
\end{aligned}
\tag{18}
$$

Here, $H_n^+, H_n^-(B_n^+, B_n^-)$ and $H_\tau^+, H_\tau^-(B_\tau^+, B_\tau^-)$ are respectively the normal component and tangential component of magnetic field (magnetic induction) in the surface of medium.

Substituting Eq. 18 into Eq. 14, the generalized magnetic forces are as follows,

$$
\mathbf{f}^{em} = \nabla[\frac{\mu_m^2}{2\mu_0}(\mathbf{H}^+)^2 - \int_0^{H^+} B^+ dH^+] \tag{19}
$$

$$\mathbf{F}^{em} = [-\frac{1}{2\mu_0}(\mu_m^2 - \mu_0^2)(H_\tau^+)^2]\mathbf{n}^+ \tag{20}$$

where, $\mathbf{f}^{em}$ is the equivalent magnetic body force density and $\mathbf{F}^{em}$ is the equivalent surface force density acting on magnetic medium.

For a linear, homogeneous and isotropic magnetic medium, χ_m and μ_m are constant, so the magnetic body and surface force densities can be simplified as,

$$\mathbf{f}^{em} = \nabla[\frac{\mu_m^2}{2\mu_0}(\mathbf{H}^+)^2 - \frac{\mu_m}{2}(\mathbf{H}^+)^2] = \frac{\mu_0\mu_r\chi_m}{2}\nabla(\mathbf{H}^+)^2 \tag{21}$$

$$\mathbf{F}^{em} = -\frac{\mu_0\chi_m(\mu_r+1)}{2}(H_\tau^+)^2\mathbf{n}^+ \tag{22}$$

where μ_r is the relative permeability. It is obvious that this model (Eqs. 21 and 22) was in full accord with the magnetoelastic model proposed by Zhou and Zheng (1999).

4 Verification and Discussion

To solve the magnetoelastic coupling problem, some basic magnetic and mechanical equations should be included to form complete equations. For instance, the magnetic equations should include the Maxwell equations, the magnetization constitutive equation, the jump conditions on the interface of medium and magneto-mechanics coupling equations (i.e. the expression of magnetic forces); the mechanical equations should include the kinetic equation, the geometrical relationship, the constitutive relationship, the edge conditions and the initial condition. For magnetoelastic medium, the most concerned equations are the constitutive relationships and the expression of magnetic forces in those equations. And in this paper, the two general equations (i.e. Eqs. 7, 8, 19 and 20) are presented from the viewpoint of energy.

Firstly, the constitutive relationships derived in this paper is used to simulate the experimental data [Kuruzar and Cullity (1971)] of magnetostrictive strain versus magnetization at seven pre-stress levels for an iron rod. Considering the experimental conditions and results, we select $\lambda_s = 4.17\times10^{-6}$, $M_{ws} = 1.0\times10^6$A / m, $\mu_0 M_s = 1.885$T, $\chi_m = 215$ and $\sigma_s = 120$MPa. The predicted magnetostrictive curves under various pre-stresses by the constitutive model and experimental results are shown in Fig. 1. As shown in this figure, when the absolute value of pre-stress is small, the calculated results are consistent with the experimental data not only qualitatively but also quantitatively.

Then, the constitutive model presented in this paper is used to simulate the experimental data [Jiles and Atherton (1984)] on magnetization versus external magnetic

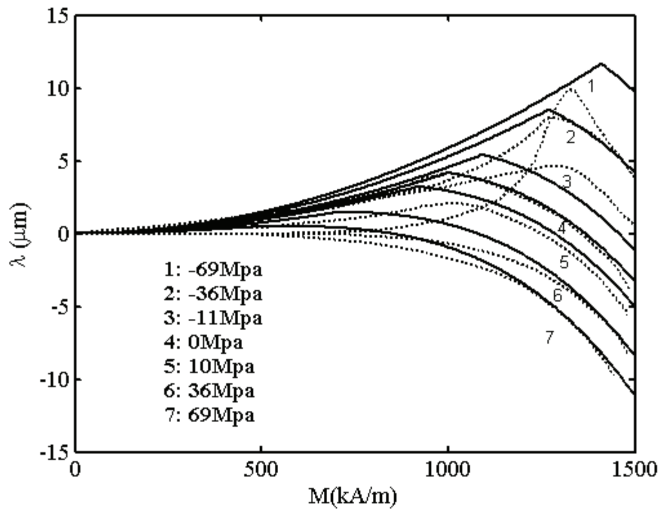

Figure 1: Comparison of the calculated magnetostrictive strain curves with the experimental data for soft ferromagnetic rods under seven different pre-stresses (Dashed lines: experimental ; solid lines: theoretical).

field at five pre-stress levels for an iron rod. Fig. 2 is the magnetic induction intensity curves of the rod at five pre-stress levels predicted by this constitutive model. From the comparison of the calculated magnetization curves with the experimental data of an iron rod under various pre-stresses (the dashed lines and the solid lines in Fig. 2 respectively), it can be found that the predicted curves and experimental curves are consistent both quantitatively and qualitatively. As shown in Fig. 1 and Fig. 2, the constitutive model adequately captures the nonlinear mechanical-magnetic coupling characteristics of magnetic, magnetostrictive and elastic properties of deformable magnetized medium.

For the expression of magnetic forces (Eqs. 19 and 20), we know that μ_m is a function of stress and magnetic field, and the calculation of magnetoelastic deformation of structure from Eqs. 19 and 20 is very difficult. But the magnetic body and surface force densities can be degenerated to the existing magnetic forces expression, i.e. Zhou and Zheng model [Zhou and Zheng (1999)] for a linear, homogeneous and isotropic ferromagnetic body, which can simulate two types of experiment results simultaneously. Some complex numerical simulation examples including magnetostrictive constitutive relationships (Eqs. 9 and 10) and magnetic forces (Eqs. 19 and 20) which can simultaneously and fully display the power of the proposed theory will be the subject of a forthcoming article.

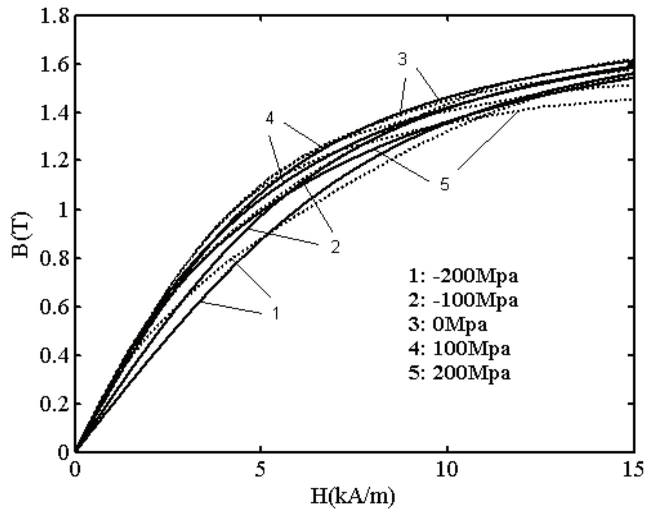

Figure 2: Comparison of the calculated magnetization curves with the experimental data of soft ferromagnetic rods at five different pre-stress levels (Dashed lines: experimental; solid lines: theoretical).

5 Conclusions

Based on thermodynamic relationships, we get a compact magnetostrictive constitutive relationship, which can well predict magnetization and magnetostrictive strain curves under various pre-stresses. Furthermore, a general magnetic forces expression for nonlinear magnetization status is presented according to the variational principle based on the above nonlinear constitutive relationship. Finally, a general nonlinear magnetoelastic coupling theory is presented for deformable magnetized medium. If some degeneration is made considering the characteristics of GMM, for example, the constitutive relationship degenerated to Liu-Zheng model [Liu and Zheng (2005)], this nonlinear magnetoelastic coupling theory will be suitable for calculating the magnetoelastic deformation of GMM.

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