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# Interval-Based Uncertain Multi-Objective Optimization Design of Vehicle Crashworthiness

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**Abstract:** In this paper, an uncertain multi-objective optimization method is suggested to deal with crashworthiness design problem of vehicle, in which the uncertainties of the parameters are described by intervals. Considering both lightweight and safety performance, structural weight and peak acceleration are selected as objectives. The occupant distance is treated as constraint. Based on interval number programming method, the uncertain optimization problem is transformed into a deterministic optimization problem. The approximation models are constructed for objective functions and constraint based on Latin Hypercube Design (LHD). Thus, the interval number programming method is combined with the approximation model to solve the uncertain optimization problem of vehicle crashworthiness efficiently. The present method is applied to two practical full frontal impact (FFI) problems.

**Keywords:** vehicle crashworthiness; uncertain multi-objective optimization; interval number programming; approximate model

## 1 Introduction

Crashworthiness design is very important in the automotive industry and transportation safety field to ensure the vehicle structural integrity and more importantly the occupant safety in the crash event. Optimization design [Lin et al. (2006), Li et al. (2006), Sapountzakis et al. (2009), Amaziane et al. (2009)] based on computer analysis has become a powerful and efficient tool for crashworthiness design of vehicles. Successive response approximate optimization is applied to solve crashworthiness problem[Kurtaran, Eskandarian, Marzougui and Bedewi (2002)]. Multi-objective optimization is applied in full frontal impact and 40% offset-frontal

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impact [Liao et al. (2008)]. Multi-cell section is investigated for crashworthiness design with multi-objective optimization [Hou et al. (2008)].In the above-mentioned works, deterministic optimization methods were used, in which all of these parameters were given certain values. However, uncertainties in loading conditions, material characteristics, geometric properties, manufacturing precision, and etc widely exist in practical vehicle crashworthiness problems. To obtain a reliable design, uncertainty should be considered in the design process of the crashworthiness. Design for six sigma through robust optimization has been employed in side impact [Koch, Yang and Gu (2004)]. Sinha (2007) used approximate moment approach and reliability index approach to perform multi-objective crashworthiness optimization. System reliability [Acar and Solanki (2008)] based vehicle design for crashworthiness is performed, and the effect of reliability allocation in different failure modes is analyzed. The above mentioned works all used the probability method, in which system parameters of uncertainty were treated as stochastic numbers based on precise probability distributions. Unfortunately, for crashworthiness problems it is generally difficult and costly to specify a precise probability distribution.

Thus to overcome the difficulty of the probability method, developing an efficient optimization design method for crashworthiness design of vehicle with uncertainty seems more and more important. In recent years, the interval method has been developed to model the uncertainty, in which the bounds of the uncertain parameters are only needed, unnecessarily knowing their precise probability distributions. Based on the interval method, a new kind of uncertain optimization method, namely interval number programming, has been obtaining more and more attentions. Most of the practical engineering problems are nonlinear, and generally cannot be expressed in an explicit form as they are often based on some simulation analysis models. Thus in recent years, some works have been published in the nonlinear interval number programming (NINP). The reference [Ma (2002)] seems the first to study the NINP problem, however, the uncertain constraints have not been investigated and furthermore the low optimization efficiency blocks its practical applications. Jiang et al. (2007a,b,c,d) proposed an NINP model to transform a general NINP problem to a deterministic optimization problem successfully, and furthermore developed several efficient algorithms to solve the transformed two-layer optimization. As the interval method is very convenient and economical in uncertainty modeling, it seems very inspiring to extend the interval optimization method into the crashworthiness design of vehicle with uncertainty in which information on uncertainty is deficient. Unfortunately, most of the above mentioned methods are focused on the uncertain single objective optimization, and there is no efficient and precise interval multi-objective optimization algorithm to deal with this class of problems currently. However, crashworthiness design of vehicle often involves uncertain multi-objective optimization such as the lightweight and safety performance etc. Therefore, it is necessary to research the interval uncertain multi-objective optimization algorithm for the crashworthiness design of vehicle.

In this paper, an uncertain multi-objective optimization method is suggested to deal with crashworthiness problem based on an interval number programming method. The uncertainties of the system parameters are described by intervals in the impact model, in which the system parameters are treated as an uncertain coefficient. The uncertainties of parameters are described by an interval, which can be easily determined through the engineering experiences and the practical crashworthiness design problem. The uncertain multi-objective optimization is created to reduce structural weight and peak acceleration together with enhance impact performance of a vehicle. To improve the crashworthiness optimization efficiency, the approximation models are constructed for the uncertain objective functions and constraint function based on the Latin Hypercube Design (LHD).The interval optimization method is combined with approximation models to form an efficient uncertain multi-objective optimization method. The present method is applied to two full-frontal impact (FFI) problems. The results are given and discussed for FFI problems.

## 2 Statement of the problem

In crashworthiness design, design variables are composed by structural considerations (e.g, sheet metal thickness, geometric shape, or other design variables, etc.). The lightweight of vehicle is of great importance to reduce the weight of the vehicle. Reduction of vehicle weight can not only save materials, but also improve fuel economy of the vehicle. Thus, for the consideration of lightweight, the weight of the vehicle is set as the first design objective. Safety performance of a vehicle can be measured by parameters such as intrusion distance, intrusion velocity, peak acceleration, and contact force, dummy response. The vehicle impact response is best described by the acceleration history with the peak acceleration typically used as an indictor of impact severity. Intrusion distance is the key performance measures for the most severe mechanical injury. Therefore, in this paper, considering safety performance of a vehicle, the peak acceleration is chosen as the second design objective, and the intrusion distance is regarded as constraint.

The uncertainty widely exists in material property, component structures, impact speed, the occupant mass etc. As a result, an uncertain multi-objective optimization for crashworthiness problem can be given in the following form:

$$\begin{aligned}
&\min_{\mathbf{x}} \{W(\mathbf{x},\mathbf{a}), A(\mathbf{x},\mathbf{a})\} \\
s.t. \quad & Intr(\mathbf{x},\mathbf{a}) \le v^I = [v^L, v^R] \\
& \mathbf{a} \in \mathbf{a}^I = [\mathbf{a}^L, \mathbf{a}^R], \\
& a_i \in a_i^I = [a_i^L, a_i^R],\ i = 1,2,\cdots,q, \\
& x_{il} \le x_i \le x_{iu},\ i = 1,2,\cdots,n,
\end{aligned} \tag{1}$$

Where $W$ and $A$ are the objective functions which represent the vehicle weight and peak acceleration, respectively. $Intr$ is constraint function, which represents intrusion distance. $\mathbf{x}$ is an $n$-dimensional decision vector. $\mathbf{a}$ is a $q$-dimensional uncertain vector which collects all of the uncertain parameters in the crashworthiness model, and its uncertainty is modeled by an interval vector $\mathbf{a}^I$. The superscripts I denote an interval, and L and R denote lower and upper bounds of the interval. $v^I$ denotes the allowable interval of the constraint.

Obviously, the objective function and constrain are both nonlinear functions of $\mathbf{x}$ and $\mathbf{a}$. For a specific$\mathbf{x}$, the possible values of $W(\mathbf{x},\mathbf{a})$, $A(\mathbf{x},\mathbf{a})$ or $Intr(\mathbf{x},\mathbf{a})$ form an interval because each uncertain parameter vector $\mathbf{a}$ belongs to an interval. Thus the above uncertain multi-objective optimization problem is much more difficulty treated than the deterministic optimization problems. In the following sections, an uncertain multi-objective optimization for crashworthiness design of vehicle will be proposed to solve above complex uncertain optimization problem.

## 3 Uncertain multi-objective optimization based on interval method for crashworthiness design of vehicle

The reference [Jiang, Han and Liu (2008a ,2008b); Zhao, Han, Jiang and Zhou (2010)] gives a definition of the satisfactory degree of interval, which represents the possibility that one interval is larger or smaller than another. Using this satisfactory degree, the uncertain constraint in Eq.(1) can be transformed into the following deterministic constraints:

$$\begin{aligned}
& P(C^I \ge v^I) \ge \lambda \\
& C^I = [Intr^L(\mathbf{x}), Intr^R(\mathbf{x})], v^I = [v^L, v^R]
\end{aligned} \tag{2}$$

Where $P(C^I \ge v^I)$ is satisfactory degree of the constraint. $\lambda$ is a predetermined satisfactory degree level. $C^I$ is interval of the intrusion distance at $\mathbf{x}$ which is caused by the uncertainty, and $Intr^L(\mathbf{x})$ and $Intr^R(\mathbf{x})$ are the lower and upper bounds of this interval, respectively:

$$Intr^L(\mathbf{x}) = \min_{a\in\Gamma} Intr(\mathbf{x},\mathbf{a}), \quad Intr^R(\mathbf{x}) = \max_{a\in\Gamma} Intr(\mathbf{x},\mathbf{a}), \tag{3}$$

$$\Gamma = \left\{ \mathbf{a} \left| \mathbf{a}^L \le \mathbf{a} \le \mathbf{a}^R \right. \right\}$$

An order relation implies that an interval number is better than another but not that one is larger than another. In reference [Han, Jiang, Gong and Huang (2008); Jiang et al. (2008c, 2008d)], an order relation $\le_{mw}$ was adopted to treat objective function. Similarly, the uncertain objective function in Eq. (1) can be transformed into a deterministic multi-objective optimization problem using the order relation $\le_{mw}$:

$$\min_{\mathbf{x}} \left[ m(A^I(\mathbf{x},\mathbf{a})), w(A^I(\mathbf{x},\mathbf{a})) \right]$$

$$\min_{\mathbf{x}} \left[ m(W^I(\mathbf{x},\mathbf{a})), w(W^I(\mathbf{x},\mathbf{a})) \right]$$

$$\begin{array}{ll} m(A(\mathbf{x},\mathbf{a})) = \frac{1}{2}(A^L(\mathbf{x}) + A^R(\mathbf{x})) & m(W(\mathbf{x},\mathbf{a})) = \frac{1}{2}(W^L(\mathbf{x}) + W^R(\mathbf{x})) \\ w(A(\mathbf{x},\mathbf{a})) = \frac{1}{2}(A^R(\mathbf{x}) - A^L(\mathbf{x})) & w(W(\mathbf{x},\mathbf{a})) = \frac{1}{2}(W^R(\mathbf{x}) - W^L(\mathbf{x})) \end{array} \tag{4}$$

Where $m$ and $w$ denote the midpoint and radius of intervals, respectively. For each specific $\mathbf{x}$, the bounds of the objective functions caused by uncertainty can be obtained:

$$\begin{aligned} &A^L(\mathbf{x}) = \min_{a\in\Gamma} A(\mathbf{x},\mathbf{a}), A^R(\mathbf{x}) = \max_{a\in\Gamma} A(\mathbf{x},\mathbf{a}), \\ &W^L(\mathbf{x}) = \min_{a\in\Gamma} W(\mathbf{x},\mathbf{a}), W^R(\mathbf{x}) = \max_{a\in\Gamma} W(\mathbf{x},\mathbf{a}), \\ &\Gamma = \left\{ \mathbf{a} \left| a_i^L \le a_i \le a_i^R, i = 1,2,...,q \right. \right\} \end{aligned} \tag{5}$$

Through Eq.(5),the uncertain vector $\mathbf{a}$ is eliminated and the deterministic objective functions are obtained.

The midpoint of objective function interval in Eq.(4) analogously minimizes the average value of the uncertain objective function, and the radius analogously minimizes the deviation. Through minimizing the deviation, the design robustness can be ensured.

Using linear combination method to deal with the multiple objectives and each objective can be transformed as following:

$$\begin{aligned} \min_{\mathbf{x}} f_1(\mathbf{x},\mathbf{a}) &= (1-\beta)(m(A(\mathbf{x},\mathbf{a})) + \xi)/\varphi + \beta(w(A(\mathbf{x},\mathbf{a})) + \xi)/\psi \\ f_2(\mathbf{x},\mathbf{a}) &= (1-\beta)(m(W(\mathbf{x},\mathbf{a})) + \xi)/\varphi + \beta(w(W(\mathbf{x},\mathbf{a})) + \xi)/\psi \end{aligned} \tag{6}$$

Where $0.0 \le \beta \le 1.0$ is a weight factor, and its different values will lead to different optimization. $\xi$ is a number making $m$ and $w$ non-negative. $\phi$ and $\psi$ are the normalization factors of objectives.

Through above treatments, the uncertain optimization problem Eq. (1) can be transformed into a following deterministic multi-objective optimization problem:

$$
\begin{aligned}
\min_{\mathbf{x}} f_1(\mathbf{x},\mathbf{a}) &= (1-\beta)(m(W(\mathbf{x},\mathbf{a}))+\xi)/\varphi+\beta(w(W(\mathbf{x},\mathbf{a}))+\xi)/\psi \\
f_2(\mathbf{x},\mathbf{a}) &= (1-\beta)(m(A(\mathbf{x},\mathbf{a}))+\xi)/\varphi+\beta(w(A(\mathbf{x},\mathbf{a}))+\xi)/\psi
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
s.t.\quad & P(C^I \geq v^I) \geq \lambda, \\
& x_{il} \leq x_i \leq x_{iu}, \quad i=1,2,\cdots,n.
\end{aligned}
$$

Applying the penalty function method to deal with the constraint, a nonconstraint optimization problem can be obtained for each objective function, and Eq.(7) can be transformed:

$$
\begin{aligned}
\min_{\mathbf{x}} f_{p1}(\mathbf{x},\mathbf{a}) &= (1-\beta)(m(W(\mathbf{x},\mathbf{a}))+\xi)/\varphi+\beta(w(W(\mathbf{x},\mathbf{a}))+\xi)/\psi \\
&\quad +\sigma\phi(P(C^I \geq v^I)-\lambda) \\
f_{p2}(\mathbf{x},\mathbf{a}) &= (1-\beta)(m(A(\mathbf{x},\mathbf{a}))+\xi)/\varphi+\beta(w(A(\mathbf{x},\mathbf{a}))+\xi)/\psi \\
&\quad +\sigma\phi(P(C^I \geq v^I)-\lambda)
\end{aligned} \tag{8}
$$

Where $f_{p1}$, $f_{p2}$ are penalty functions. Penalty factor is $\sigma$ which is usually specified as a large value. The function $\phi$ has the following form:

$$
\phi(P(C^I \geq v^I)-\lambda) = \left(\max\left(0,-\left(P(C^I \geq v^I)-\lambda\right)\right)\right)^2 \tag{9}
$$

## 4 Uncertain multi-objective optimization of crashworthiness based on approximation models

Crash simulations with acceptable accuracy are computationally very expensive. To improve optimization efficiency, approximation models are widely used in most crashworthiness optimization, Eq.(8) can be formulated as a following approximation optimization problem:

$$
\begin{aligned}
\min_{\mathbf{x}} \tilde{f}_{p1}(\mathbf{x},\mathbf{a}) &= (1-\beta)(m(\tilde{W}(\mathbf{x},\mathbf{a}))+\xi)/\varphi+\beta(w(\tilde{W}(\mathbf{x},\mathbf{a}))+\xi)/\psi \\
&\quad +\sigma\phi(P(\tilde{C}^I \geq v^I)-\lambda) \\
\tilde{f}_{p2}(\mathbf{x},\mathbf{a}) &= (1-\beta)(m(\tilde{A}(\mathbf{x},\mathbf{a}))+\xi)/\varphi+\beta(w(\tilde{A}(\mathbf{x},\mathbf{a}))+\xi)/\psi \\
&\quad +\sigma\phi(P(\tilde{C}^I \geq v^I)-\lambda)
\end{aligned} \tag{10}
$$

where

$$
(\tilde{C}^I = [Intr^L(\mathbf{x},\mathbf{a}), Intr^R(\mathbf{x},\mathbf{a})] \tag{11}
$$

Where $\tilde{W}(\mathbf{x},\mathbf{a})$ and $\tilde{A}(\mathbf{x},\mathbf{a})$ are approximation models of vehicle weight and acceleration, respectively. $I\tilde{n}tr(\mathbf{x},\mathbf{a})$ is approximation model of intrusion distance constraint on crashworthiness design, respectively. $\tilde{f}_{p1}(\mathbf{x},\mathbf{a})$ and $\tilde{f}_{p2}(\mathbf{x},\mathbf{a})$ are the penalty function based on the approximation models of the objective functions (termed as "approximate penalty function"), respectively. $\tilde{C}^I$ is interval of approximate intrusion distance constraint. Here, the design vector $\mathbf{x}$ and the uncertain vector $\mathbf{a}$ are both used as input variables when creating the approximation models, and hence $\tilde{W}(\mathbf{x},\mathbf{a})$, $\tilde{A}(\mathbf{x},\mathbf{a})$ and $I\tilde{n}tr(\mathbf{x},\mathbf{a})$ are explicit functions with respect to both of $\mathbf{x}$ and $\mathbf{a}$, instead of only $\mathbf{x}$ as we usually do for deterministic optimization problems.

The Latin Hypercube Design (LHD) technique [Morris and Mitchell (1995)] is employed to select the sampling points in the space of input variables when creating the approximation models. LHD is capable of capturing the higher order of nonlinearity with relatively fewer design points.

The flowchart of uncertain crashworthiness optimization is shown in Fig.1. In the uncertainty space and current design space, $\mathbf{x}$ and $\mathbf{a}$ are both used as the input variables, one set of sampling points are obtained by LHD. After inputting the sampling points into the actual crash simulation models, the samples can be obtained to construct the approximation models of the objective function $\tilde{W}(\mathbf{x},\mathbf{a})$, $\tilde{A}(\mathbf{x},\mathbf{a})$ and constraint $I\tilde{n}tr(\mathbf{x},\mathbf{a})$. Then the actual crash simulation models can be discarded temporally, and the optimization process can be performed only based on these approximation models. Obviously, it is a two-layer nesting optimization problem. Here, the Non-dominated Sorting Genetic Algorithm II (NSGA-II) [Deb (2001); Deb, Pratap, Agarwal and Meyarivan (2002)] and sequential quadratic programming (SQP) are used as the outer layer and inner layer optimization solver, respectively. In the outer layer, an amount of individuals of the design vector $\mathbf{x}$ are generated by multi-objective genetic algorithm named NSGA-II, NSGA-II is employed to optimize the design vector. In the inner layer, the SQP method for each individual will be called two times to obtain the intervals of objective functions and constraint based on these approximation models. Then the approximate penalty function can be calculated based on these intervals. As a result, the Pareto set can be obtained.

## 5 The application

In this section, two application examples are investigated, which are frontal impact problems.

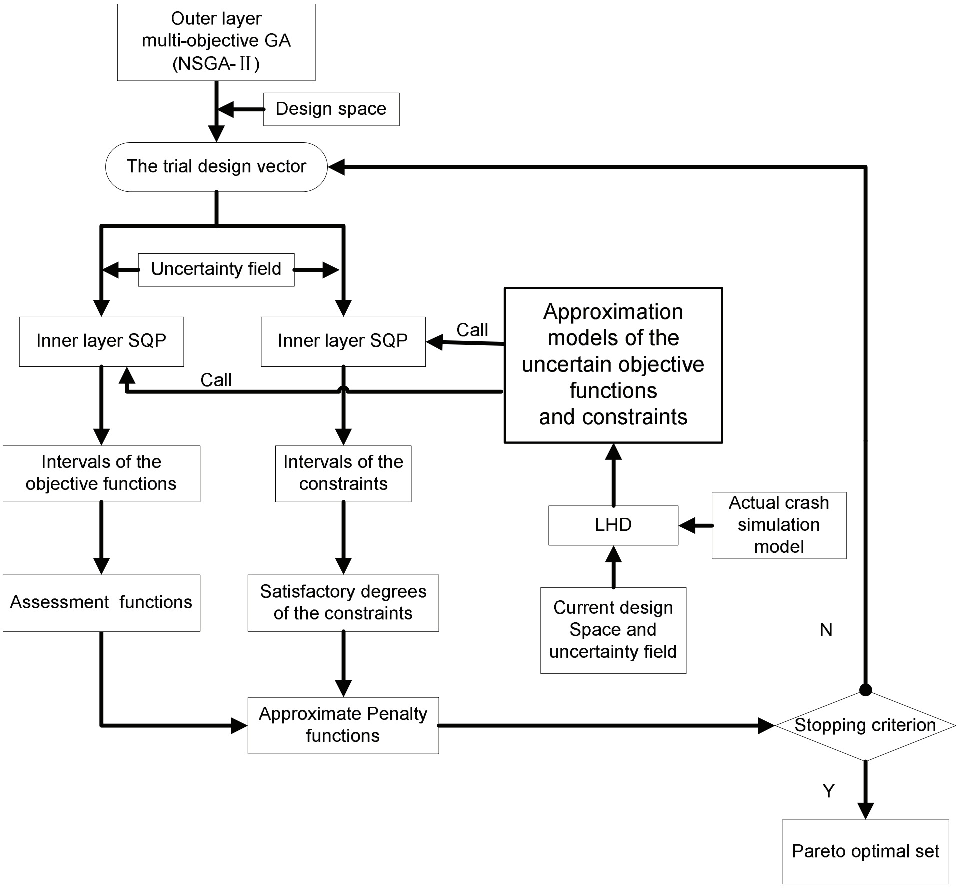


Figure 1: Nesting optimization based on the approximation models

### *5.1 Application 1*

As shown in Fig.2, the thickness of four reinforced members around the frontal structure is chosen as the design variables which could significantly affect the crash safety. For the weight which is the first objective, we only generate linear polynomial response surface, because the weight must be linearly related to component thickness. Peak acceleration of engine bottom is chosen as the second objective. Radial basis function (RBF) produces good results for the nonlinear function [Fang, Raus-Rohani, Liu and Horstemeyer (2005)], so RBF [Hon, Ling and Liew (2005), Amaziane, Naji and Ouazar(2004), Le, Mai-Duy, Tran-Cong and Baker (2007), Mai-Duy, Khennane and Tran-Cong (2007)] is constructed for the peak acceleration of engine bottom and intrusion distance, respectively. The intrusion distance of the fire wall is shown in Fig.3.

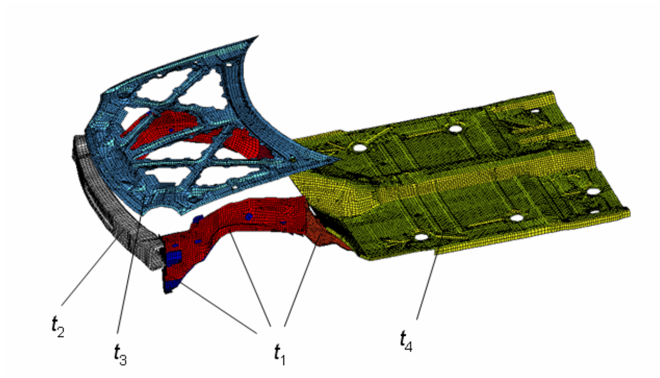


Figure 2: Design variables of the vehicle model

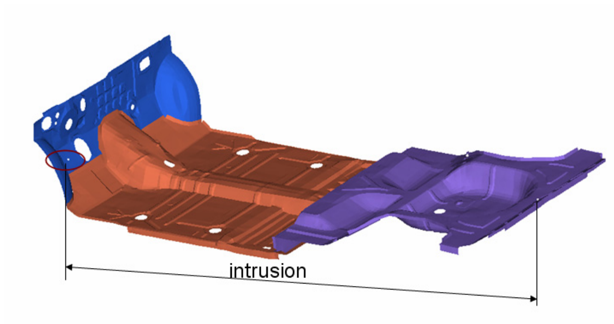


Figure 3: Approximate locations for intrusion measurement

Table 1: Material properties considering uncertain

| Material | Young's Modulus $E$ (MPa) | Poisson's ratio $\nu$ | Density $\rho$ (kg/mm$^3$) | Yield stress$\sigma_s$ (MPa) |
|---|---|---|---|---|
| 1 | $2.07 \times 10^5$ | 0.3 | $7.85 \times 10^{-6}$ | 400 |
| 2 | $2.07 \times 10^5$ | 0.3 | $7.85 \times 10^{-6}$ | 330 |
| 3 | $2.07 \times 10^5$ | 0.3 | $7.85 \times 10^{-6}$ | 570 |

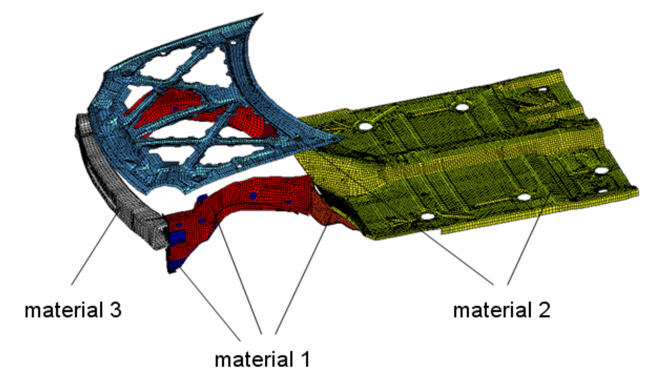


Figure 4: The material of considering uncertainty

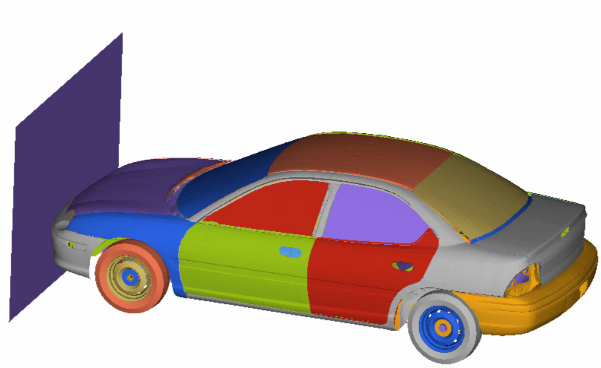

Figure 5: Finite element model of vehicle

Materials are complex, hierarchical, heterogeneous systems, it is not reasonable or sufficient to adopt a deterministic approach to materials design, due to the measuring and manufacturing errors and model errors, the uncertainty of material in crashworthiness design is more complicate. In this study, the material of considering uncertainty is shown in Fig.6. Their nominal values are given in Table 1. Yield stress $\boldsymbol{\sigma}_{\mathbf{s}}$ is treated as uncertain parameters, and the uncertain level is $\pm 10\%$ off from their nominal values, namely $\sigma_{s1} \in [360\text{Mpa}, 440\text{Mpa}]$, $\sigma_{s2} \in [297\text{Mpa}, 363\text{Mpa}]$, $\sigma_{s3} \in [513\text{Mpa}, 627\text{Mpa}]$. As a result, a following optimization problem can be formulated:

$$\min_{\mathbf{x}} \{W(\mathbf{x}), A(\mathbf{x}, \mathbf{a})\}$$

$$s.t \quad Intr(\mathbf{x}, \mathbf{a}) \leq [265\text{mm}, 270\text{mm}]$$

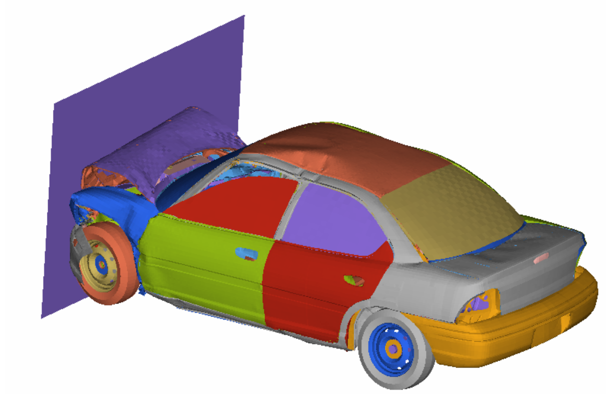

Figure 6: The deformation of the full frontal impact

$$\mathbf{a} \in \mathbf{a}^I = [\boldsymbol{\sigma}_\mathbf{s}^L, \boldsymbol{\sigma}_\mathbf{s}^R], \tag{12}$$

$$a_i \in a_i^I = [\sigma_{si}^L, \sigma_{si}^R],\ i = 1, ..., 3,$$

$$\mathbf{x}^T = (t_1, t_2, t_3, t_4)$$

$$0.5\text{mm} \le \mathbf{x} \le 2.0\text{mm}$$

$$\sigma_{s1} \in [360\text{Mpa}, 440\text{Mpa}],\ \sigma_{s2} \in [297\text{Mpa}, 363\text{Mpa}],\ \sigma_{s3} \in [513\text{Mpa}, 627\text{Mpa}]$$

The FEM simulation is carried out on the commercial software LS-DYNA. The finite element analysis (FEA) model of a Dodge Neon was developed by NCAC (National Crash Analysis Center). It has 283859 nodes and 270768 (mostly shell) elements. The total vehicle mass is 1,333kg. The initial velocity is 56.5km/h. This model was used for full-frontal impact (FFI) simulations and found the results to be consistent with physical crash test data by NCAC. A simulation of 100ms FFI takes approximately seven hours with 4 processors p4 2.40 GHz. In this study, we used FEA model in simulations of FFI. Fig.5 shows the original FEA model .The deformation of the full frontal impact is given in Fig.6.

50 sampling points of crash simulations are executed through LHD, thus the approximation models are created within the uncertainty material parameters $\boldsymbol{\sigma}_\mathbf{s}$ and current design variable $\mathbf{x}$. Approximation model of mass is a linear function of design variable $\mathbf{x}$, but approximation models of acceleration and intrusion are non-linear function with respect to $\mathbf{x}$ and $\boldsymbol{\sigma}_\mathbf{s}$, then in outer layer using NSGA-II, design variable $\mathbf{x}$ can be generated, in inner layer intervals of the objective functions and constraint can be obtained based on approximation models by calling the SQP.

Thus the penalty function can be calculated based on these intervals. Therefore the Pareto optimal points of weight and peak acceleration can be achieved.

NSGA-II specific parameters are given in table 2. The possibility degree level $\lambda$ of the constraint and the penalty factor $\sigma$ are set to 0.95 and 1000, respectively. $\varphi$ and $\psi$ are specified as 1000 and 65.

Table 2: Details of NSGA-II specific parameters used

| GA parameter name | Value |
|---|---|
| Population size | 100 |
| Number of generation | 100 |
| Probability of crossover | 0.9 |
| Probability of mutation | 0.1 |
| Distribution index for crossover | 2.0 |
| Distribution index for mutation | 2.0 |

Table 3: Typical Pareto front points of optimization result with $\beta = 0.5$

| | Weight(Kg) | Acceleration penalty function | Acceleration interval($m/s^2$) |
|---|---|---|---|
| 1 | 1326.013 | 1.0259 | [1807.012, 1836.059] |
| 2 | 1327.000 | 0.9813 | [1730.807, 1759.108] |
| 3 | 1328.016 | 0.9698 | [1720.300, 1747.086] |
| 4 | 1329.084 | 0.9609 | [1695.669, 1723.292] |
| 5 | 1330.196 | 0.9506 | [1688.187, 1714.192] |
| 6 | 1331.108 | 0.9417 | [1675.613, 1700.998] |
| 7 | 1332.089 | 0.9341 | [1661.111, 1686.405] |
| 8 | 1333.045 | 0.9280 | [1626.374, 1654.410] |
| 9 | 1334.138 | 0.9186 | [1595.845, 1625.312] |
| 10 | 1335.174 | 0.9113 | [1570.716, 1601.464] |
| 11 | 1336.167 | 0.9057 | [1544.493, 1577.023] |
| 12 | 1337.223 | 0.9016 | [1517.486, 1552.366] |
| 13 | 1338.060 | 0.8993 | [1500.085, 1536.505] |
| 14 | 1339.119 | 0.8983 | [1475.632, 1514.832] |

In Fig.7 and Fig.8, 100 Pareto front points are achieved, and Table 3 and Table 4 show the typical Pareto front points of optimization result. From the table, when $\beta = 0.5$, the midpoint of the acceleration objective function at the optimum is relatively bigger than the one at $\beta = 0.2$, while the radius is relatively smaller than

Table 4: Typical Pareto front points of optimization result with $\beta = 0.2$

| | Weight(Kg) | Acceleration penalty function | Acceleration Interval($m/s^2$) |
|---|---|---|---|
| 1 | 1326.025 | 1.5091 | [1808.402, 1840.588] |
| 2 | 1327.173 | 1.4365 | [1694.910, 1736.510] |
| 3 | 1328.127 | 1.4162 | [1660.455, 1705.792] |
| 4 | 1329.035 | 1.4028 | [1616.314, 1672.961] |
| 5 | 1330.039 | 1.3853 | [1571.713, 1637.720] |
| 6 | 1331.120 | 1.3677 | [1530.716, 1604.561] |
| 7 | 1332.105 | 1.3542 | [1497.498, 1578.115] |
| 8 | 1333.367 | 1.3345 | [1450.177, 1540.140] |
| 9 | 1334.148 | 1.3247 | [1423.460, 1518.852] |
| 10 | 1335.078 | 1.3142 | [1400.988, 1500.787] |
| 11 | 1336.121 | 1.3003 | [1371.497, 1476.276] |
| 12 | 1337.065 | 1.2901 | [1351.675, 1459.415] |
| 13 | 1338.208 | 1.2731 | [1331.116, 1438.544] |
| 14 | 1339.195 | 1.2612 | [1307.669, 1418.623] |
| 15 | 1340.023 | 1.2499 | [1306.712, 1412.228] |
| 16 | 1341.049 | 1.2386 | [1301.668, 1403.469] |
| 17 | 1342.065 | 1.2290 | [1297.963, 1396.342] |
| 18 | 1343.002 | 1.2242 | [1293.829, 1391.415] |
| 19 | 1344.043 | 1.2191 | [1298.275, 1391.383] |
| 20 | 1345.279 | 1.2116 | [1304.440, 1390.079] |
| 21 | 1346.274 | 1.2058 | [1304.274, 1388.069] |

$\beta = 0.2$. With the decrease of $\beta$, the midpoint of the acceleration objective function also decreases, while the radius increases. It means that the average value of the objective function becomes better but the design robustness becomes worse. As a result, the midpoint and radius of the objective function behave two different trends with the variation of $\beta$. If we pay more attention to the average crashworthiness performance of the vehicle under the uncertain material property, a large weighting factor $\beta$ can be selected; if the robust of crashworthiness performance of the vehicle is cared more, a relatively small weighting factor $\beta$ is preferred.

According to the decision make, when robust of crashworthiness performance is considered, $\beta = 0.5$ is selected, the designer may choose the $1th$, the $2th$, *or* the $3th$ as the solution, considering the lightweight of vehicle design; while the designer would like to care for the safety performance, they may choose the $12th$, the $13th$, *or* the $14th$ solutions. When average crashworthiness performance is considered,

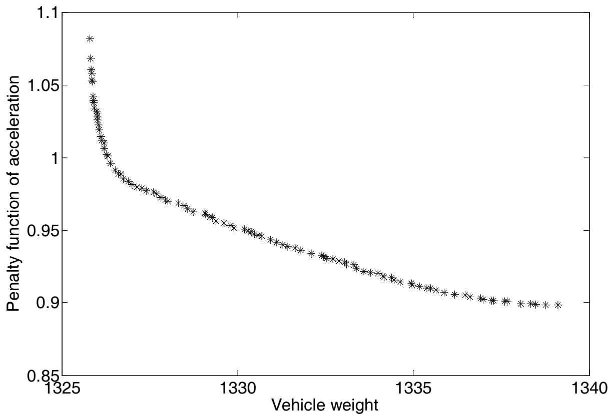


Figure 7: Pareto optimal front with $\beta = 0.5$

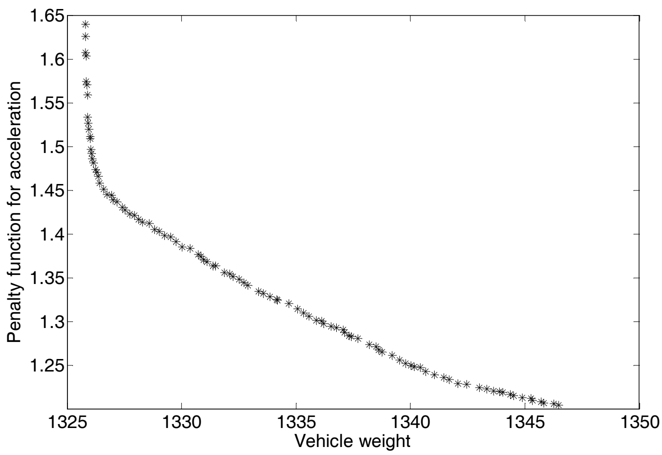


Figure 8: Pareto optimal front with $\beta = 0.2$

$\beta = 0.2$ is selected, the designer can select the $1th$, the $2th$, *or* the $3th$ as the solution, paying attention to the lightweight of vehicle design; while the designer would like to care for the safety performance, they may choose 14 *th*-$21th$ solutions.

### *5.2 Application 2*

There are three design variables used for this optimization study (see Fig.9).The yield stress $\sigma$ is treated as the uncertain parameter (see Fig.10). Their nominal values are given in Table 5. The weight and the peak acceleration of B-pillar are

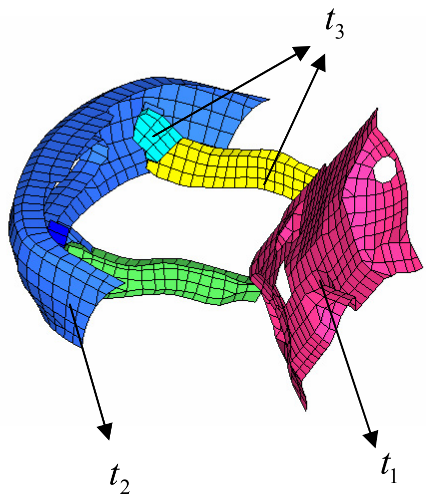


Figure 9: Design variables of the vehicle model

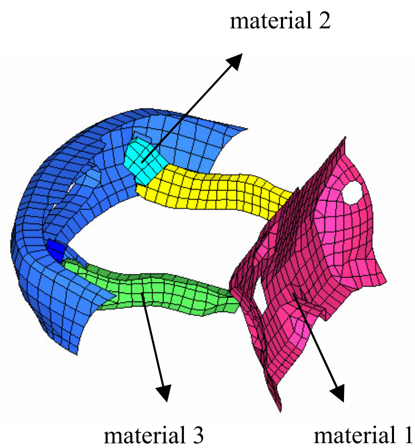


Figure 10: The material of considering uncertainty

selected as objectives. The instruction of the fire wall is chosen as constraint (see Fig.11). Thus, the uncertain multi-objective optimization can be formulated as follow£º

$$\min_{\mathbf{x}} \{W(\mathbf{x}), A(\mathbf{x}, \mathbf{a})\}$$

$$s.t \quad Intr(\mathbf{x}, \mathbf{a}) \leq [150\text{mm}, 170\text{mm}]$$

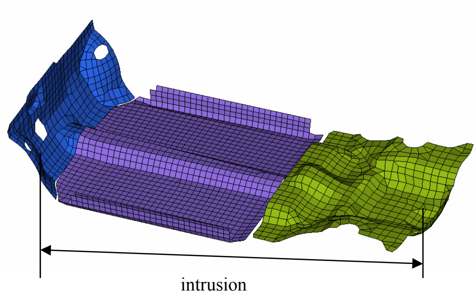


Figure 11: Approximate locations for intrusion measurement

$$\mathbf{a} \in \mathbf{a}^I = [\boldsymbol{\sigma}_\mathbf{s}^L, \boldsymbol{\sigma}_\mathbf{s}^R], \tag{13}$$
$$a_i \in a_i^I = [\sigma_{si}^L, \sigma_{si}^R],\ i = 1,...,3,$$
$$\mathbf{x}^T = (t_1, t_2, t_3)$$
$$0.8\text{mm} \le t_1 \le 2.5\text{mm}$$
$$2.0\text{mm} \le t_2 \le 4.0\text{mm}$$
$$0.8\text{mm} \le t_3 \le 2.5\text{mm}$$
$$\sigma_{s1} \in [206.1\text{Mpa}, 251.9\text{Mpa}],$$
$$\sigma_{s2} \in [299.7\text{Mpa}, 366.3\text{Mpa}],\ \sigma_{s3} \in [225\text{Mpa}, 275\text{Mpa}]$$

Table 5: Material properties considering uncertain

| Material | Young's Modulus $E$ (MPa) | Poisson's ratio $\nu$ | Density $\rho$ (kg/mm$^3$) | Yield stress $\sigma_s$ (MPa) |
|---|---|---|---|---|
| 1 | $2.10 \times 10^5$ | 0.3 | $7.85 \times 10^{-6}$ | 229 |
| 2 | $2.10 \times 10^5$ | 0.3 | $7.85 \times 10^{-6}$ | 330 |
| 3 | $2.10 \times 10^5$ | 0.3 | $7.85 \times 10^{-6}$ | 250 |

The finite element analysis (FEA) model was developed by NCAC (National Crash Analysis Center). Fig.12 shows the original FEM model. The total number of the elements in this model is about 17554 while total number of nodes is around 19217. The initial velocity is 50km/h with impact duration time of 100ms. The FEM simulation is carried out in the explicit non-linear finite element coded Ls-dyna. The deformation of the full frontal impact is given in Fig.13.

Table 6: Details of NSGA-II specific parameters used

| GA parameter name | Value |
|---|---|
| Population size | 50 |
| Number of generation | 100 |
| Probability of crossover | 0.9 |
| Probability of mutation | 0.1 |
| Distribution index for crossover | 20 |
| Distribution index for mutation | 20 |

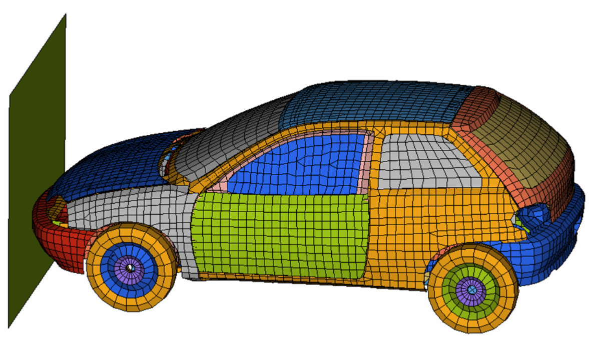

Figure 12: The finite element model of the vehicle

Figure 13: A typical deformation of the model

The LHD is used to generate the sample points for building the approximation models by the LHD. The number of sample points is 40. The approximation model of weight of vehicle is built, which is the linear response surface. The Kriging

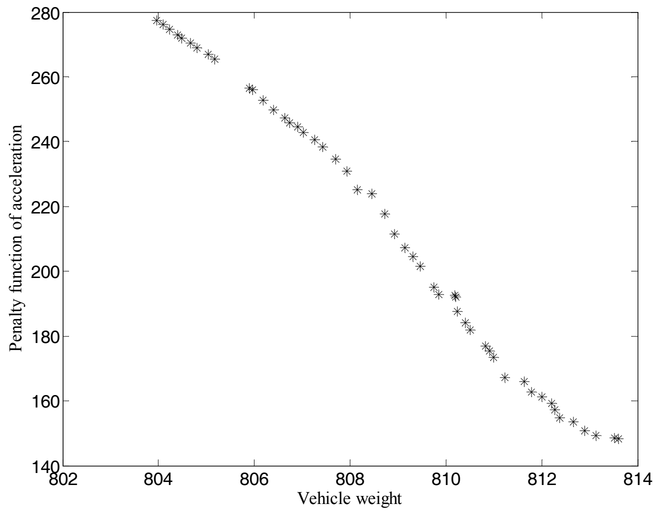


Figure 14: The Pareto set obtained

approximation models [Simpson, Peplinski, Koch, Allen (2001)] are constructed for peak acceleration of B-pillar and intrusion distance of the fire wall.

The parameters in NSGA-II are specified as the Table 1. The weighting factor $\beta$ is set 0.5, and the satisfactory degree level k of the constraint is specified as 0.9. The factors $\sigma$, $\varphi$ and $\psi$ are specified as $1.0 \times 10^{10}$, 2000 and 1800, respectively.

The flowchart of the present method is as shown in Fig.1. The obtained Pareto set is given in Fig.14. The penalty value of the peak acceleration is from 148.4 to 277.4 and the weight is from 804 to 813.6.

## 6 Conclusion

This paper presents an uncertain multi-objective optimization problem based on interval number programming method for the design of vehicle crashworthiness, and interval is used to model the parameter uncertainty. It is typically nesting optimization, in outer layer, multi-objective genetic algorithm (NSGA-II) is used to generate design variable; in inner layer, SQP is employed to obtain the intervals of objective and constraint functions. To improve efficiency, the approximation models are generated by LHD. The interval number programming method combined with the approximation model to form an efficient and effective design optimization approach for crashworthiness applications of vehicle.

Two automotive application-vehicle structural designs for full frontal impact are demonstrated the effect of this method. In these applications, the uncertain parameters of materials are treated as interval number. Two objectives and one constraint are taken into account simultaneously, where objectives of acceleration and weight are constructed, and the constraint of intrusion distance is formulated. The Pareto optimal points can be obtained, and the design engineers can select a set of solution points on the Pareto font according to their decision making.

However, full frontal impact is only one crash scenario. Future investigations are necessary to combine various crash modes (side impact, 40% offset frontal impact, rear impact, roof crush). Furthermore, it seems possible to extend the present method to analyze other practical engineering problem for the uncertain optimization, such as sheet metal forming, etc. The more uncertain parameters and relatively larger uncertainty level would be considered in future.

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