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The Effective Material Properties of a Steel Plate Containing Corrosion Pits

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Abstract: Corrosion pits on a steel plate can reduce the strength of the plate. However, it is difficult to calculate the corrosion effect analytically since the pits are normally distributed on the plate's surface randomly. In this manuscript, a simple approach is proposed to convert the corroded plate into a perfect one. By this method, the corrosion pits are treated as inclusions embedded in the plate. Then the analytical mechanics model used for composite material can be adopted in the calculation of the steel plate's effective material properties. To verify the proposed approach, numerical simulation is conducted using finite element method.

Keywords: Steel plate, corrosion pit, effective properties

1 Introduction

Corrosion is the disintegration of an engineered material into its constituent atoms due to chemical reactions with its surroundings. Corrosion damage induced by pitting is commonly observed in a wide range of steel structures such as ships and offshore platforms [Wang, Duan, Li, Zhang, Ma and Hou (2009); Nakai, Matsushita and Yamamoto (2006); Melchers (1999)]. Therefore, steel plates have been become a great importance of study. In engineering applications, the thickness of a steel plate may have a variable thickness mainly resulted from corrosion induced metal loss on its surface. In a general corrosion, the thickness of the plate varies uniformly but in a localized corrosion, the thickness changes in a contrary way. However, it is obvious that both general and localized corrosion will affect the strength, serviceability and stability etc. of a steel plate structure. To evaluate the effect of corrosion, quite a number of related research have been conducted in the fields of material science and engineering mechanics [Pidaparti, Puri, Palakal and Kashyap (2005)]. For the former, the focus is more likely on the investigation

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of the corrosion processes. The objective is to find a way to improve the corrosion resistance of steel [Sofronkov, Vivdenko, Magdenko and Chivireva (1986)]. On the other hand, the main purpose of the study in the latter area is to assess the residual service life of corroded steel structural members [Almusallam (2001)].

Up to today, many methods have been developed to make strength assessment for pitted steel plates [Paik, Lee and Ko (2003); Paik, Lee and Ko (2004)]. However, it is hard to have an analytical model since the distribution of the corrosion pits is normally non-uniform. To overcome this difficulty, numerical methods, including finite element analysis and boundary element method are frequently used in relevant studies [Araujo, Gray (2008); Pidaparti, Koombua and Rao (2009); Sidharth (2009)].

In this manuscript, a simple analytical approach is proposed to investigate the bending resistance of a corroded steel plate. Instead of using the concept of equivalent thickness, this method treats the corrosion pits as inclusions embedded in the steel matrix. Based on this fictitious conversion, the analytical model for composite can be adopted and extended to further calculate the effective material properties of the steel plate. The correctness of the proposed method is verified by a series of finite element simulation.

2 Methodology

As shown in Figure 1, a steel plate contains many corrosion pits on its two surfaces. In this study, it is assumed that the pits have different shapes but their depths are the same. In the figure, the Young's modulus and the Poisson's ratio of the steel are denoted by E and ν, respectively. The original thickness of the steel plate is t and the depth of the corrosion pits is $t-t^*$.

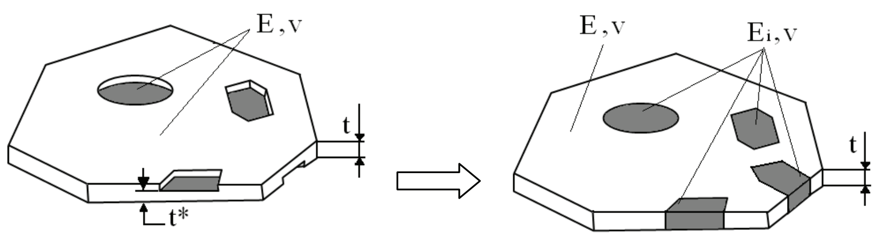

Figure 1: A steel plate with many corrosion pits is treated as a composite plate

The methodology proposed in this paper can be described by the following two steps:

(a) the original plate is converted into a composite plate, as illustrated in Figure 1.

In reality, the thickness of the plate is reduced to t^*at each corrosion pit. However, to use the proposed approach, it is assumed that the whole plate has a constant thickness and the material at the location of each pit is no longer steel. In other words, the corroded parts are treated as plates made of a fictitious material. Denoting by E_i, the Young's modulus of the fictitious material can be calculated by Eq. (1). Obviously, Eq. (1) is based on the assumption that the thinner plate (at each corrosion pit) and its substitute have the same bending resistance. The Poisson's ratio of the new material remains v.

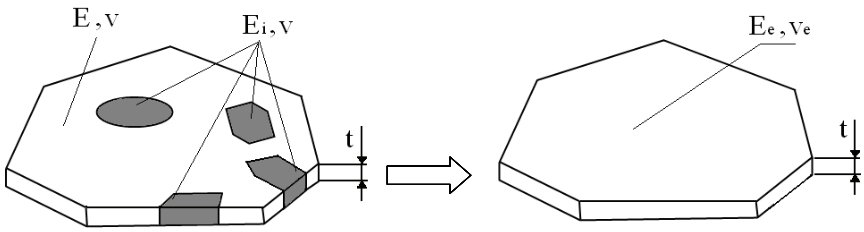

Figure 2: A composite plate is converted into an isotropic plate

$$\frac{E_i}{E} = \left(\frac{t^*}{t}\right)^3 \tag{1}$$

(b) as shown in Figure 2, the composite plate is further converted into to an isotropic plate that has effective homogeneous material properties.

Through step (a), the original steel plate can be treated as a composite one. In this composite, steel and the fictitious material are regarded as the matrix and the inclusion, respectively. At this stage, numerical analysis can be conducted for this kind of composite but a number of difficulties have to be overcome since the number of inclusions may be numerous. Although some novel approaches have been developed to improve the efficiency of the simulation of composite [Yao, Kong, Wang and Wang (2004); Wang, Yao and Lei (2006); Yuan and Tan (2009)], the composites are usually homogenized into isotropic materials in many cases to simplify the modelling [Yang and Becker (2004); Takashima, Nakagaki and Miyazaki (2007)]. Since E, v and E_i are all known, the material properties of the isotropic plate can also be evaluated. According to the existing theory, E_e and v_e, the effective Young's modulus and Poisson's ratio, are dependant on the volume fraction of the inclusion [Huang, Hwang, Hu and Chandra (1995)]. Although various averaging schemes have been proposed to estimate the overall properties of a composite material, the effective self-consistent method (ESCM) is adopted in this study [Zheng and Du

(1998)]. Based on the ESCM, it can be derived that:

$$K_e = \frac{1}{2(1+v)} \frac{E}{1+\frac{2c(1-e^3)}{2e^3+(1-c)(1-e^3)}} \tag{2}$$

$$G_e = \frac{1}{2(1+v)} \frac{E}{1+\frac{4c(1-e^3)}{4e^3+(1-c)(1-e^3)(1+v)}} \tag{3}$$

where K_e and G_e are the effective bulk modulus and shear modulus. The term c is the volume fraction of the inclusion. It is equal to the area fraction of the inclusion in a 2-dimensional problem. It should be noted that $e = t^*/t$ is used in the derivation.

Therefore, one can further obtain Eq. (4) and (5).

$$v_e = \frac{K_e - G_e}{K_e + G_e} \tag{4}$$

and

$$E_e = 2(1+v_e)G_e \tag{5}$$

As examples, some results of conversion are listed in Table 1.

Table 1: Effective material properties for selected cases

	$c = 0.2$		$c = 0.3$		$c = 0.5$		$c = 0.75$	
	E_e/E	v_e/v	E_e/E	v_e/v	E_e/E	v_e/v	E_e/E	v_e/v
$e = 0.15$	0.600	1.240	0.467	1.319	0.274	1.433	0.114	1.520
$e = 0.25$	0.610	1.226	0.479	1.300	0.287	1.402	0.127	1.461
$e = 0.50$	0.689	1.134	0.575	1.176	0.398	1.220	0.240	1.199
$e = 0.75$	0.838	1.032	0.769	1.041	0.648	1.049	0.524	1.038

3 Validation

Figure 3 shows a simply supported rectangular corroded plate under a uniformly distributed load. It is assumed that there are a large number of irregular corrosion pits on the plate's surfaces. To verify the proposed approach, such a plate is analyzed by both finite element method and analytical approach. In the former, the physical geometry of the corrosion pits is considered in the modelling whilst in the latter, the analysis is based on making use of effective material properties.

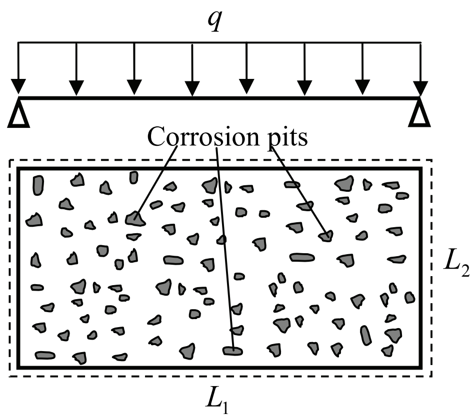

Figure 3: A simply supported plate under uniformly distributed load

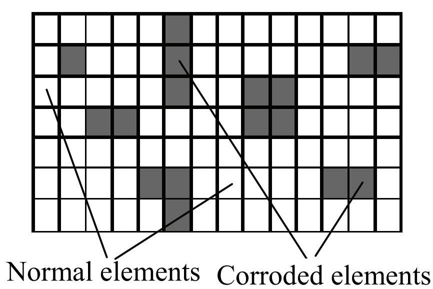

Figure 4: FEA model for a plate with randomly distributed corrosion pits

The typical FEA model for the plate is shown in Figure 4. The plate is meshed into many shell elements. In the area where corrosion pits locate, the thickness of the shell elements is thinner than that of the normal elements.

It is understandable that the distribution of the corrosion pits will affect the FEA modelling. The results corresponding to different pit patterns are normally different either, even though the fraction of inclusion remains unchanged. However, from statistics point of view, the diversity of the results induced by different distribution patterns of the pits will reduce to a low level if the number of corroded elements is large enough. Therefore, it is necessary to conduct trial runs in a typical case study to determine the minimum number of corroded elements in FEA models. Theoretically, this minimum number can be calculated by $N_{\min} = cN$, where N is the total number of elements used in the FEA model by which convergent result can be obtained.

In order to find out the relationship between the convergence of the modelling results and the number of the corroded elements, it is assumed without loss of generality that $L_1 = 2.0m$, $L_2 = 1.0m$, $q = 1.0 \times 10^5 Pa$, $t = 10mm$, $E = 2.0 \times 10^{11} Pa$, $v = 0.3$, $e = 0.25$ and $c = 0.2$. Based on four meshing schemes, this problem is studied by ABAQUS. For each meshing scheme, eight trial runs based on random distribution of corroded elements are conducted to calculate the maximum deflection of the plate. The results are compared with the closed-form solution calculated using the effective material properties given in Table 1. From Figure 5, it can be seen that the eight results are quite scattered if the plate is divided into 5×10 elements. However, the convergence of the eight results can be improved significantly when 10×15, 10×20 or 15×30 meshing scheme is used. As shown in Figure 6, the mean square error of the trial results is less than 6.4 which is just about 7.0% of the average value. If such a level of accuracy is accepted, the minimum number of corroded elements can be determined accordingly, which is $N_{\min} = 10 \times 15 \times 0.2 = 30$.

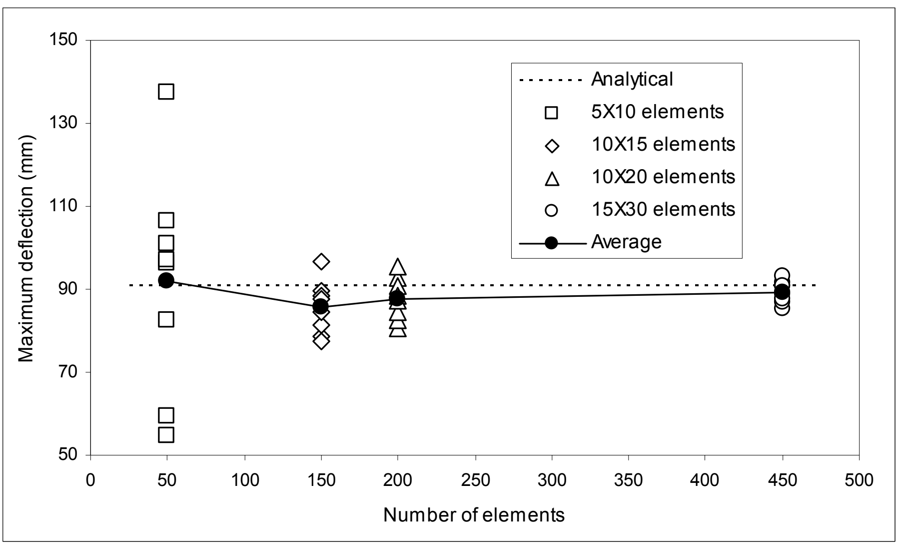

Figure 5: The convergence of the results and the number of elements

Sixteen more cases are simulated by finite element method to show the correctness of the proposed approach. The term c (the fraction of inclusion) varies from 0.2 to 0.75, whilst e (the thickness ration) varies from 0.15 to 0.75. In each case, the total number of elements is set to 200 (10×20 scheme) so that the number of corroded elements is always larger than 30 (10×20×0.20=40) in all FEA models. To avoid

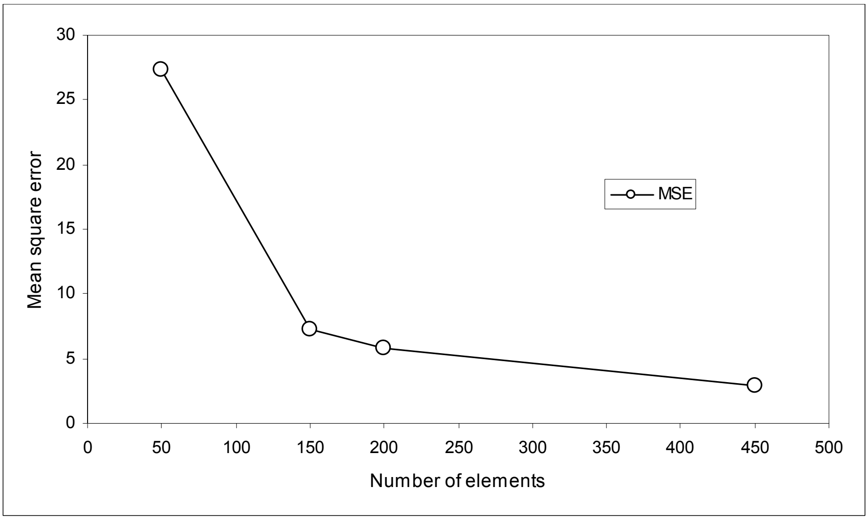

Figure 6: The relationship between mean square error and the number of elements

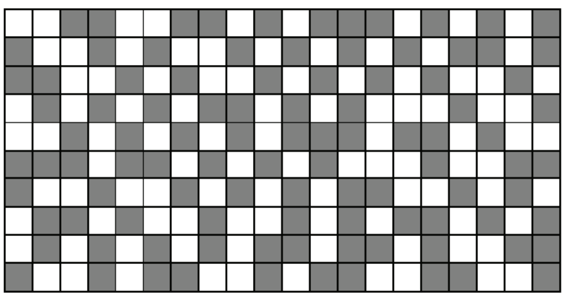

Figure 7: An example of the FEA model for $c = 0.5$

that the calculation violates the small displacement assumption, the UDL is always set to $q = 1.0 \times 10^4 Pa$. For each case, three FEA models are analyzed for the consideration of the randomicity of the distribution of corroded elements. Figure 7 is an example of a FEA model for $c = 0.5$. The comparison between the numerical and the analytical results is shown in Figure 8. It should be noted that the analytical maximum deflections of the plates is calculated by Eq. (6). In this formula, the term D_e denotes the effective bending stiffness of a corroded plate which is given in Eq. (7).

$$d = \frac{16p}{\pi^6 D_e} \frac{L_1^4 L_2^4}{L_1^4 + L_2^4 + 2L_1^2 L_1^2} \tag{6}$$

$$D_e = \frac{E_e t^3}{12(1 - v_e^2)} \tag{7}$$

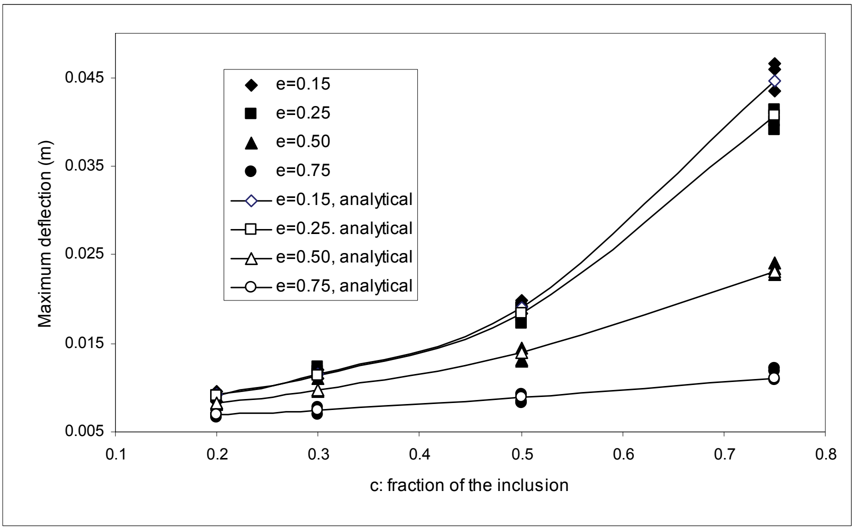

Figure 8: Comparison between numerical and analytical results

From Figure 8, it can be seen that the numerical results have a good agreement with the closed-form solutions. If each plate is divided into finer mesh, for instance 15×30=450 elements, the convergence of the numerical results can be further improved since the distribution of the corroded elements is more likely uniform.

4 Conclusion

This study proposes a simple approach to calculate the effective material properties of steel plates containing a large number of corrosion pits. By this approach, a corroded plate can be converted into a composite one. Using the effective material properties, the study of the plate's overall behaviour can be simplified tremendously. However, it must be mentioned that this method should not be applied to obtain the detailed information such as the stress concentration near a corrosion pit. In addition, the methodology for conversion stands on the concept of equivalent bending resistance. Hence the formulae presented in this manuscript are only valid for steel plate under bending. On the other hand, it may be predictable that the methodology used in this study can be extended to the modelling of corroded plates under in-plane loading. Since a flat-shell element in finite element analysis can be treated as the combination of a plane-stress element and a plate-bending element, the present approach can be easily incorporated into FEA modelling for corroded plates subjected to complex forces. Moreover, the depths of the corrosion pits on a steel plate may be various, but this case can also be dealt with using the proposed methodology since ESCM can be applied to a composite that consists of single matrix and different types of inclusions.

References

Almusallam, A. A. (2001): Effect of degree of corrosion on the properties of reinforcing steel bars. *Construction and Building Materials*, vol. 15, no. 8, pp. 361-368.

Araujo, F. C.; Gray, L. J. (2008): Evaluation of effective material parameters of CNT-reinforced composites via 3D BEM. *CMES: Computer Modeling in Engineering & Sciences*, vol. 24, no. 2, pp. 103-122.

Huang, Y.; Hwang, K. C.; Hu, K. X.; Chandra, A. (1995): A unified energy approach to a class of micromechanics models for composite materials. *Acta Mechanica Sinica*, vol. 11, pp. 59-75.

Melchers, R. E. (1999): Corrosion uncertainty modelling for steel structures. *Journal of Constructional Steel re Research*, vol. 52, no. 1, pp. 3-19.

Nakai, T.; Matsushita H.; Yamamoto, N. (2006): Effect of pitting corrosion on the ultimate strength of steel plates subjected to in-plane compression and bending. *Journal of Marine Science and Technology*, vol. 11, no. 1, pp. 52-64.

Paik, J. K.; Lee, J. M.; Ko, M. J. (2003): Ultimate compressive strength of plate elements with pit corrosion wastage. *Proceedings of the Institution of Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment*, Vol. 217, no. 4, pp. 185-200.

Paik, J. K.; Lee, J. M.; Ko, M. J. (2004): Ultimate shear strength of plate elements with pit corrosion wastage. *Thin-Walled Structures*, vol. 42, no. 8, pp. 1161-1176.

Pidaparti, R. M.; Puri, A.; Palakal, M. J.; Kashyap, A. (2005): Two-dimensional corrosion pit initiation and growth simulation model. *CMC: Computers, Materials & Continua*, vol. 2, no. 1, pp. 65-76.

Pidaparti, R. M.; Koombua, K.; Rao, A. S. (2009): Corrosion pit induced stresses prediction from SEM and finite element analysis. *International Journal for Computational Methods in Engineering Science and Mechanics*, vol. 10, no. 2, pp. 117-123.

Sidharth, A. P. (2009): Effect of pitting corrosion on ultimate strength and buckling strength of plates-a review. *Digest Journal of Nanomaterials and biostructures*, vol. 4, no. 4, pp. 783-388.

Sofronkov, N.; Vivdenko, N. I.; Magdenko, E. I.; Chivireva, N. A. (1986): Corrosion of ship hull steels under the action of liquid chemical cargoes. *Materials Science*, vol. 21, no. 4, pp. 381-382.

Takashima, S.; Nakagaki, M.; Miyazaki, N. (2007): An elastic-plastic constitutive equation taking account of particle size and its application to a homogenized finite element analysis of a composite material. *CMES: Computer Modeling in Engineering & Sciences*, vol. 20, no. 3, pp. 193-202.

Wang, P. B.; Yao, Z. H.; Lei, T. (2006): Analysis of solids with numerous microcracks using the fast multipole DBEM. *CMC: Computers, Materials & Continua*, vol. 3, no. 2, pp. 65-76.

Wang, X. T.; Duan, J. Z.; Li, Y.; Zhang, J.; Ma, S. D.; Hou, B. R. (2005): Corrosion of steel structures in sea-bed sediment. *Bulletin of Materials Science*, vol. 28, no. 2, pp. 81-85.

Yang, Q. S.; Becker, W. (2004): Comparative investigation of different homogenization methods for prediction of the macroscopic properties of composites. *CMES: Computer Modelling in Engineering & Sciences*, vol. 6, no. 4, pp. 319-332.

Yao, Z. H.; Kong, F. Z.; Wang, H. T.; Wang, P. B. (2004): 2D simulation of composite materials using BEM. *Engineering Analysis with Boundary Elements*, vol. 28, no. 8, pp. 927-935.

Yuan, W. F.; Tan, K. H. (2009): Heat transfer in composite beams using combined cellular automaton and fibre model. *CMC: Computers Materials & Continua*, vol. 13, no.1, pp. 49-62.

Zheng, Q. S.; Du, D. X. (1998): Closed-form interaction solutions for overall elastic moduli of composite materials with multi-phase inclusion, hole and microcracks. *Key Engineering Materials*, vol. 145-149, pp. 479-488.