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Fire Safety Analysis of Plastic Steel Frames

Rong-gang Zhang[1,2], Hong-tao Zhang[3], Yu-xing Bai[3], Jian-ling Gao[3], Lai-yong Zhang[2],Bing-ye Xu[1]

Abstract: Based on the upper bound theorem, the fire resistance is studied using the combination of element collapse mechanisms of steel frames, where the element collapse mechanisms are automatically determined from independent mechanisms. The fire limit load is calculated by solving a nonlinear mathematical programming. The computing procedure is programmed by FORTRAN language. Results show that this method is useful to find the collapse mechanism with the lowest fire limit load, which can provide a theoretical and practical way for the fire design of steel frame structure.

Keywords: upper bound, nonlinear mathematical programming, steel frame, fire limit load,element collapse mechanism

1 INTRODUCTION

Until now the fire resistance design of steel frames is still using the 'Prescriptive Method', especially in the design of steel frames of petrochemical equipments in China.According to the Chinese Petrochemical Enterprises Code for Fire Protection (GB50160-2008), it demands fire protection for the first layer of steel frame, all beams and columns for tubular below 4.5 meters. But other international companies have their different demands for fire protection. such as SHELL in Netherland below 12 meters, BP in Britain below 9 meters, KBR in USA below 8 meters. This requires the performance based approach of fire resistance design in steel frame structures, which can consider external load conditions, fire loads and material nonlinearity.

Now the fire resistance design for steel frames can be reduced to two methods: finite element method [Chan et al (2010); Jiang et al (2002); Li et al (2007); Najjar and Burgess(1996); O'Connor and Martin (1998)] and direct method [Toh, Fung,

Tsinghua University,Beijing,100084,China
HQCEC,Beijing,100029,China
North China University of Technology,Beijing,100144,China)

Tan(2001);Wong (2001); Zhang et al (2008); Zhang, Zhang, Bai(2007)]. The finite element method has been widely used suitable for the material and geometrical nonlinearity. But it can only solve some special problems and be difficult for engineering applications. The direct method,such as the plastic limit analysis method, can obtain the limit load factor of structures and be used more and more in fire resistance design.

This paper proposes an upper bound limit analysis method for the fire resistance design of steel frames. Firstly, the independent mechanisms of steel frames are obtained by solving a linear equation. Then the element collapse mechanisms are automatically determined from the independent mechanisms, the same as Deeks' method [Deeks (1996)]. Then the real collapse mechanisms are found by using the element mechanisms combination, and the limit load factor is the objective function and the elevated temperature of fire is a variable, which is a nonlinear mathematical programming and can be solved by the Newton method. The lowest load factor is related with the lowest bearing capacity of steel frame, which is the fire resistance design parameter. Some examples are shown to indicate this method's validity.

Similar with classical limit analysis theorem,following assumptions are made: the constitutive relation is the rigid perfectly plastic, and the displacement is small and the frame is stable.Except that, the temperature in steel frames is assumed to be uniform and the material degradation of steel in high temperature is time independent.

2 YIELD STRESS OF STEEL WITH TEMPERATURE

According to EC3-1.2[EC3(2001)], the effective yield stress f_{YT} of steel with temperature has following relation:

$$f_{YT} = \left[1 + \frac{T_s}{767\ln(T_s/1750)}\right] \cdot f_{Y20} \qquad 0 \leq T_s \leq 600°C \tag{1a}$$

$$f_{YT} = \left[\frac{108(1-\frac{T_s}{1000})}{T_s - 440}\right] \cdot f_{Y20} \qquad 600 \leq T_s \leq 1000°C \tag{1b}$$

Here f_{Y20} is the yield strength at temperature $20°C$, and T_s is the temperature of steel.

3 UPPER BOUND APPROACH FOR FIRE RESISTANCE DESIGN

The upper bound method for fire resistance analysis is: when fire happens, temperature arises.Although the loads acting on the steel frames constant, the bearing capacity of frame structure is decreased because of the decreasing of yield strength of steel. With the temperature elevated, the plastic hinges are formulated until they are many enough to make the frame collapse.

The temperature at that time is named 'the fire limit load'. The internal plastic work of steel frame collapse mechanism at the elevated temperature is:

$$W_i = \sum M_{uj}(t) \cdot \theta_j \tag{2}$$

Here $M_{uj}(t)$is the yield moment of plastic hinge at temperature t, θ_j is the rotation angel of corresponding plastic hinge.

The external work of frames is expressed as:

$$W_e = \int_{ST} P_i \cdot \dot{u}_i ds = P_i u_i \tag{3}$$

Here P_i is the generalized external load and $\dot{u}_i$ is the displacement rate of collapse frame.

Based on the virtual work of collapse mechanism:

$$W_i = W_e \tag{4}$$

Here W_i is the total internal work with the collapse mechanism and W_e is the total external work.

Let $\eta = \frac{W_i}{W_e}$, here η is critical limit load factor. The frames are in critical collapse state when $\eta = 1$ and in safe when $\eta > 1$, in failure when $\eta < 1$.

4 DETERMINATION OF COLLAPSE MECHANISMS

There have been many ways to find the collapse mechanism for the arbitral steel frames. As Deeks' method is suitable for mechanism combination methods, this method is used and listed.

Firstly, the independent mechanisms for an assembly should be found where the joint rotations and axial deformations are neglected. A compatibility matrix **C** is formulated relating the elongations **e** of the frames to the displacements **d** of the joints. That is

$$\mathbf{e} = \mathbf{Cd} \tag{5}$$

And the valid mechanism condition is $e = 0$. As the number of independent mechanism is equal to the number of independent displacements d^i, while the number of dependent displacements d^d is equal to the number of rows of **C**, the linear equations are changed as:

$$[\mathbf{C}^d|\mathbf{C}^i][\frac{d^d}{d^i}] = 0 \tag{6}$$

By using Gauss elimination, the C matrix reduces $\mathbf{C}^d$ to the identity marix $\mathbf{I}$ and $\mathbf{C}^i$ to $\mathbf{C}^{ri}$.

$$\mathbf{I}\mathbf{d}^d + \mathbf{C}^{ri}\mathbf{d}^i = 0, \qquad \mathbf{d}^d = -\mathbf{C}^{ri}\mathbf{d}^i \tag{7}$$

So the independent mechanism can be found and by setting the independent displacement to unity the dependent displacements can be computed using Eqn.(7).

Next the element collapse mechanism can be obtained from the set of independent mechanism by using linear combination to eliminate the common active hinges belonging to each other. And the process is given in detail in [Deeks (1996)].

As soon as the element collapse mechanisms found, the collapse combination method is used to obtain the collapse load and fire limit load. Assumed the internal plastic work of element collapse mechanism n is $\mathbf{W}_{in}$, corresponding external work $\mathbf{W}_{en}$,the total number of element collapse mechanisms $\mathbf{N}$, the limit load factor is

$$\eta = \frac{\sum_{1}^{n} x_n \mathbf{W}_{in}}{\sum_{1}^{n} x_n \mathbf{W}_{en}}, \qquad (x_n \geq 0) \tag{8}$$

Here x_n is the rotation scale of element collapse mechanism. Here a nonlinear mathematical programming algorithm is used to obtain the fire limit load, and the process is stated following.

The limit load factor η in normal temperature is obtained by using Newton method to solve Eqn.(8). If $\eta > 1$, frames temperature arise to a new value with corresponding plastic moments changed. η is recalculated using Eqn.(8) and compared with 1. So the fire limit load (temperature) is obtained until the limit load factor $\eta = 1$ and the combined collapse mechanism is got with the rotation factor x_n.

After solving Eqn.(8), the plastic displacement of the mechanism can be obtained by the rotation scale x_n and element collapse mechanism n. For example, a single plane frame is shown in Fig.1. The element collapse mechanisms are shown in Fig.1(b, c), with the rotation scale x_1, x_2. So the displacement $\mathbf{d}_i$of joint i can be calculated as $\sum_{n=1}^{2} x_n \mathbf{d}_{in} \quad (i = 1,2,3,4,5)$.

Here an alternative upper bound method for solving the displacements $\mathbf{d}$ is proposed and expressed as:

$$\begin{cases} \lambda = \min\limits_{\mathbf{d}} \sum\limits_{i \in I} \mathbf{M}_i \sqrt{\mathbf{d}_i^T \mathbf{d}_i}/l_i \\ s.t. \qquad \mathbf{Q}^T \mathbf{d} = 1 \end{cases} \tag{9}$$

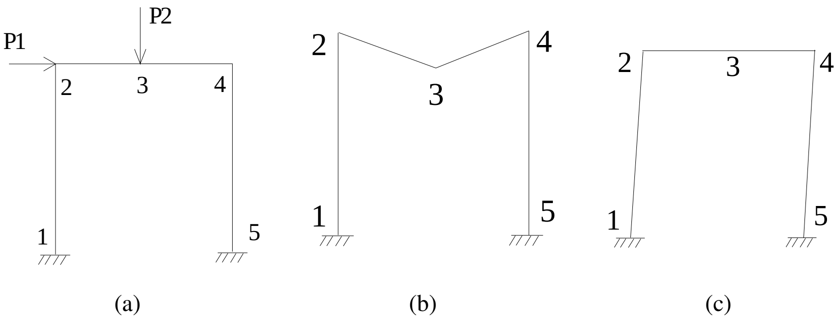

Figure 1: single plane frame structure (a) potential plastic hinges and load condition (b) element collapse mechanism 1 (c) element collapse mechanism 2

$\mathbf{Q}$ is the external load condition. Equation (9)'s objective function is nonlinear and non-smooth. The rigid region need be first determined, because the objective function is non-derivative when $\mathbf{d}^T\mathbf{d} = 0$. Here a directly iterative algorithm is used to solve it.

Before solving it, rigid area is unknown. Iteration starts from the full plastic state, the rigid area of objective function is identified continuously during iterative process, and objective function is updated. The constraint $\mathbf{Q}^T\mathbf{d} = 1$ is introduced by Lagrange multiplier into the objective function. Load factor is decreased and converged to the true solution. Algorithm procedure is listed as follows:

Step 0: solve the following minimization problem to initialize the node displacements $\mathbf{d}$.

$$\begin{cases} \lambda = \min\limits_{\mathbf{d}} \sum\limits_{i \in I} \mathbf{M}_i \mathbf{d}_i^T \mathbf{d}_i / l_i \\ s.t. \qquad \mathbf{Q}^T \mathbf{d} = 1 \end{cases} \tag{10}$$

After the initial node displacement $\mathbf{d}_0$ is obtained, the initial limit load factor λ_0 can be gotten by:

$$\lambda_0 = \frac{1}{V} \sum_{i \in I} \mathbf{M}_i \mathbf{d}_{0i}^T \mathbf{d}_{0i} / l_i \tag{11}$$

Step $k+1 (k = 0, 1, 2, \cdots)$:judge if $\mathbf{d}^T\mathbf{d} (i \in I)$ are equal to zero so as to determine the rigid set $\mathbf{R}_{k+1}$. The plastic set is $\mathbf{Y}_{k+1}$. The problem is updated as:

$$\begin{cases} \lambda = \min\limits_{\mathbf{d}} \frac{1}{V} \sum\limits_{i \in Y_{k+1}} \mathbf{M}_i \frac{\mathbf{d_i}^T \mathbf{d_i}}{\sqrt{\mathbf{d_{ki}}^T \mathbf{d_{ki}}}} / l_i \\ s.t. \quad \mathbf{Q}^T \mathbf{d} = 1 \\ \qquad \mathbf{d}_i^T \mathbf{d}_i = 0 \quad (i \in R_{k+1}) \end{cases} \tag{12}$$

And the node displacement $\mathbf{d}_{k+1}$is obtained in this step. The limit load factor in this step is:

$$\lambda_{k+1} = \sum_{i \in I} \mathbf{M}_i \mathbf{d}_{k+1}^T \mathbf{d}_{k+1} / l_i \tag{13}$$

Following such iteration step, a sequential decreased limit load factor is obtained, and they are converged to the real value. The convergence criterion is determined by:

$$\begin{cases} \frac{|\lambda_{k+1} - \lambda_k|}{\lambda_{k+1}} \leq \eta_1 \\ \frac{\|\mathbf{d}_{k+1} - \mathbf{d}_k\|}{\|\mathbf{d}_{k+1}\|} \leq \eta_2 \end{cases} \tag{14}$$

Here η_1 and η_2 are the convergence parameters.

5 VERIFICATION

(1)Two-story-single-span frame structure

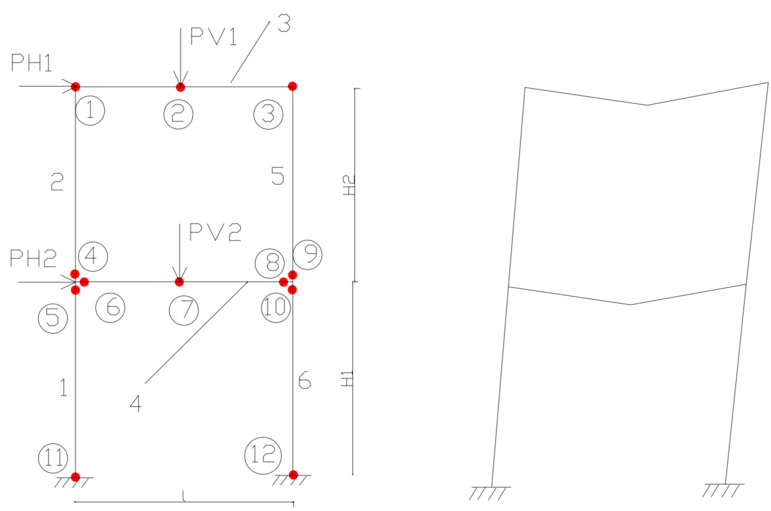

Figure 2: Two-story-single-span frame structure and the collapse mechanism under fire

Two-story-single-span frame structure is analyzed here, as shown in Fig.2. $PV1 = 0.8, PV2 = 2, PH1 = 0.2, PH2 = 0.4, L = 4, H1 = H2 = 3$. In normal temperature,the plastic moments of beam $2,3,5$ are all 1, and those of beam 1, 4, 6 are all

2. If no fire happens, the limit load factor is 2.0, and the plastic hinges are (6 7 8) or (2 3 7 8 11 12). The results agree with Deeks.Now if fire happens in all area, the fire limit load is 548°C, and the plastic hinges are (2, 3, 7, 8, 11, 12), with the corresponding displacement velocity of members shown in Fig. 2. And ANSYS result is 481°C.

(2)Four-story-Four-span plane frame

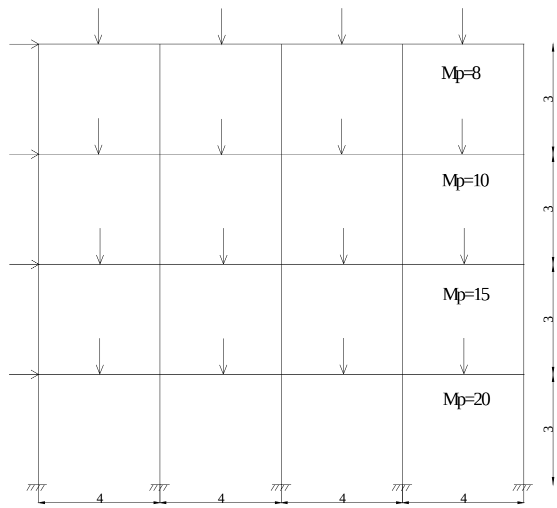

Figure 3: Four-story-Four-span plane frame structure (All forces 5)

Four-story-Four-span plane frame structure (All forces 5) Four-story-Four-span plane frame structure is analyzed, as shown in Fig.3. In normal temperature,the limit load factor is 2.70, and the results agree with Deeks. Now if fire happens in all area, the fire limit load is 613°C and the collapse mechanism is a sway mechanism. ANSYS result is 554°C.

6 CONCLUSIONS

Based on the upper bound theorem, the fire resistance is studied using the combination of element collapse mechanisms of steel frames, where the element collapse mechanisms are automatically determined from independent mechanisms. The fire limit load is calculated by solving a nonlinear mathematical programming. Results show that this method is useful to find the collapse mechanism with the lowest fire limit load. Comparison with the FEM analysis shows that this method provides a theoretical and practical way for the fire design of steel frame structures.

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