 CMC, vol.22, no.3, pp.261-273, 2011

Estimation of the Residual Stiffness of Fire-Damaged Concrete Members

J.M. Zhu[1], X.C. Wang[1], D. Wei[2], Y.H. Liu[2] and B.Y. Xu[2]

Abstract: The residual stiffness of concrete member after fire is a very important parameter of the load-bearing ability and seismic performance of fire-damaged concrete structures. It is also one of the most important criteria for repairing and reinforcing the fire-damaged concrete structures. Based on the equivalent elastic modulus method, improved segment model method and parameter inversion method developed in this paper, the residual stiffness of concrete members exposed to standard fire is calculated and the effects of fire duration, steel ratio and section size on the stiffness are also presented in detail. The results show that these three methods can easily and effectively estimate the residual stiffness of fire-damaged concrete members. These methods and their findings can be useful for designing and assessing the fire resistance of concrete structures.

Keywords: fire, concrete members, residual stiffness, equivalent elastic modulus method, improved segment model method, parameter inversion method

1 Introduction

The residual stiffness of concrete member after fire is one of the most important parameters for evaluating the safety and reliability of fire-damaged concrete structures. Therefore, to obtain its accurate figure is an important basis for analyzing and assessing the stress state, damage level and residual mechanical properties of concrete structures after fire. Based on the amended traditional method and piecewise linear representation of concrete materials during fire, Deng [Deng (2006)] investigated the equivalent stiffness of a concrete member subjected to 3-face heating with the 300°-700° two-step method. This method, however, has flaws, since it only simulates the non-linear changes of elastic parameters with two-section constant and fails to take the contributions of steel bars into consideration. Wu [Wu (2003)] presented the fitting formula to calculate residual equivalent stiffness of concrete mem-

[1] College of Architecture , North China University of Technology, Beijing 100144, China
[2] Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China

bers with rectangular cross-section exposed to 2-face symmetrical heating or 4-face heating. This formula is the function of maximal and minimal temperatures of the cross section provided that the neutral axis of fire-damaged concrete member keeps unchanged. In the literature review [Shen, Shen and Huang (1997)], the amended traditional method and finite element method were adopted respectively to analyze the residual stiffness of typical simply-supported beams. The results show the residual stiffness is overestimated by using the amended traditional method due to the concrete degradation. Given different geometric shapes of concrete members' cross-section, different heating modes and uneven material and mechanic properties along the cross-section, to calculate the stiffness is quite difficult. Usually it requires an enormous computational work using the finite element method to get a precise figure, so it is impossible to use the finite element method for every concrete member's fire-resistance design and post-disaster evaluation. Therefore, for the analysis of inner force and deformation of concrete members it is of great significance to develop a simple and clear, accurate and practical method to estimate the residual equivalent stiffness of concrete members [Deng (2006)].

In this paper, three methods, which are the equivalent elastic modulus method, improved segment model method and parameter inversion method, are developed to estimate the residual equivalent stiffness of concrete members according to the different heating modes. The cross-section of concrete member is firstly divided into several layers, in which each layer is considered as uniform media according to the theory of "zoning". Based on the equivalent elastic modulus method of composite materials, the residual stiffness of beams and plates exposed to one-face heating can be equivalent to the stiffness of laminated beam and shell. Then since the stiffness of cross section under high and normal temperature is the same, the improved segment model method transforms the stiffness of high-temperature cross section with complicated mechanical property to that under normal temperature. The above two methods dramatically simplifies the stiffness calculation of damaged concrete members. While, the parameter inversion method developed in this paper can effectively estimate the residual equivalent stiffness of concrete members by considering the effects of different loadings, boundary conditions and local fire, which agrees well with the real situation.

2 Analysis models

According to the damages or failure sustained by reinforced concrete members after fire, three kinds of stiffness should be estimated, which are the bending stiffness (*EI*), compressive stiffness (*EA*) and shearing stiffness (*G*A). Among them, two most important basic variables are the equivalent elastic modulus and effective cross-section area. While the two variables are fixed, the section equivalent rigidity can be determined [Deng (2006)].

The basic elements of reinforced concrete structure include beam, column and plate, etc. Take the different heating modes of a rectangle cross-section of an indoor reinforced concrete pillar in fire as an example, which is shown in Fig. 1, fire exposure cases are classified into one-sided fire, two-sided fire£¨two adjacent sides or two opposite sides£©, three-sided fire and four-sided fire. According to the deviations of neutral axis in the cross section of fire-damaged members, the calculation of the equivalent stiffness can be divided into three categories in different situations: (1) fire on four sides and fire on opposite sides; (2) one-sided fire; (3) three-sided fire and fire on two adjacent Sides [Franssen, Pintea and Dotreppe (2007), Tan and Yao (2004)].

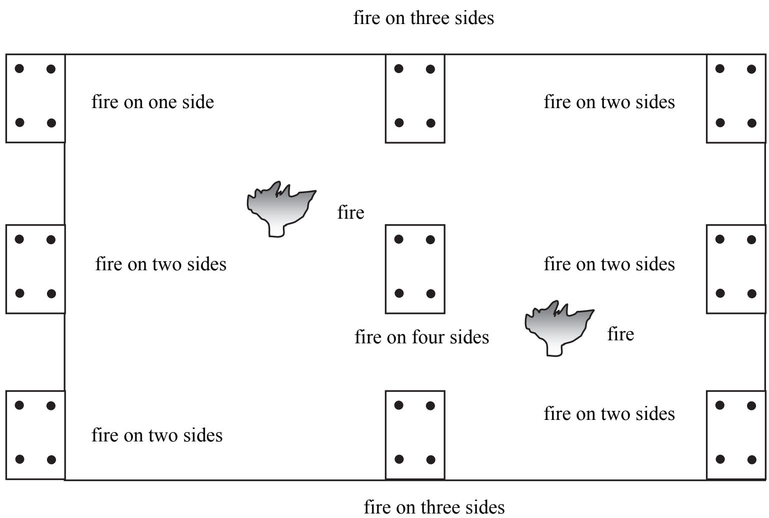


Figure 1: Fire exposure cases of concrete columns with indoor fire

The calculation of the section equivalent stiffness involves three basic processes: firstly, determining the distribution of temperature field on the section; secondly, calculating the neutral axis position and obtaining information about equivalent cross-section; thirdly, achieving the section equivalent stiffness of fire-damaged concrete members. In the calculation, the following assumptions are made: (1) the temperature field on the section is known by field inspections or calculated from other methods; (2) the effects of cracks and concrete falling on fire temperature are ignored.

3 The equivalent elastic modulus method

3.1 Fundamental model

Material properties of concrete structures under high temperature show non-linear changes along the direction of fire. According to the concept of "zoning", the temperature gradient can be expressed as piecewise linear. It means that dividing the cross-section of concrete members into different small areas along the direction of fire, the temperature of each area is assumed to be symmetrical and equals to that of each area's central point. Therefore, concrete material behavior after fire can be expressed as piecewise linear and the material behavior of each area can be replaced by that of its central point, which enables fire-damaged concrete to possess characteristics of laminated composite or functionally graded material.

Based on the principle of stiffness equivalence of virtual laminated beam in mechanics of material [Ou, Xiao and Zhong (1985)], the equivalence of elastic modulus of concrete beam and column after fire is valid. Compressive stiffness (*EA*) and bending stiffness (*EI*) in the stiffness matrix of a beam element are calculated with the following two equations:

$$EA = \sum_{i=1}^{N} E_i A_i \tag{1}$$

$$EI = \sum_{i=1}^{N} E_i A_i d^2 \tag{2}$$

where N is the number of layers virtually divided in the cross-section, E_i and A_i are respectively the elastic modulus and the sectional area of the i virtual layer, d is the distance from the center of every layer to the neutral axis.

The equivalent elastic modulus of plate and shell structures can be obtained with the equivalent strain energy principle [Sun and Lin (1993)]. Under the assumption that the thickness of each layer is the same, the equivalent stiffness is

$$D_{ij} = \frac{4}{h^3} \sum_{i}^{N} (z_{i+}^3 - z_{i-}^3) G_{ij} = \frac{1}{h^3} \sum_{i}^{N} (12 z_i^2 t_i + t_i^3) G_{ij} \tag{3}$$

In the above equation, h and N indicate total thickness and total number of layers respectively; $G_{ij}$is the stiffness of each layer plate; $t_i = h/N$ is the thickness of each layer; and the $z_i = |h\,[(N+1)/2 - i]\,/N|$ is the distance from the center of the i layer to neutral axis.

It is clear that the increase in the number of layers will involve slightly larger computational work, but the computational accuracy will be improved greatly.

3.2 The calculation of neutral axis

Concrete members after fire, especially beam and plate structures exposed to one-sided fire along the fire direction, their mechanical properties will be badly destroyed and neutral axis of members under axial force and bending moment will be deflected. In the calculation of the offset of neutral axis, it is assumed that there is no interaction between longitudinal fibers and the cross-section of members in the bending process is still plane.

Based on the plane cross-section assumption, we can obtain

$$X = \frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2} h \tag{4}$$

where h is the height of the cross section, and X is the distance between the neutral axis and the top of cross section. ε_1 and ε_2 are the maximal strain in tension zone and compression zone respectively.

With the establishment of local coordinate shown in Fig. 2 and under the static equilibrium conditions, the following equations of static equilibrium can be derived:

$$\int_0^X \sigma_{1i} B dx = \int_X^h \sigma_{2i} B dx \tag{5}$$

$$\int_0^X \sigma_{1i} B(X - x) dx + \int_X^h \sigma_{2i} B(x - X) dx = M \tag{6}$$

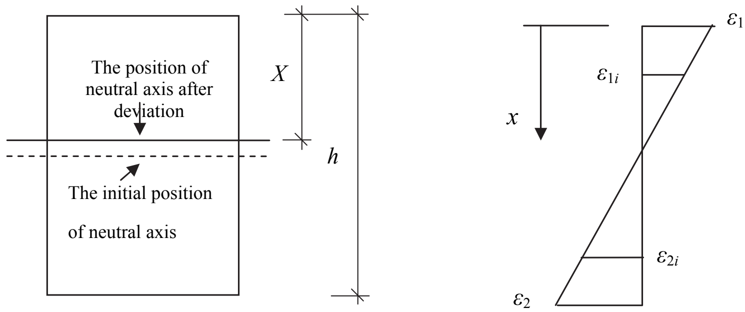
where σ_{1i} and σ_{2i} are respectively the stress of random point in tension and compression zone.*B* is the width of cross section and *M is* bending moment imposed by external force.

4 Improved segment model method

Similar with the calculation process in the segment model, the paper firstly presents the distribution pattern of temperature field, and then calculates the width and length of equivalent cross-section with the integration method, finally obtains the equivalent stiffness and neutral axis. By the same token, take a case exposed to three-sided fire as an example, the coordinate as indicated in Fig. 3 can be established based on symmetry.

As a result of degradation layer-by-layer, material parameters along the heating course can be expressed as a function of coordinates, such as quadratic function or power function. Equivalent cross-section information on rectangular cross section of concrete material is calculated as follows:

$$b_T = \frac{2}{E_c} \int_0^{b/2} E(x) dx \tag{7}$$



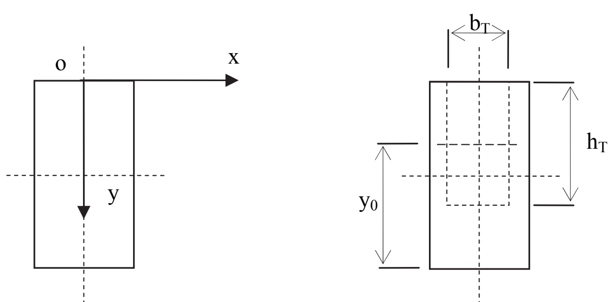
(a) the offset of neutral axis

(b) the assumption of strain distribution in cross section

Figure 2: The calculation of neutral axis offset

$$h_T = \frac{1}{E_c}\int_0^h E(y)dy \tag{8}$$

It is evident that the equivalent cross-section calculated with the integration method is still a rectangle , and according to Mechanics of Materials its equivalent section area and moment of inertia can be easily obtained.



(a) The original cross-section

(b) The equivalent cross-section after fire

Figure 3: Calculation diagram of the cross-section with equivalent stiffness exposed to three-sided fire

The neutral axis doesn't change when cases are exposed to four-sided fire or to fire on opposite sides, so the gross equivalent flexural rigidity of cross-section can be expressed as:

$$G = Q + E_{sr} I_s \tag{9}$$

In the equation, E_{sr} is the elastic modulus of steel bars while I_s is the cross-section moment of inertia relative to neutral axis; Q is the equivalent flexural stiffness of concreter cases.

When exposed to one-sided fire or three-sided fire, the above equation reckons the contribution of steel bars to equivalent stiffness, but this would make the calculation of central axis of cross section complicated. To get an approximate figure, the deviation of neutral axis can be obtained by calculating the height of compression zone as in reference [Su (2004)], that is

$$h_c = (h - c)\frac{\varepsilon_c}{\varepsilon_s + \varepsilon_c} \tag{10}$$

In the above equation, h is the height of cross-section and c is the thickness of concrete protective cover; meanwhile ε_s and ε_c are the yield strain of steel bars and the compressive ultimate strain of concrete respectively. After the position of neutral axis is determined based on concrete material, the equivalent stiffness of steel bars can be solved according to the equation of moving axis. The way to select relevant parameters can be found in the literature [Li, Han, Lou and Jiang (2006)].

5 The parameter inversion method

Material property degradation of concrete caused by high temperature brings additional deformation to concrete structure. Under the assumption that additional deformation excludes residual deformation of creep or plasticity, damages resulted from this additional deformation can be expressed as changes in some structural properties (such as stiffness). Hence, the equivalent stiffness of structure after fire can be calculated by using the method of parametric identification. It is apparent that stiffness of member is somewhat affected by such factors as quantity and position of steel bars in reinforced concrete case, different types of fire exposure, cracks and concrete falling. By dividing cases longitudinally into different areas, the degree of their influence can be reflected and expressed as the equivalent stiffness of each area by using the inversion model.

In the inverse estimation of equivalent stiffness, the objective function $\Gamma(p)$ sets the unknown identification parameter p as equivalent flexural rigidity EI (for beam) or equivalent compression stiffness EA (for column) or EI and EA (for the compressive and flexural member). When the function $\tilde{A}(\mathbf{p})$ is established, the inverse estimation of parameters can be considered as solving the following nonlinear optimization problem:

$$\text{Minimize: } \Gamma()s \tag{11a}$$

$$\text{Subject to: } K(p,u) = F$$

$$p \in D_p, f \in D_f \tag{11b}$$

where $\Gamma(p)$ is the objective function, **p** is the unknown model parameter, u_s is measured displacement, **u** is actual displacement, and **S** is the selective matrix. It is assumed that errors in measured displacement are additive, the following equation can be obtained, $u_s = Su + w$, where w is additive error; ***f*** is load input, **K** and **F** are corresponding differential operators. There are many optimization algorithms for solving the objective function. Since this paper is based on elastic postulate, here Gauss-Newton algorithm is used to calculate the objective function of ordinary least squares, whose detailed solving process is available in the reference [Xiang (2002), Snanyei, Imbaro, Jennifer and Linfield (1997)].

With a view of conducting finite element modeling of concrete cases exposed to one-side and many-side heating modes, and obtaining relatively meticulous deformation distribution, the paper adopts 3D beam element to simulate the equivalent stiffness of fire-damaged concrete beam and column and their bending and axial deformation due to external loads.

6 Numerical results and discussions

Example 1

It is based on the reference [Shen, Shen and Huang (1997)]. Suppose there is a concrete simply supported beam subjected to 1-face heating (see Fig. 4), its size and reinforcement are shown in Tab. 1. According to the literature, the residual stiffness of the simple beam is the ratio of uniform load q to cross-deflection (Δ), that isq/Δ. In light of the mechanical model of fire-damaged concrete materials under the ISO-834 standard fire curve, the residual stiffnesses of simply supported beam in different fire duration calculated by such methods as finite element, equivalent elastic modulus and parameter inversion are presented in Tab. 2.

Note: C_R and $T_R$respectively refer to the residual stiffness of compression zone and tension zone under heating.

As indicated in Tab. 2, the parameter inversion method can derive the residual stiffness directly and since the input displacement is the calculation result of the FEM, so their results are almost the same. As shown in the calculation, the results obtained with FEM, equivalent elastic modulus method and parameter inversion

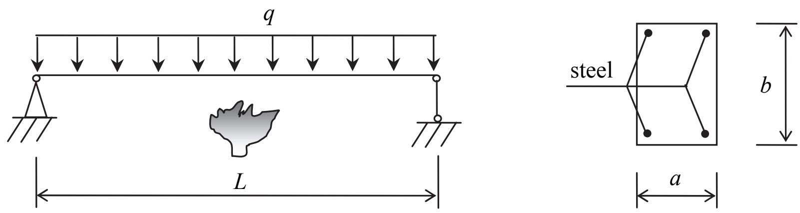


Figure 4: The diagram of concrete beam subjected to uniform load

Table 1: Size and Reinforcement of Simply Supported Beam

Case	Size (a mm × b mm × L mm)	Reinforcement ratio in tension zone	Reinforcement ratio in compression zone.
Case 1	300× 500 × 6000	1.0 %	0.5 %
Case 2	300× 500 × 6000	2.0 %	1.0 %
Case 3	400× 700 × 6000	1.0 %	0.5 %
Case 4	400× 700 × 6000	2.0 %	1.0 %
Case 5	500× 800 × 6000	1.0 %	0.5 %
Case 6	500× 800 × 6000	2.0 %	1.0 %

method are in consistence with those in the literature except for case 3 and case 5. The differences lie in the fact that thermal parameter and mechanical property parameter of concrete and steel are different between this paper and literature. This is because although the same standard time-temperature curve is adopted, inside temperature field, residual stiffness and deformation after fire calculated in the paper are still slightly different from that in the literature.

The following conclusions can be drawn from the findings. With the increase of fire duration, mechanical properties will deteriorate more seriously and the residual stiffness will decrease gradually. Cross-Section size and reinforcement ratio have less influence on residual stiffness of fire-damaged simply supported beam, but different types of fire exposure have greater impact, for example, compared to tension zone, fire exposure in compression zone exerts bigger influence.

Example 2

A case along its longitudinal direction is usually exposed to heterogeneous heating under local fire, so the residual stiffness of each area can not be accurately depicted based on q/Δ. The change in mechanical property of different areas can be reflected with parameter inversion method, which facilitates the estimation of residual stiff-

Table 2: Residual stiffness of fire-damaged simply supported beam (%)

Case	fire duration /h	Results from the reference [Shen, Shen and Huang (1997)]		FEM		Equivalent elastic modulus method		Inversion method	
		C_R	T_R	C_R	T_R	C_R	T_R	C_R	T_R
Case 1	1	82.0	67.1	75.1	63.7	74.8	71.2	75.1	63.7
	2	70.0	57.7	64.9	52.4	65.6	60.9	64.9	52.4
	3	60.6	50.5	57.1	45.1	57.2	52.2	57.1	45.1
Case 2	1	80.6	68.6	78.9	74.7	80.2	74.3	78.9	74.7
	2	68.4	59.8	70.5	64.4	72.7	64.9	70.5	64.4
	3	58.5	53.2	63.0	57.6	65.3	57.0	63.0	57.6
Case 3	1	88.0	74.6	78.2	75.6	78.2	75.2	78.2	75.6
	2	79.8	66.2	69.5	66.1	70.1	66.1	69.5	66.1
	3	72.3	60.2	63.7	59.7	63.3	58.7	63.7	59.7
Case 4	1	86.5	75.8	82.0	78.1	82.4	77.5	82.0	78.1
	2	78.0	67.6	74.5	69.3	75.9	69.2	74.5	69.3
	3	68.7	61.5	69.3	63.4	69.9	62.4	69.3	63.4
Case 5	1	89.1	76.4	80.6	78.5	80.1	77.6	80.6	78.5
	2	83.1	69.2	72.4	69.6	72.4	69.1	72.4	69.6
	3	77.8	63.9	66.6	63.4	65.9	62.2	66.6	63.4
Case 6	1	87.9	77.5	83.9	80.7	83.8	79.6	83.9	80.7
	2	81.4	70.9	76.7	72.5	77.3	71.9	76.7	72.5
	3	74.5	65.5	71.2	66.6	71.6	65.6	71.2	66.6

ness and reinforcement after fire. Here suppose a simply supported concrete beam exposed to fire on the underside, whose Load conditions and boundary constraints are the same with that of case (1) in example 1. The difference is damages suffered in $L/3$ length of beam's left-half end, $L/3$ length of beam's middle piece and $L/3$ length of beam's right-half end (see Fig. 5). Different levels of damage are expressed by fire durations. Due to minimal thermal conductivity, local thermal conduction among different areas can be neglected in calculation. In the example, FEM is firstly used to calculate the displacement of fire-damaged concrete case, and then the displacement is taken as input parameter of parameter inversion method; finally the residual stiffness is obtained, which are shown in Tab. 3.

Example 3

Suppose a concrete beam exposed to 3-face heating, whose boundary and load con-

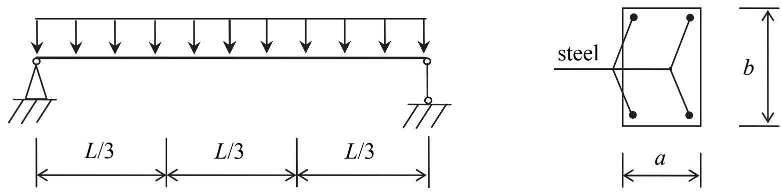


Figure 5: The diagram of concrete beam under the local fire

Table 3: Residual stiffness of simply supported beam after fire by using parameter inversion method

Case	Fire duration / *h*			Residual stiffness		
	left	middle	right	left	middle	right
Case 1	1	1	1	75.1	75.1	75.1
Case 2	2	1	1	58.7	76.1	74.9
Case 3	3	2	1	52.3	66.25	78.66

ditions are the same with case 1 and case 2 in example 1. The residual stiffness of the simply supported beam is calculated with finite element method, improved segment model method and parameter inversion method. The results are shown in Tab. 4. Compared to the finite element method, errors of results obtained with the improved segment model method are acceptable. This shows the residual stiffness of concrete case exposed to fire on many sides can be estimated quickly and effectively by using the improved segment model method.

Table 4: Residual stiffness of simply supported beam subjected to 3-face heating (%)

Case	Fire duration /*h*	FEM	improved segment model method	inversion method
Case 1	1	48.0	31.9	48.0
	2	31.5	19.8	31.5
	3	22.2	17.0	22.2
Case 2	1	53.8	49.7	53.8
	2	37.0	32.2	37.0
	3	26.3	28.0	26.3

Equivalent elastic modulus method and improved segment model method are two ways of calculating the section equivalent stiffness. The two methods assume equivalence between heterogeneous cross section after fire and the section under normal temperature. The underlying principle for the equivalence lies in the fact that the stiffness of cross section under high and normal temperatures is the same. This simplifies the calculation of residual stiffness of fire-damaged concrete members with complicated mechanical property by only computing the stiffness of concrete cross section under normal temperature. It is shown that results obtained with the two methods are conservative, which is conducive for the project. The parameter inversion method gives thorough consideration to various boundaries, load conditions and local fire exposure of concrete cases which helps to accurately estimate the residual stiffness. Results obtained with the three simple and effective methods, with the finite element method and existing findings are quite consistent.

7 Conclusions

Based on the findings of the research, following conclusions can be drawn:

The equivalent elastic modulus, improved segment model and parameter inversion methods developed in this paper can simply, exactly and effectively estimate the residual stiffness of fire-damaged concrete beams. Furthermore, the parameter inversion method can investigate the influences of local fire exposure on the residual stiffness of concrete members after fire. Parameter analysis indicates that fire duration and different heating modes play important roles in the residual stiffness, while the effects of section dimension and reinforcement ratio of simply-supported beam are very small.

References

Deng J. Y. (2006), *Study on the Internal force calculation and fire resistance of rc frame structure subjected to high temperature*. The master's degree thesis of Hunan University.

Franssen J. M., Pintea D., Dotreppe J. C. (2007), Considering the effects of localized fires in the numerical analysis of a building structure. *Fire Safety Journal*, vol. 42, pp. 473–481.

Gang, L.; Liu, Y.H.; Xiang, Z. H. (2009), Computational Framework for Durability Design and Assessment of Reinforced Concrete Structures Exposed to Chloride Environment. *CMES: Computer Modeling in Engineering & Sciences*, Vol. 47, No. 3, pp. 217-252.

Li G. Q., Han L. H., Lou G. B., Jiang S. C. (2006), *Fire resistance design of steel structure and steel-concrete composite structures*. China Building Industry Press.

Ou M. C., Xiao M. X, Zhong L. X. (1985), *Mechanics of materials*. China Building Industry Press.

Shen J. F., Shen D. X., Huang S. J. (1997), *Advance in research on safety assessment process for Reinforced concrete structures after Fire*, Study of Taiwan University of Science and Technology, Taiwan province of China.

Snanyei M., Imbaro G. R., Jennifer A. S. M., Linfield C. B. (1997), Structural model updating using experimental static measurements. *Journal of Structural Engineering*, ASCE, vol. 123, no. 6, pp. 792-798.

Su J. (2004), *Fire Resistant performance research about reinforced concrete columns*. Engineering Master's degree thesis of Harbin Engineering University.

Sun H. D., Lin Y. X. (1993), Computation and analysis of equivalent moduli for composite laminates. *Journal of Taiyuan Institute of Machinery*, vol. 14, no. 2, pp. 113-119.

Tan K. H., Yao Y. (2004), Fire resistance of reinforced concrete columns subjected to 1-, 2-, and 3-face heating. *Journal of Structural Engineering*, vol. 130, no. 11, pp. 1820-1828.

Xiang Z. H. (2002), *Studies on rock and soil parameter identification based on displacement measurements*. Doctoral Dissertation of Tsinghua University.

Wei, D.; Liu, Y.H.; Xiang, Z. H. (2009), A Numerical Method for Estimating the Maximal Temperature Gradients Reached in Fire-Damaged Concrete Structures Based on the Parameter Identification. *CMES: Computer Modeling in Engineering & Sciences*, Vol. 46, No. 1, pp. 77-106.

Wu B. (2003), *Mechanical property of reinforced concrete structure after fire*. Science Press.