 CMC, vol.25, no.1, pp.1-17, 2011

Fracture Mechanics Based Model for Fatigue Remaining Life Prediction of RC beams Considering Corrosion Effects

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Abstract: This paper presents methodologies for crack growth study and fatigue remaining life prediction of reinforced concrete structural components accounting for the corrosion effects. Stress intensity factor (SIF) has been computed by using the principle of superposition. At each incremental crack length, net SIF has been computed as the difference of SIF of plain concrete and reinforcement. The behaviour of reinforcement has been considered as elasto-plastic. Uniform corrosion rate has been assumed in the modeling. Corrosion effect has been accounted in the form of reduction in the diameter and modulus of elasticity of steel. Numerical studies have been carried out to validate the methodologies. It is observed that the predicted remaining life for RC beam without corrosion effects is significantly larger compared to plain beams. The predicted remaining life decreases with increase in percentage of corrosion.

Keywords: Plain concrete, Reinforced Concrete, Stress Intensity Factor, Crack growth, Remaining life, Corrosion.

1 Introduction

Concrete is a widely used material that is required to withstand a large number of cycles of repeated loading in structures such as highways, dams, airports, bridges and offshore structures. Concrete has numerous flaws, such as holes or air pockets, pre-cracked aggregates, lack of bond between aggregate and matrix, etc., from which cracks may originate. Cracks generally propagate in a direction, which is perpendicular to the maximum tensile stress. The fracture behavior of concrete is greatly influenced by the fracture process zone. Few analytical and experimental investigations on fatigue and fracture analysis of concrete structural components

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have been reported [Prasad and Krishnamoorthy (2002); Bazant and Xu (1991)]. The size effect on fatigue in bending is strongly dependent on the structural member size. The characteristics such as fatigue crack growth history as well as critical crack length at failure are also dependent on the structural member size [Bazant and William (1993); Bazant and Kazemi (1990)].

Under cyclic loading, the reinforcement in concrete in?uences residual strength and subsequent crack propagation. The stability and the residual strength of a cracked RC member under fatigue loading, depends on a number of factors such as, reinforcement ratio, specimen size, grade of concrete, fracture properties, and on the tension-softening behavior of concrete. The de-bonding of the reinforcement is attributed to the horizontal crack. Simultaneously, the vertical crack propagation is driven by the externally applied force. The reinforcement behaves as elasto-plastic spring that generally decreases SIF and increases the fatigue life of beam.

When the structural components are subjected to repetitive live loads of high-stress amplitude, according to classical theory, applied loads result in in-plane tensile stresses at the bottom of the components. The stress-state in such structures is often simulated with three-point bending tests. Plain concrete subjected to flexural loading fails owing to crack propagation. Repeated loading results in a steady decrease in the stiffness of the structure, eventually leading to failure. It is of interest to characterize the material behavior subjected to such loading and study the crack propagation and remaining life. The current approaches used to evaluate fatigue performance of structural components are mainly empirical. Fatigue equations based on the well known S-N approach have been developed. Implementation of the conventional S-N approach requires considerable effort and time for experimental data collection for a given design case followed by statistical analysis. A severe limitation of the S-N approach is that it does not use fundamental material parameters that can be determined for use in design or evaluation. The resulting information is not applicable to other design cases with different loading configurations or boundary conditions. Mechanics based approaches that utilize the concept of fracture mechanics to study fatigue crack propagation have also been proposed. For example, Perdikaris and Calomino (1987) showed that compliance measurements provide a convenient method for estimating the traction-free crack length of fatigue loaded concrete specimens. Since then, few experimental investigations on fatigue crack propagation in concrete have been reported [Baluch, et al. 1987, Ramsamooj 1994, Stuart 1982, Subramanian, et al. 2000, Matsumoto and Victor 1999, Toumi and Turatsinze 1998, Slowik, et al. 1996, Bazant and Oh 1991, Ingraffea 1977, Mu, et al. 2004]. The rate of fatigue crack growth in concrete exhibits an acceleration stage that follows an initial deceleration stage. In the deceleration stage the rate of crack growth decreases with increasing crack length, whereas in the acceleration stage there is a steady increase in crack growth rate up to failure. Also, an attempt has been made (Mu, et al. 2004) to apply the fracture mechanics principles to describe the crack growth during the acceleration stage of fatigue crack growth in concrete.

Analysis and design of RC structures require better understanding of the behaviour of the constituent materials, their combinations, and material degradation procedures to ensure safety and serviceability. Reinforcement corrosion is the major cause of damage and early failure of RC structures worldwide with subsequent enormous costs for maintenance and repair.The stress and strain in a RC structure are subject to change with long period of time; during which corrosion of steel develop gradually. Most design codes over simplify design procedures for determining the mechanical response of RC structures under service loads and focus mainly on instantaneous behavior. Corrosion of steel reinforcements in RC structures diminishes the load carrying capacity of RC structures, not only by means of rebar cross-sectional area reduction, but also by bond deterioration as reported by Amleh et al. (2002). In most cases, degradation due to reinforcement corrosion causes decreased serviceability and can in the worst case lead to a loss of structural stability. Therefore, cracking and spalling of concrete cover due to corrosion of the reinforcement define important limit states of durability and stability of RC structures. Although research on reinforcement corrosion in concrete has been extensive in the last few decades, research on the corrosion effect on structural serviceability is scanty (Li and Melchers (2005). Also it was reported by Toongoenthong and Maekwa (2005) that there is a considerable increase in corrosion mass loss with creep consideration. So far, most of the analytical investigations in this field were carried out by the use of finite element analyses based on models, which express the process of concrete cover cracking only in a qualitative way. Aspects of time dependent material properties were mostly neglected. Thus, it is required to develop realistic and quantitative representation, as well as a reliable prediction of corrosion effects on remaining life. To the best of author's knowledge limited work/research was carried out related to crack growth study and remaining life prediction of RC structural components accounting for corrosion effects.

This paper presents methodologies for fracture analysis and remaining life prediction of plain and RC structural components accounting for corrosion effects. A novel method for calculation of SIF and for the prediction of remaining life for RC beams accounting for corrosion has been proposed. SIF has been computed by using the principle of superposition. The effect of corrosion has been modeled in the form of reduction in the diameter and modulus of elasticity of steel and remaining life has been predicted for RC structural components using fracture mechanics principles. Numerical studies have been carried out on plain and RC beams considering corrosion effects to validate the proposed methodologies.

2 SIF calculation and remaining life prediction

In the proposed approach, fracture mechanics principles are used to describe the crack growth phenomena during the acceleration stage of fatigue crack growth in concrete. The fatigue mechanism in plain concrete may be attributed to progressive bond deterioration between aggregates and matrix or by development of cracks existing in the concrete matrix.

Modelling assumptions

i. Plane sections of the beam remain plane after deformation

ii. Fictitious crack surface remains plane after deformation

iii. Normal closing fractions acting on the fictitious crack follow linear stress crack opening displacement

iv. Bottom fiber bending stress in the concrete along the beam is equal to the traction normal to the crack mouth at the bottom of the beam.

2.1 SIF Calculation

To incorporate the reinforcement, based on the principle of superposition SIF has to be calculated as,

$$K_I = K_I^P - K_I^X \tag{1}$$

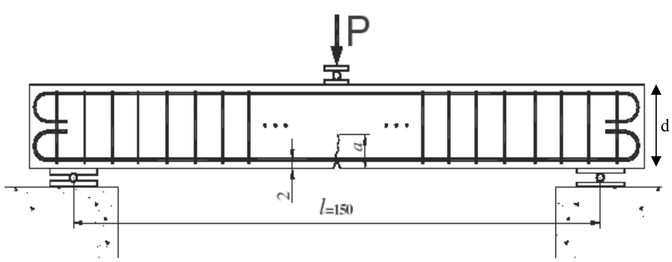

Figure 1: Three point bending of notched RC beam

where K_I^P is SIF for the concentrated load P in a three point bending beam. Schematic representation of RC beam under center point load is shown in Figure 1. SIF at the crack tip for three point bending plain concrete beam with the vertical crack [Tada, et al. (1985)] can be computed by using

$$K_I^P = \frac{6PlF_p(\alpha)}{4bd^2} \tag{2}$$

where

$$F_p(\alpha) = 1.107 - 2.120\alpha + 7.71\alpha^2 - 13.55\alpha^3 + 14.25\alpha^4 \tag{3}$$

and α = a /d

SIF due to force of intensity X acting at the location of reinforcement is [Sumarac et al; 2003]:

$$K_I^X = \frac{2X}{b\sqrt{\pi a}} F_X(\alpha,\beta), \tag{4}$$

where

$$\begin{aligned} F_X(\alpha,\beta) = {} & \frac{3.52(1-(\beta/\alpha))}{(1-\alpha)^{3/2}} - \frac{4.35-5.28(\beta/\alpha)}{(1-\alpha)^{0.5}} \\ & + \left[0.83 - 1.76(\beta/\alpha) + \left(1.3 - 0.3(\beta/\alpha)^{3/2}\right)/\sqrt{1-(\beta/\alpha)^2}\right](1-\alpha+\beta) \end{aligned} \tag{5}$$

and α = a /d, β = c/d and c is the cover of reinforcement from the bottom edge of beam. From the eqns. (2) and (4) and by using eqns. (3) and (5), the crack opening displacement at the location of reinforcement, attributed to the external force P, can be obtained as

$$\delta_{10} = \frac{6Pl}{bdE_b} \int_{\beta}^{\alpha_i} F_F(\alpha) \,.\, F_X(\alpha,\beta)\, d\alpha \tag{6}$$

where d = depth and b = width

E_b is theYoung's modulus of concrete, $\alpha_i = a_i/d$

The crack opening displacement, due to force of intensity X= 1 acting at the location of reinforcement can be computed by using eqns. (4) and (5)

$$\delta_{11} = \frac{8}{\pi b E_b} \int_{\beta}^{\alpha_i} \frac{[F_X(\alpha,\beta)]^2}{\alpha} d\alpha \tag{7}$$

Compatibility condition at the location of the reinforcement can be obtained by using eqns. (6) and (7) and the debonding length l_0. Compatibility condition requires that the crack opening displacement, given by eqns. (6) and (7), multiplied by X, must be equal to the total elongation of debonded reinforcement of length l_0 due to force X. This condition can be written as

$$\delta_{10} - \delta_{11} = \frac{l_0 X}{F_a E_a} \tag{8}$$

In the above eqn. $E_a$stands for Young's modulus of reinforcement and F_a is the area of reinforcement. The unknown force in the reinforcement can be expressed using eqn. (8) in the form [Sumarac et al., 2003].

$$X = \frac{3Pl\pi}{4d} \frac{F_F(\alpha)}{F_X(\alpha) + (\pi b l_0 E_b / 8 F_a E_a)} \tag{9}$$

where $F_F(\alpha)$ is the integral which appears in eqn. (6), and $F_X(\alpha)$ is the integral in eqn. (7). Once the force in the reinforcement X is known, it is possible to determine the elongation at the plane of reinforcement, which also represents the crack opening displacement as given by

$$\delta_A = \frac{X l_0}{F_a E_a} \tag{10}$$

After X is determined from eqn. (9), the net SIF at the crack tip can be expressed as

$$K = K_I^P - K_I^X \tag{11}$$

2.2 Remaining Life Prediction

Remaining life can be predicted by using any one of the standard crack growth equations (such as Paris, Erdogan-Ratwani, etc.)

$$\frac{da}{dN} = f(\Delta K) \tag{12}$$

Here ΔK can be computed by using following expression

$$\Delta K = K_{max} - K_{min}$$

$$dN = \sum \frac{da}{C(\Delta k)^m} \tag{13}$$

where C and m are crack growth constants.

2.3 Modeling of Corrosion

In the present study, the effect of corrosion is accounted for as reduction in diameter and modulus of elasticity of steel. A two-phase model composed of corrosion product with non-corroded steel has been employed to represent the corrosion aspects [Toongoenthong and Maekawa, 2005]. Uniform corrosion accumulation is assumed for simplification of computation. The parent steel exhibits a smaller diameter due to corrosion loss. The remaining diameter of the uncorroded steel can be expressed as

$$D_{Corroded-Steel} = D\sqrt{1-\gamma} \tag{14}$$

where D is the original steel diameter (m) and γ is the effective volume fraction loss of steel per unit length ranging from 0 to 1 wherein 0 means no corrosion and 1 means complete loss of reinforcement bars. The corroded product, which effectively creates internal stress, consequently forms around the parent steel. The mean diameter of the corroded reinforcement system, which consists of parent steel and its corrosive product, is obtained in the case of stress free and geometrically uniform expansion as

$$D_{Corroded-Layer} = D\sqrt{1-\gamma(\alpha-1)} \tag{15}$$

Here, is defined as the coefficient of expansion of corroded substance (assumed α=2) suggested by Morikawa et al.(1988).Then, the reinforcement free expansion strain is considered as

$$\varepsilon_{\text{s,free}} = \sqrt{1+\gamma(\alpha-1)} - 1 \tag{16}$$

The above equation is a function of specific level of corrosion 'γ'. The stress induced by the restraint of surrounding concrete is computed by multiplying the total strain of corroded system with its average stiffness strain. The average stiffness of corroded system $E_{s,eq}$, is given by

$$E_{\text{S,eq}} = \frac{1+\gamma(\alpha-1)}{\left(1-\frac{\gamma}{E_S}\right)+\left(\frac{\gamma\alpha}{G}\right)} \tag{17}$$

E_S =modulus of steel (210 Gpa) and G is the stiffness of corroded substance (assumed to be 14.7 GPa as suggested by Lundgren et al. (2002)).

There are ceratin limitations for the proposed model such as the increase in corrosion with time has not been considered, the effect of stress in concrete developed due to corrosion of reinforcement has not considered and effect of corrosion induced cracking and loss of bond due to corrosion has neglected. These aspects will be addressed through future research.

3 Numerical Studies

Crack growth analysis and remaining life prediction studies have been carried out by using LEFM principles for concrete three point bending specimens made of concrete under constant amplitude loading. The details of the studies are presented below.

3.1 Problem 1

This problem was studied by [Toumi and Turatsinze 1998] for three point bending concrete specimen (refer Fig. 2).

Length (S) = 320 mm

Depth (b) = 80 mm

Thickness (t) = 50 mm

Initial crack length (a_o) = 2 to 4 mm

Compressive strength = 57 MPa

Tensile strength = 4.2 MPa

Fracture toughness = 0.63 MPa$\sqrt{m}$

Crack growth equation = Paris

Stress ratio = 0.2

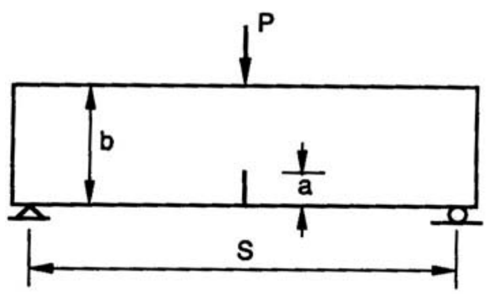

Figure 2: Three point bending problem

The bending tensile stress (f_b) can be calculated by using the formula given below
$f_b = 3PS / 2tb^2$

Crack growth analysis and remaining life prediction has been carried out by using LEFM principles. Remaining life has been predicted for the different loading cases. Table 1 shows the predicted remaining life values for various loading cases along with the respective experimental values presented by [Toumi and Turatsinze 1998]. From Table 1, it can be observed that there is maximum difference of about 12%

between the predicted and experimental values. Figure 3 shows the variation of predicted remaining life with crack length for differed loading cases.

Table 1: Predicted remaining life values using LEFM principles

S. No.	Max. Stress (MPa)	Crack growth constants		Remaining life (Cycles)		% diff.
		C (μm/cycle)	m	Present study	Exptl. [Toumi and Turatsinze 1998]	
1	1.125	6.45	4.18	28689	32222	10.96
2	1.05	0.33	2.31	57251	63611	9.98
3	0.975	0.26	2.25	62603	69444	9.82
4	0.9	2.04	2.6	16188	18333	11.7

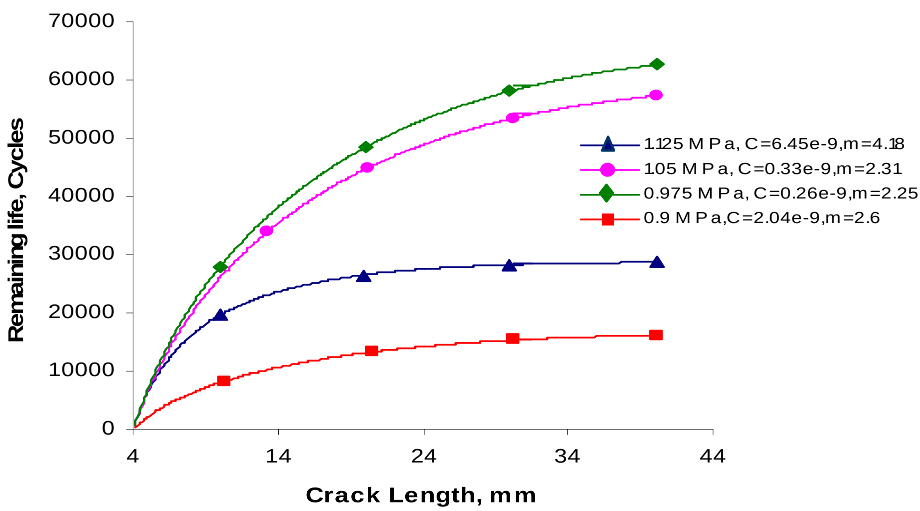

Figure 3: Crack Length vs remaining Life

3.2 Problem 2

Numerical studies have been carried out for the beams under three point bending to compute SIF and remaining life accounting for corrosion effects. The details are given in Table 2. The results are presented only for typical cases.

3.2.1 SIF Comparison

SIF for medium and long beams is calculated using the eqns. given in Section 2.1. The plot of SIF v/s crack depth for M30 grade long beam and medium beam are

Table 2: Description of Beams

Beam/Details	Medium	Long
Length (mm)	400	800
Width (mm)	50	50
Depth (mm)	100	400
Reinforcement (mm)	2 Nos.10 Φ	2 Nos. 10 Φ
Load (N)	2240	4473

Crack growth constants: C = 6.45e-3, m = 4.18

shown in Figures. 4 and 5 corresponding to plain and RC beams.

From Figures 4 and 5, it can be observed that SIF for RC beams is much lesser (reduces to approximately 50%) than the SIF for the corresponding plain concrete beams.

3.2.2 Crack growth study and remaining life prediction of RC beams

The remaining life is predicted for plain and RC beams for M30 grade concrete. Figures 6 and 7 show the plot of remaining life vs crack depth for long and medium beams.

From Figures 6 and 7, it can be observed that the predicted remaining life is significantly higher for RC beams compared to plain concrete beams. It can be observed that predicted remaining life for the case of RC beam is about 10 times higher compared to plain concrete beams. Figures 8 & 9 shows the variation of remaining life for medium and long RC beams for M20 and M30 grades of concrete. It can be observed that remaining life increases by about 15% with increase in grade of concrete under similar loading conditions.

3.2.3 Crack Growth study and Remaining Life Prediction accounting for Corrosion

The remaining life of all the three types of beams is predicted in accordance to 40% and 80% corrosion in reinforcement for M30 concrete. The diameter of the corroded bars have been calculated using eqn. (14). The remaining life comparisons between plain concrete beam, RC beam and corroded beams are shown in Figures 10 and 11 for long and medium beams respectively. From figures 10 and 11, it can be observed that the predicted remaining life decreases significantly due to corrosion. Further, it can be observed that remaining life decreases with the increase of percentage of corrosion.

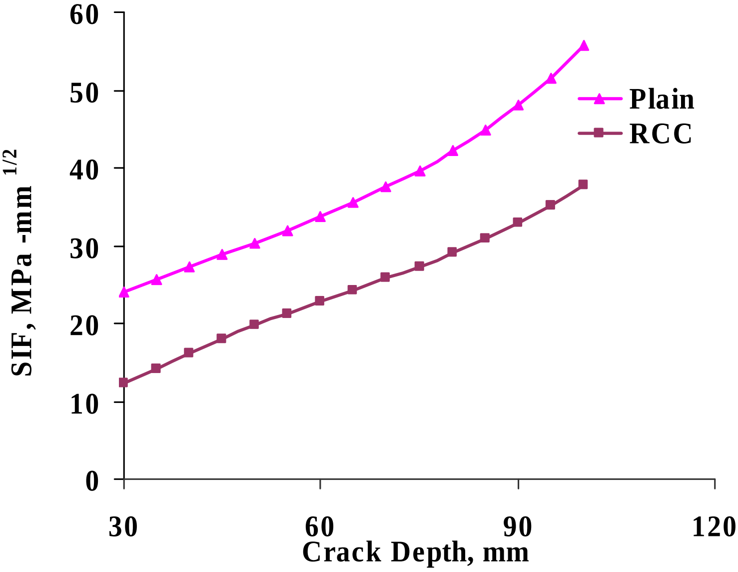

Figure 4: SIF vs crack depth variation for M30 grade long beam

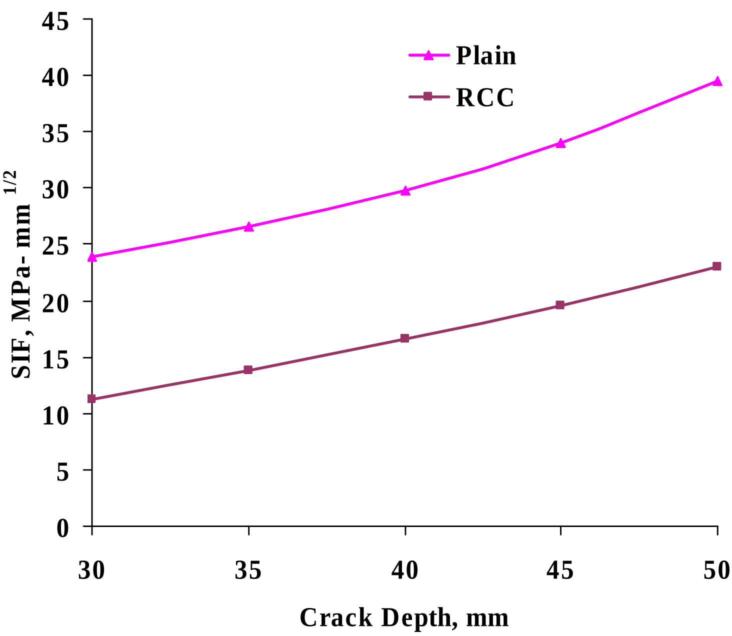

Figure 5: SIF vs crack depth variation for M30 grade medium Beam

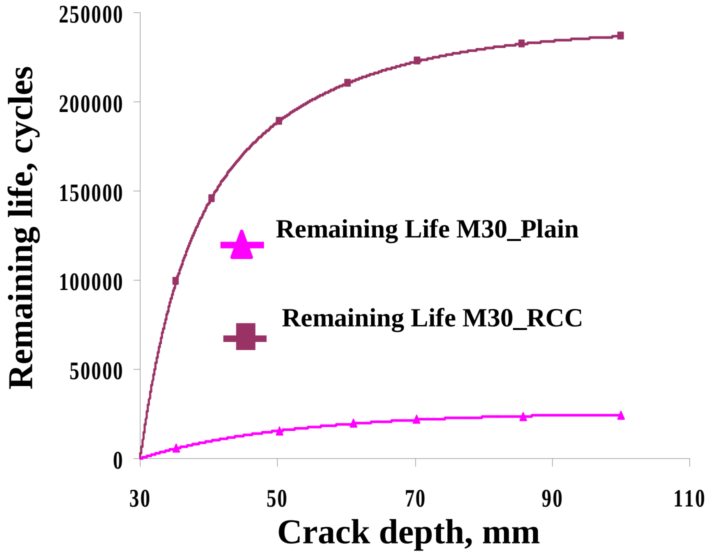

Figure 6: Remaining life vs crack depth variation for long beam

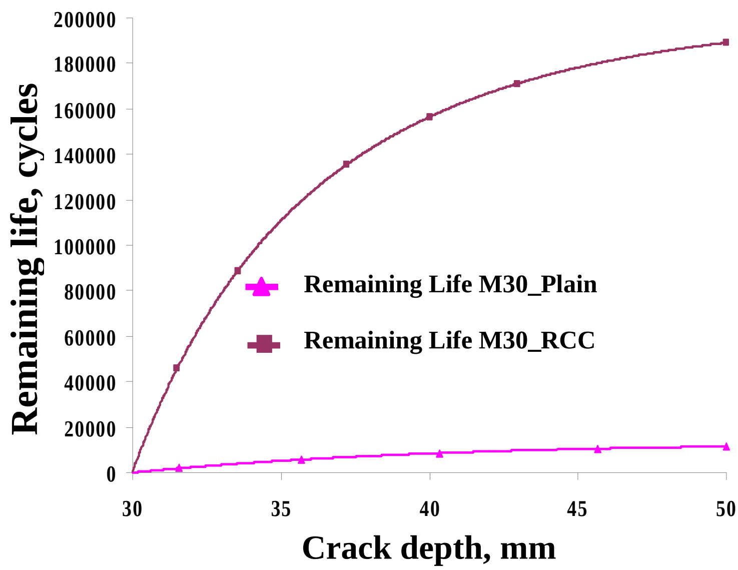

Figure 7: Remaining life vs crack depth variation for medium beam

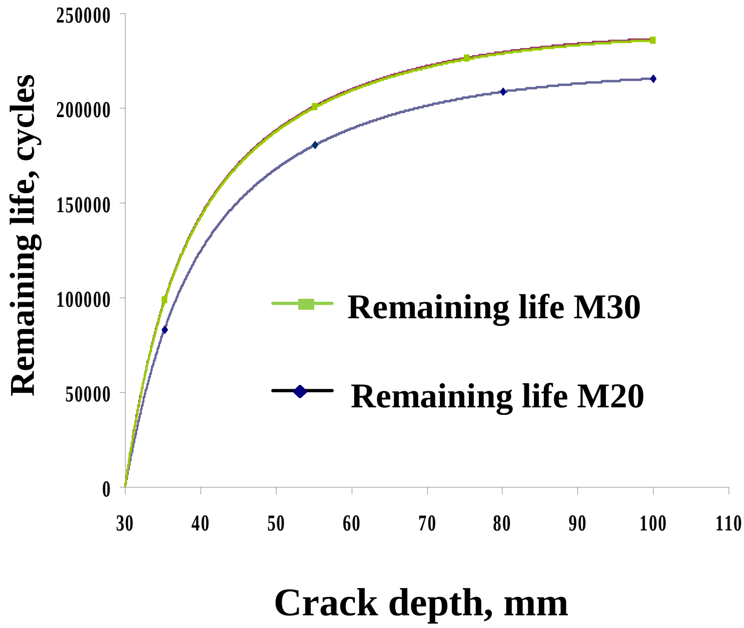

Figure 8: Remaining life vs crack depth variation for long RCC beam

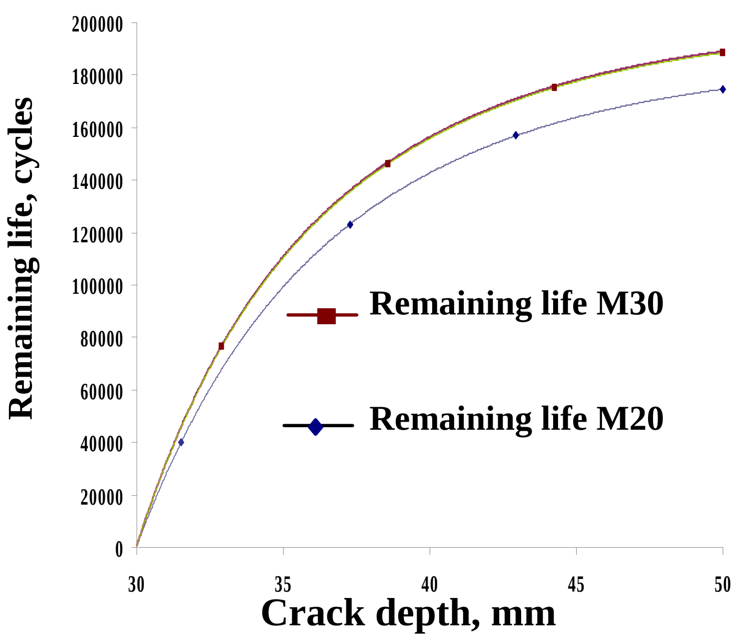

Figure 9: Remaining life vs crack depth variation for medium RCC beam

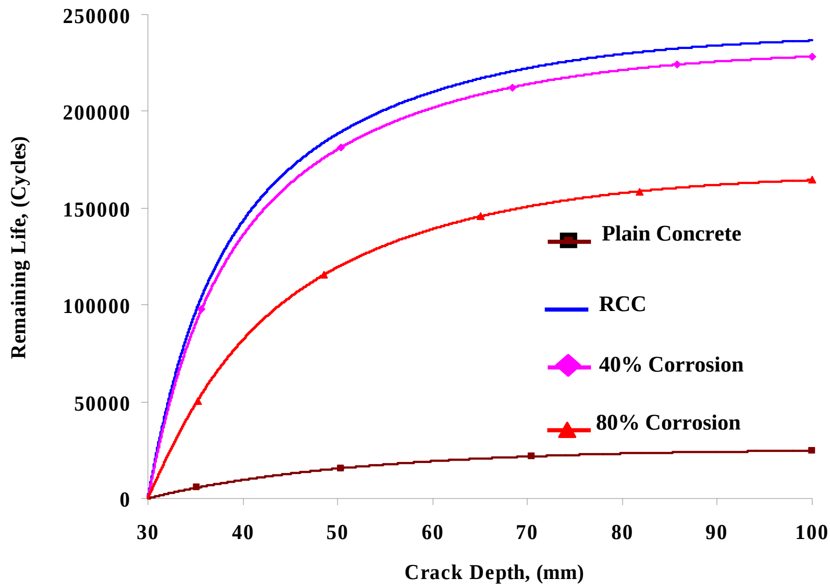

Figure 10: Remaining life vs crack depth for M30 Long Beam

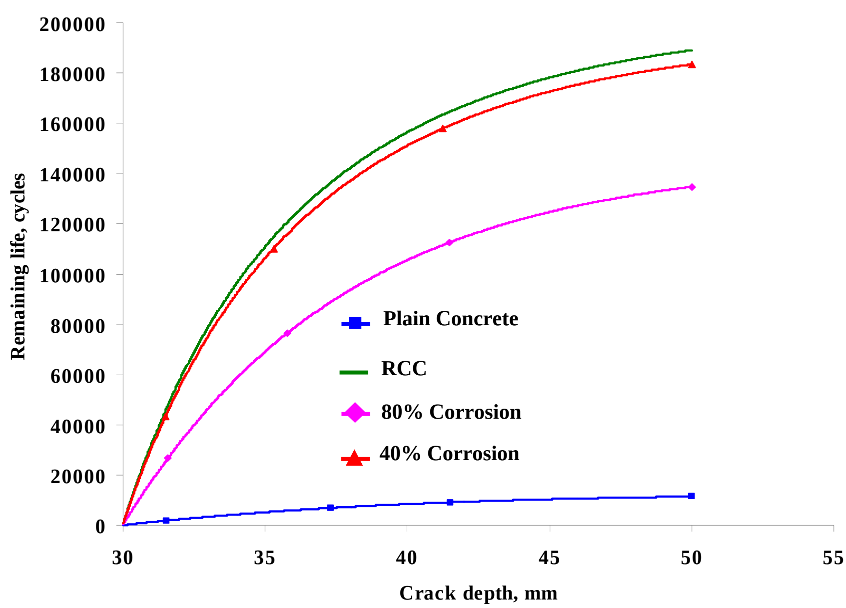

Figure 11: Remainng life vs crack depth for M30 medium Beam

4 Summary and Conclusions

In the present study, fracture mechanics based model has been developed to compute SIF and remaining life of RC beams. SIF has been computed by using the principle of superposition. At each incremental crack length, the net SIF has been computed by taking the difference between SIF due to plain concrete and the reinforcement. The reinforcement behaviour has been assumed as elasto-plastic. The study shows that SIF for RC beams reduces to about 50% of that of the plain concrete beams. It is observed that the remaining life of beams increases with increase in grade of concrete. The predicted remaining life of RC structural component is significantly higher about 10 times compared to plain concrete beams. Further, a model has been developed to predict the remaining life of corroded RC structural component. It is observed that there is a significant decrease in the remaining life of the RC beam with severity of corrosion compared to uncorroded one.

Acknowledgement: Authors thank Shri S. Maheswaran and Shri V. Ramesh kumar, Scientists and Ms. B. Bhuvaneswari, Quick Hire Fellow for fruitful discussions during the investigation. This paper is being published with the kind permission of the Director, CSIR-SERC, India.

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