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A Computational Inverse Technique to Determine the Dynamic Constitutive Model Parameters of Concrete

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Abstract: In this paper, a computational inverse technique is presented to determine the constitutive parameters of concrete based on the penetration experiments. In this method, the parameter identification problem is formulated as an inverse problem, in which the parameters of the constitutive model can be characterized through minimizing error functions of the penetration depth measured in experiments and that computed by forward solver LS-DYNA. To reduce the time for forward calculation during the inverse procedure, radial basis function approximate model is used to replace the actual computational model. In order to improve the accuracy of approximation model, a local-densifying method combined with RBF approximation model is adopted. The intergeneration projection genetic algorithm is employed as the inverse solver. Through the application of this method, the parameters of HJC constitutive model are determined. Results show that the identified constitutive parameters' computational penetration depth and projectile deceleration-time curves are closely in accordance with experimental data. The proposed inverse approach is a potentially useful tool to effectively help identify material parameters.

Keywords: inverse problem; parameters identification; radial basis functions; local-densifying.

1 Introduction

Concrete has been widely used as the fundamental building construction material for civil and military purposes. Concrete is a composite material which consists of aggregate particles dispersed in a porous cement paste. As the differences of the properties of the aggregate particles and cement paste in the physics and mechanics,

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the irregular shape and distribution of aggregate particles, and a lot of air voids contained, the dynamic behavior of concrete is very complex. To describe the dynamic behavior of concrete, a number of dynamic constitutive models have been developed recently, such as HJC model [Holmquist, Johnson and Cook (1993)], RHT model [Riedel, Thoma, Hiermaier and Schmolinske (1999)], TCK model [Taylor, Chen and Kuszmaul (1986)]. In general, the constitutive model for concrete contains a lot of material constants. The determination of material parameters is an important step in modeling and the need for precise material parameters is crucial in both simulations and experimental results.

The material constitutive parameters are usually determined by using physical and mechanical tests with specimens. For example, unconfined compressive strength can be identified by uniaxial compression test. However, several parameters cannot be directly determined from these experiments. They are often identified implicitly by performing a fitting method through sets of different experimental data. In fact there are not enough test data for concrete material due to the limitations of testing machines and difficulty of specimen design. Such as it usually can obtain the test data for the strain rates lower than $10^4 s^{-1}$, while it is difficult to obtain the data for higher strain rate. Additionally, it is difficult to obtain available triaxial compression test data with different confining pressure levels. Even though these parameters are determined, it is essential to validate their availability. Generally, the validation should be carried out from two aspects. Firstly, the constitutive model based on these parameters should fit the available test data well. Secondly, the results from numerical computation for penetration of concrete slab should be in good agreement with the experimental data. Thus it is time-consuming and requiring expensive installations and equipments to determine accurately these parameters. To overcome these difficulties, it is obviously valuable to seek a reliable method to identify these parameters of the constitutive model. Among those proposed methods, the method by employing inverse techniques appears more promising.

The computational inverse technique for determination of material constitutive parameters utilizes the complex relationships between the material responses and material parameters. The relationships are often represented by a known mathematical model, which defines the forward problem. So if a set of accurately measured experimental response data is available, the material constitutive parameters may be identified by solving an inverse problem properly formulated. The material constitutive parameters can be determined by minimizing a particular norm of the difference between the calculated and the experimental response data. Many studies have been performed to determine the material constitutive parameters by the inverse method. Markiewicz et al. [Markiewicz, Ducrocq and Drazetic (1998)] developed an inverse method based on quasi-static and dynamic experiments to determine the elastic-plastic and viscoplastic parameters of the power and Cowper-Symonds' constitutive model. Fairbairn et al. [Fairbairn, Ebecken, Paz and Ulm (2000)] used the inverse method for determination of probabilistic parameters of concrete. Forestier et al. [Forestier, Massoni and Chastel (2002)] proposed an inverse approach coupled to a 3D finite element software to estimate the parameters of Norton-Hoff constitutive law. Husain et al. [Husain, Sehgal and Pandey (2004)] presented an inverse finite element procedure for determination of constitutive tensile behavior of materials based on small punch test. Qu et al. [Qu, Jin and Xu (2008)] used GA-based global optimization method for identification of superplastic constitutive model parameters. Sedighi et al. [Sedighi, Khandaei and Shokrollahi (2010)] adopted an inverse approach to determine the parameters for Johnson-Cook and Zerilli-Armstrong constitutive models based on split Hopkinson pressure bar data.

In this paper, a computational inverse technique is presented to determine the parameters of HJC concrete constitutive model based on penetration experiments. In this work, the input data used for inverse procedure is penetration depth, which can be obtained from the penetration experiments. LS_DYNA solver is adopted for forward analysis to obtain responses with given constitutive parameters. Considering the forward calculations may be called for too many times during the inverse process, the approximation model is applied to replace the actual one. RBF method is adopted to construct the approximation model because of its fine performance on computational efficiency, numerical stability and capacity of capturing nonlinear behavior. In order to improve the accuracy of approximation model, a local-densifying method combined with RBF approximation model is adopted. The constitutive parameters can be identified by minimizing an objective function, which is chosen as the error function defined by the sum of the squares of the deviations between the numerical results and the experimental measurements. A computational optimization technique, intergeneration projection genetic algorithm (IP-GA), is employed as an inverse operator to determine the constitutive parameters. The results of this combined computational-experimental investigation are presented in the current paper.

2 Parameters of HJC concrete model

HJC constitutive model [Holmquist, Johnson and Cook (1993)] is widely used to numerical computations for prediction of penetration and impact problems [Johnson , Beissel, Holmquist and Frew (1998); Beissel and Johnson (2000); Dawson, Bless, Levinsion, Pedersen and Satapathy (2008); Liu, Ma and Huang (2009); Lian, Zhang , Zhou and Ma (2010)], because this model describes the compressive behavior of concrete under large strains, high strain rates and high pressure. It can be expressed in the following form:

$$\sigma^* = [A(1-D) + BP^{*N}][1 + C\ln\varepsilon'^*] \tag{1}$$

in which σ^*, P^* and $\varepsilon'^* = \dot{\varepsilon}/\dot{\varepsilon}_0$ are the normalized equivalent stress, the normalized pressure and the dimensionless strain rate, respectively, where σ, P and $\dot{\varepsilon}$ are the actual equivalent stress, hydrostatic pressure and strain rate, respectively. The normalizing parameters are the quasi-static uniaxial compressive strength fc and the reference strain rate $\dot{\varepsilon}_0$. Further, A, B, N and C represent the normalized cohesive strength, normalized pressure hardening coefficient, pressure hardening exponent and the strain rate coefficient, respectively. Material degradation is described by the damage variable $D(0 \le D \le 1.0)$, resulting in reduction of the cohesive strength.

As described in the literature [Holmquist, Johnson and Cook (1993)], the HJC concrete model contains 19 parameters which should be determined. Some parameters can be determined by simple tests. Some can be identified by calculating classical formulations based on the basic experiments; while some parameters should be determined indirectly from a great number of data, which are obtained through different experiments. According to these methods for determination of these parameters above mentioned, these can be divided into three groups, as follow:

In the first group, the value of parameters can be directly determined through physical and mechanical tests (compression, tensile, torsion test, et al.). For example, the value of fc can be identified by quasi-static uniaxial compressive test. These parameters include initial density ρ_0, fc, T, $S_{\max}$, where T and $S_{\max}$ are the maximum tensile hydrostatic pressure and the normalized maximum strength, respectively.

For the second group, these parameters can be determined by calculating classical formulations based on the tests or using recommended values from the literature [Holmquist, Johnson and Cook (1993)]. They are as follows:

(a) Shear modulus G and elastic bulk modulus $K_{elastic}$ are determined from elasticity theory using Yong's modulus and Poisson's rate.

(b) As defined be the difference between the undamaged strength and the completely fractured strength at a given pressure, the cohesive strength A can be determined by Eq. (1) for quasi-static conditions combining with experiments.

(c) The literature [Holmquist, Johnson and Cook (1993)] assumed that the rate effects were independent of initial strength and were constant for all concrete. So the recommended value is used for C, which is 0.007.

(d) The damaged model constants, D_2 and $E_{f\min}$, use the recommended values for low sensitivity. Therefore, D_2 is set to 1.0 and $E_{f\min}$ is set to 0.01. The other constant, D_1, can be determined using equation $\varepsilon_p^f + \mu_p^f = D_1(P^* + T^*)^{D_2}$ based on the uniaxial compression test [Holmquist, Johnson and Cook (1993)].

(e) The pressure constants, the crushing pressure P_{crush}, the crushing volumetric strain μ_{crush} and the locking volumetric strain μ_{lock} can be calculated through the computational formulations, $p_{crush} = fc/3$, $\mu_{crush} = p_{crush}/K_{elastic}$, $\mu_{lock} = \rho_{grain}/\rho_0 - 1$, respectively [Holmquist, Johnson and Cook (1993)]. Where ρ_{grain} is the grain density.

(f) The constants, K_1, K_2, K_3, which are represented the concrete in the full crushed state, can also be obtained from shock Hugoniot data for granite and quartz [Marsh (1980)].

For the third group, the parameters cannot be directly measured or determined by the computational formulations. They can be identified implicitly from sets of data obtained through different experiments. These parameters include B, N and the locking pressure P_{lock}, which will be identified by the inverse method in this study.

3 Statement of the problem

It is aimed to inversely determine the material constitutive parameters for concrete from the measured penetration depths, which were measured from the penetration experiments into concrete targets. In this study, the penetration into concrete targets with 3.0 caliber-radius-head (CRH) steel penetrators are used as forward problem and have been conducted by Forrestal et al.[Forrestal, Frew, Hickerson and Rohwe (2003)] , as shown in Fig. 1. The projectiles were machined from 4340 Rc45 steel, and had a nominal mass of 13 kg. Targets were cast in corrugated steel culverts with a diameter of 1.83 m. In these experiments, the initial projectile velocity, the projectile deceleration and the penetration depth were measured. These data are very important for parameters identification by the inverse method.

Using the forward problem solver, the penetration depth of numerical calculation can be obtained with the assumed constitutive parameters. In general, the obtained results are different from the experimental data. The inverse procedure can then be formulated by an optimization technique, which minimizes the sum of squares of deviations between numerical calculated and experimental results. The optimization problem can be stated as follow:

Minimize the objective function of error defined by

$$Err(\mathbf{r}) = \sum_{j=1}^{N} \left\| d_{mj} - d_{cj}(\mathbf{r}) \right\|^2 \tag{2}$$

where, $\mathbf{r}$ is a vector which collects the trial constitutive parameters, d_{mj} indicates penetration depth measured from experiments, d_{cj} indicates computational penetration depth, m denotes the “measured” value, c denotes the “calculated” value, and N is the number of points of measured data.

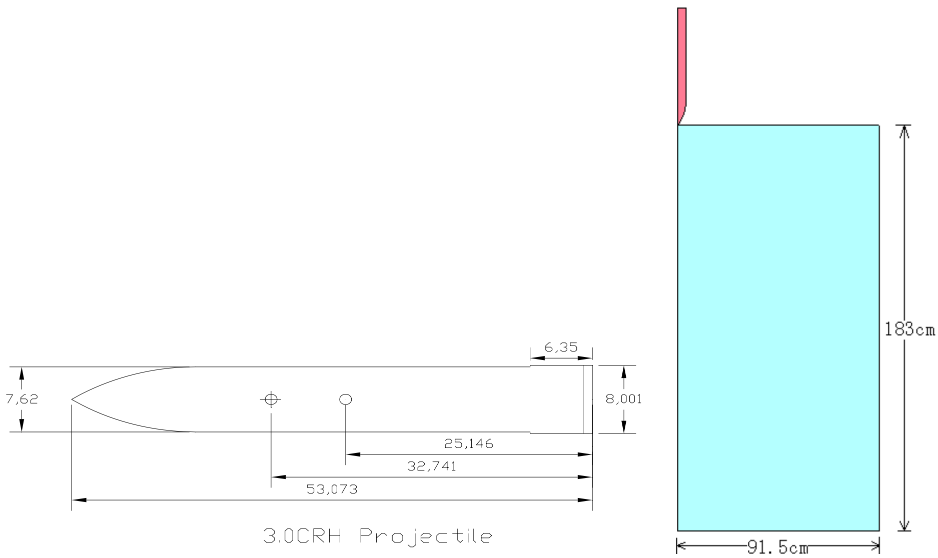


Figure 1: Projectile and target geometries. Dimensions in centimeter [Forrestal, Frew, Hickerson and Rohwe (2003)]

In this paper, three Forrestal's test (SNL-00-03/1, SNL-00-02/2, SNL-00-05/3) data [Forrestal, Frew, Hickerson and Rohwe (2003)] are applied to determine the parameters in the inverse procedure. There are three group data including striking velocity, penetration depth and projectile deceleration. So N is set to 3, and the Eq. (2) can be rewritten as follow:

$$Err(\mathbf{r}) = \sum_{j=1}^{3} \left\| u_{mj} - u_{cj}(\mathbf{r}) \right\|^2 \tag{3}$$

The constitutive parameters can be determined by solving the optimization problem to minimize the objective function defined in Eq. (3).

4 Computational inverse technique for constitutive parameters identification

An inverse process for determination of the material constitutive parameters for concrete is outlined in Fig. 2. Here LS-DYNA solver is used as the forward calculation technique. Latin Hypercube design method is used to obtain a uniform allocation inside the design domain of inversed parameters, and RBF method combined with a local-densifying method is used to construct the response surface as the input data for inverse approach. And the IP-GA is employed as the inverse operator to find the minimal *Err* by using the response surface. The detail of each part will be given in the following sections.

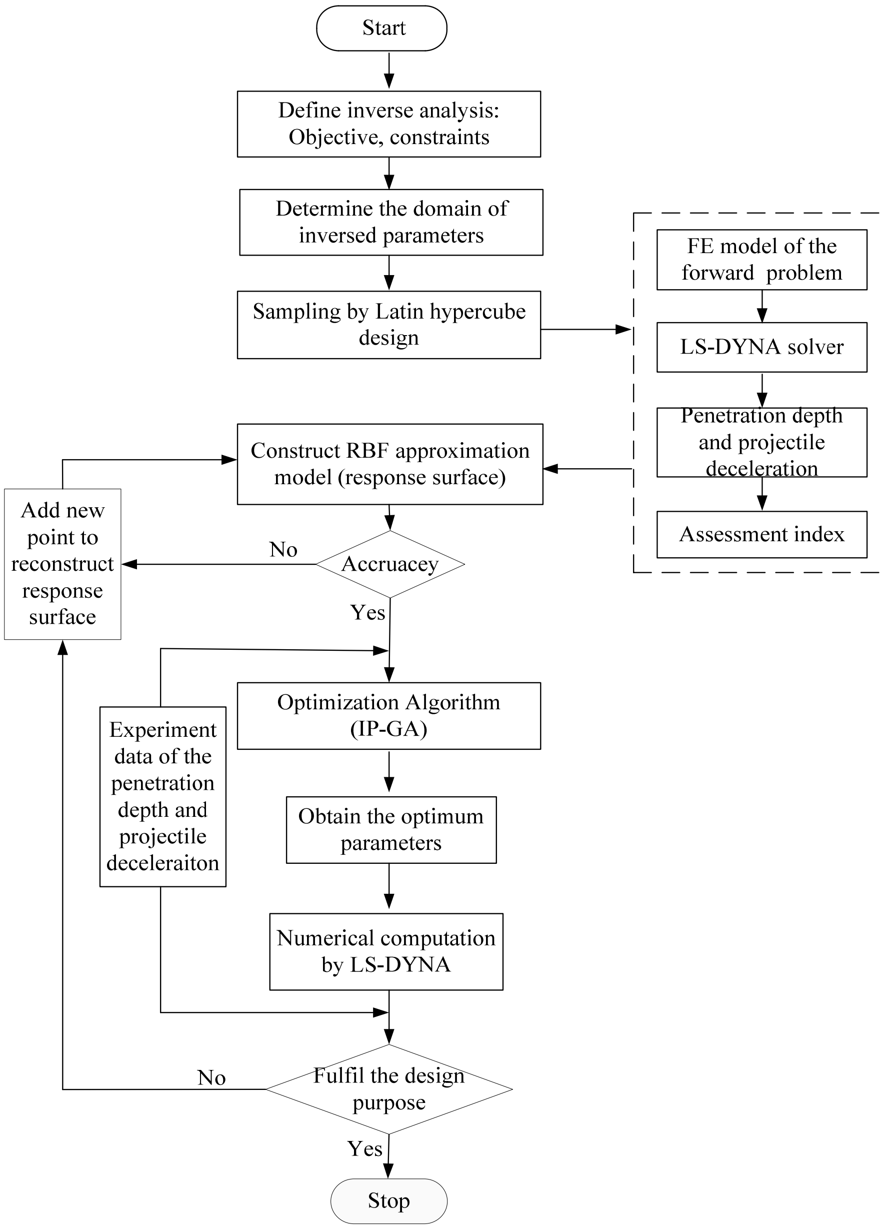


Figure 2: Flowchart of the presented inverse technique

4.1 FEM model

In a forward calculation, one needs to calculate penetration depth and the projectile deceleration with trial constitutive parameters. Both efficiency and accuracy are very important, as many times of forward computations may have to be carried out in the later work and the process of the penetration of concrete is very complex. LS-DYNA solver is used in this work as the forward calculator because of its outstanding efficiency and accuracy for the penetration problem.

For forward computation, a 2D axisymmetric finite element model is established to represent the penetration into concrete target, shown in Fig. 3. The projectile and the concrete target were modeled as Lagrange meshes. A total of 208 square elements are used for the steel projectile. The projectile is modeled with a von Mises material model (Mat_003 in LS-DYNA [Hallquist (2006)]) with linear isotropic hardening. The main data used for the projectile are Young's modulus , Poisson's ratio , yield stress , tangent modulus $G = 15.0GPa$. No strain rate effect is considered. The original density of the projectile ($7830kg/m^3$) is slightly modified to $6730kg/m^3$ in order to obtain the total projectile mass of 13.0 kg. The concrete target is modeled with HJC constitutive model. For parameters of HJC concrete model, the first and second group parameters, which can be obtained based on Section 2 and the reference papers [Forrestal, Frew, Hickerson and Rohwe (2003); Bush (2010)], are shown in Tab. 1. While, the third group parameters will be determined by the inverse method. For the concrete slab, the mesh size is very important as there is a problem of mesh size dependency of numerical results. In order to reduce the mesh size dependency, the "Nonlocal" option is adopted. The model bases on the concept of non-localization by Pijaudier-Cabot and Bazant [Cabot and Bazant (1987)], and it depends on the state of the material within a radius of influence which surrounds the integration point.

Table 1: Material parameters for concrete

$\rho_0\ (kg/m^3)$	$G\ (GPa)$	A	C	$T\ (GPa)$	$S_{\max}$
2040	7.917	0.79	0.007	0.00255	30
$fc\ (GPa)$	E_{spo}	D_1	D_2	$E_{f\min}$	
0.023	10^{-6}	0.036	1.0	0.01	
$P_{crush}\ (GPa)$	μ_{crush}	μ_{lock}	$K_1\ (GPa)$	$K_2\ (GPa)$	$K_3\ (GPa)$
0.007667	0.00128	0.299	85	-171	208

In the present calculations, a reduced integration scheme with hourglass control is adopted. The 2D_automatic_single_surface contact option of LS-DYNA is used to define the contact behavior between projectile and target without friction. And

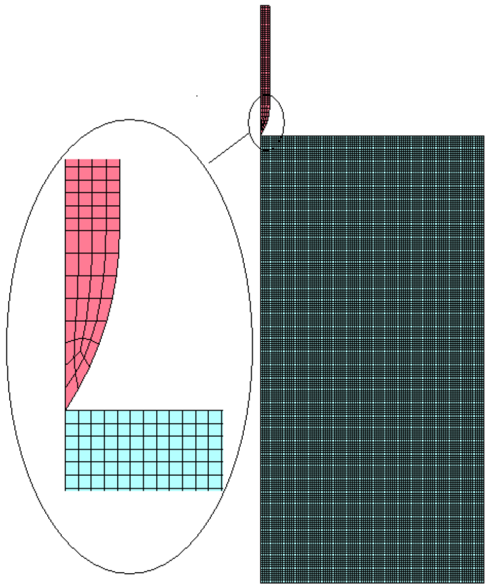
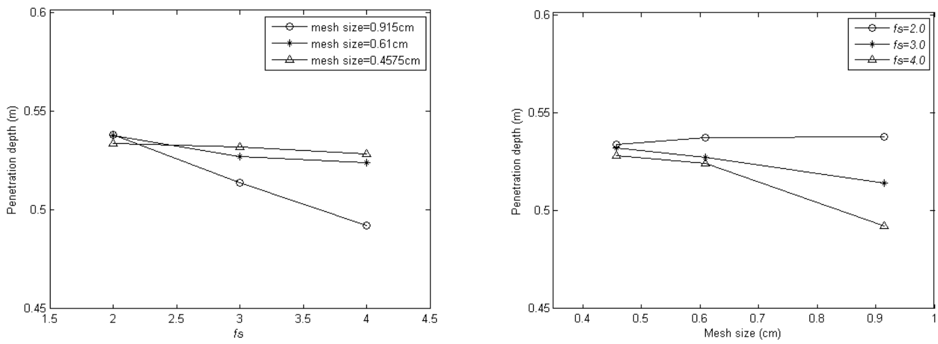


Figure 3: 2D axisymmetric finite element model used in the penetration analysis

we adopt the element erosion option of LS-DYNA with a criterion based on the equivalent plastic strain fs, which allows the integration time increment to remain larger. The erosion strain is also sensitive to the computed results.

In order to make the finite element model available and reliable, it is important to use the feasible mesh size and erosion strain value. Hence, the mesh size and erosion strain are varied independently to analysis the sensitivity of the computed results. First of all, it is necessary to set initial values for the inversed parameters, $B = 1.6$, $N = 0.8$, $P_{lock} = 0.9GPa$. And the initial striking velocity of projectile is set to 336.6 m/s. Additionally, the range of erosion strain is recommended from 2 to 4 [Johnson, Beissel, Holmquist and Frew (1998)]. So three cases are discussed of the effect of the erosion strain and mesh size, respectively. The analysis results are shown in Fig. 4. It is found that the computed results converge gradually as the mesh size decreases. And when the mesh size is bigger, the difference of the calculated results for different erosion strain is larger. It also can be seen that when the mesh size is small enough, the computed results converge gradually with the increase of the erosion strain fs and the relative error of computed results is less than 3%. Simultaneously, it is found that the cycle of integration and the CPU time multiply as the mesh size decreases and the erosion strain increases. Thus, in order to obtain an available and reliable finite element model and fulfill the computation efficiency, the concrete target was formed from 150×300 square elements with an element size of 0.61 cm and the erosion strain of 3.0 was used.



(a) (b)

Figure 4: Analysis of the effect of mesh size and erosion strain fs

4.2 Sensitivity analysis

In order to reduce ill-posedness in the later stage of inverse analysis, the input vector and output vector should have a strong causal-effect relationship. In other word, it is necessary to evaluate the influence of the inversed parameters on the response data. This is accomplished using a sensitivity analysis combined with forward calculations.

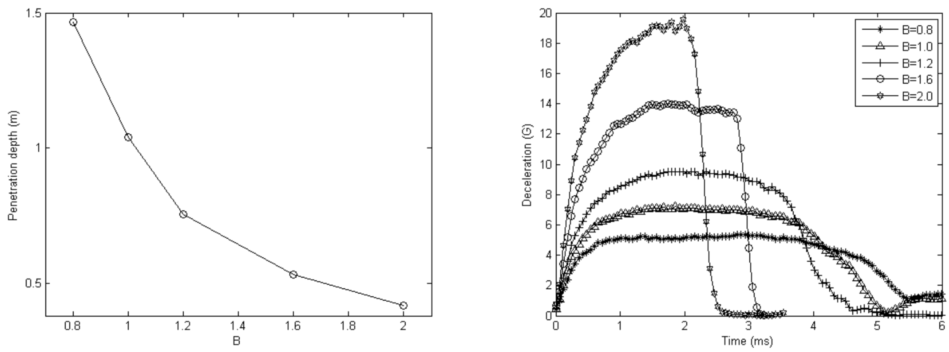
At the beginning of sensitivity analysis, it is necessary to set initial values for the inversed parameters as set in Section 4.1. Then the shot and projectile number SNL-00-05/3 test [Forrestal, Frew, Hickerson and Rohwe (2003)] is applied to sensitivity analysis, in which the initial striking velocity is 336.6 m/s and the penetration depth is 0.93 m. For the purpose of sensitivity analysis, five values are given for each parameter, as listed in Tab. 2. And the three inversed parameter are varied independently to do the sensitivity analysis. Hence, the forward calculation will be carried out for 13 times, which are run on a personal computer (eight 2.81GHz Intel Core i7 processors with 4 GB of RAM running Windows XP).

The results of the sensitivity analysis for these parameters are shown in Figs. 5-7. In every figure, the figure (a) depicts the penetration depth versus parameter data. It is found that the penetration depth decreases gradually with the increase of the value of parameter. Figure (b) shows the projectile deceleration-time data with

Table 2: Five values of each parameter for sensitivity analysis

No.	B	N	P_{lock} (GPa)
1	0.8	0.1	0.08
2	1.0	0.3	0.2
3	1.2	0.6	0.5
4	1.6	0.87	0.9
5	2.0	1.0	1.5

different values of parameter. It is shown that the peak deceleration increases and the penetration time decreases gradually as the value of parameter increases.



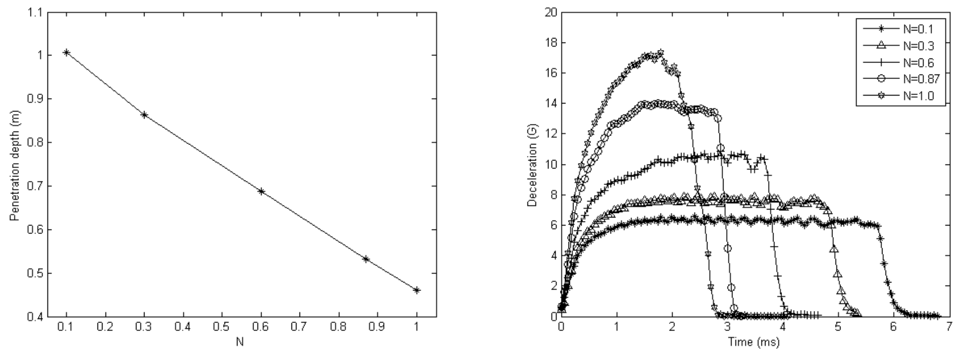
(a) (b)

Figure 5: The sensitivity analysis for parameter B at $Vs = 336.6m/s$. (a) Penetration depth versus parameter B; (b) Deceleration versus time data with different values for parameter B

From the sensitivity analysis, it can be found that the inversed parameters are sensitive to both penetration depth and projectile deceleration. Thus, these parameters can be identified by inverse method for the next step. Simultaneously, we can initially design domains which cover the possible range of the inversed parameters based on the sensitivity analysis and experimental data. The domains for parameters are listed as follow: B[1.0, 2.0], N[0.1, 1.0], and P_{lock}[0.08, 1.0]. These ranges define a feasible domain of the parameters to be identified.

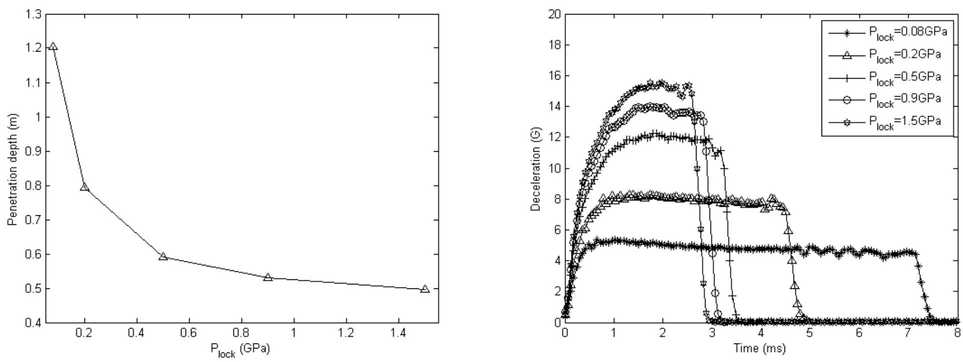
4.3 Local-densifying method based on RBF approximation model

A response surface identified through RBF method [Dyn, Levin and Rippa (1986)] is proposed to find the optimal constitutive parameters by inverse operator. Consid-



(a) (b)

Figure 6: The sensitivity analysis for parameter N at $Vs = 336.6m/s$. (a) Penetration depth versus parameter N; (b) Deceleration versus time data with different values for parameter N



(a) (b)

Figure 7: The sensitivity analysis for parameter P_{lock} at $V_s = 336.6m/s$. (a) Penetration depth versus parameter P_{lock}; (b) Deceleration versus time data with different values for parameter P_{lock}

ering the large range of the inversed parameters to be investigated, it is necessary to reduce the number of forward computations before constructing the RBF approximation model. To overcome the difficult, the Design of Experiments (DOE) method is adopted. The DOE method, which is aimed at minimizing the number of runs while simultaneously acquiring as much information as possible, generates samples using uniform distributions in the entire sampled space. There are several DOE methods that have been proposed, such as the full factorial, D-optimal and Latin Hypercube design. The Latin hypercube design (LHD) method [Mckay, Beckman and Conover (1979)], which has been used extensively, is adopted in this work. This technique is a space-filling design with constrainedly stratified sampling method. The key advantage of this technique is that the number of samples does not increase exponentially with the number of variables, and at the same time it ensures that a small number of computer experiments with multiple levels will be sufficient to investigate the potentially nonlinear relationships between input variables and output variables.

The number of simulations in LHD method is determined by the total number of design variables involved. In this study, there are three design variables involved. To construct a reasonably accurate approximation model, 30 sample points are conducted initially within design space, as listed in Tab. 3.

After generating the LHD sample points, numerical computations are completed by using LS-DYNA solver. The corresponding assessment indices, calculated from these computations, are used as the response to construct the approximated model. This approximation model can be constructed by RBF approximation technique, in which the Gaussian radial function is adopted in the current study. The approximation model can be expressed as:

$$F(d) = f(B, N, P_{lock}) \tag{4}$$

where d represents the penetration depth.

Due to three measured data of penetration depth applied to parameters identification, three RBF approximation models are constructed at three different striking velocities.

While the accuracy of approximation is one of the most important issues in all kinds of approximation assisted optimization methods. To improve the accuracy of approximation model, it is necessary to increase sample size over some key local regions.

Consequently, local-densifying method is an updating strategy of sampling method focusing the limited sample resources on the concerned local regions. In the local-densifying method, it usually adds one sample at the largest distance of specimen on the basis of the second-order derivative of response surface. What's more, the current best combinations consisted of the current best design and its corresponding boundary inversed parameters are sequentially added to the local regions where the minimal and maximal responses of current approximation models take place.

Then the RBF approximation models are reconstructed using these densified samples for next step until the stop criteria are reached. The parameter, R, used as

Table 3: Samples generated by LHD method

Number	B	N	P_{lock}(GPa)
1	1.8500	0.2647	0.25
2	1.0830	0.2953	0.65
3	1.8170	0.8947	0.28
4	1.0170	0.5050	0.40
5	1.8830	0.4753	0.86
6	1.5170	0.8353	0.16
7	1.2170	0.3547	0.95
8	1.1170	0.8650	0.52
9	1.2500	0.4447	0.13
10	1.3170	0.5653	0.68
11	1.7500	0.1747	0.80
12	1.3830	0.6247	0.37
13	1.7830	0.7750	0.92
14	1.9500	0.3250	0.56
15	1.2830	0.9253	0.77
16	1.6500	0.6850	0.59
17	1.9170	0.5950	0.31
18	1.6170	0.1153	0.46
19	1.6830	0.5347	0.10
20	1.5500	0.4150	0.89
21	1.4830	0.2350	0.19
22	1.1500	0.8047	0.22
23	1.7170	0.9847	0.71
24	1.3500	0.1450	0.74
25	1.0500	0.6553	0.83
26	1.1830	0.2053	0.34
27	1.5830	0.3853	0.49
28	1.9830	0.7453	0.62
29	1.4500	0.9550	0.43
30	1.4170	0.7147	0.98

an error indicator to gauge the accuracy of the RBF approximation model for stop criteria, is represented as follow:

$$R = 1 - \frac{\sum_{i=1}^{n} (f_i - \tilde{f})^2}{\sum_{i=1}^{n} (f_i - \bar{f})^2} \tag{5}$$

Where f_i is the actual value, $\tilde{f}$ is the value predicted by the RBF approximation model, and $\bar{f}$ is the average of all actual values. When R calculated from the RBF approximation model is sufficiently close to one, the approximated model has high accuracy.

In this work, this procedure is continued until the parameter R is greater than 0.9. The parameters R are below 0.9 when the initial response surfaces are constructed with initial 30 specimens. Then the local-densifying method is used. All of the parameters R are greater than 0.9 until 35 specimens are used. It indicates that the response surfaces are accurate enough to determine the inversed parameters.

4.4 Intergeneration projection genetic algorithm

The error function is given as the sum of nonlinear squares, as presented in Eq. (3). The constitutive parameters identification problem can be solved as an optimization problem. IP-GA [Xu, Liu and Wu (2001)] is adopted as the optimization strategy to minimize the objective function in this paper. This method is one of the most powerful nonlinear programming algorithms for solving differentiable nonlinear programming problems in an efficient and reliable way.

In the IP-GA, the child generation is produced using information from the parent and grandparent generations. IP-GA is a modification based on micro GA (μGA) [Krishnakumar (1989)], to make use of its feature of small population size per generation so as to maximize the efficiency. The intergeneration projection (IP) operator aims to find a better individual by jumping along the move direction of the best individuals at two consecutive generations so as to improve the convergence rate [Xu, Liu and Wu (2001)]. The IP-GA combines the μGA with IP operator and whereby has a better global convergence performance.

The search space of the three inversed variables have been given, which are discretized and translated into a chromosome of 16 bits length according to the binary coding procedure in IP-GA. Thus there are a total of 2^{16} possible combinations of individuals. The population size and the probability of crossover for IP-GA are set to 4 and 0.6, respectively. During this optimization algorithm, the stopping criterion is imposed to limit each IP-GA run to a maximum of 1000 generations. The iterative process will be terminated until the stopping criterion is reached.

At the end of the optimization, the optimal material constitutive parameters can be obtained from this procedure. Then, it is essential to validate the availability and reliability of these parameters. This should be considered from two aspects. For one thing, the value of objective function should be less than 0.01. For another thing, the relative errors of penetration depth between the experimental test and numerical calculation based on these parameters are below 8%, and the curves of the projectile deceleration-time between the experimental test and numerical calculation should be in good agreement. If these demands are fulfilled, these parameters are available and reliable. Otherwise, these parameters are used as a new sampling point, which is added to update the approximation model, as shown in Fig. 2.

5 Results and discussion

As an application of the above described computational inverse technique, the constitutive parameters of concrete were determined and listed in Tab. 4. It was found that the optimal values were achieved with the total number of 38 samples. And the value of the objective function was just 0.00743, which satisfied the demand.

Table 4: Inversed parameters and results

Inversed parameters	Search range	Inversed results
B	[1.0, 2.0]	1.3871
N	[0.1, 1.0]	0.37625
P_{lock}(GPa)	[0.08, 1.0]	0.596

Numerical verifications are also performed with the identified parameters. Fig. 8 and Tab. 5 show penetration depth versus striking velocity data, compared with experiment data. As shown in Fig. 8, the line with empty symbols represent the experimental data from Forrestal et al. [Forrestal, Frew, Hickerson and Rohwe (2003)], and the line with solid symbols are obtained from the numerical computations. It can be seen that the results from the identified parameters coincide with the experiment results well. From Tab. 5, it can be found that the minimum relative error for the penetration depth is just 1.6483%, while the maximum is 4.352%, which is in the acceptable error range. It is noted that the relative error of the last test (SNL-00-04/4) is 2.3135%. This test is not applied to identify the parameters.

Figures 9∼12 present the projectile deceleration-time data at the four different velocities, compared with the experimental data. In these figures, the real lines represent the experimental data from Forrestal et al. [Forrestal, Frew, Hickerson and Rohwe (2003)], and the lines with solid symbols are obtained through numerical calculations. Form these figures, it is found that the projectile deceleration-time

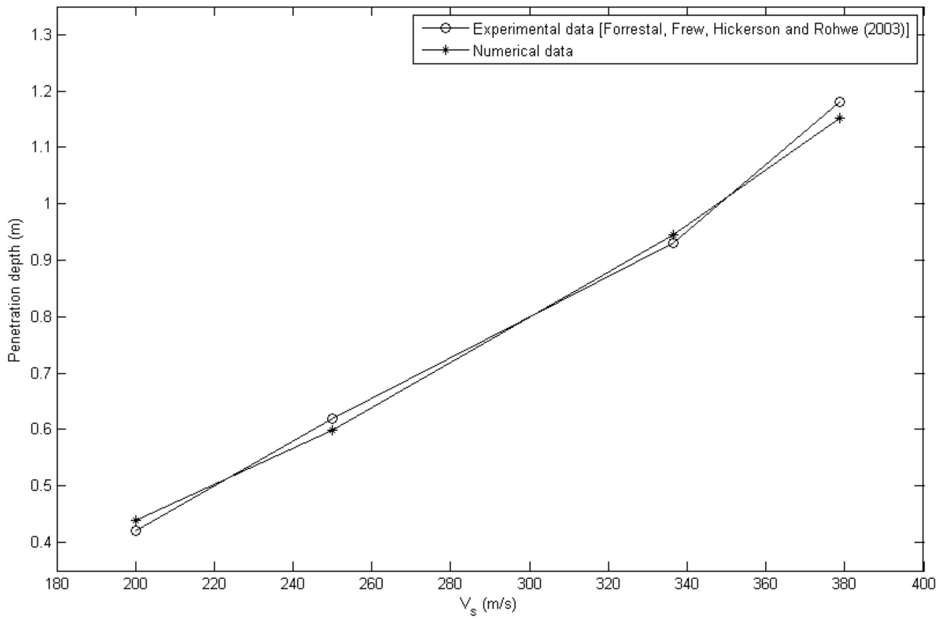


Figure 8: Penetration depth versus striking velocity for the concrete, $\psi = 3.0$

Table 5: Penetration data with 3.0 CRH projectile at different initial velocity

Shot and projectile number	Striking velocity (m/s)	Penetration depth of experiment (m)	Penetration depth of numerical computation (m)	Relative error (%)
SNL-00-03/1	200.0	0.42	0.43828	4.352
SNL-00-02/2	250.0	0.62	0.598524	3.4638
SNL-00-05/3	336.6	0.93	0.94533	1.6483
SNL-00-04/4	378.6	1.18	1.1527	2.3135

data from numerical computations are in good agreement with experimental measurements for the rise times, peak plateau responses and the declined times. Especially, it can be seen that the curve of deceleration-time is the best fitting at 336.6 m/s from Fig. 11.

In order to validate these inversed parameters available and reliable deeply, we perform the numerical computations for the penetration into the same concrete targets with 6.0 CRH steel projectile, which were conducted by Forrestal et al. [Forrestal, Frew, Hickerson and Rohwe (2003)]. The finite element model is established as

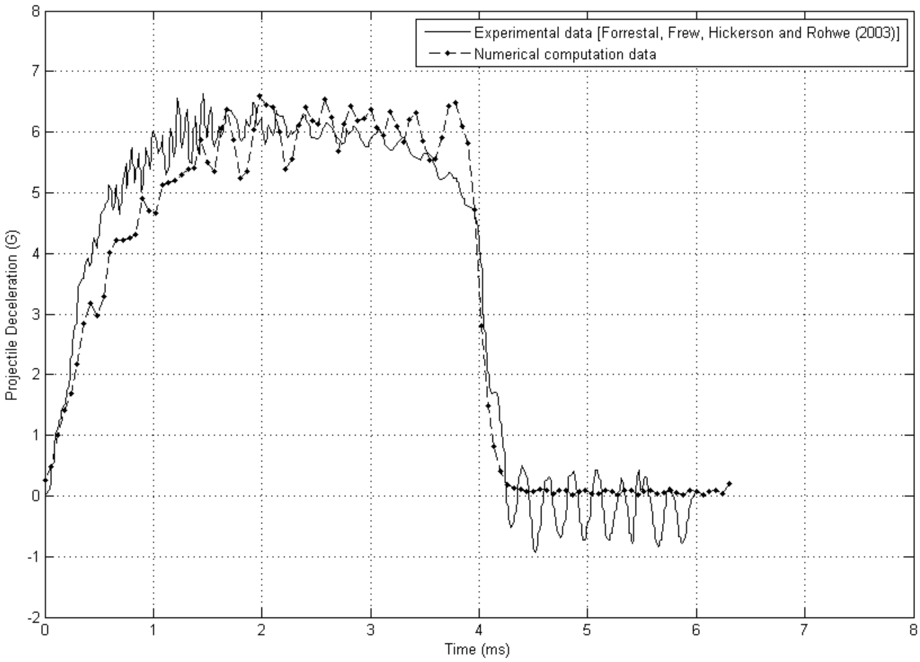


Figure 9: Deceleration versus time data. Shot Number SNL-00-03/1, $\psi = 3.0$, $V_s = 200m/s$

Section 4.1. The computed results are shown in Tab. 6. It can be found that the computed results are in good agreement with experimental data. The relative errors are less than 4%. Fig. 13 presents the projectile deceleration-time data at the velocity of 378.6 m/s, compared with the experimental data. It is found that the deceleration is in very good agreement with measurement. It is very important for the coincidence between the computed deceleration data and measured data. The deceleration data indicate the resisting force which the target exerts on the projectile, and the dynamical behavior of the target.

With regard to these results, it is noted that the constitutive parameters identified through the inverse method based on a set of experiment data are available and reliable.

The presented inverse method is a helpful tool to efficiently and reliably identify material constitutive parameters, providing scientific basis for FE model development. Before the inverse computation, it is essential to analysis the sensitivity of parameters of material constitutive model in order to make sure the inversed parameters. During the analysis, some parameters are not determined directly by

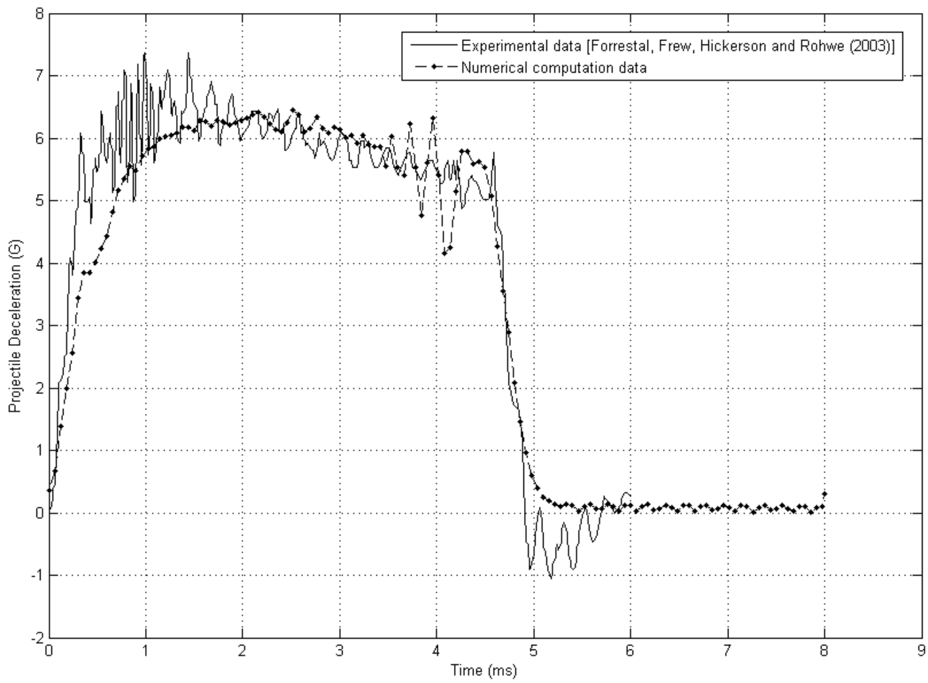


Figure 10: Deceleration versus time data. Shot Number SNL-00-02/2, $\psi = 3.0$, $V_s = 250m/s$

Table 6: Penetration data with 6.0 CRH projectile at different initial velocity

Shot and projectile number	Striking velocity (m/s)	Penetration depth of experiment (m)	Penetration depth of numerical computation (m)	Relative error (%)
SNL-00-08/2	238.4	0.58	0.59058	1.824
SNL-00-07/1	378.6	1.25	1.2004	3.968

experiment, but set recommended value derived from the literature, because these parameters are not sensitive to the experimental data. In other words, the inversed parameters must be sensitive to the response data. Then considering the number and varying range of the inversed parameters, the DOE method is adopted to generate the sample points in order to reduce the number of forward calculations. While the distribution of sample points affects the accuracy of the approximation model. Therefore, a more feasible DOE method will be considered in future work.

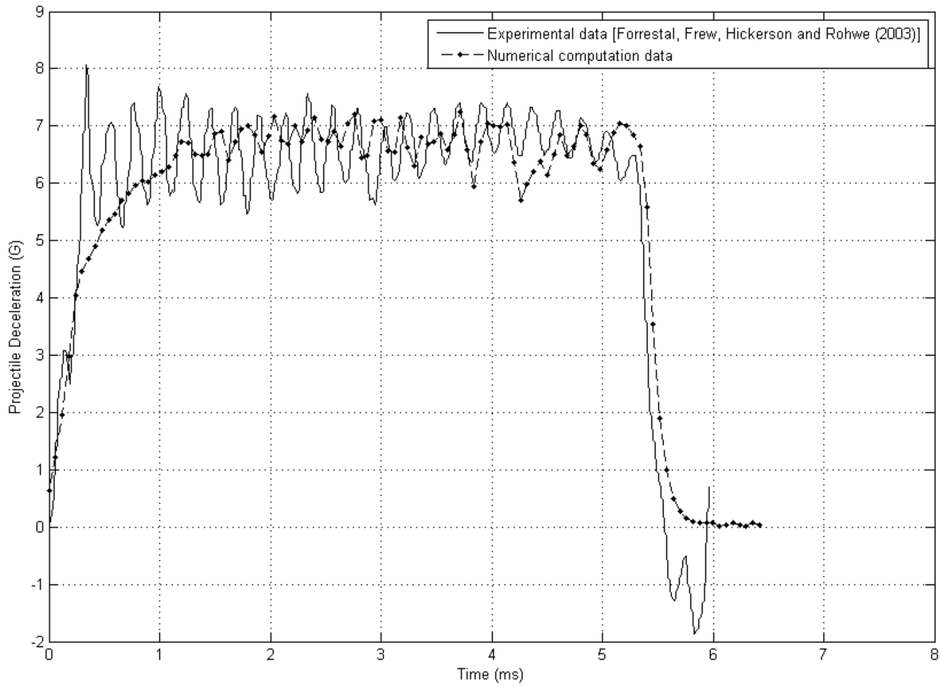


Figure 11: Deceleration versus time data. Shot Number SNL-00-05/3, $\psi = 3.0$, $V_s = 336.6m/s$

6 Conclusions

This paper presents a computational inverse technique to determine the constitutive model parameters of concrete based on the penetration experiments. LS-DYNA solver is employed as the forward solver to calculate the response data for given constitutive parameters. Using RBF method with the local-densifying method to construct the response surface model, the relationships between the constitutive parameters and penetration depth can be mapped accurately, simultaneously, reducing the total computational time and improving accuracy of the constitutive parameters. IP-GA is used as the inverse operator to determine the constitutive parameters. Through this work, the parameters of HJC concrete model were obtained. The numerical computations of penetration of concrete target with the identified constitutive parameters, give good results compared with experimental data. This demonstrates the availability of this inverse technique.The presented method can be adapted to other material constitutive model to obtain accurate parameters.

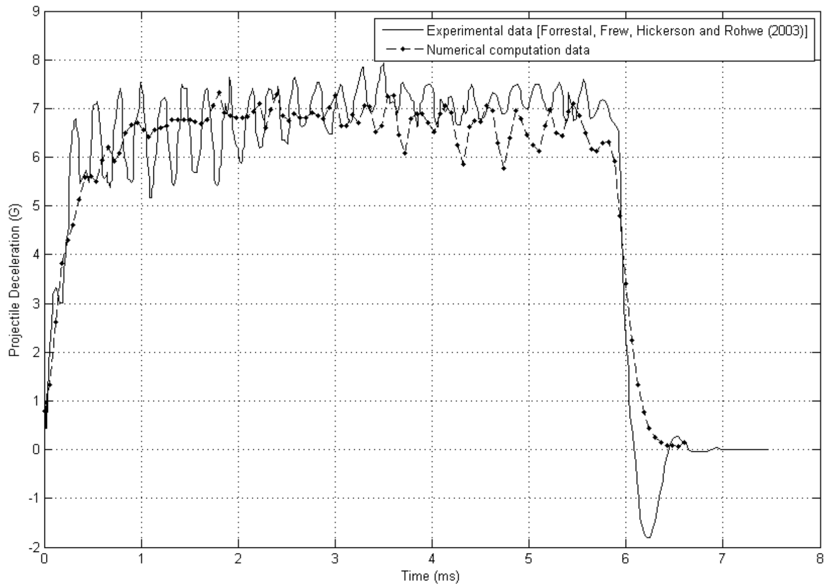


Figure 12: Deceleration versus time data. Shot Number SNL-00-04/4, $\psi = 3.0$, $V_s = 378.6 m/s$

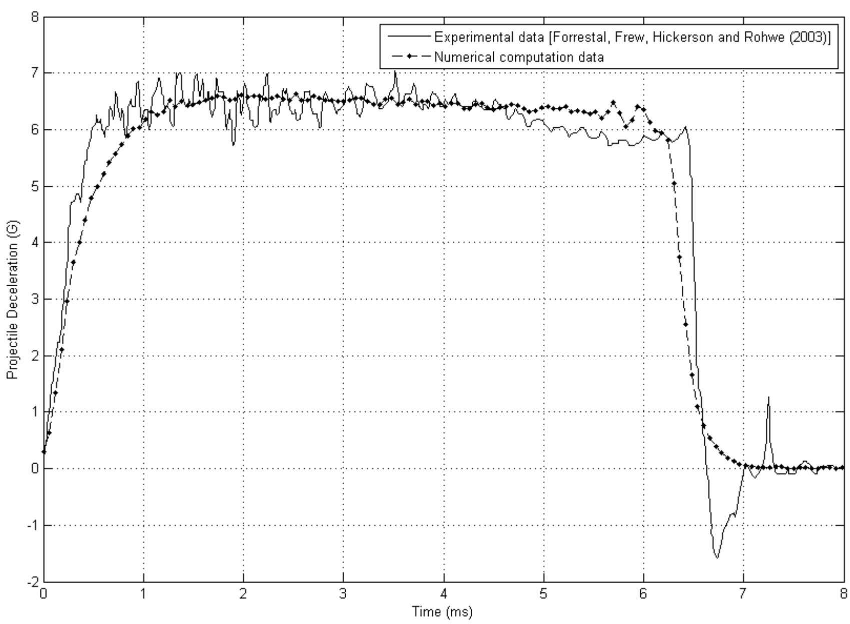


Figure 13: Deceleration versus time data. Shot Number SNL-00-07, $\psi = 6.0$, $V_s = 378.6 m/s$

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