

A Multi-Criteria Topology Optimization for Systematic Design of Compliant Mechanisms

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Abstract: This paper attempts to present a new multi-criteria topological optimization methodology for the systematic design of compliant micro-mechanisms. Instead of employing only the strain energy (SE) or the functional specifications such as mechanical efficiency (ME), in this study an alternative formulation representing multiple design requirements is included in the optimization to describe the performance of compliant mechanisms. In most conventional designs, SE is used to only measure the design requirement from the point of view of structures, while ME is usually applied to describe the mechanical performance of mechanisms. However, the design of a compliant mechanism is required to comprehensively consider both the structural and mechanical performance quantities. Displacement, material usage and dynamic response are imposed as three external constraints to narrow the searching domain. In doing so, the multi-criteria optimization problem involving the SE and ME can reasonably embody the mechanical structural characteristics of compliant mechanisms. A sequential convex programming, the method of moving asymptotes (MMA), is applied to solve the topological optimization problem, which can not only ensure numerical accuracy but also both the monotonous and non-monotonous structural behaviors. SIMP model (solid isotropic material with penalization) is used to indicate the dependence of elastic modulus upon regularized element densities. Several typical numerical examples are used to demonstrate the effectiveness of the proposed methodology, and the prototype of a resulting mechanism has also been manufactured to validate the design of the compliant mechanism.

Keywords: Topology optimization; Compliant mechanisms; Multi-criteria

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1 Introduction

Compliant mechanisms [Howell (2001); Ananthasuresh and Howell (2005)] are a relatively new breed of mechanical devices that transmits motion and energy from specified input ports to output ports via the flexibility of its structural components rather than from hinges, bearings, and slides movement. Compared to rigid-body mechanisms, in compliant mechanisms, the elastic deformation is intended as a source of motion, and the flexibility of structural components permits the structure to fulfill the required functions analogous to rigid-link mechanisms. The primary advantage of compliant mechanisms is that fewer parts, fewer assembly processes and no lubrication are needed, which are beneficial for improving reliability, performance and manufacturing. The disadvantage of compliant mechanisms is that some of the input energy is stored as elastic work in the mechanism, which will reduce the efficiency. Fatigue analysis is typically another issue for compliant mechanisms than for rigid-body counterparts when undergoing the prescribed functions cyclically. Due to member links and flexural joints involved, the design of compliant mechanisms is more complex than rigid-body mechanisms, remarkable development has occurred in recent years after introducing the topology optimization.

Topology optimization has experienced considerable progress and has been extended to a wide range of engineering areas, such as mechanisms, functional materials, and micro and nano-scaled structures [Bendsøe & Sigmund (2003)]. The topology optimization consists of determining the best arrangement of a prescribed volume of material within a given design space to iteratively eliminate and redistribute material in the space in order to achieve the best performance of the conceptual design [Eschenauer and Olhoff (2001)]. Several typical approaches have been developed for topology optimization of continuum structures, such as the element-based optimization methods characterized by the material distribution, typically the homogenization approach [Bendsøe and Kikuchi (1988)], the density variable approach [Bendsøe and Sigmund (1998)], and the geometry-oriented topology optimization introducing a function that describes the shape and topology of the structure implicitly, typically the level set-based method [e.g. Sethian and Wiegmann (2000); Wang, Wang and Guo (2003); Allaire, Jouve and Toader (2004)].

Due to their great promise in providing better solutions to mechanical and structural design problems, compliant mechanisms have recently experienced considerable development in a variety of areas [e.g. Larsen, Sigmund and Bouwstra (1997); Sigmund (1997); Wang, Chen, Wang and Mei (2005); Luo, Chen, Yang, Zhang and Abdel-Malek (2005); Luo and Tong (2008)]. The systematic design of compliant mechanisms falls into two categories: lumped and distributed compliant mechanisms. Lumped compliant mechanisms are based on rigid-body kinematic

synthesis [Howell and Midha (1994); Howell (2001)], and distributed compliant mechanisms are based on topology optimization [e.g. Ananthasuresh, Kota and Gi-anchandani (1994); Sigmund (1997); Frecker, Ananthasuresh, Nishiwaki, Kikuchi and Kota (1997)].

If the elastic deformation in a mechanism is only limited to local regions as in mechanisms with flexural or notch hinges, such mechanisms are called lumped compliant mechanisms and they can be typically synthesized by the pseudo-rigid-body method (PRBM) [Howell (2001)]. Lumped compliant mechanisms are first accomplished by the synthesis of a conventional rigid-link mechanism to explore basic topological patterns of the mechanism and then produce the pseudo-rigid-body mechanism, after which flexibility is introduced to formulate some small-length local flexural pivots to formulate a lumped compliant mechanism. Therefore, the PRBM mechanism involves rigid-body mechanism theory and compliant mechanism analysis. However, lumped compliant mechanisms may not be suitable for micro-scale mechanisms due to the approximation of the design approach. The mechanism mainly bends around concentrated flexural hinges where the material around the hinges is easily subject to overstress and overstrain, which will speed up fatigue breakage. Although under certain specific loading configurations, the PRBM is available for modeling beams with continuous deflection [Nathan and Howell (2003)].

Topological optimization methods [Bendsøe and Kikuchi (1988)] were first extended to the optimal design of compliant mechanisms by Ananthasuresh [1994]. This method can engender jointless distributed compliant mechanisms bending throughout the structure and obtaining their flexibility from topology and shape of the material continuum. The distributed compliant mechanism is particularly suitable for micro-scale structures [Ananthasuresh and Howell (2005)] due to its continuous and monolithic characteristics. It is difficult to fabricate the micro-structured compliant mechanisms as the conventional macro-scale mechanisms with hinges and links because of the difficulty associated with manufacture, lubrication, and friction processes under micro-scale. The fully compliant mechanism is in essence a kind of jointless mechanical device, which is capable of producing distributed compliance from the elastic deformation of the structure due to continuity and monolithic of comprising materials.

With respect to the optimal design of compliant mechanisms via topology optimization, there are two typical methods in defining the optimization objectives, focusing on structural quantities or/and mechanical measurements as single objective optimization, or as multi-objective optimization. For instance, Yin and Ananthasuresh (2003) formulated the optimization problem for compliant mechanisms where the objective is in the flexibility-strength formulation to maximize

the structural mutual strain energy. Saxena and Ananthasuresh (2001) proposed the homogenization-based approach for nonlinear and curved path problems with an efficiency-strength objective. The work of [Nishiwaki, Frecker, Min and Kikucki (1998)] studied the multi-objective optimization problem of compliant mechanisms using the homogenization-based method, but found that utilization of this method is somewhat troublesome. [Sigmund (1997), Wang, Chen, Wang and Mei (2005), Luo, Tong, Wang and Wang (2007)] have studied the approaches for optimal design of compliant mechanisms using mechanical advantage or geometrical advantage. Frecker, Ananthasuresh, Nishiwaki, Kikuchi and Kota (1997) suggested a multi-criterion formulation, including mutual strain energy (or mutual potential energy) and strain energy, to ensure the prescribed flexibility and enough stiffness of the structure. Luo, Chen, Zhang, Yang and Abdel-Malek (2005) presented a multi-objective programming method using a compromise programming scheme, in which a kinematic function and a structural function are considered, simultaneously. Luo, Tong, Wang and Wang (2007) employed the level set method to study the distributed compliant mechanisms using the objective function of mechanical efficiency.

Most above mentioned formulations for topology optimization of compliant mechanisms have primarily been concerned with the optimization formulations which only use structural or mechanical functionalities from mechanism problems. However, topology optimization of compliant mechanisms is required to own the characteristics of both mechanism performance and structural ability. Therefore, the aim of this paper is to present a new multi-criterion formulation for the topology optimization of distributed compliant mechanisms, where the mechanical efficiency is applied to meet motion and force requirements of the mechanism while structural strain energy is considered to satisfy structural stiffness requirement.

2 Multi-criteria optimization problem

2.1 SIMP material interpolations

Topological optimization of continuum structures is essentially the integer programming problem with 0-1 discrete design variables. However, the optimization problem is ill-posed and difficult to solve directly using gradient-based optimization approaches. Therefore, the original optimization problem is usually relaxed to allow elemental densities to take intermediate values from 0 to 1. Homogenization approach [Bendsøe and Kikuchi (1988)] and variable density method [Bendsøe and Sigmund (1999)] are two typical schemes for the relaxation. Homogenization-based approaches were once the most commonly adopted interpolation scheme for compliance topology optimization where the effective material properties are char-

acterized by the homogenized process. However, it is difficult to determine the microstructure pattern for compliant mechanisms. Consequently, solid isotropic material with penalization (SIMP) [Zhou and Rozvany (1991)], a simple but effective interpolation scheme belonging to the variable density method, is used to indicate the dependence upon some power of the element relative densities and material modulus. Bendsøe and Sigmund (1999) proved the existence of physical meaning of the SIMP model when some simple conditions on the exponent are satisfied (such as penalty exponent $p \geq 3$ for Poisson's ratio $1/3$). SIMP was adopted as the density-stiffness interpolation scheme in this work.

2.2 Multi-criteria formulation

In the multi-criteria formulation, the multiple objective functions are considered in the design, both from structural problems such as and strain energy (SE), and mechanical function specification such as mechanical efficiency (ME). The multi-criteria objective is considered to be a ratio of the ME to the SE. Three constraints (displacement, stress, and dynamic) are imposed to limit the searching domain of a compliant mechanism. The proposed multi-criteria optimization problem is defined by

$$\left\{ \begin{array}{l} \text{Minimize} : f(X) = \left\{ -\frac{ME(X)}{SE(X)} \right\} \\ X=(x_1, x_2, \dots, x_n)^T \\ \text{Subject to: } \left\{ \begin{array}{l} u_{in}(x) \leq u_{in}^* \\ \lambda_i(X) \geq [\beta], \quad i = 1, 2, \dots, M \\ \sum_{j=1}^n x_j V_j - \bar{V} \leq 0, \quad j = 1, 2, \dots, n \\ 0 < x_{\min} \leq x_j \leq 1, \quad j = 1, 2, \dots, n \end{array} \right. \end{array} \right. \quad (1)$$

where $u_{in}(x)/u_{in}^* \leq 1$ is the constraint introduced to limit the largest input displacement to control the maximum stress level in the optimum mechanism. ME is used to indicate the functionality of the mechanism, and SE is applied to measure the structural stiffness. \bar{V} is the amount of allowable material usage in the design domain. n is the number of finite elements, x is the design variable, and $x_{\min}=0.001$ is introduced to avoid numerical singularity of the stiffness matrix, as well as to prevent possibility of localized modes in low density areas.

β is a newly introduced design variable that posed a lower bound on each eigenvalue, which can be obtained by solving the following dynamic optimization prob-

lem to narrow the feasible search domain

$$\left\{ \begin{array}{l} \text{Maximize} \quad : [\beta] \\ \beta, \Delta X = \{\Delta x_1, \Delta x_2, \dots, \Delta x_n\}^T \\ \text{Subject to : } \left\{ \begin{array}{l} \lambda_j(X) + f_{jj}^T(X) \Delta X \geq \beta, \quad j = r_m, \dots, R_m, m = 1, \dots, M \\ \left(\sum_{i=1}^n V_j x_j \right) + \nabla^T V(X) \Delta X \leq \bar{V}, \quad i = 1, 2, \dots, n \\ 0 < x_{\min} \leq x_i < 1, \quad i = 1, 2, \dots, n \end{array} \right. \end{array} \right. \quad (2)$$

where f_{jj}^T represents gradient vector components. ΔX denotes the design increment vector. The multiple eigenvalues are numbered from r_m to R_m , where r_m , and R_m indicate the first and the last eigenvalues ($r_m = R_m$ for simple eigenvalue). M is the total number of eigenvalues. For further details of solving this dynamic optimization problem, readers are referred to the relevant works [Seyanian, Lund and Olhoff (1994); Luo, Yang, Chen Zhang and Abdel-Malek (2005)].

2.3 Sensitivity analysis

The spring model [Sigmund (1997)] has been widely applied to characterize the relationship between work-piece and compliant mechanism. A spring model consists of a design domain and a spring with known stiffness attached at the output port in conformity to the desired direction, where the spring is used to model the work-piece. The spring stiffness can be estimated from the elastic behavior of the work-piece, and the interaction between the mechanism and the work-piece is captured properly. The spring model is subsequently used to express functional specifications as well as structural stiffness. In this context, based on the work [Sigmund (1997)], a piezoelectric actuated micro compliant amplifier, as shown in Fig. 1, is used to explain the spring model in Fig. 2.

Only for the sake of numerical simplicity, the following analysis is based on the assumption of linear finite elements and a stiff work-piece with no gap between the output port and the work-piece. Objective functions and constraints of the compliant mechanism can be evaluated by employing finite element analysis based on the spring model. The geometrical advantage is measured by the ratio of displacements at the output and input ports $GA = \Delta_{out}/\Delta_{in}$, and the mechanical advantage is the ratio of reaction (output) force at the output port and the input force $MA = F_{out}/F_{in}$. The mechanical efficiency can then be determined by the ratio of output work to input work $ME = \text{sgn}(GA) \times MA \times GA$, which involves MA and GA to compliant mechanism design, where $\text{sgn}(GA)$ indicates the desired direction of the output displacement. The mechanical advantage (MA) is re-written as $MA = F_{out}/F_{in} = k_s \cdot \Delta_{out}/F_{in}$, where k_s represents the stiffness of the spring model, and Δ_{out} is the displacement at the output port of the spring model.

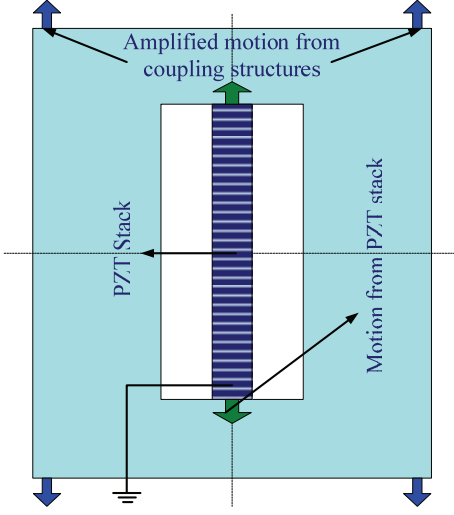


Figure 1: Compliant micro-amplifier

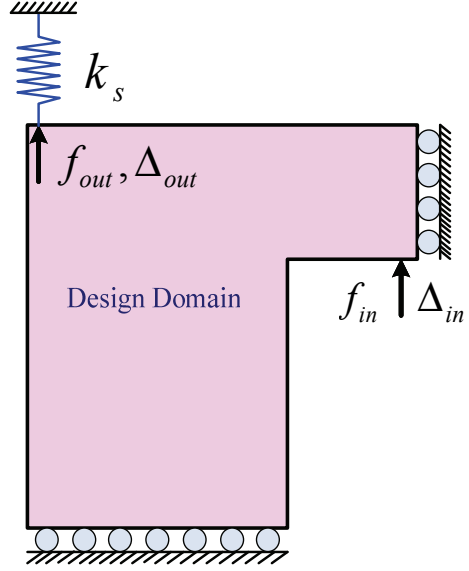


Figure 2: Spring model of quarter symmetry

The mechanical efficiency (ME) is then represented as

$$ME = \text{sgn}(GA) \times MA \times GA = \text{sgn}(GA) (k_s \cdot \Delta_{out}^2 / F_{in} \cdot \Delta_{in}) \quad (3)$$

After getting the ME , the objective function is defined as

$$f(X) = \left\{ -\frac{ME}{SE} \right\} = \left\{ -\frac{\text{sgn}(GA) \times MA \times GA}{SE} \right\} = -\frac{\text{sgn}(GA) (k_s \cdot \Delta_{out}^2 / F_{in} \cdot \Delta_{in})}{SE} \quad (4)$$

where $\Delta_{out} = \Delta_{21} - c\Delta_{22}$ can be measured by superposition of displacements at the output port caused by load case 1 and load case 2, respectively, where $c = (\Delta_{21} - \Delta_{out})/\Delta_{22}$. c is the factor with which the displacement Δ_{22} , induced by the unit dummy load applied at the output port, must be considered to obtain the displacement Δ_{out} . Reaction force $F_{out} = cf_2$ at the output port is rewritten as $F_{out} = f_2(\Delta_{21}/(\Delta_{22} + f_2/k_s))$. MA , GA and ME can be calculated by using displacements (Δ_{11} , Δ_{21} , Δ_{12} and Δ_{22}) and load cases (f_1 and f_2).

Here the mechanical advantage (MA) and the geometrical advantage (GA) are given as:

$$MA = \frac{F_{out}}{F_{in}} = \frac{f_2}{f_1} \frac{\Delta_{21}}{\Delta_{22} + f_2/k_s}$$

$$GA = \frac{\Delta_{out}}{\Delta_{in}} = \frac{f_2 \Delta_{21}}{(k_s \Delta_{22} + f) \Delta_{11} - k_s \Delta_{21}^2} \quad (5)$$

Based on MA and GA, the mechanical efficiency ME is then stated as

$$ME = \text{sgn}(GA) \times MA \times GA = \frac{f_2}{f_1} \left(\frac{f_2 \Delta_{21}}{(k_s \Delta_{22} + f_2) \Delta_{11} - k_s \Delta_{21}^2} \right) \left(\frac{\Delta_{21}}{\Delta_{22} + f_2/k_s} \right) \quad (6)$$

Similarly, the displacements at the output and input ports can be expressed as follows:

$$\Delta_{out} = \frac{F_{out}}{k_s} = f_2 \frac{\Delta_{21}}{k_s \Delta_{22} + f_2}$$

and

$$\Delta_{in} = \Delta_{11} - c \Delta_{12} = \Delta_{11} - \Delta_{21}^2 \left(\frac{k_s}{k_s \Delta_{22} + f_2} \right) \quad (7)$$

The sensitivity of objective function is expressed by

$$\frac{\partial f(x)}{\partial x_i} = - \frac{\partial \left(\frac{ME}{SE} \right)}{\partial x_i} = \frac{- \frac{\partial(ME)}{\partial x_i} (SE) + (ME) \frac{\partial(SE)}{\partial x_i}}{(SE)^2} \quad (8)$$

where the sensitivity of mechanical efficiency ME is derived as

$$\frac{\partial ME}{\partial x_j} = \text{sgn}(GA) \left(MA \frac{\partial GA}{\partial x_j} + GA \frac{\partial MA}{\partial x_j} \right) \quad (9)$$

where the sensitivities of GA and MA are respectively described by

$$\frac{\partial GA}{\partial x_j} = \left(\Delta_{in} \frac{\partial \Delta_{out}}{\partial x_j} - \Delta_{out} \frac{\partial \Delta_{in}}{\partial x_j} \right) / \Delta_{in}^2 \quad (10)$$

$$\frac{\partial MA}{\partial x_j} = \frac{f_2}{f_1} \left[\left\{ (\Delta_{22} + f_2/k_s) \frac{\partial \Delta_{21}}{\partial x_j} - \Delta_{21} \frac{\partial \Delta_{22}}{\partial x_j} \right\} / (\Delta_{22} + f_2/k_s)^2 \right] \quad (11)$$

In terms of the sensitivities of Δ_{ij} , sensitivities of Δ_{out} and Δ_{in} can be specified as

$$\frac{\partial \Delta_{out}}{\partial x_j} = f_2 \left[\left((k_s \Delta_{22} + f_2) \frac{\partial \Delta_{21}}{\partial x_j} - \Delta_{21} \frac{\partial \Delta_{22}}{\partial x_j} \right) / (k_s \Delta_{22} + f_2)^2 \right] \quad (12)$$

$$\frac{\partial \Delta_{in}}{\partial x_j} = \frac{\partial \Delta_{11}}{\partial x_j} - \frac{\partial c}{\partial x_j} \Delta_{21} - c \frac{\partial \Delta_{12}}{\partial x_j} \quad (13)$$

where the sensitivity of c can be expressed by Δ_{out} and Δ_{in} together with their sensitivities, which is also dependent on MA and its sensitivities, and it is written as

$$\frac{\partial c}{\partial x_j} = \left\{ \Delta_{22} \left(\frac{\partial \Delta_{21}}{\partial x_j} - f_2/k_s \cdot \frac{\partial MA}{\partial x_j} \right) - \frac{\partial \Delta_{22}}{\partial x_j} (\Delta_{21} - MA \cdot f_1/k_s) \right\} / \Delta_{22}^2 \quad (14)$$

Hence, the sensitivities analysis of ME can be directly performed in terms of the sensitivities of displacement Δ_{ij} , where Δ_{ij} and its sensitivity will be derived based on the following discussion. Since the spring model is assumed to be linear in this work, the equivalent spring model can therefore be implemented by superposition of two load cases (Fig. 3), and the continuum structure is discretized into N elements for subsequent finite-element analysis. Therefore, displacements of Δ_{ij} can be calculated by solving a finite element problem with two load cases, respectively. The first load case consists of the input load f_1 defined by finite element load vector F_1 , and the second load case is a dummy load f_2 defined by finite element load vector F_2 , in terms of the direction of intended motion.

Using the dummy load method, we have the following forms:

$$\begin{aligned} \Delta_{11} &= (\{V\}^T [K] \{V\}) / f_1, & \Delta_{21} &= (\{V\}^T [K] \{U\}) / f_2 \\ \Delta_{12} &= (\{U\}^T [K] \{V\}) / f_1, & \Delta_{22} &= (\{U\}^T [K] \{U\}) / f_2 \end{aligned} \quad (15)$$

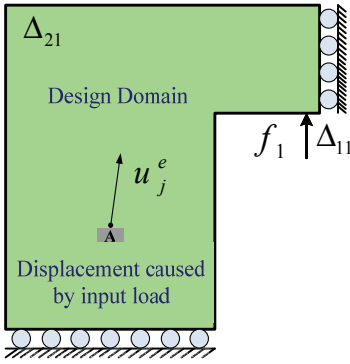
where the indices i, j in Δ_{ij} denote the displacement at port i induced by the exerted load case at port j . If there is no resistance from a work-piece, Δ_{11} and Δ_{21} (or Δ_{22} and Δ_{12}) will evolve to the calculated displacements at the input and output ports, respectively. $[K]$ denotes the global stiffness matrix. Displacement vectors $\{V\}$ and $\{U\}$ related to two load cases $\{F_1\}$ and $\{F_2\}$ can be found by solving the following two sets of linear equations:

$$[K] \{U\} = \{F_1\}, \quad [K] \{V\} = \{F_2\} \quad (16)$$

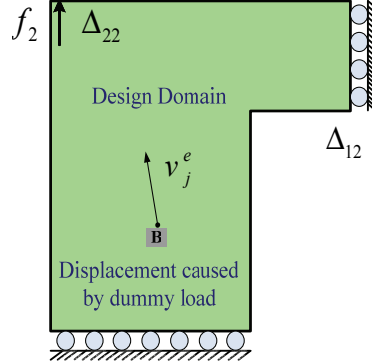
Using the SIMP interpolation scheme [Bendsøe and Sigmund (1999)], the sensitivities analysis of displacements for all $\Delta_{ij}(i, j = 1, 2)$ can be worked out by using the adjoint method. For instance, the sensitivity of Δ_{11} can be given by

$$\Delta_{11} = (\{U\}^T [K] \{U\}) / f_1 \quad (17)$$

$$\frac{\partial \Delta_{11}}{\partial x^e} = \left(\frac{\partial \{U\}^T}{\partial x^e} ([K] \{U\}) + \{U\}^T \frac{\partial ([K] \{U\})}{\partial \partial x^e} \right) / f_1 \quad (18)$$



(a) Input force F1



(b) Output dummy load F2

Figure 3: Dummy load method with superposition of two load cases for mechanism design

Since the load vectors in $[K]\{U\} = \{F\}$ and $\{U\}^T[K] = \{F\}^T$ are actually independent of the design variables, we can obtain the following equations:

$$\frac{\partial (\{U\}^T [K])}{\partial x^e} = \{U\}^T \frac{\partial [K]}{\partial x^e} + \frac{\partial \{U\}^T}{\partial x^e} [K] = 0 \tag{19}$$

$$\frac{\partial ([K]\{U\})}{\partial x^e} = [K] \frac{\partial \{U\}}{\partial x^e} + \frac{\partial [K]}{\partial x^e} \{U\} = 0 \tag{20}$$

Substituting (18) and (19) into (17) can lead to

$$\frac{\partial \Delta_{11}}{\partial x^e} = -\{U\}^T \frac{\partial [K]}{\partial x^e} \{U\} \tag{21}$$

Combining SIMP interpolation model, we can get

$$\frac{\partial \Delta_{11}}{\partial x^e} = -\sum_{e=1}^N [p(x^e)^{p-1} \{u_1^e\}^T [k^e] \{u_1^e\}] / f_1 \tag{22}$$

Following a similar way, the general formulation of sensitivity of displacement is given as

$$\frac{\partial \Delta_{ij}}{\partial x^e} = -\sum_{e=1}^N [p(x^e)^{p-1} \{u_j^e\}^T [k^e] \{u_i^e\}] / f_i \tag{23}$$

where x^e is the design variable (relative density of element), p is the penalty exponent, the subscripts i and j denote the displacement at port i induced by load j . $[k^e]$ is the element matrix, $\{u_j^e\}$ and $\{u_i^e\}$ are the displacement vectors.

If ME is only maximized, a mechanism with very weak structure might be obtained. Therefore, the structural requirements can be remedied by minimizing the strain energy, which is equivalent to minimizing the mean compliance or maximizing the stiffness. The strain energy is then calculated by considering the load case characterized in Fig. 4 where the boundary of the applied input load is restrained and a dummy load is applied at the output port in the opposite direction, to account for the resistance of the work-piece. Thus, the SE is incorporated into the optimization formulation.

The structural strain energy (SE) can be defined by:

$$SE = \sum_{j=1}^{N_e} (E^{\min} + x_j^p \Delta E) \{\tilde{u}_j^e\}^T [\tilde{k}_j^e] \{\tilde{u}_j^e\} \quad (24)$$

$$\frac{\partial SE}{\partial x_j} = - \sum_{j=1}^{N_e} p x_j^{p-1} \{\tilde{u}_j^e\}^T [\tilde{k}_j^e] \{\tilde{u}_j^e\} \quad (25)$$

where, N_e is the element number, \tilde{u}_j^e represents the displacement exerted by the applied force $-f_2$ in Fig. 4, $[\tilde{k}_j^e]$ is the stiffness matrix for the mechanism design problem which is different from $[k_j^e]$ due to the different boundary condition induced by the extra specified restraint at the input port. The following optimal results based

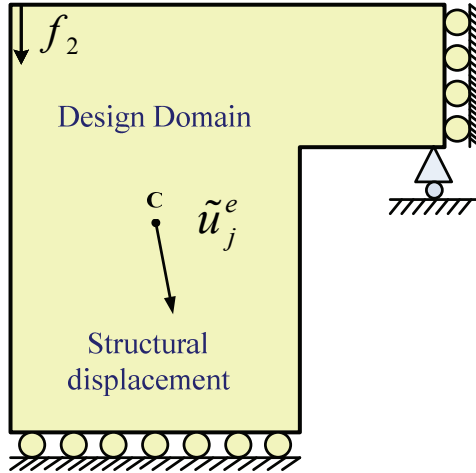
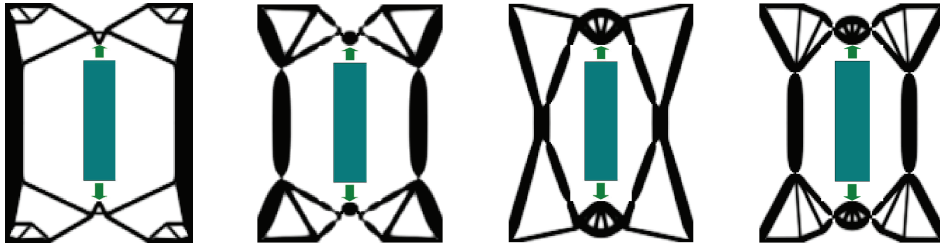


Figure 4: Strain energy (SE) calculation for structure design

on design domain shown in Fig. 1 are briefly sketched, showing that the different objectives, such as SE, MA, GA and ME, can obtain different optimal results. For simplicity, the reaction force provided by vertical motion of PZT amplifier to the coupled compliant mechanism is equalized by two vertical forces. The mechanism topologies are shown in Fig. 5.



- (a) Optimum mechanism topology of SE , iterations is 138, optimal results is 2.15
- (b) Optimum mechanism topology of MA , iterations is 262, optimal results is 0.31
- (c) Optimum mechanism topology of GA , iterations is 214, optimal results is 2.74
- (d) Optimum mechanism topology of ME , iterations is 313, optimal results is 0.85

Figure 5: Optimal solutions using different objective functions

2.4 Numerical instabilities

Numerical instabilities, checkerboards, mesh-dependency, and one-node connected hinges, will occur when performing the design of compliant mechanisms using topological optimization. The reasons for checkerboards and mesh-dependency are well understood [Diaz and Sigmund (1995)] and many methods such as filtering scheme can be applied to overcome these difficulties [Sigmund and Petersson (1998)]. Similar to checkerboards, one-node connected hinges are also caused by improper computational modeling in the numerical procedure, as well as improper modeling of stress variations, i.e., when stresses and strains are inadequately modeled using low-order finite elements. In other words, the hinges can be regarded as one form of the building blocks of checkerboard patterns where the hinge is modeled by an artificially stiff corner-to-corner connection of two four-node elements. However, compared to checkerboards, the hinge pattern is not repeated within the mechanism. Deformation takes place almost entirely within hinge regions where the shape is subject to infinity and the mechanism is easy to break. Although this scheme can eliminate checkerboards in the mechanism, they are not always appropriate or suitable for preventing one-node connected hinges.

The sensitive filtering method [Sigmund (2001)] can improve smoothness and differential properties of the objective function and it can regularize the topology optimization problem. However, it may not be effective in removing one-node hinges and cannot guarantee a hinges-free design. Although the checkerboards and hinges problem are relevant, but the formulation of the optimization problem should favor the distribution of compliant hinges from one-node connected hinges. Luo, Chen, Zhang, Yang and Abdel-Malek (2005) suggested a hybrid-filtering approach and a density filtering method, respectively. Practically, they are beneficial for alleviating the one-node connected hinges to some extent, and for efficiently eliminating checkerboards and preventing mesh-dependency. In this work, the density filtering method is applied to settle numerical instabilities. However, the hinge in compliant mechanisms is an open topic, and other approaches are still being explored. Some recent developments on this topic can refer to [Luo, Luo, Chen, Tong and Wang (2008); Sigmund (2009); Wang (2009); Kang and Wang (2011)].

3 Convex Programming Approach

Mathematical programming schemes, such as Sequential Linear Programming (SLP), Sequential Quadratic Programming (SQP), and Sequential Convex Programming (SCP) are the commonly used gradient-based approaches for solving topological optimization problems. Considering such large-scale optimization problems with multi-criterion objective functions and multiple constraints, mathematical programming methods based on sequential convex programming are more flexible and theoretically well-founded for advanced optimization applications. The convex programming methods based on approximations of the MMA family [Bruyneel and Fleury (2002)] have been applied to solve topology optimizations of compliant mechanisms. There is no explicit updating scheme for the design variables. The MMA approach solves a sequence of linearized, convex, and separable sub-problems, and the approximation of a design function is computed based on the function value and the first or second derivatives at the current design point. In each step, several approaches, such as the dual approach and primal-dual interior-point approach, can be applied for solving the sub-problem.

The original MMA approximation is based on the CONLIN (Convex Linearization Method) of Fleury and Braibant (1986), which was developed based on the first-order Taylor expansions at current design point. Svanberg (1987) had generalized CONLIN to MMA by introducing two moving sets of lower and upper asymptotes to adjust the convexity of the approximations. The process of MMA approximation has the monotonous characteristic because for each design point only one asymptote is used in terms of the sign of the first-order derivative. In addition, the approximations for all the design functions use the same asymptote,

which will limit the flexibility for adjusting the approximation of each of the structural response functions according to their natural characteristics. Svanberg (2002) extended the original MMA to GCMMA method, where two asymptotes are used at the same time to generate the approximation by using a heuristic non-monotonous parameter, and the approximation thus has a non-monotonous character. The curvature of MMA approximation is approximated by using non-mixed second-order derivatives instead of non-monotonous parameter.

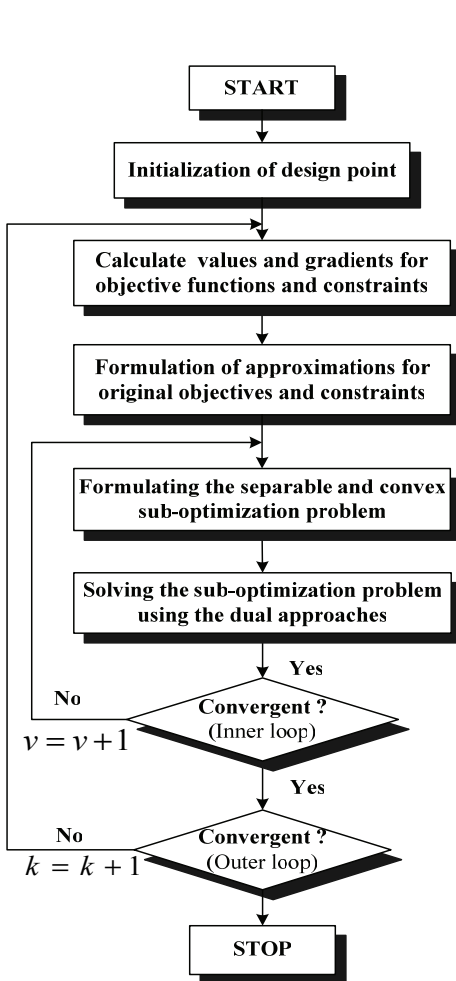


Figure 6: GCMMA approximation

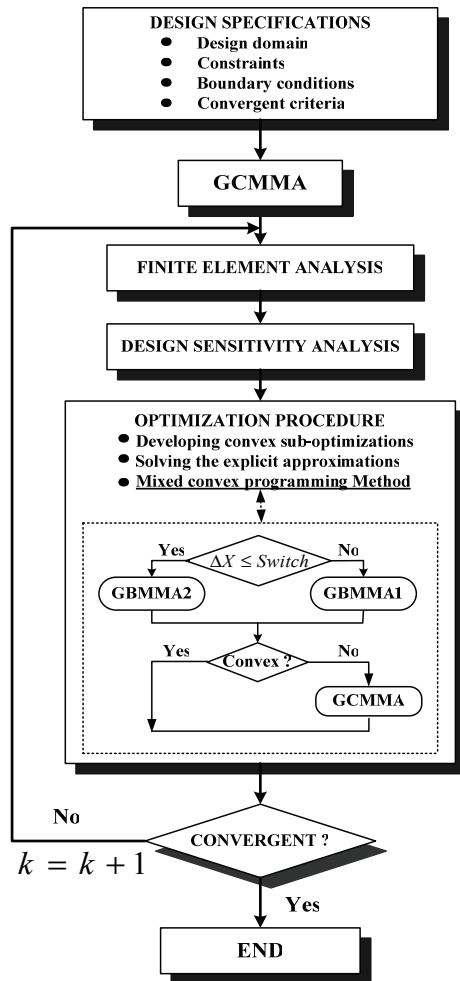


Figure 7: GCMMA and GBMMA approximation

The use of second sensitivity information may enhance the reliability and the efficiency of the optimization process. Instead of using the non-monotonous parameter or non-mixed second-order derivatives in GCMMA, [Bruyneel and Fleury (2004); Bruyneel and Duysinx (2004)] developed the gradient-based GBMMA1 to improve the MMA by matching the first-order information from two successive iterations, and GBMMA2 method to employ a kind of approximated scheme by replacing the second-order derivatives used in the MMA method. In this context, we use a GCMMA-GBMMA approximation based on the works of [Bruyneel and Duysinx (2004)] and [Svanberg (2002)]. It is expected that the computational efficiency can be improved when the solution is carried out using the GBMMA instead of the GCMMA approximation. In the work of Svanberg (2002), the GCMMA method has revealed several characteristics, such as some theoretical properties concerning existence of optimal solutions and optimality conditions, a strictly feasible solution with global convergence, and is reasonably efficient compared with the other MMA methods. Here, the mixed approximation approach is expected to characterize the advantages of both the GCMMA method and the GBMMA approximation. Readers can refer to the relevant works for more details about GCMMA and GBMMA approximations. The flowchart of this scheme is sketched as Fig. 7.

4 Numerical Applications

Two typical examples are used to demonstrate the optimal design of compliant micro-mechanisms using the proposed multi-criteria topology optimization formulation. In the first example, the filtering effect of the density method [Luo, Chen, Zhang, Yang and Abdel-Malek (2005)] upon the multi-criteria is illuminated by using a micro-inverter mechanism with different input and output displacements. In the second example, the influence of dynamic constraint on the optimum mechanism is displayed by a micro-gripper mechanism, where both mechanical efficiency and strain energy objectives are considered by using the mixed convex approach with or without a dynamic constraint.

Example one: micro-inverter mechanisms

The design domain of a micro-inverter is shown in Fig. 8 with the volume constraint 27.5%. Applying symmetry, the lower half of the design domain is considered with 5000 elements. Supposing the material Young's Modulus is 210GPa, the Poisson Ratio is 0.31. $f_{in} = 0.1mN$ and $f_{out} = 0.25mN$. An artificial spring with stiffness K_{in} is attached to the input port with an actuation force f_{in} simulating the input work, and an artificial spring with stiffness K_{out} at the output port simulates the resistance from a work-piece. The input displacement is limited to $U_{in} \leq 35\mu m$.

The optimization problem is illustrated as Eq.1 with three constraints and solved

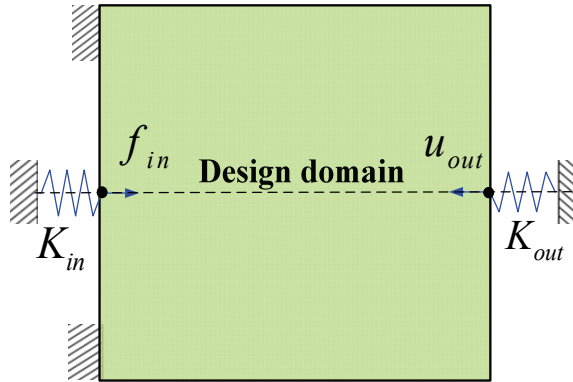


Figure 8: Design domain of compliant micro-inverter

using the mixed convex programming method. The optimal results in Fig. 9 show that different stiffness amounted at the input port can be used to adjust the allowable input displacements. The stress will reach to a higher level when a larger input displacement is produced. For a distributed compliant micro mechanism, a higher stress level will speed up the fatigue failure. Similarly, the spring attached to the output port can also be used to simulate the action of the work-piece (different stiffness) at the output port. Smaller spring stiffness can lead to a larger output displacement and a higher stress level in the resulting mechanism, as shown in Fig. 10. Therefore, in a practical design, there is just a matter of experience to determine the appropriate maximal input and output displacement levels. If the original sensitivity filtering scheme [Sigmund (2001)] is used with the new multi-criteria formulation. The results in Figs 9 and 10 show that checkerboards can actually be eliminated, but the one-node connected hinges still occur when a larger output is expected.

The density filtering method (Luo and et al. 2005) has been introduced into the multi-criteria formulation. The results in Figs. 11 and 12 show that the density filtering method is more capable of preventing one-node connected hinges appearing in optimum mechanisms than the sensitivity filtering method. In addition, the stress level in the optimum mechanisms is also further reduced compared to the stress level regularized by the sensitivity filtering method. Therefore, the new multi-criteria formulation coupled with the density filtering approach can be beneficial for preventing numerical instabilities as well as for controlling the maximal stress level. However, as denoted in the previous research (Luo and et al. 2005), the density filtering scheme cannot be used to totally eliminate the de-facto hinges, although the improvement can be actually expected compared to the widely used

sensitivity filtering scheme.

For simplicity, in this section we limit our discussion only to the cases show in Fig. 11. Here the values, mechanical efficiency, and the strain energy obtained by using the multi-criteria topology formulation are compared with the results gained by using only the traditional optimization formulation, in which the functional specification such as mechanical efficiency is mostly adopted as the objective function (Sigmund 1997; Wang and et al. 2005; Luo and et al. 2007). Within the proposed multi-criteria formulation, one key factor is related to the new multi-criterion objective of the mechanical efficiency and the strain energy. On the one hand, the single individual objective of mechanical efficiency in the proposed formulation for the cases in Fig. 11, which are calculated as 0.1046, 0.3257, 0.6181 and 0.7211, respectively, compared to 0.1305, 0.4701, 0.7103 and 0.8152 obtained by only using the mechanical efficiency as the objective. Although the performance of the mechanical efficiency in the multi-criteria formulation is slightly decreased, it still remains in a reasonable and acceptable scope. On the other hand, structural strain energy related to the cases expressed in Fig. 11 is characterized by the value 10.3418 compared to 55.7046 obtained only by using mechanical efficiency to formulate the problem, where the SE has not been considered in the optimization problem, which reveals the fact that the structural stiffness is obviously enhanced by using the new multi-criteria formulation. During the optimization, it can be found that the convergence of the algorithm is good and the volume constraint is conservative.

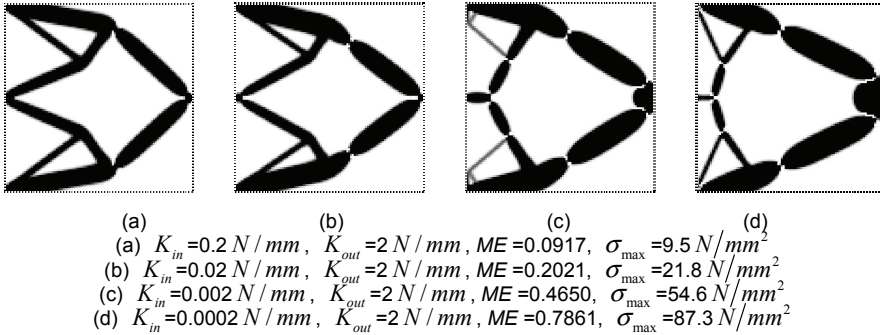
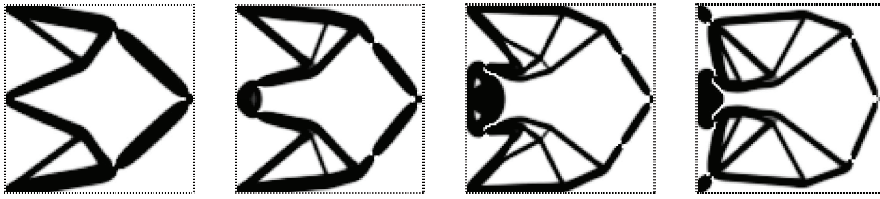


Figure 9: Different input stiffness using the sensitivity filtering method

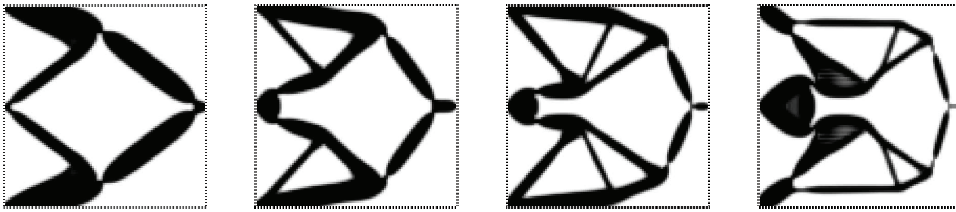
Example two: micro-gripper mechanisms

The size of the design domain is $800\mu m \times 600\mu m$, as shown in Fig. 13. Applying symmetry, the lower half of the design domain is initially discretized using 4800 elements. The input displacement is limited to $u_{in} \leq 40\mu m$ and the volume



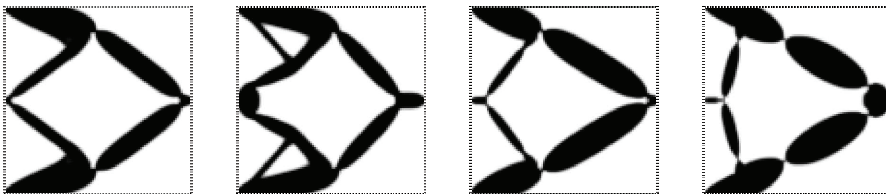
- (a) $K_{in}=2\text{ N/mm}$, $K_{out}=0.2\text{ N/mm}$, $ME=0.0883$, $\sigma_{max}=9.5\text{ N/mm}^2$
 (b) $K_{in}=2\text{ N/mm}$, $K_{out}=0.02\text{ N/mm}$, $ME=0.3890$, $\sigma_{max}=26.8\text{ N/mm}^2$
 (c) $K_{in}=2\text{ N/mm}$, $K_{out}=0.002\text{ N/mm}$, $ME=0.5181$, $\sigma_{max}=58.1\text{ N/mm}^2$
 (d) $K_{in}=2\text{ N/mm}$, $K_{out}=0.0002\text{ N/mm}$, $ME=0.7571$, $\sigma_{max}=89.6\text{ N/mm}^2$

Figure 10: Different output stiffness using the sensitivity filtering method



- (a) $K_{in}=2\text{ N/mm}$, $K_{out}=0.2\text{ N/mm}$, $ME=0.1046$, $\sigma_{max}=9.3\text{ N/mm}^2$
 (b) $K_{in}=2\text{ N/mm}$, $K_{out}=0.02\text{ N/mm}$, $ME=0.3257$, $\sigma_{max}=16.8\text{ N/mm}^2$
 (c) $K_{in}=2\text{ N/mm}$, $K_{out}=0.002\text{ N/mm}$, $ME=0.6181$, $\sigma_{max}=45.1\text{ N/mm}^2$
 (d) $K_{in}=2\text{ N/mm}$, $K_{out}=0.0002\text{ N/mm}$, $ME=0.7211$, $\sigma_{max}=58.2$

Figure 11: Different output stiffness using the density filtering method



- (a) $K_{in}=0.2\text{ N/mm}$, $K_{out}=2\text{ N/mm}$, $ME=0.0975$, $\sigma_{max}=9.2\text{ N/mm}^2$
 (b) $K_{in}=0.02\text{ N/mm}$, $K_{out}=2\text{ N/mm}$, $ME=0.2921$, $\sigma_{max}=21.8\text{ N/mm}^2$
 (c) $K_{in}=0.002\text{ N/mm}$, $K_{out}=2\text{ N/mm}$, $ME=0.5344$, $\sigma_{max}=49.6\text{ N/mm}^2$
 (d) $K_{in}=0.0002\text{ N/mm}$, $K_{out}=2\text{ N/mm}$, $ME=0.6605$, $\sigma_{max}=61.4\text{ N/mm}^2$

Figure 12: Different input stiffness using the density filtering method

constraint is 25%. Young's Modulus is $200MPa$, Poisson Ratio is 0.31, and applied force at the input port is $f_{in}=0.25mN$. An artificial spring with stiffness K_{in} attached to the input port to simulate the input work, and K_{out} simulates the resistance from a work-piece.

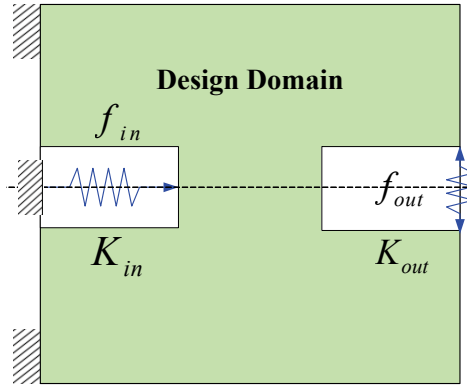


Figure 13: Design domain of compliant micro-gripper

In this example, the optimization problem is first developed by using the proposed multi-criteria formulation without imposing the dynamic constraint, and the density filtering method is also applied to handle numerical instabilities occurring in the optimal compliant mechanism. The computational model is solved by using the proposed convex programming method. In Fig. 14, when the spring stiffness attached at the output port is reduced, such as from $0.2N/mm$ to $0.0002N/mm$, the mechanical efficiency increases from 0.0981 to 0.7271. Although the maximal stress level also increases from $10.5N/mm^2$ to $58.8N/mm^2$, it still remains in an acceptable scope when the input displacement is bounded. From the point of view of optimization, a larger mechanical efficiency is achieved, but a too large mechanical efficiency may result in a higher stress level in the resulting mechanism. In practical designs, the trade-off of the mechanical efficiency of mechanism and the strain energy of structure are required to be considered at the same time. Mechanical efficiencies without dynamic constraint are calculated (Fig. 14), and strain energies corresponding to all cases in Fig. 14 have the same value 30.1165. For the designs given in Fig. 15, the similar conclusion can be obtained.

In Figs. 16 and 17, the optimal mechanisms are obtained by employing the new formulation expressed in Eq. 1 with the dynamic constraint, and the maximal stress levels with different K_{in} and K_{out} are also calculated. When the spring stiffness simulating the reaction of the work-piece at the output port is reduced (e.g.

from $0.2N/mm$ to $0.0002N/mm$), the mechanical efficiency gradually increases from 0.0823 to 0.7512 (Fig. 16), and the stress level increases from $9.3N/mm^2$ to $40.1N/mm^2$, simultaneously. Both ME and the stress level stay within a reasonable scope; as a result, the new multi-criteria formulation can actually meet the design requirements of compliant mechanisms.

To investigate more detailed information about the mechanical efficiency in the new problem formulation, the mechanical efficiencies listed in Fig. 16 are given as 0.0823, 0.2232, 0.5651 and 0.7512 compared to 0.1026, 0.2904, 0.6011 and 0.8089 calculated via the traditional optimization formulation where only ME is considered as the sole objective. Strain energy in Fig. 16 is 4.8908 compared to the value 13.6637 obtained by only using the traditional optimization formulation. It is obvious that the structural stiffness increases using the new objective formulation. With careful investigation, the optimal topologies shown in Figs. 16 and 17 are somewhat different from those topologies without using the dynamic constraint in Fig. 14 and 15. The performance characteristics of the designed mechanisms (mechanical efficiencies and maximal stress levels) might be different for cases with and without eigenvalue constraint, respectively. For the case with $K_{in}=0.2N/mm$ and $K_{out}=0.2N/mm$, the proposed GCMMA-GBMMA algorithm is converged after 245 iterations, compared to 501, 372, 302 iterations for MMA, GCMMA and GBMMA approximations, respectively, with the change tolerance 0.001. From this example, the proposed method has shown to be beneficial for producing compliant mechanisms that simultaneously satisfy the requirements of mechanism problems as well as structure problems.

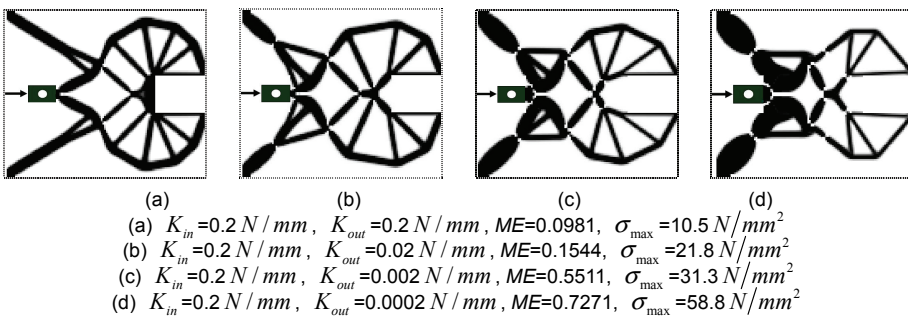


Figure 14: Different output stiffness without dynamic constraint

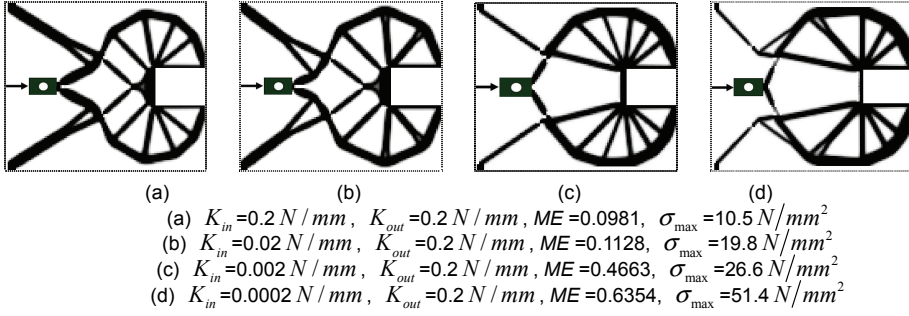


Figure 15: Different input stiffness without dynamic constraint

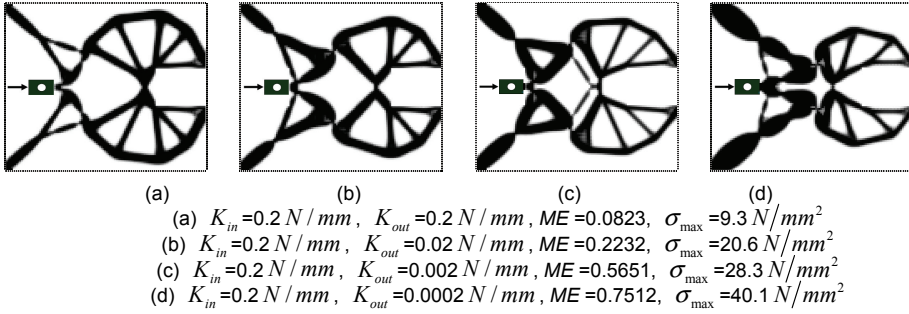


Figure 16: Different output stiffness with dynamic constraint

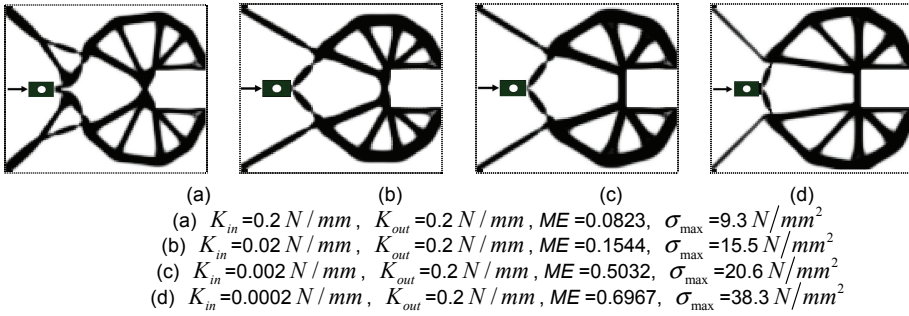


Figure 17: Different input stiffness with dynamic constraint

5 Prototype Manufacturing

A procedure has been developed to perform prototype manufacturing according to the optimal design compliant mechanism using the proposed topological optimization

tion. Here the resulting mechanisms in Fig. 14 (d) and Fig. 16 (d) are used to explore the detailed prototype manufacturing procedure.

The optimal topological contours are actually the grey-white plotting of material density distribution. Reconstruction technologies can be used to convert them into the parametric models that can be read by a general CAD/CAM/CAE system, such as UG-NX, HYPERWORKS or NASTRAN. If needed, the more detailed designs such as shape and size optimization can also be carried out based on these parametric models to further improve the structural performance. Generally, the solid model can be obtained either by the procedure of converting the topologies into a solid model with the designer redrawing these topologies using a CAD software, or by using an automatic conversion procedure with software packages. Here, the former case is adopted to convert topologies into solid models, and the corresponding solid models achieved by using UG-NX are show in Fig. 18. Then the parametric models are imported into the HYPERWORKS through the IGES data file, and the finite element models are formulated in Fig. 19.

The macro size for prototype manufacture is $80\text{mm} \times 60\text{mm} \times 2.5\text{mm}$ with input force $f_{in}=0.25\text{mN}$, the density $1.95 \times 10^{-9}\text{kg}/\text{mm}^3$, and the elastic modulus 200MPa . The maximal allowable stress is 70-80MPa. The mechanical efficiency, strain energy, and maximal stress level and eigenvalues of free vibration are the key factors in the prototype design. The finite element analysis of prototype design is then carried out to show these characteristics. It can be found that the maximum displacements are, $26.1\mu\text{m}$ and $44.5\mu\text{m}$ (Fig. 20), respectively, and the Von Mises stresses inside these models are 19.5 MPa and 14.6 MPa (Fig. 21), respectively, which are well below the maximum allowable stress. In addition, eigenvalues (1~7 order) corresponding to two prototypes are obtained via the calculation of the finite element model. The eigenvalues are listed in Table 1, and the corresponding modes are given in Fig. 22. From Table 1, it can be concluded that the mechanical performance of compliant mechanisms can be improved with the dynamic constraint. The solid models are imported in STL formats, and two compliant grippers have been manufactured in macro-scale at the Molding Centre of HUST, the prototype pictures are shown in Fig. 23.

6 Conclusions

This paper proposes a new topology optimization method for the optimal design of distribute compliant mechanisms. The design problem is expressed by using an alternative multi-criteria objective function considering the strain energy of structures and mechanical efficiency of mechanisms, simultaneously, together with three external design constraints. It can be found that it is reasonable to explore design requirements of compliant mechanisms by using the continuum topological op-

Table 1: Eigenvalues of the mechanism

	First	Second	Third	Fourth	Fifth	Sixth	Seventh
Without eigenvalue constraint (HZ)	5.05E+01	5.94E+01	5.99E+01	6.12E+01	7.20E+01	7.84E+01	8.34E+01
With eigenvalue constraint (HZ)	5.87E+01	1.04E+02	1.58E+02	1.74E+02	2.19E+02	2.49E+02	3.09E+02

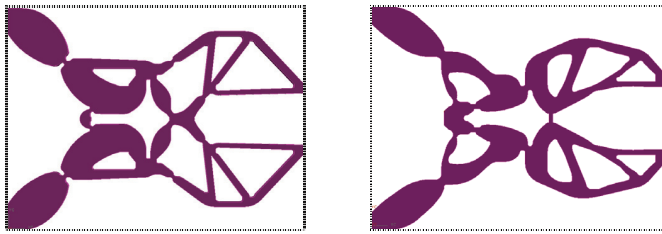
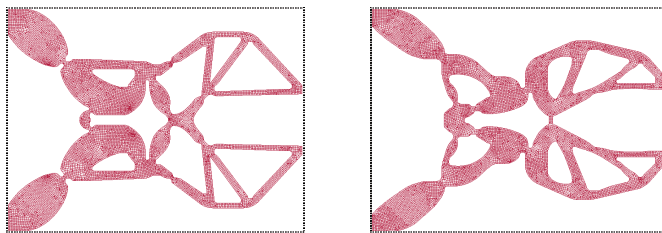


Figure 18: Reconstruction solid model



(a) With 8744 elements (b) With 9072 elements

Figure 19: Finite element model

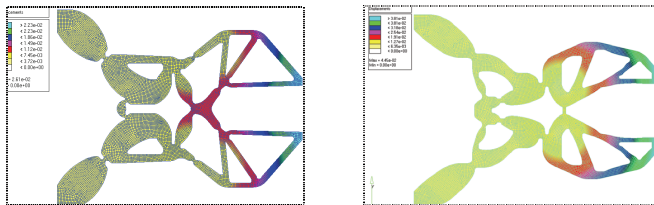


Figure 20: Displacement fields

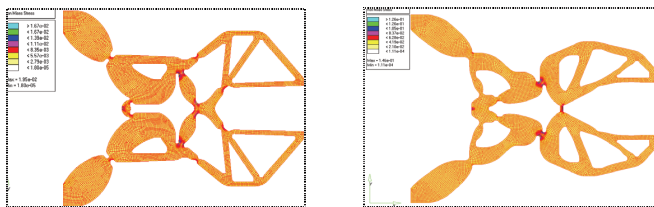


Figure 21: Von Mises stress fields

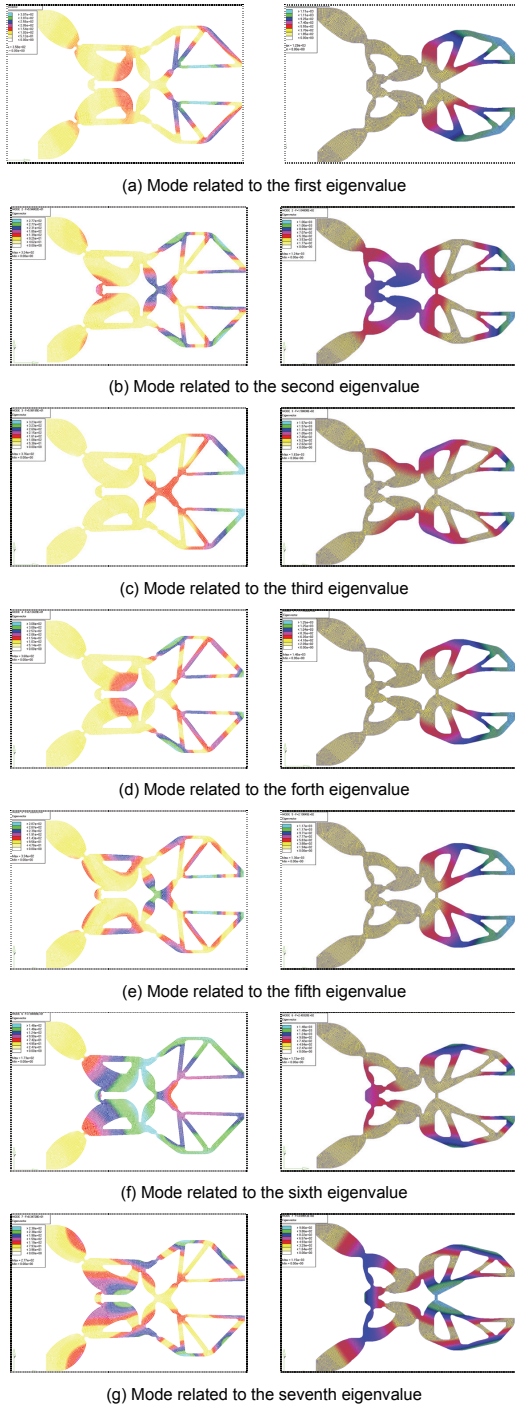


Figure 22: Left side: without eigenvalue constraint, Right side: with eigenvalue constraint

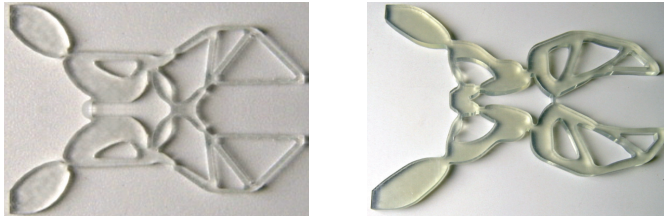


Figure 23: Prototype design

timization approach. The advantages of the proposed methodologies have been demonstrated via numerical applications, and the optimal mechanisms developed by adopting the new formulation can meet the demands of both maximizing the work ratio of mechanism designs and minimizing of the structural compliance. The mixed mathematical programming approximation is desirable for solving the topology optimization problem of compliant mechanisms. It is straightforward to extend the proposed method to more advanced structural optimization problems [e.g. Li, Li, Sun, Luo and Zhang (2007); Luo and Tong (2008); Kang and Luo (2009)].

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