 CMC, vol.28, no.3, pp.261-280, 2012

A Local Adaptive Differential Quadrature Method for Multi-Dimensional Inverse Scattering Problem of Wave Propagation

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Abstract: In this paper, we use the Local adaptive differential quadrature method (La-DQM) to solve multi-dimensional inverse scattering problem (ISP) of wave propagation. The La-DQM uses fictitious points to tackle the high-order differential equations with multi-boundary conditions and numerical results can be obtain directly in the calculation process. Six examples show the effectiveness and accuracy of the La-DQM in providing excellent estimates of unknown wave propagation from the given data. We think that the scheme is applicable to the ISP of wave propagation. Numerical results show that the La-DQM is powerful method for solving the inverse scattering problem of wave propagation.

Keywords: Local adaptive differential quadrature method (La-DQM), Differential quadrature method (DQM), Wave propagation, Inverse scattering problem, Multi-boundary conditions.

1 Introduction

An Inverse problems is important in many practical application fields such as materials science, it needs to find the material parameter from the known data. However, it is usually difficult to get the material parameter. The inverse problem is complicated for solving process because it is often unstable and produces an ill-pose matrix. Therefore, many researchers put forward a plan for improving various kinds of ill-posed inverse problems. Alifanov, Artyukhin and Rumyantsev (1995) employed the iterative regularization method to resolve the inverse heat transfer problems. Then, the inverse problems can consult from Gottfried (1990) proposed a book for Inverse problems in differential equations. There have been many numerical methods to tackle the identification of parameters in inverse problems. Various nu-

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merical methods have been developed for determining the material properties. For example, Bond, Punjani and Saffari (1988) use explicit finite difference method to solve Ultrasonic wave propagation and scattering. Tadi (1997) apply an explicit method for inverse wave scattering in solids. Later, Tadi (1998) uses an explicity method to solve elastic property. Tadi (1999) used an iterative algorithm for inverse wave scattering in 2-D elastic solids and obtained good results. Guzina, Fata and Bonnet (2003) use a regularized boundary integral equation method to study the inverse scattering problem for elastic half-space housing an internal void. Later, Telejko and Malinowski (2004) employed the finite element to tackle the thermal conductively identification. Thereafter, Char, Chang, Tai (2008) apply the DQM to resolve the inverse determination of thermal conductivity in one-dimensional slab and obtain good results. Liu (2010) propose a Lie-group adaptive method to solve inverse scattering problem through iterations and then the scheme is special character that it needn't any extra information from the wave equation. Recently, Wu and Chang (2011) use the DQM to solve multi-dimensional inverse heat conduction problem of heat source and obtain good results. Even thoug the noise is added to the exact temperature, the DQM is still robust against disturbance.

The differential quadrature method (DQM) was proposed by [Bellman and casti (1971); Bellman, Kashef and Casti (1972)], this method is widely used in solving various engineering and scientific problems, such as vibration mechanics [Choi, Wu and Chou (2000); Malekzadeha and Vosoughic (2009)], fluid mechanics [Shu, Chew and Richards (1995); Tai and Char (2010)]. The DQM has some oscillation problem when the domain uses a lot of grid points. Then, Wang, Zhao and Wei (2003) proposed to improve the DQM by using fictitious points. The La-DQM is mainly for solving high-order differential equations that arise in an eigenvalue problem and a boundary value problem. Recently, Char and Tai (2009) use the La-DQM to solve the effects of viscous dissipation on slip-flow heat transfer in a micro annulus.

In this paper, we considered with an inverse problem for multi-dimensional wave equation and employed the La-DQM to solve multi-dimensional inverse scattering problem of wave propagation. We find that the scheme is applicable to the multi-dimensional inverse scattering problem of wave propagation and obtain accuracy of the results. We thought that the La-DQM has been successfully to solve the inverse wave scattering problem. The paper is summarized as follows. In section 2, we presented the multi-dimensional inverse wave scattering problems (IWSP). Then, we explain the La-DQM theory in Section 3, and use the La-DQM to discretize the governing equation. Section 4 shows six examples to estimate the unknown wave propagation item. Finally, we draw some important conclusions in Section 5.

2 Formulation of the wave scattering problems

First, we consider the one-dimensional inverse scattering problem (ISP) is respectively given by the following equations:

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial}{\partial x}(\alpha(x)\frac{\partial u(x,t)}{\partial x}) + h(x,t) \text{ in } \Omega,\ (x,t) \in \Omega := [0,\ell] \times [0,T]. \tag{1}$$

Second, we contemplate the two-dimensional IWSP:

$$\frac{\partial^2 u(x,y,t)}{\partial t^2} = \frac{\partial}{\partial x}(\alpha(x,y)\frac{\partial u(x,y,t)}{\partial x}) + \frac{\partial}{\partial y}(\alpha(x,y)\frac{\partial u(x,y,t)}{\partial y}) + h(x,y,t) \tag{2}$$

in Ω, $(x,y,t) \in \Omega := [0,a] \times [0,b] \times [0,T]$.
Third, the following three-dimensional IWSP is deliberated:

$$\begin{aligned}\frac{\partial^2 u(x,y,z,t)}{\partial t^2} = \frac{\partial}{\partial x}(\alpha(x,y,z)\frac{\partial u(x,y,z,t)}{\partial x}) + \frac{\partial}{\partial y}(\alpha(x,y,z)\frac{\partial u(x,y,z,t)}{\partial y})\\ + \frac{\partial}{\partial z}(\alpha(x,y,z)\frac{\partial u(x,y,z,t)}{\partial z}) + h(x,y,z,t)\end{aligned} \tag{3}$$

in Ω, $(x,y,z,t) \in \Omega := [0,a] \times [0,b] \times [0,c] \times [0,T]$.

$$u = u_B \text{ on } \Gamma_{\mathrm{B}}, \tag{4}$$

$$u = u_i \text{ on } \Gamma_{\mathrm{i}}, \tag{5}$$

where $h(x,t)$ and $u(x,t)$are given functions, and $\alpha(x)$ is to be determined. We take a bounded domain D in R^j, j = 1, 2, 3 and a spacetime domain $\Omega = D \times (0,t)$ in R^{j+1} for a time $t > 0$, and write two surfaces $\Gamma_B = \partial D \times [0,t]$ and $\Gamma_i = \partial D \times \{t\}$ of the boundary $\partial\Omega$. While Eqs. (1)-(5) constitute a j-dimensional HCP for the given boundary data u_B: $\Gamma_B \mapsto R$ and the initial data u_i: $\Gamma_{\mathrm{i}} \mapsto R$.

3 Differential quadrature method

Pondering a one-dimensional function $f(x)$ on the area $a \le x \le b$. To approximate the derivate of a smooth function at a discrete point x_i in the domain, the DQM employs the weighted linear sum of all function values at all discrete points in the x direction. Then, the mth-order derivatives $f(x)$ with respect to x_i at point i can be formulated as

$$\frac{d^m f(x_i)}{dx^m} = \sum_{i=1}^{N} C_{i,j}^m f(x_j), \quad i = 1,...,N, \tag{6}$$

where $f(x_j)$ are the function values at the jth sampling point x_j, N is the number of discrete points, and $C_{i,j}^{m}$ are the unknown weighting coefficients of the mth order derivative at discrete point x_i, in which $m \leq N-1$.

Shu and Richards (1995) provided a convenient and recurrent formula for determining the following these derivative weighting coefficients:

$$C_{i,j}^{1} = \frac{M(x_i)}{(x_i - x_j)\cdot M(x_j)}, \text{ for } i \neq j, \text{ and } i,j = 1,....,N, \tag{7}$$

$$C_{i,j}^{m} = m\cdot\left[C_{i,j}^{m-1}\cdot C_{i,j}^{1} - \frac{C_{i,j}^{m-1}}{(x_i - x_j)}\right], \text{ for } 2 \leq m \leq N-1,\ i \neq j, \text{ and } i,j = 1,....,N, \tag{8}$$

$$C_{i,j}^{m} = -\sum_{\substack{j=1 \\ i\neq j}}^{N} C_{i,j}^{m}, \text{ for } 1 \leq m \leq N-1 \text{ and } i = 1,....N, \tag{9}$$

where

$$M(x_i) = \prod_{j=1, i\neq j}^{N} (x_i - x_j). \tag{10}$$

Note that in accordance with the principle of the DQM, the locations of the sampling grid point x_i can be arbitrarily determined. The La-DQM of conception is as follows:

The weighting coefficients of the first derivatives is given by

$$C_{i,j}^{[1]} = g_{i,j}^{[1]}(x_i) \text{ for } j = -L_i,....,R_i;\ j \neq 0. \tag{11}$$

and for $j = 0$

$$C_{i,0}^{[1]} = -\sum_{j=-L_i, j\neq 0}^{R_i} C_{i,j}^{[1]} \tag{12}$$

where

$$g_{i,j}(x) = \prod_{k=i-L_i, k\neq i+j}^{i+R_i} \frac{x - x_k}{x_{i+j} - x_k} \text{ for } j = -L_i,....,R_i \tag{13}$$

We can computer the weighting coefficients of the higher-order derivatives by a recurrence formula:

$$C_{i,j}^{[m]} = m(C_{i,j}^{[1]} C_{i,j}^{(m-1)} - \frac{C_{i,j}^{(m-1)}}{x_i - x_{i+j}}),\ j = -L_i, ..., R_i;\ j \neq 0 \quad (14)$$

and for j = 0

$$C_{i,0}^{[m]} = -\sum_{j=-L_i, j\neq 0}^{R_i} C_{i,j}^{[m]} \quad (15)$$

4 Numerical examples

We employ the DQM to solve multi-dimensional ISP with wave propagation through six examples. We apply the quadrature rule to get a vector matrix form:

$$\{A\}\{\alpha\} = \{B\}. \quad (16)$$

4.1 Example 1

Let us ponder another one-dimensional ISP [Liu (2010)]:

$$\alpha(x) = 1 + \sin(3\pi x) \quad 0 < x < \ell, \quad (17)$$

$$h(x,t) = e^{(x+t)} - 3\cos(3\pi x)\pi e^{(x+t)} - \{1 + \sin(3\pi x)\} e^{(x+t)}, \quad (18)$$

with the boundary conditions

$$\alpha(0) = 1,\ \alpha(1) = 1, \quad (19)$$

The exact solution is given by

$$u(x,t) = e^{(x+t)}\ 0 < x < \ell,\ 0 < t < T. \quad (20)$$

we use chain rule for the equation (1) and obtain as follow

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial \alpha(x)}{\partial x}\frac{\partial u(x,t)}{\partial x} + \alpha(x)\frac{\partial^2 u(x,t)}{\partial x^2} + h(x,t) \quad (21)$$

Using the quadrature rule for the equation (1) is to obtain the following algebraic equations:

$$\sum_{m=1}^{M} D_{j,m}^{[2]} u(x_i,t_m) = \sum_{n=1}^{N} C_{i,n}^{[1]} \alpha(x_i) \sum_{n=1}^{N} C_{i,n}^{[1]} u(x_n,t_m) + \alpha(x_i) \sum_{n=1}^{N} C_{i,n}^{[2]} u(x_n,t_m) + h(x_i,t_j) \quad (22)$$

Under the following parameters: $\ell = 1$, $N = 31$, $\Delta x = 1/30$, $T = 1$, and $\Delta t = 1/20$. Fig. 1 displays the numerical and the exact solution. The maximal absolute error is about 6.52×10^{-7} and the RMSE is about 1.96×10^{-7}. The present results are also better than that calculated by Liu (2010), of which the maximum error is about 5.02×10^{-2}. To the authors' best knowledge, there has been no open literature that the numerical methods can calculate this inverse problem well as the La-DQM.

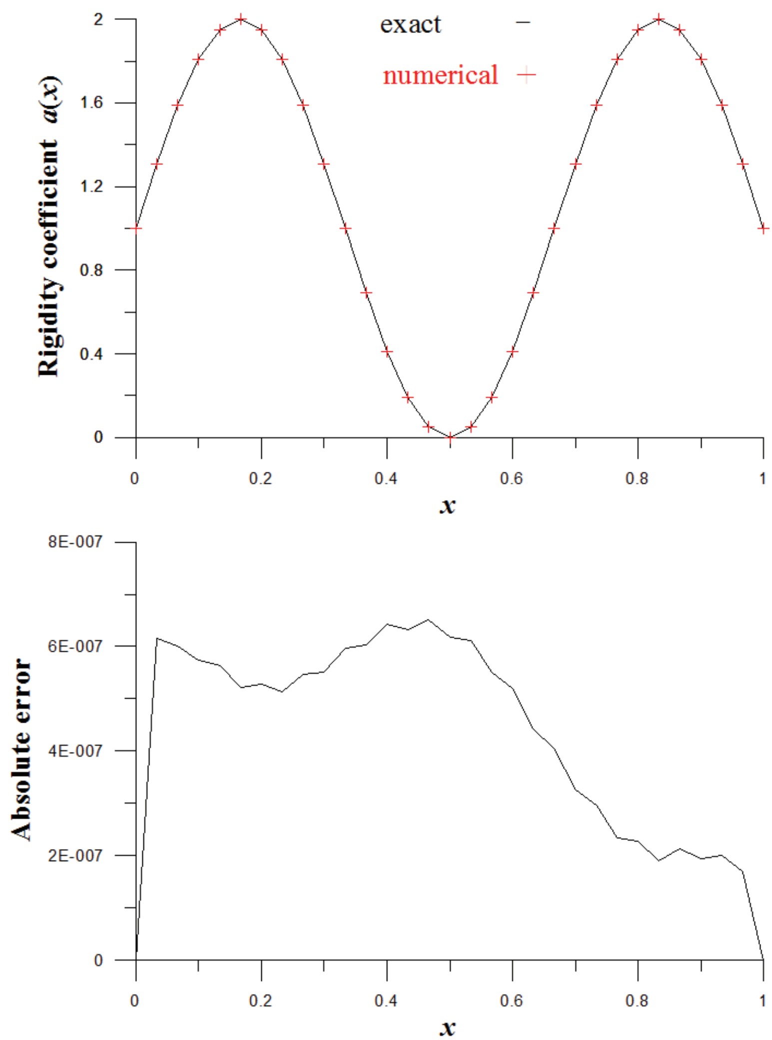


Figure 1: Comparisons of the exact solutions and numerical solutions for Example 1 and the corresponding numerical errors.

4.2 Example 2

We considered the one-dimensional ISP with one hump function [Liu (2010)]:

$$\alpha(x) = 2 + e^{\{-\frac{(x-0.5)^2}{0.05}\}}\ 0 < x < \ell, \tag{23}$$

$$h(x,t) = (x-3)^2 e^{-t} - 2(-40x+20)\ \mathrm{e}^{-20(x-0.5)^2}(x-3)e^{-t} - 2(2+e^{-20(x-0.5)^2})e^{-t}, \tag{24}$$

with the boundary conditions

$$\alpha(0) = 2 + e^{-5},\ \alpha(\ell) = 2 + e^{\{-\frac{(\ell-0.5)^2}{0.05}\}}\ , \tag{25}$$

The exact solution is given by

$$u(x,t) = (x-3)^2 e^{-t}\ 0 < x < \ell,\ 0 < t < T. \tag{26}$$

Under the following parameters: ℓ = 1, N = 31, Δx = 1/30, T = 1, and Δt = 1/20. Fig. 2 displays the numerical and the exact solution. The maximal absolute error is about 2.22×10^{-6} and the RMSE is about 5.10×10^{-7}. The present results are also better than that calculated by Liu (2010), of which the maximum error is about 5.89×10^{-2}. We think that the La-DQM is accuracy and effectiveness of numerical method by this example.

4.3 Example 3

We considered the one-dimensional ISP with two hump functions [Tadi (1998); Liu (2010)]:

$$\alpha(x) = 1 + e^{\{-\frac{(x-0.26)^2}{0.02}\}} + e^{\{-\frac{(x-0.74)^2}{0.02}\}}\ 0 < x < \ell, \tag{27}$$

$$\begin{aligned} h(x,t) &= (x-3)^2 e^{-t} \\ &\quad - 2\{(-100x+26)e^{-50(x-0.26)^2} + (-100x+74)e^{-50(x-0.74)^2}\}(x-3)e^{-t} \\ &\quad - 2(1+e^{-50(x-0.26)^2} + e^{-50(x-0.74)^2})e^{-t}, \end{aligned} \tag{28}$$

with the boundary conditions

$$\alpha(0) = 1 + e^{\{-\frac{(-0.26)^2}{0.02}\}} + e^{\{-\frac{(-0.74)^2}{0.02}\}}\ ,\ \alpha(\ell) = 1 + e^{\{-\frac{(\ell-0.26)^2}{0.02}\}} + e^{\{-\frac{(\ell-0.74)^2}{0.02}\}}\ , \tag{29}$$

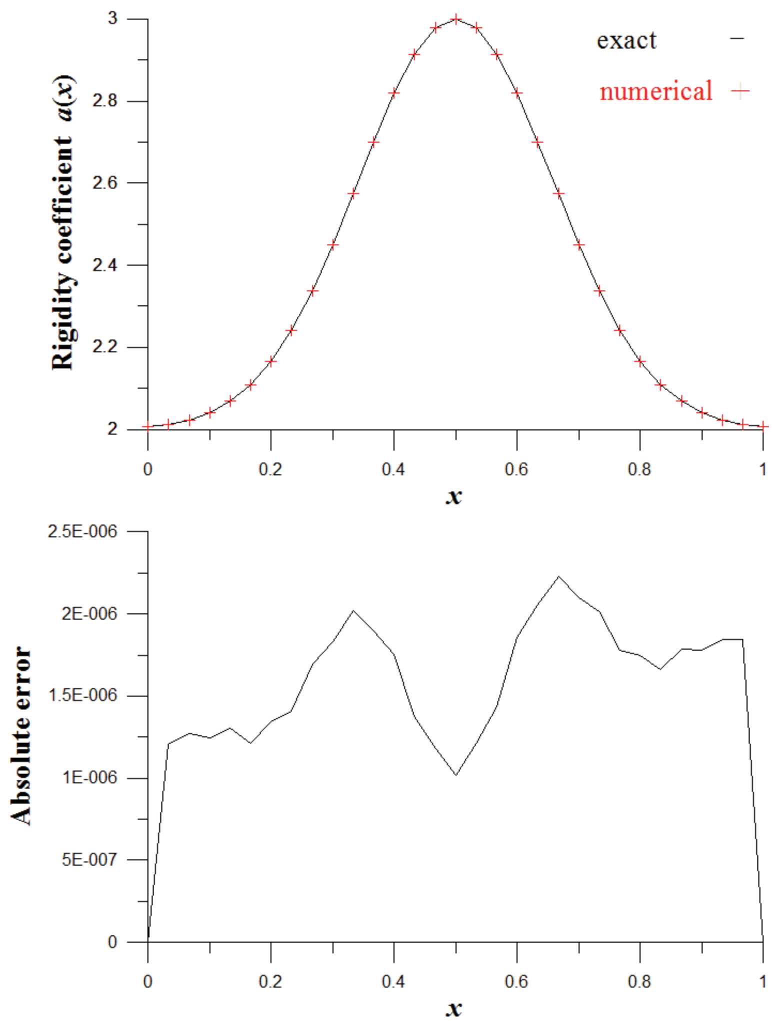


Figure 2: Comparisons of the exact solutions and numerical solutions for Example 2 and the corresponding numerical errors.

The exact solution is given by

$$u(x,t) = (x-3)^2 e^{-t} \; 0 < x < \ell, \; 0 < t < T. \tag{30}$$

Under the following parameters: $\ell = 1$, $N = 31$, $\Delta x = 1/30$, $T = 1$, and $\Delta t = 1/20$. Fig. 3 displays the numerical and the exact solution. The maximal absolute error is about 2.77×10^{-4} and the RMSE is about 6.28×10^{-5}. The present results are also better than that calculated by [Tadi (1998); Liu (2010)]. We get good results even though the problem has two hump functions.

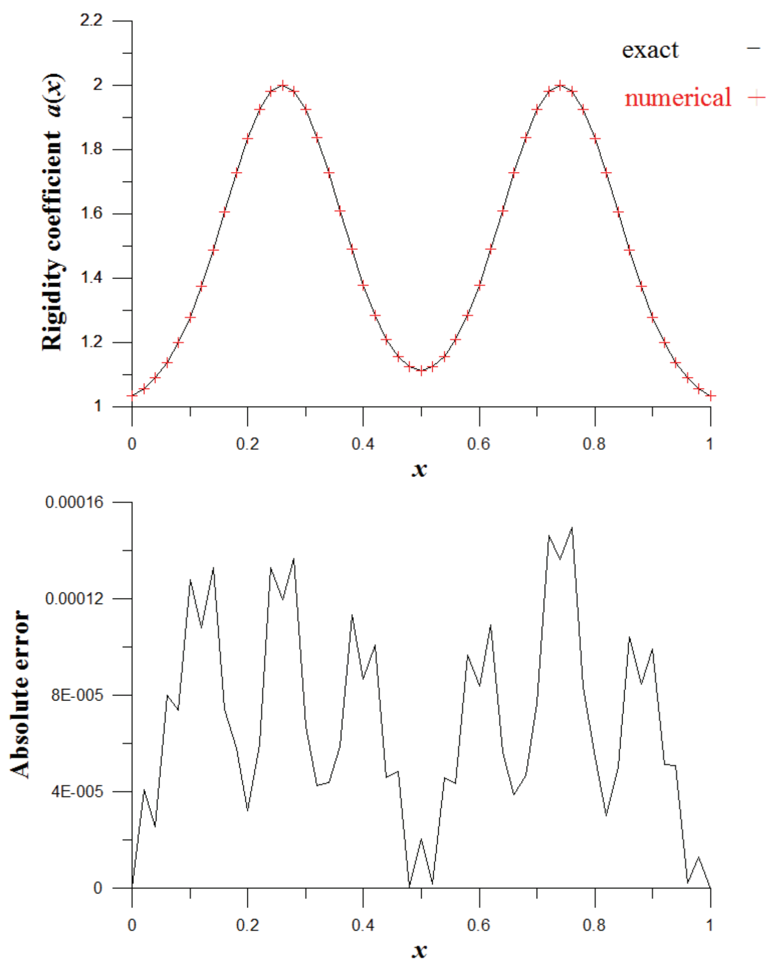


Figure 3: Comparisons of the exact solutions and numerical solutions for Example 3 and the corresponding numerical errors.

4.4 Example 4

The following two-dimensional ISP is pondered:

$$\alpha(x) = 1 + y\cos(\pi x)\ 0 < x < a,\ 0 < \mathrm{y} < \mathrm{b}, \tag{31}$$

$$\begin{aligned} h(x,t) = {} & (x-3)^2(y-3)^2e^{-t} \\ & + 2y\sin(\pi x)\pi(x-3)(y-3)^2e^{-t} - 2(1+y\cos(\pi x))(y-3)^2e^{-t} \\ & - 2\cos(\pi x)(x-3)^2(y-3)e^{-t} - 2(1+y\cos(\pi x))(x-3)^2e^{-t}, \end{aligned} \tag{32}$$

with the boundary conditions

$$\begin{aligned} & \alpha(0,y) = 1 + y\ ,\ \alpha(a,y) = 1 + y\cos(\pi a), \\ & \alpha(x,0) = 1\ ,\ \alpha(x,b) = 1 + b\cos(\pi x) \end{aligned} \tag{33}$$

The exact solution is given by

$$u(x,t) = (x-3)^2(y-3)^2e^{-t}\ 0 < x < a,\ 0 < \mathrm{y} < b, 0 < t < T. \tag{34}$$

We use chain rule for the equation (2) and obtain as follow:

$$\begin{aligned} \frac{\partial^2 u(x,y,t)}{\partial t^2} = {} & \frac{\partial \alpha(x,y)}{\partial x}\frac{\partial u(x,y,t)}{\partial x} + \alpha(x,y)\frac{\partial^2 u(x,y,t)}{\partial x^2} \\ & + \frac{\partial \alpha(x,y)}{\partial y}\frac{\partial u(x,y,t)}{\partial y} + \alpha(x,y)\frac{\partial^2 u(x,y,t)}{\partial y^2} + h(x,y,t) \end{aligned} \tag{35}$$

Using the quadrature rule for the equation (3) is to obtain the following algebraic equations:

$$\begin{aligned} \sum_{p=1}^{P} E_{k,p}^{[2]} u(x_i,y_j,t_p) = {} & \sum_{n=1}^{N} C_{i,n}^{[1]}\alpha(x_n,y_j)\sum_{n=1}^{N} C_{i,n}^{[1]} u(x_n,y_j,t_k) + \alpha(x_i,y_j)\sum_{n=1}^{N} C_{i,n}^{[2]} u(x_n,y_j,t_k) \\ & + \sum_{m=1}^{M} D_{j,m}^{[1]}\alpha(x_i,y_m)\sum_{m=1}^{M} D_{j,m}^{[1]} u(x_i,y_m,t_k) + \alpha(x_i,y_j)\sum_{m=1}^{M} D_{j,m}^{[2]} u(x_i,y_m,t_k) \end{aligned} \tag{36}$$

Under the following parameters: a = b = 1, $N = M = 21$, $\Delta x = \Delta y = 1/20$, $T = 1$, and $\Delta t = 1/20$. Fig. 5 displays the numerical and the exact solution. The maximal absolute error is about 1.17×10^{-8}. In addition, at the point $y = 0.8$, the error is plotted with respect to x in Fig. 4(a), and at the point $x = 0.2$, the error is plotted with respect to y in Fig. 4(b). The exact solutions and numerical solutions are drawn in Figs. 5(a)-(b) sequentially.

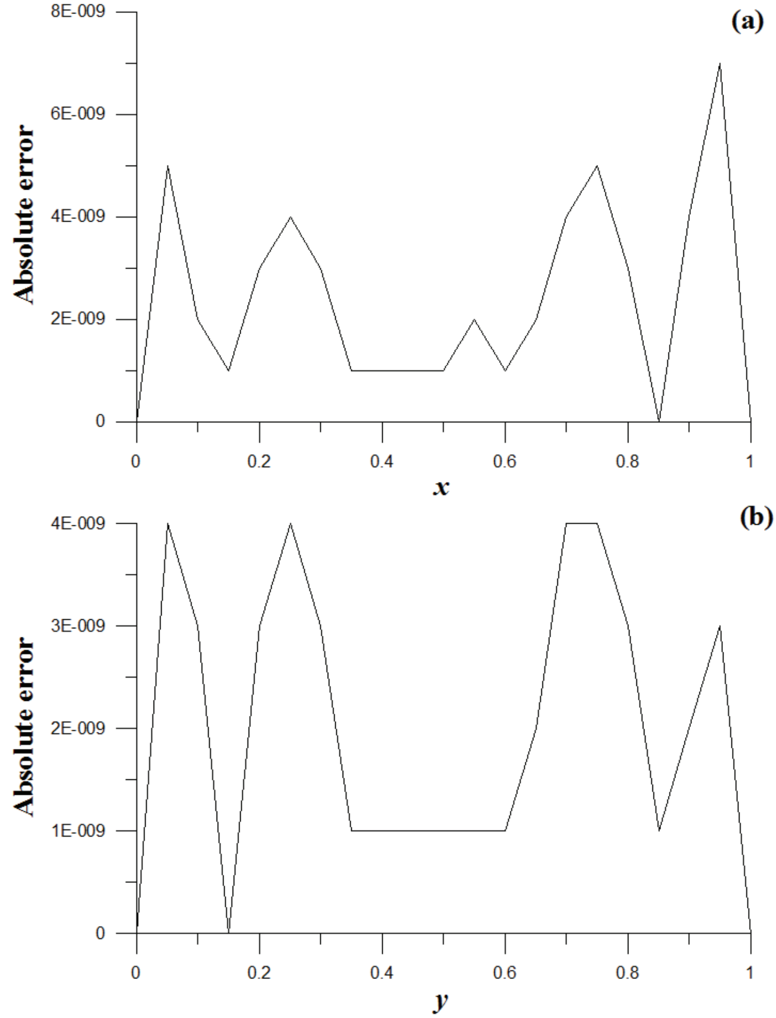


Figure 4: The numerical errors of La-DQM solutions for Example 4 are plotted in (a) with respect to x at fixed $y = 0.8$, and in (b) with respect to y at fixed $x = 0.2$.

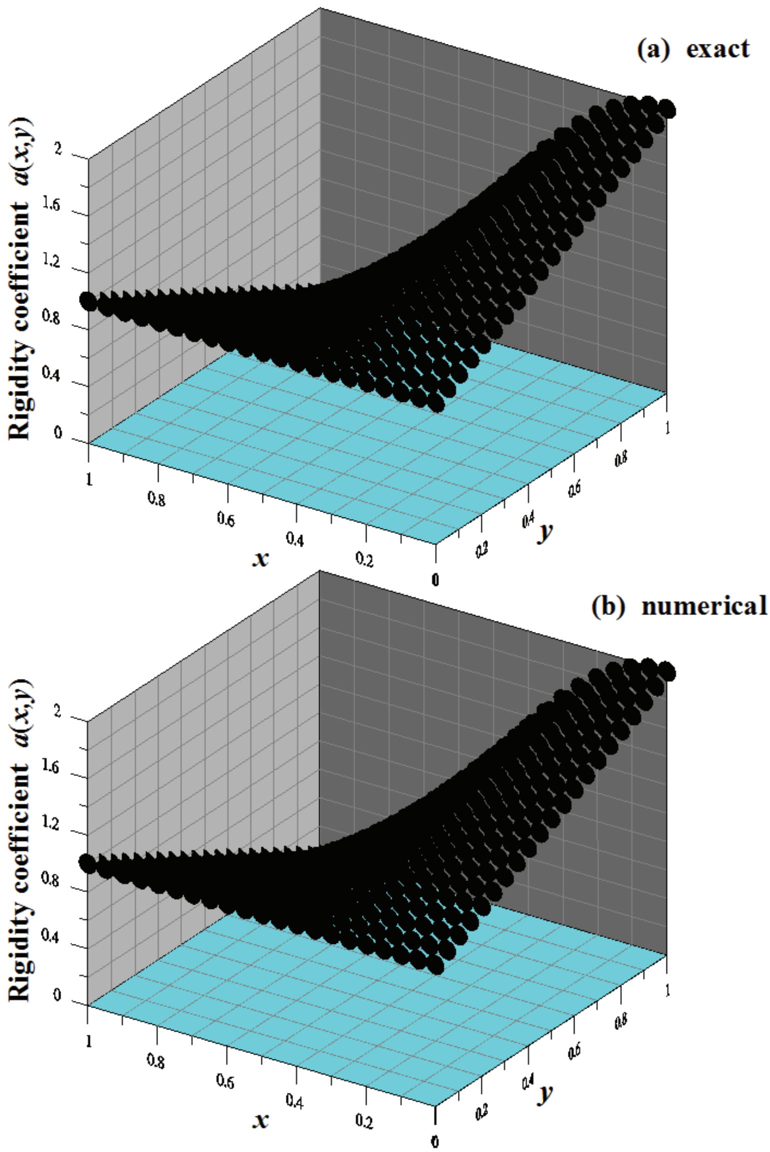


Figure 5: The exact solution for Example 4 of two-dimensional inverse problem is shown in (a), and in (b) the La-DQM solution result.

4.5 Example 5

Let us further consider the two-dimensional IWSP:

$$\alpha(x) = x + y \; 0 < x < a, 0 < \mathrm{y} < \mathrm{b}, \tag{37}$$

$$h(x,t) = (x-3)^2(y-3)^2e^{-t} - 2(x-3)(y-3)^2e^{-t} - 2(x+y)(y-3)^2e^{-t} \\ - 2(x-3)^2(y-3)e^{-t} - 2(x+y)(x-3)^2e^{-t}, \tag{38}$$

with the boundary conditions

$$\alpha(0,y) = y \;,\; \alpha(a,y) = a + y,$$

$$\alpha(x,0) = x \;,\; \alpha(x,b) = x + b \tag{39}$$

The exact solution is given by

$$u(x,t) = (x-3)^2(y-3)^2e^{-t} \; 0 < x < a, \; 0 < \mathrm{y} < b, 0 < t < T. \tag{40}$$

Under the following parameters: a = b = 1, $N = M = 21$, $\Delta x = \Delta y = 1/20$, $T = 1$, and $\Delta t = 1/20$. Fig. 7 displays the numerical and the exact solution, with the maximal absolute error is about 3.6×10^{-7}. In addition, at the point $y = 0.8$, the error is plotted with respect to x in Fig. 6(a), and at the point $x = 0.2$, the error is plotted with respect to y in Fig. 6(b). For this difficult problem, the La-DQM proposed here is still a good result.

4.6 Example 6

We deliberate a three-dimensional IWSP:

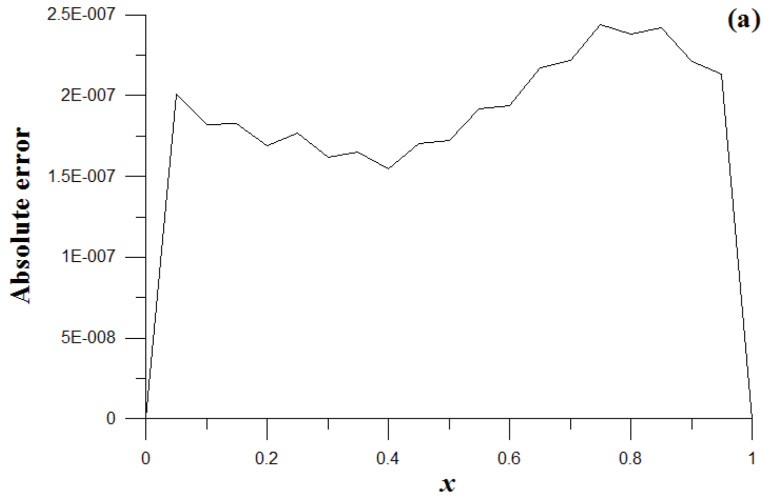
$$\alpha(x,y,z) = x + y + z \; 0 < x < a, \; 0 < \mathrm{y} < b, 0 < z < c, 0 < t < T \tag{41}$$

$$h(x,t) = (x^2 + y^2 + z^2)e^{-t} - 2xe^{-t} - 6(x+y+z)e^{-t} - 2ye^{-t} - 2ze^{-t} \tag{42}$$

with the boundary conditions

$$\alpha(0,y,z) = y + z \;,\; \alpha(a,y,z) = a + y + z,$$

$$\alpha(x,0,z) = x + z \;,\; \alpha(x,b,z) = x + b + z,$$



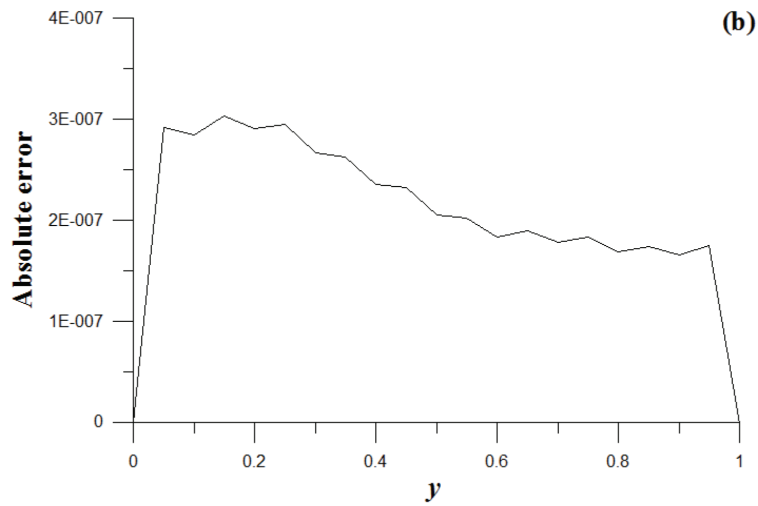


Figure 6: The numerical errors of La-DQM solutions for Example 5 are plotted in (a) with respect to x at fixed $y = 0.8$, and in (b) with respect to y at fixed $x = 0.2$.

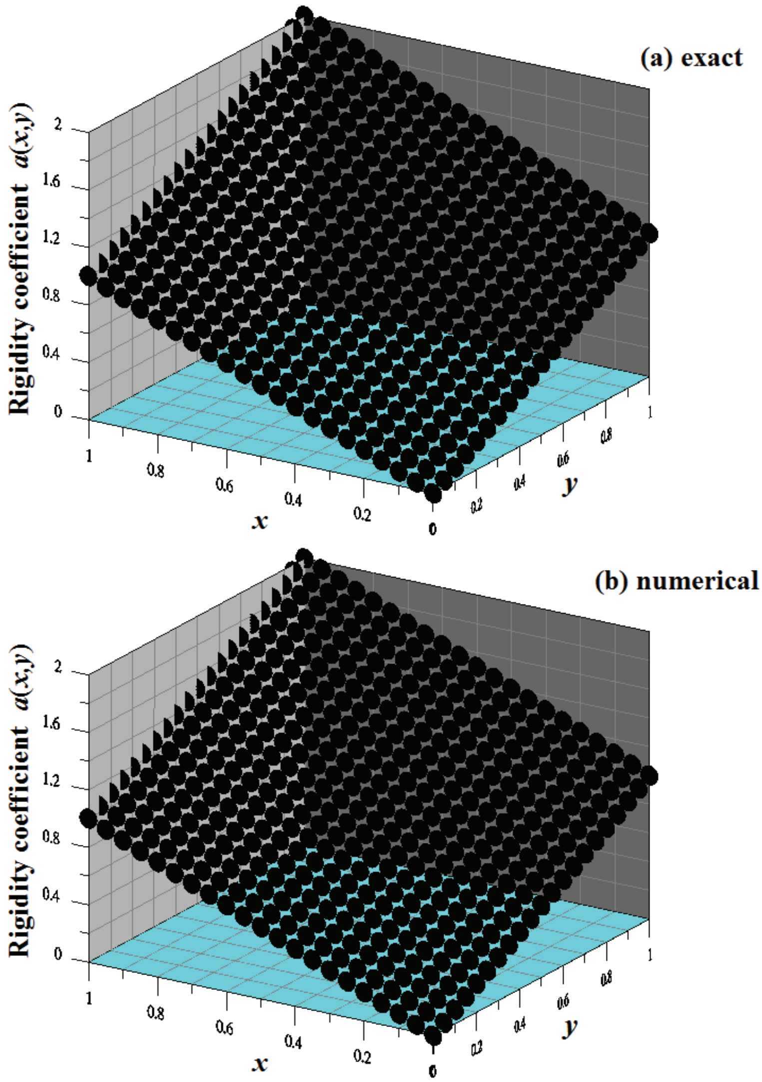


Figure 7: The exact solution for Example 5 of two-dimensional inverse problem is shown in (a), and in (b) the La-DQM solution result.

$$\alpha(x,y,0) = x+y,\ \alpha(x,y,c) = x+y+c, \tag{43}$$

The exact solution is given by

$$u(x,y,z,t) = (x^2+y^2+z^2)e^{-t}, \tag{44}$$

We use chain rule for the equation (3) and obtain as follow:

$$\begin{aligned}\frac{\partial^2 u(x,y,z,t)}{\partial t^2} &= \frac{\partial \alpha(x,y,z)}{\partial x}\frac{\partial u(x,y,z,t)}{\partial x} + \alpha(x,y,z)\frac{\partial^2 u(x,y,z,t)}{\partial x^2} \\ &+ \frac{\partial \alpha(x,y,z)}{\partial y}\frac{\partial u(x,y,z,t)}{\partial y} + \alpha(x,y,z)\frac{\partial^2 u(x,y,z,t)}{\partial y^2} \\ &+ \frac{\partial \alpha(x,y,z)}{\partial z}\frac{\partial u(x,y,z,t)}{\partial z} + \alpha(x,y,z)\frac{\partial^2 u(x,y,z,t)}{\partial z^2} + h(x,y,z,t),\end{aligned} \tag{45}$$

Using the quadrature rule for the equation (3) is to obtain the following algebraic equations:

$$\begin{aligned}&\sum_{q=1}^{Q} F_{s,q}^{[2]} u(x_i,y_j,z_k,t_q) = \\ &\sum_{n=1}^{N} C_{i,n}^{[1]}\alpha(x_n,y_j,z_k)\sum_{n=1}^{N} C_{i,n}^{[1]}u(x_n,y_j,z_k,t_s) + \alpha(x_i,y_j,z_k)\sum_{n=1}^{N} C_{i,n}^{[2]}u(x_n,y_j,z_k,t_s) \\ &+ \sum_{m=1}^{M} D_{j,m}^{[1]}\alpha(x_i,y_m,z_k)\sum_{m=1}^{M} D_{j,m}^{[1]}u(x_i,y_m,z_k,t_s) + \alpha(x_i,y_j,z_k)\sum_{m=1}^{M} D_{j,m}^{[2]}u(x_i,y_m,z_k,t_s) \\ &+ \sum_{p=1}^{P} E_{k,p}^{[1]}\alpha(x_i,y_j,z_p)\sum_{p=1}^{P} E_{k,p}^{[1]}u(x_i,y_j,z_p,t_s) + \alpha(x_i,y_j,z_k)\sum_{p=1}^{P} E_{k,p}^{[2]}u(x_i,y_j,z_p,t_s).\end{aligned} \tag{46}$$

Under the following parameters: $a = b = c = 1$, $N = M = H = 11$, $\Delta x = \Delta y = \Delta z = 1/10$, $T = 1$, and $\Delta t = 1/10$. Fig. 9 exhibits the numerical and exact solution. In addition, at fixed points $y = 0.8$ and $z = 0.5$, the error is plotted with respect to x in Fig. 8(a), and at fixed points $x = 0.3$ and $z = 0.5$, the error is plotted with respect to y in Fig. 8(b), and at fixed points $x = 0.3$ and $y = 0.8$, the error is plotted with respect to z in Fig. 8(c). For this high dimensional problem, the La-DQM proposed here is still good with a maximum error 1.13×10^{-7}. To the authors' best knowledge, there has been no open report that the numerical methods can calculate this inverse problem well as the La-DQM.

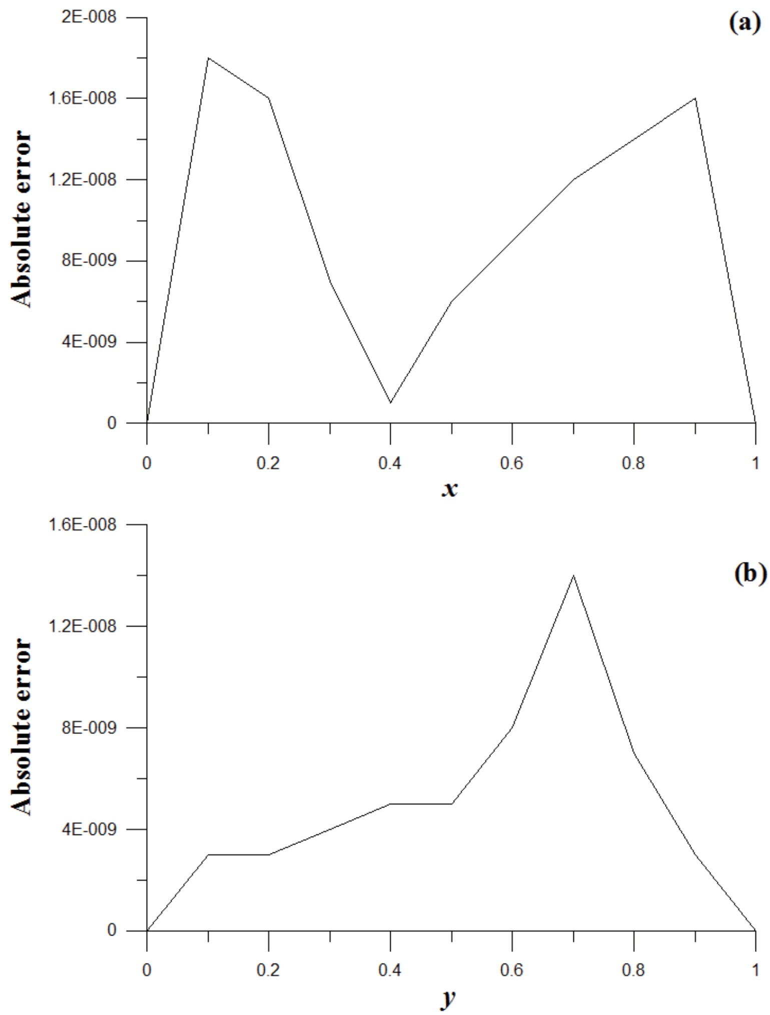


Figure 8: The numerical errors of La-DQM solutions for Example 6 are plotted in (a) with respect to x at fixed $y = 0.8$ and $z = 0.5$, (b) with respect to y at fixed $x = 0.3$ and $z = 0.5$, and (c) with respect to z at fixed $x = 0.3$ and $y = 0.8$.

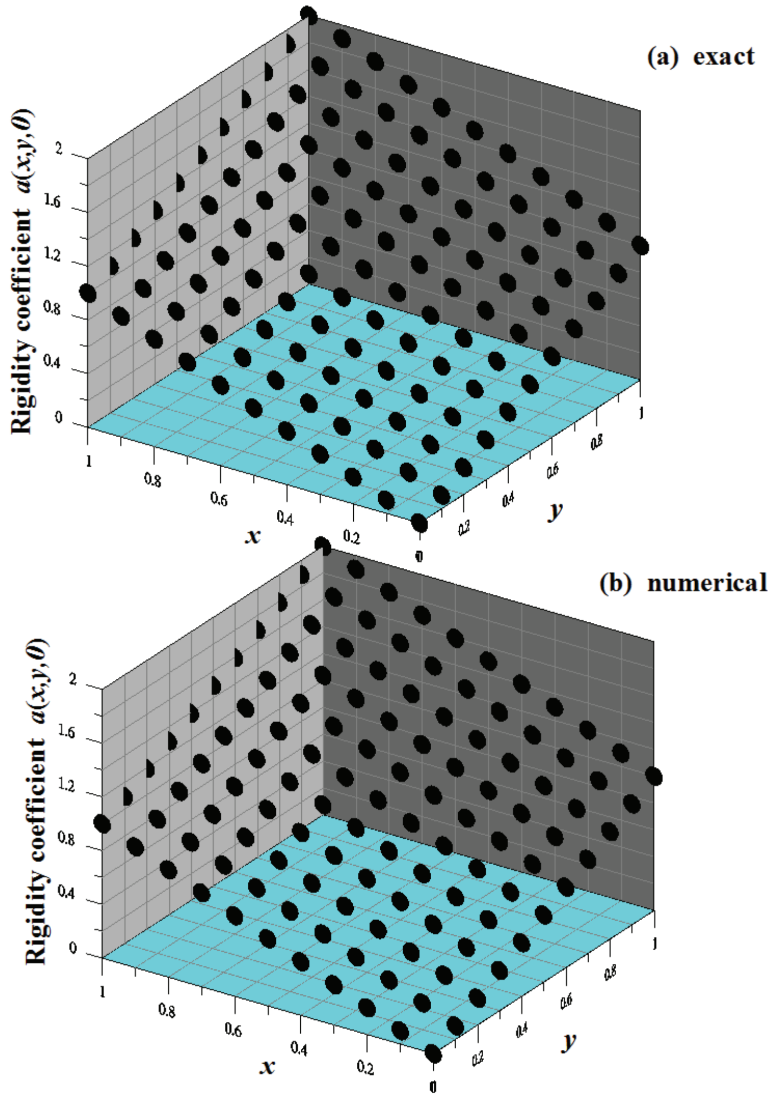


Figure 9: The exact solution for Example 6 of three-dimensional inverse problem is shown in (a), and in (b) the La-DQM solution result.

5 Conclusions

In this paper we estimate inverse scattering problem of wave Propagation by employing the La-DQM. The La-DQM is quite simple and straightforward that is no literature process, no regularization process and can determine directly to the inverse problem. While we work through these examples, we think that the La-DQM is powerful numerical method for solving the multi-dimensional inverse scattering problem. From the present study, we can estimate the unknown parameter of wave propagation that is very well with high order accuracy. The numerical errors of our scheme are in the order of $O(10^{-4})$£$O(10^{-8})$. Therefore, it can be concluded that the La-DQM is stable, accurate, and effective.

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