# Modal Characteristics of Five-Layered Slab Waveguides with Double-Clad Metamaterials

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**Abstract:** The guided modes are investigated in a five-layered slab waveguide with double claddings of metamaterials using a new graphical method. The dispersion equations in the symmetric and asymmetric five-layered slab waveguide are derived from the presented graphical method, and corresponding field distributions are plotted for the oscillating guided and surface guided modes. The energy flux distribution along the axis is plotted for the surface TE<sub>1</sub> mode.

Keywords: metamaterials, double clad, fiber layer, slab, mode.

## 1 Introduction

Recently, the metamaterials (MTMs) with both negative permittivity ( $\varepsilon$ ) and permeability  $(\mu)$ , referred to as left-handed materials (LHMs), have received considerable attention in the scientific and engineering communities [Veselago (1968); Smith, Padilla, Vier, Nemat-Nasser, and Schultz (2000)]. Many abnormal properties such as negative refraction, reverse Cerenkov radiation, and reverse Goos-Hanchen shift [Berman (2002)] have been demonstrated and investigated widely. Slab waveguides using LHMs have potentials in device applications and deserve to investigate because of some different properties than conventional right-handed materials (RHMs) waveguides, such as the absence of the fundamental mode, double degeneracy of modes, and slow propagation sign-varying energy flux [Dong, Li, and Yang (2010)]. Due to such unique properties, LHMs waveguides have been widely applied to perfect superlenses [Liu, Fang, Yen, and Zhang (2003)], spatial filters [Schurig and Smith (2003)], beam shaping [Shadrivov, Zharov, and Kivshar (2003)], and sensors [Qing and Chen (2004); Taya, Shabat, and Khalil (2009)]. In LHM waveguides, there exists a sort of unique electromagnetic waves termed as surface waves [Wu, Grzegorczyk, Zhang, and Kong (2003)] when the wave number becomes purely imaginary.

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The surface waves existing near the boundary of a LHMs slab decay exponentially from the interface to outer medium. Due to the different signs of  $\varepsilon$  and  $\mu$  across the interface, the asymmetric slab waveguides with LHMs and RHMs can support surface modes not by the index difference. In [Qing and Chen (2004)], it was verified that the optical waveguide using LHMs as a cladding layer can enhance the evanescent fields of slab waveguides.

Since optical waveguide sensors using evanescent waves have been widely used for humidity sensing, bio sensing, chemical sensing, and physical sensing, LHMsclad waveguide sensor offers a highly effective solution to the enhancement of the evanescent field. In [Taya, Shabat, and Khalil (2009)], the sensitivity of a slab waveguide sensor was enhanced by using a single LHMs-clad. The increased sensitivity of the sensor was analyzed as a function of the thickness, the negative permittivity, and the negative permeability of the single LHMs-clad Hence, in this study, we present modal characteristics of a five-layered slab waveguide with double-clad LHMs, including the dispersion properties and field distribution of the guided and surface modes of the five-layered slab waveguide with double LHMs-clad. Double LHMs cladding layers can enhance the sensitivity of the RHMs-core slab waveguide sensors because the evanescent surface modes are generated on both end sides of LHMs layers. The four- and five-layered LHMs-core waveguide has been proposed and their properties of guided modes were analyzed [Lee and Lee (2012); He, Zhang, and Li (2008)]. However, modal characteristics of five-layered RHMscore waveguides with double-LHMs clad waveguide have not been studied, and we adopted a graphical method to determine the guided optical modes in the presented five-layered slab waveguide.

Figure 1 shows an infinite-y-axis slab waveguide with five layers, including a first semi-infinite external-medium layer, a LHMs-cladding layer with thickness of  $(d_1 - d_2)$ , a RHMs-guiding layer with a thickness of  $d_2$ , a LHMs-cladding layer with thickness of  $d_4$ , and a second semi-infinite external-medium layer.

### 2 Theory

The RHMs-guiding core layer with positive permeability  $\mu_3$  and permittivity  $\varepsilon_3$  is inserted between LHMs-clad layers with double negative  $\mu_c$  and  $\varepsilon_c$ , which are surrounded by air. For simplicity, our investigation was confined to only TE guided modes,  $E_y(x) \exp[-j(\beta z - \omega t)]$ , where  $\omega$  is the angular frequency of the field and  $\beta$  is the propagation constant in z-direction. By using one-dimensional Helmholtz wave equations,

$$\frac{\partial^2 E_y}{\partial x^2} + \left(\omega^2 \mu \varepsilon - \beta^2\right) E_y = 0, \tag{1}$$



Figure 1: Structure of the presented slab waveguide

 $E_{y}$  fields at each layer can be described as

$$E_{y}(x) = \begin{cases} Ae^{-\alpha_{1}(x-d_{1})}, & d_{1} \leq x \\ B\cosh\left[\alpha_{2}(x-d_{2}) + \phi_{2}\right], & d_{2} \leq x < d_{1} \\ C\cos\left(\kappa_{3}x\right) + D\sin\left(\kappa_{3}x\right), & 0 \leq x < d_{2} \\ E\cosh\left[\alpha_{4}(x+d_{4}) + \phi_{4}\right], & -d_{4} \leq x < 0 \\ Fe^{\alpha_{5}(x+d_{4})}, & x \leq 0 \end{cases}$$
(2)

where the transverse wave numbers of  $\alpha_1$ ,  $\alpha_2$ ,  $\kappa_3$ ,  $\alpha_4$ , and  $\alpha_5$  correspond to the 1<sup>st</sup>-, 2<sup>nd</sup>-, 3<sup>rd-,</sup> 4<sup>th</sup>-, and 5<sup>th</sup>-layer, respectively, and, $\alpha_1^2 = \beta^2 - k_0^2 \mu_1 \varepsilon_1$ ,  $\alpha_2^2 = \beta^2 - k_0^2 \mu_2 \varepsilon_2$ ,  $\kappa_3^2 = k_0^2 \mu_3 \varepsilon_3 - \beta^2$ ,  $\alpha_4^2 = \beta^2 - k_0^2 \mu_4 \varepsilon_4$ ,  $\alpha_5^2 = \beta^2 - k_0^2 \mu_5 \varepsilon_5$ .

Using the boundary conditions, the tangential component  $E_y$  is continuous at  $x = d_1$ ,  $d_2$ , 0, and  $-d_4$ , the coefficients of  $E_y$  are determined as follows:

$$A = \frac{1}{\cos \phi_2} \left[ C \cos \left(\kappa_3 d_2\right) + D \sin \left(\kappa_3 d_2\right) \right] \cosh \left[\alpha_2 \left(d_1 - d_2\right) + \phi_2\right]$$

$$B = \frac{1}{\cos \phi_2} \left[ C \cos \left(\kappa_3 d_2\right) + D \sin \left(\kappa_3 d_2\right) \right]$$

$$E = \frac{1}{\cosh \left[ \left(\alpha_4 d_4\right) + \phi_4 \right]} C$$

$$F = \frac{\cosh \left(\phi_4\right)}{\cosh \left[ \left(\alpha_4 d_4\right) + \phi_4 \right]} C$$
(3)

Using the boundary conditions, the tangential component  $H_z$  is continuous at x =

 $d_1$ ,  $d_2$ , 0, and  $-d_4$ , the values of  $\phi_2$ ,  $\phi_4$ , C, and D are determined as follows:

$$D = qC, \quad q = \frac{\mu_3}{\mu_4} \frac{\alpha_4}{\kappa_3} \tanh\left[(\alpha_4 d_4) + \phi_4\right], \quad p = \frac{\mu_3}{\mu_2} \frac{\alpha_2}{\kappa_3}$$
$$\tanh\left(\phi_4\right) = \frac{\mu_4}{\mu_5} \frac{\alpha_5}{\alpha_4}, \quad \tanh\left[\alpha_2\left(d_1 - d_2\right) + \phi_2\right] = -\frac{\mu_2}{\mu_1} \frac{\alpha_1}{\alpha_2}$$
(4)

When  $\kappa_3$  is real and  $k_0^2 \mu_3 \varepsilon_3 > \beta^2 > k_0^2 \mu_2 \varepsilon_2 > k_0^2 \mu_1 \varepsilon_1$ , oscillating guided modes can exist in the RHMs-core layer, and we can obtain the dispersion equation of guided modes from Eq. 4 as following:

$$\kappa_3 d_2 = m\pi + \tan^{-1} q + \tanh^{-1} \left[ -p \tanh(\phi_2) \right].$$
(5)

Dispersion Eq. 5 is a transcendental equation and cannot be solved analytically, therefore, we use a graphical method to determine  $\kappa_3$  for the guided modes.

The left- and right-hand sides of the characteristic Eq. 5 of a symmetric five-layered slab waveguide are plotted in Fig. 2(a) with a red solid line and blue dashed lines, which correspond with the first-, second-, third-mode numbers of the right-hand side of Eq. 5. The crossing points of the characteristic curves in Fig. 2(a) give the eigenvalues of  $\kappa_3$ , while the first-, second-, and third-crossing points represent the  $TE_2$ ,  $TE_3$ , and  $TE_4$  modes, respectively. The constitutional parameters were used as follows:  $\lambda = 1.6 \ \mu m$ , ( $\mu_1 = 1$ ,  $\varepsilon_1 = 1$ ), ( $\mu_2 = -2$ ,  $\varepsilon_2 = -2$ ), ( $\mu_3 = 2.5$ ,  $\varepsilon_3 = 2.5$ ), ( $\mu_4 = -2$ ,  $\varepsilon_4 = -2$ ), and ( $\mu_5 = 1$ ,  $\varepsilon_5 = 1$ ). The thicknesses of the  $2^{nd}$ - and  $3^{rd}$ - and  $4^{th}$ -layers are 0.2  $\mu m$ , 2  $\mu m$ , and 0.2  $\mu m$  respectively.

Figure 2(b) shows the field distributions of guided  $TE_2$ ,  $TE_3$ , and  $TE_4$  modes of the symmetric five-layered RHMs slab waveguide. The mode number means the number of intersections between the fields and the x-axis. As the figures show, the electric fields are oscillatory in the RHMs-core layer while becoming evanescent outside the LHMs-clad layers.

The left- and right-hand sides of the characteristic Eq. 5 of an asymmetric fivelayered slab waveguide are plotted in Fig. 3(a). The crossing points of the characteristic curves in Fig. 3(a) give the eigenvalues of  $\kappa_3$ , while the first-, second-, and third-crossing points represent the  $TE_2$ ,  $TE_3$ , and  $TE_4$  modes, respectively. The constitutional parameters were used as follows:  $\lambda = 1.6 \ \mu m$ , ( $\mu_1 = 1$ ,  $\varepsilon_1 = 1$ ), ( $\mu_2 = -2$ ,  $\varepsilon_2 = -2$ ), ( $\mu_3 = 2.5$ ,  $\varepsilon_3 = 2.5$ ), ( $\mu_4 = -2$ ,  $\varepsilon_4 = -2$ ), and ( $\mu_5 = 1$ ,  $\varepsilon_5 = 2$ ). The thicknesses of the  $2^{nd}$ - and  $3^{rd}$ - and  $4^{th}$ -layers are 0.2  $\mu m$ , 2  $\mu m$ , and 0.2  $\mu m$  respectively. Figure 3(b) shows the field distributions of guided  $TE_2TE_3$ , and  $TE_4$ modes of the asymmetric five-layered RHMs slab waveguide. As the figures show, the electric fields are oscillatory in the RHMs-core layer while becoming evanescent outside the LHMs-clad layers.



Figure 2: (a) Dispersion curves and (b) Field distributions of the symmetric fivelayered slab waveguide for the oscillating  $TE_2$ ,  $TE_3$ ,  $TE_4$  guided modes

The special class of the guided optical modes, surface guiding modes, exists under certain special conditions at the interface separating two different dielectrics. The negative permittivity and permeability of a LHMs slab allows the existence of surface guiding waves at the interface with a RHMs layer [Shadrivov, Sukhorukov, and Kovshar (2004)]. In particular, TM-polarized surface waves exist when the dielectric permittivity constants of two dielectric materials have different signs at the interface, while TE-polarized surface waves exist when the magnetic permeability constants of the materials have different signs [Nkoma, Loudon, and Tilley



Figure 3: (a) Dispersion curves and (b) Field distributions of the asymmetric fivelayered slab waveguide for the oscillating  $TE_2$ ,  $TE_3TE_4$  guided modes

(1974)]. In this study, we show the existence of surface waves of TE polarizations in the presented slab waveguide.

$$\tanh(\alpha_3 d_2) = -\frac{q + p\left[-\tanh(\phi_2)\right]}{1 + q p\left[-\tanh(\phi_2)\right]},$$
(6)  
where  $\alpha_3^2 = \beta^2 - k_0^2 \mu_3 \varepsilon_3.$ 



Figure 4: (a) Dispersion curve and (b) Field distribution of the presented slab waveguide for the surface  $TE_1$  guided mode

Figure 4 shows that the dispersion curve and field distribution of the surface guiding  $TE_1$  mode, which are calculated by the characteristic Eq. 6 with the following constitutional parameters:  $\lambda = 1.6 \ \mu m$ , ( $\mu_1 = 1$ ,  $\varepsilon_1 = 1$ ), ( $\mu_2 = -1.12$ ,  $\varepsilon_2 = -2$ ), ( $\mu_3 = 2$ ,  $\varepsilon_3 = 2$ ), and ( $\mu_4 = -1.12$ ,  $\varepsilon_4 = -2$ ), ( $\mu_5 = 1$ ,  $\varepsilon_5 = 1$ ), ( $d_1 = 0.18 \ \mu m$ ), and ( $d_1 - d_2$ 

= 1  $\mu$ m). In Fig. 3(a), a red dashed line and black solid lines correspond to the right-hand side and left-hand side of the characteristic Eq. 6, respectively, and the crossing point of the characteristic curves gives the eigenvalue of  $\alpha_3$  of the surface  $TE_1$  guiding mode.



Figure 5: Energy distribution along the x-axis of the presented slab waveguide for the surface  $TE_1$  guided mode

In Fig. 5, the energy distribution along the x-axis for the surface  $TE_1$  guided mode is plotted with the same waveguide parameters of Fig. 4. The Poynting vetor,  $S_z = \beta E^2(x)/(2\omega\mu)$ , is directed to along the z-axis, and the directions of energy flux in LHMs layers are opposite to that in RHMs-core layer because of negative permeability,  $\mu$ . We can get the net power flux in slab waveguide by using the normalized power introduced in [Ahadrivov, Sukhorukov, and Kivshar (2004)]. The normalized power flux can be expressed as

$$\overline{P} = \frac{P_1 + P_2 + P_3 + P_4 + P_5}{|P_1| + |P_2| + |P_3| + |P_4| + |P_5|}$$
(7)

When the normalized power flux is negative, the net total power flow of the guided mode is opposite to the direction of phase flow and this wave is called the backward wave. Equation (8) shows the power flux in each layers by which the unknown amplitude A of guided modes at Eq. 2 can be calculated using the condition ( $P_1$  +

$$P_2 + P_3 + P_4 + P_5 = 1$$
W).

$$P_{1} = \frac{1}{4} \frac{\beta}{\omega \mu_{1}} \frac{A^{2}}{\alpha_{1}}$$

$$P_{2} = \frac{1}{4} \frac{\beta}{\omega \mu_{2}} B^{2} \left[ (d_{1} - d_{2}) + \frac{1}{2\alpha_{2}} \left\{ \sinh \left[ 2\alpha_{2} (d_{1} - d_{2}) + 2\phi_{2} \right] - \sinh \left( 2\phi_{2} \right) \right\} \right]$$

$$P_{3} = \frac{1}{4} \frac{\beta}{\omega \mu_{3}} \left[ \left( C^{2} d_{2} + \frac{CD}{2\kappa_{3}} + D^{2} d_{2} \right) + \frac{C^{2} - D^{2}}{2\kappa_{3}} \sin \left( 2\kappa_{3} d_{2} \right) - \frac{CD}{2\kappa_{3}} \cos \left( 2\kappa_{3} d_{2} \right) \right]$$

$$P_{4} = \frac{1}{4} \frac{\beta}{\omega \mu_{4}} E^{2} \left[ d_{4} + \frac{1}{2\alpha_{4}} \left\{ \sinh \left( 2\alpha_{4} d_{4} + 2\phi_{4} \right) - \sinh \left( 2\phi_{4} \right) \right\} \right]$$

$$P_{5} = \frac{1}{4} \frac{\beta}{\omega \mu_{5}} \frac{F^{2}}{\alpha_{5}}$$
(8)

#### 3 Conclusion

A graphical method was presented for the analysis of symmetric and asymmetric five-layered RHMs-core slab waveguides with double LHMs-clad layer for TE guided modes. The new dispersion curves of symmetric and asymmetric guiding modes were used, and the effective indices were graphically obtained from the crossing points of the curves. The symmetric and asymmetric oscillating guided modes,  $TE_2$  and  $TE_3$  and  $TE_4$  modes, were plotted from the calculated effective indices of the five-layered slab waveguide as well as surface guided modes,  $TE_1$  mode. The energy flux distribution along the axis is plotted for the surface  $TE_1$  mode.

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#### References

**Ahadrivov, I. V.; Sukhorukov, A. A.; Kivshar, Y. S.** (2004): Guided modes in negative refractive index waveguides. *Phys. Rev. E.*, vol. 67, pp. 057602-1~3.

**Berman, P. R**. (2002): Goos-Hanchen shift in negatively refractive media. *Phy. Rev. E.*, vol. 66, pp. 067603-1~3.

**Dong, P.; Li, J.-G.; Yang, H. W.** (2010): A slab optical waveguide with negative effective thickness. *J. of Mod. Optics*, vol. 57, no. 21, pp. 2137-2140.

He, Y.; Zhang, J.; Li, C. (2008): Guided modes in a symmetric five-layer lefthanded waveguide. J. Opt. Soc. Am. B, vol. 25, no. 12, pp. 2081-2091. Lee, C.-H.; Lee, J. (2012): Enhancement of evanescent field of metamaterialscore slab waveguide using metamaterials-buffer layer. *Opt. Eng.*, vol. 51, no. 8, pp. 084601-1~4.

Liu, Z.; Fang, N.; Yen, T.-J.; Zhang, X. (2003): Rapid growth of evanescent wave by a silver superlens. *Appl. Phys. Lett.*, vol. 83, pp. 5184-5186.

Nkoma, J.; Loudon, R.; Tilley, D. R. (1974): Elementary properties of surface polaritons. *J. Phys. C: Solid State Phys.*, vol. 7, no. 1974, pp. 3547-3559.

**Qing, D.-K.; Chen, G.** (2004): Enhancement of evanescent waves in waveguides using metamaterials of negative permittivity and permeability. *Appl. Phys. Lett.*, vol. 84, pp. 669-671.

Schurig, D.; Smith, D. R. (2003): Spatial filtering using media with indefinite permittivity and permeability tensors. *Appl. Phys. Lett.*, vol. 82, pp. 2215-2217.

Shadrivov, I. V.; Zharov, A. A.; Kivshar, Y. S. (2003): Giant Goos-Hanchen effect at the reflection from left-handed metamaterials. *Appl. Phys. Lett.*, vol. 83, pp. 2713-2720.

**Shadrivov, I. V.; Sukhorukov, A. A.; Kovshar, Y. S.** (2004): Nonlinear surface waves in left-handed materials. *Phys. Review E*, vol. 69, pp. 016617-1~3.

Smith, D. R.; Padilla, W. J.; Vier, D. C.; Nemat-Nasser, S. C.; Schultz, S. (2000): Composite medium with simultaneously negative permeability and permittivity. Phys. Rev. Lett., vol. 84, no. 18, pp. 4184-4187.

Taya, S. A.; Shabat, M. M.; Khalil, H. M. (2009): Enhancement of sensitivity in optical waveguide sensors using left-handed materials. *Optik*, vol. 120, pp. 504-508.

**Veselago, V. G.** (1968): The electrodynamics of substances with simultaneously negative values of  $\varepsilon$  and  $\mu$ . *Soviet Physics Uspekhi*, vol. 10, no. 4, pp. 509-514.

**Wu, B.-I.; Grzegorczyk, T. M.; Zhang, Y.; Kong, J. A.** (2003): Guided modes with imaginary transverse wave number in a slab waveguide with negative permittivity and permeability. *J. Appl. Phy.*, vol. 93, no. 11, pp. 9386-9388.