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A Theoretical Analysis on Elastic and Elastoplastic Stress Solutions for Functionally Graded Materials Using Averaging Technique of Composites

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Abstract: Functionally Graded Materials (FGMs) are being used in an ever-expanding set of applications. For better applications, an analytical methodology using averaging technique of composites is developed to describe the thermo-elastic and thermo-elastoplastic behaviors of a three-layered FGM system subjected to thermal loading Solutions using averaging technique of composites for the stress distributions in a generic FGM system subjected to arbitrary temperature loading conditions are presented. The power-law strain hardening behaviour is assumed for the FGM metallic phase and the stress of the metallic phase are calculated to judge the plastic in this work The stress distributions within the FGM systems are compared with accurate numerical solutions obtained from finite element analyses and good agreement is found.

Keywords: Analytical solution; Averaging technique of composites; Elasto-plastic

1 Introduction

Over the last two decades, a new class of materials, known as Functionally Graded Materials (FGMs) has emerged, in which the material properties vary gradually with location to optimize the response (Giannakopoulos and Pallot, 2000; Lin et al, 2001). For instance, the distribution of metal and ceramic phases in an FGM can be optimized such that the FGM retains the thermal properties of the ceramic phase with the metallic phase added to increase the mechanical properties of the composite – specifically the strength (Han et al, 2002; Shen, 2002). Due to the ability of FGMs to solve such complicated problems, these materials have been

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explored for engineering applications in thermal, structural, optical and electronic materials (Miyamoto et al, 1999). Recently, a number of researchers have explored these materials (Chen and Tong, 2004; Guler and Erdogan, 2004; Yin et al, 2005; Shao and Wang, 2006).

The real advantages of using an FGM as an alternative to two dissimilar materials joined directly together include: smoothing of thermal stress distributions across the layers (Choules and Kokini, 1996), reduce the thermal stress in such structures working in high temperature environment (Wetherhold et al., 1996), minimisation or elimination of stress concentrations and singularities at free edges (Lee and Erdogan, 1994/1995; Yang and Munz, 1997), increase in the bimaterial bonding strength (Howard et al., 1994a, b), and improved fracture toughness compared to that of monolithic ceramics as a result of the plastic deformation of the metallic phase (Erdogan, 1995; Jin and Batra, 1996; Shaw, 1998). Typical applications of FGMs are in high temperature coatings for combustion chambers and airfoils in the aerospace and power generation industries, coatings in microelectronics and optoelectronics applications, orthopaedic implants and wear-resistant coatings in bearings, gears, cams and machine tools (Miyamoto, 1996).

The FGMs have already been designed and fabricated to achieve unique microstructures (Miyamoto et al., 1999) However, very limited analytical investigations are available to predict the effective properties of FGMs Many analytical works describing the thermo-elastic behaviors have been carried out (Freund, 1993; Giannakopoulos et al., 1995; Dao et al., 1997; Berrabah et al., 2010), while for a metal–ceramic FGM, yielding in the metallic phase can greatly affect the stress distribution within the FGM and thus a stress analysis based only on elastic material behaviour will not be accurate (Olsson et al., 1995) In such cases, it is worth noting that plastic deformation play an important role and can occur solely as a result of thermal loads. Numerical investigations have also been carried out to study the thermo-elastoplastic behaviour of metal–ceramic FGMs using finite element (FE) techniques (Giannakopoulos et al., 1995; Finot and Suresh, 1996; Weissenbek et al., 1997). Although accurate, involve extensive and costly computations. We want to use analytical solutions so that many porosities and compositions can be considered without having to make a mesh that is specific to geometry. Therefore there is a strong need for accurate analytical formulations to predict the elastoplastic properties of FGM behaviors

In most of the obtained papers for the analysis of the FGMs (e.g. Pitakthapanaphong and Busso, 2002), the mixture rule was used and the elastoplastic stress distribution was calculated by viewing the FG-layer as a whole material. However, there is a great variation in the physical properties of the different phases in the composite and they should not yield at the same time. Although the constitutive models for each phase are easily obtained, the effective yield behavior of the composite is very complicated and it is hard to judge the plastic by the whole stress state. To obtain the analytical solution of system, the averaging technique of composites for heterogeneous materials is utilized in this work The effective stresses of the metallic phase are calculated to judge the plastic of metallic phase and the ceramic phase is perfectly elastic The power-law strain hardening behavior that describes the plastic of the FGM metallic phase is adopted and then combined with averaging technique of composites so that the thermo-elastic and thermo-elastoplastic behavior of a three-layered FGM system subjected to thermal loading are presented Finally, the analytical results of the system are obtained.

2 Thermo-elastic analyses under an arbitrary temperature loading

To analyze the thermo-elastic behaviors of FGMs, Solutions for the stress distributions of the composite under thermo loading are determined here. From these considerations, the analytical expressions for the stresses in an active plate consisting of a three-layered system with a middle FGlayer are then derived according to the averaging technique of composites The physical reason for such behaviors is explained and the average stresses are then determined to get the behaviors of such composites. Consider a three-layered system made up of a compositionally grade layer in between a homogeneous material and a metallic material. As shown in Fig. 1, the up layer is considered as the homogeneous ceramic and the down layer is considered as the metallic phase. The system is stress free at the initial case and the stress free is kept on all boundary surfaces.

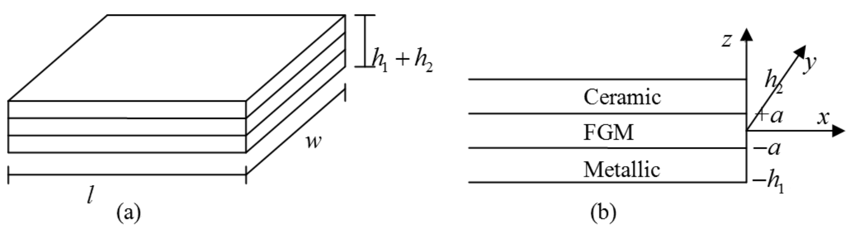

Figure 1: (a) Three-layered structural model and (b) coordinate axes and dimensions of the three-layered plate system

Let the total length, width and height of the multilayer system be given by l, w and$h_1 + h_2$, respectively, see Fig. 1(a). Assume that$l = w >> h_1 + h_2$, the geometry of the model allows it to be idealized as an equal biaxial stress plate, and the variables of interest depend only on the out of plane coordinate$z$. The in-plane geometry of the layered structure is shown in Fig. 1(b). The FGlayer extends from

$z = -a$to$z = +a$and for continuous property assumptions to be valid and the thickness of this layer must be significantly larger than its dominant micro-structural length scale. The interfaces between the different layers are assumed to be perfectly bonded at all times.

Let the volume fractions of the ceramic phase vary in the FGlayer as a function of 'z' coordinate, as described by the function$V(z)$, which satisfies the following conditions at layers' interfaces.

$$V(z) = \begin{cases} 0 & z = -a, \\ 1 & z = a. \end{cases} \tag{1}$$

The compositional gradation of the FG-layer is defined by the volume fraction of the ceramic phase. Here, the following function of $V(z)$will be considered:

$$V(z) = \begin{cases} \frac{z+a}{2a} & \text{Linear} \\ (\frac{z+a}{2a})^2 & \text{Quadratic} \\ 1-(\frac{z-a}{2a})^2 & \text{Inverse quadratic} \\ 3(\frac{z+a}{2a})^2 - 2(\frac{z+a}{2a})^3 & \text{Cubic} \end{cases} \tag{2}$$

where means that the gradation of the FGlayer is such that it changes from 100% of the ceramic phase at the top interface, to 100% of the metallic phase at the bottom interface.

In the thermo-elastic phase, by assuming small strain kinematics, the total strain,ε, applied in the plane of the multilayer system, can be decomposed into an elastic and a thermal component

$$\varepsilon = \varepsilon^e + \varepsilon^{the} \tag{3}$$

Under equal biaxial stress condition

$$\sigma_{xx}(z) = \sigma_{yy}(z) = \sigma(z); \varepsilon_{xx}(z) = \varepsilon_{yy}(z) = \varepsilon(z) \tag{4}$$

Then the stress tensor and the strain tensor can be described as

$$\sigma_{ij} = \begin{pmatrix} \sigma(z) & 0 & 0 \\ 0 & \sigma(z) & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \varepsilon_{ij} = \begin{pmatrix} \varepsilon(z) & 0 & 0 \\ 0 & \varepsilon(z) & 0 \\ 0 & 0 & \frac{-2\upsilon}{1-\upsilon}\varepsilon(z) \end{pmatrix} \tag{5}$$

Following the standard linear relation of Euler-Bernoulli beam theory between the total in-plane strain and the laminate curvature (K), the thickness-wise variation of the in-plane strain can be expressed as follows

$$\varepsilon(z) = \varepsilon_0 + Kz \tag{6}$$

where ε_0 is the strain at the mid-plane of the FGlayer at $z = 0$.

The only non-zero stress component $\sigma(z)$ is given by

$$\sigma(z) = \frac{E(z)}{1-\upsilon}[\varepsilon_0 + Kz - a(z)\Delta T] \tag{7}$$

where $E(z)$ is the Young's modulus, $a(z)$is the coefficient of thermal expansion and $\upsilon = \upsilon_1 = \upsilon_2$ is the Poisson's ratio for different layers, respectively. The subscripts 1 and 2 refer to the respective homogeneous material layers (see Fig. 1). Here$\Delta T = T - T_0$, where T_0 is the initial temperature and T is the homogeneous temperature in the multilayer system.

According to averaging technique of composites, the system is analyzed by using different constitutive models in different phases and it leaves the thermo-elastic biaxial stress solution for an arbitrary FGM characterized by a generic composition profile function$V(z)$

$$\sigma(z) = \begin{cases} \frac{E_1}{1-\upsilon}(\varepsilon_0 + Kz - a_1\Delta T) & -h_1 \le z \le -a \\ \frac{E_1}{1-\upsilon}(\varepsilon_0 + Kz - a_1\Delta T)[1 - V(z)] + V(z)\frac{E_2}{1-\upsilon}(\varepsilon_0 + Kz - a_2\Delta T) & -a \le z \le a \\ \frac{E_2}{1-\upsilon}(\varepsilon_0 + Kz - a_2\Delta T) & a \le z \le h_2 \end{cases} \tag{8}$$

The expressions for ε_0 and K are derived on the basis that the resultant internal force and moment arising from the stress distribution through the thickness must balance any externally applied in-plane force or bending moment.

$$\Sigma F_x = 0 \quad and\ also \quad \Sigma M_x = 0 \tag{9}$$

which lead to

$$\int_{-h_1}^{h_2} \sigma(z)dz = 0 \tag{10}$$

$$\int_{-h_1}^{h_2} \sigma(z)zdz = 0 \tag{11}$$

After the function $\sigma(z)$ in (11) corresponding to the assumed volume fraction $V(z)$over the thickness are substituted into (13) (14), these can be integrated to produce:

$$\varepsilon_0 I_0 + KI_1 - J_0 = 0 \tag{12}$$

$$\varepsilon_0 I_1 + K I_2 - J_1 = 0 \tag{13}$$

where

$$(I_0, I_1, I_2) = \int_{-h_2}^{h_1} (1, z, z^2) Q(z) dz \tag{14}$$

$$(J_0, J_1) = \int_{-h_2}^{h_1} (1, z) R(z) dz \tag{15}$$

where

$$Q(z) = \begin{cases} \frac{E_1}{1-\upsilon} & for \quad -h_1 \le z \le -a \\ \frac{E_1}{1-\upsilon}[1 - V(z)] + \frac{E_2}{1-\upsilon} V(z) & for \quad -a \le z \le a \\ \frac{E_2}{1-\upsilon} & for \quad a \le z \le h_2 \end{cases} \tag{16}$$

$$R(z) = \begin{cases} \frac{E_1}{1-\upsilon} a_1 \Delta T & for \quad -h_1 \le z \le -a \\ \frac{E_1}{1-\upsilon} a_1 \Delta T \; [1 - V(z)] + \frac{E_2}{1-\upsilon} a_2 \Delta T V(z) & for \quad -a \le z \le a \\ \frac{E_2}{1-\upsilon} a_2 \Delta T & for \quad a \le z \le h_2 \end{cases} \tag{17}$$

As a simple example for cases of linear quadratic, inverse quadratic and cubic variations of $V(z)$, results of stress distributions can be obtained by (8-17). Consider the system made up of Ni-FGM- AL_2O_3 layers, with all layers assumed to be isotropic elastic material, free of damage and having the temperature independent properties in Table 1 (Giannakopoulos et al. 1995; Weissenbek et al. 1997).

Table 1: Properties for the metallic (Ni) and ceramic (AL_2O_3) phases (Giannakopoulos et al. 1995; Weissenbek et al. 1997)

Material	$E_1(E_2)$	$a_1(a_2)$	$h_1(h_2)$	a	υ
AL_2O_3	380Gpa	$7.4{*}10^{-6\circ}C^{-1}$	0.405	0.285	0.25
Ni	214Gpa	$15.4{*}10^{-6\circ}C^{-1}$	0.655	0.285	0.25

To verify the accuracy of the analytical solutions, finite element analyses of the system were performed using 1000 equal sized linear plane stress elements. The length-to-height ratio for the model was chosen to be $l/2a = 2.5$so as to minimize edge effects. The FGM was modeled with continuously varying composition and properties through the thickness. To implement that, an implicit user defined material subroutine was developed and used in the FE calculations.

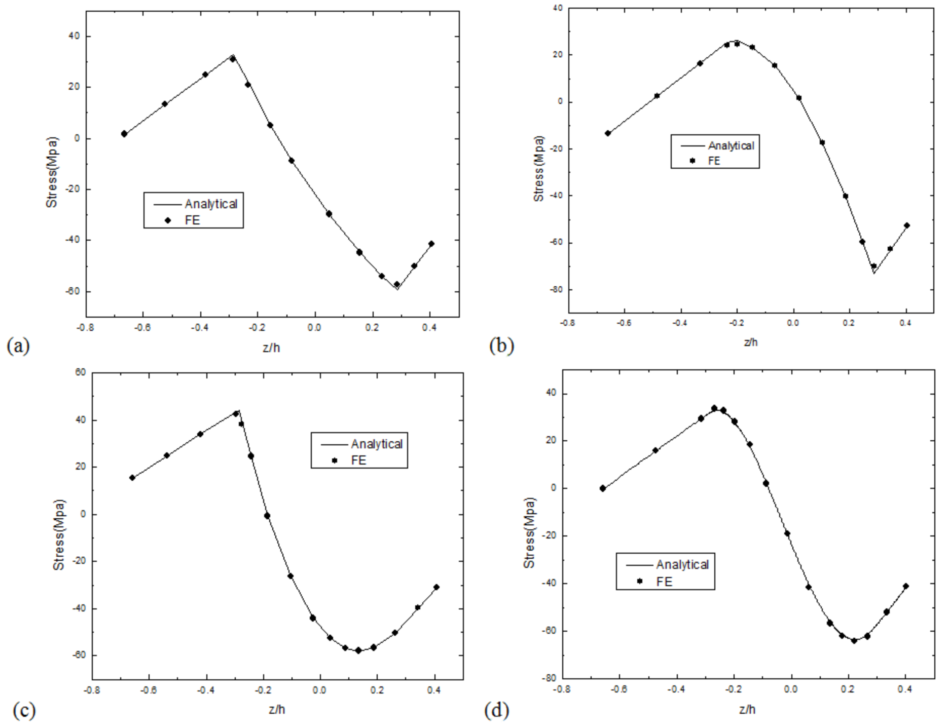

Figure 2: Analytical thermo-elastic stress distributions and corresponding FE predictions using an FGM with a (a) linear, (b) quadratic, (c) inverse quadratic and (d) cubic compositional variation with $\Delta T = -50°C$

Fig. 2 (a-d) shows the stress distributions through the thickness of the three-layered system for the different composition profiles with$\Delta T = -50°C$. Solid curves represent the analytical predictions and the dot results are the FE results. As seen in Fig. 2, the analytical predictions are in good agreement with the FE results which proves that the model using averaging technique of composites is reasonable. Furthermore, the stresses of both the homogeneous austenite and ceramic layers are linear, whereas the stress distribution in the FGlayer depends on the type of compositional gradation. It is also worth nothing that the stresses at the ceramic/FG-layer interface are always compressive whereas those at the metal/FG-layer interface are tensile in all cases. This is due to the thermal expansion coefficients in different layers are different, the bend of the system happens with the changing temperature It can also observe that by using the cubic gradation, the stresses across both interfaces vary smoothly and the interfacial stress magnitudes are comparable to the linear case

3 Thermo-elastoplastic analyses

In this section, the elastoplastic behavior of the metallic phase within the FGM in the three-layered system illustrated in Fig. 1 is considered. Layer 1 is assumed to be an elastoplastic metal and Layer 2 an elastic ceramic material. Plane stress conditions will be considered as an example. The metal is assumed to have its constitutive behavior described by a power-law hardening law, namely

$$\varepsilon_{ij} = \begin{cases} \frac{\sigma_{ij}}{E_1} & \text{if } \sigma_{\text{ij}} \leq \sigma_{y1} \\ \varepsilon_{y1}(\frac{\sigma_{ij}}{\sigma_{y1}})^n & \text{if } \sigma_{\text{ij}} > \sigma_{y1} \end{cases} \tag{18}$$

where $\sigma_{y1}$and $\varepsilon_{y1}$are the metal yield stress and strain, respectively, and $n$is the strain hardening exponent. Then the stress distributions of the metallic phase can be obtained by

$$\sigma_1(z) = \begin{cases} E_1(\varepsilon_1 + K_1 z - a_1 \Delta T) & \text{if } \sigma(z) \leq \sigma_{y1} \\ \sigma_{y1}(\frac{\varepsilon_1 + K_1 z - a_1 \Delta T}{\varepsilon_{y1}})^{\frac{1}{n}} & \text{if } \sigma(z) > \sigma_{y1} \end{cases} \tag{19}$$

where$\varepsilon_1 K_1$are there differential forms.

In the same way, according to averaging technique of composites, the system is analyzed by using different constitutive models in different phases and it leaves the thermo-elastoplastic biaxial stress solution for an arbitrary FGM characterized by a generic composition profile function$V(z)$

$$\sigma(z) = \begin{cases} \sigma_1(z) & -h_1 \leq z \leq -a \\ \sigma_1(z)[1 - V(z)] + V(z)\frac{E_2}{1-\upsilon}(\varepsilon_1 + K_1 z - a_2 \Delta T) & -a \leq z \leq a \\ \frac{E_2}{1-\upsilon}(\varepsilon_1 + K_1 z - a_2 \Delta T) & a \leq z \leq h_2 \end{cases} \tag{20}$$

Then, the elastoplastic stress distributions for cases of linear quadratic, inverse quadratic and cubic variations of $V(z)$ can be easily obtained. The thermo-elastic properties used for the individual Ni and ceramic phases are those given in Table 1. The Ni yield stress was taken to be temperature-independent and equal to 138Mpa (Weissenbek et al., 1997), and its strain hardening exponent is assumed $n = 5$ (Frost and Ashby, 1982), while when $n = \infty$the model are degenerate to the perfectly elastoplastic case Then the stress distributions within the Ni-FGM-AL_2O_3 system are then obtained with $\Delta T = -300°C$.

Fig. 3 (a), (b), (c) and (d) show the different cases of analytical thermo-elastoplastic stress distributions with$\Delta T = -300°C$ using the linear, quadratic, inverse quadratic

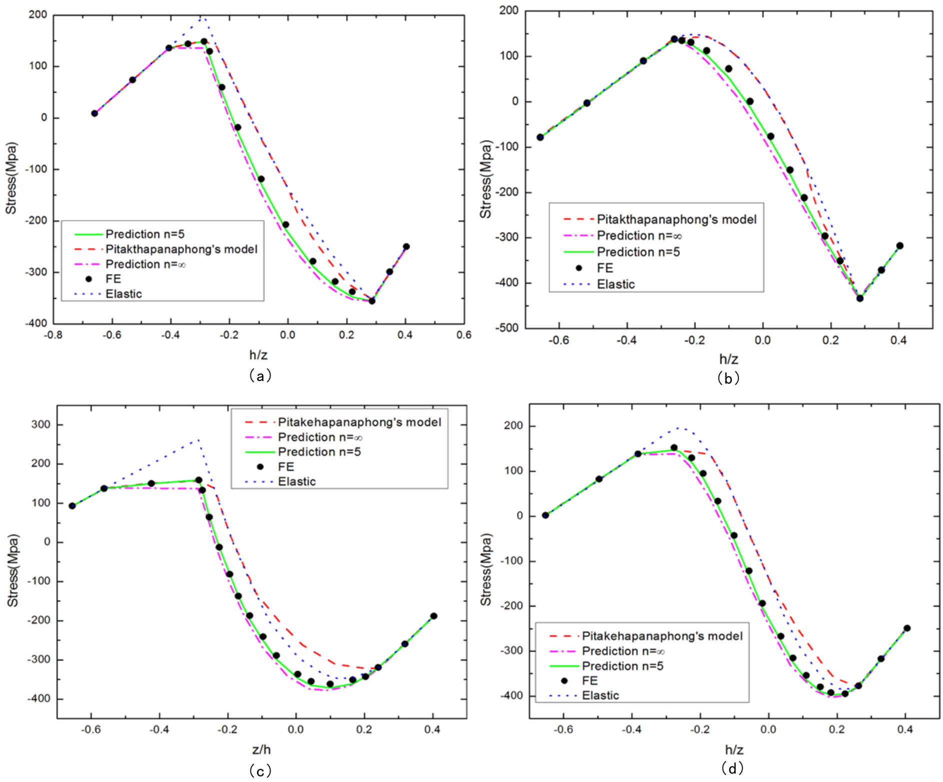

Figure 3: Analytical thermo-elastoplastic stress distributions using an FGM with a (a) linear, (b) quadratic, (c) inverse quadratic and (d) cubic compositional variation with $\Delta T = -300°C$

and cubic composition profiles, respectively For each case, the stress distributions corresponding to an elastic, a perfectly plastic $n = \infty$, and a strain hardening $(n = 5)$ metallic phase using averaging technique of composite are respectively shown and compared with the Pitakthapanaphong's model (2002) and the FE (Weissenbek et al., 1997) results. The blue dot curve represents the elastic case, the red dash curve is Pitakthapanaphong's result, the magenta dot dash curve is the perfectly plastic$n = \infty$, the black dot is the FE result, and the green solid curve is the prediction of the elastoplastic stresses of this work with$n = 5$

Several results can be observed from Fig. 3. After the plastic deformation occurs, the average stresses in the Ni rich regions show a considerable reduction. This shows that under the same material strength, the temperature resistant of the system will increase. So, there will be better mechanical properties for the composites and can adjust to a higher temperature In Pitakthapanaphong's model (2002), the elastoplastic stress distribution was calculated by viewing the FG-layer as a whole material and judge the yield by the average stress of FGM, while it needs to be discussed the reasonability of using the metallic phase's properties to analysis the plastic of the FGM layer. In the present work, it is assumed that the yielding only takes place at the metallic phase when the stress of the metallic phase is bigger than the yielding stress. The new stress distributions of the FGM are calculated by the average technique of composite after the yielding happens. And the prediction results of this work agree well with the FE results (Weissenbek et al., 1997). Futhermore, from the choices of the compositional gradation considered here, the cubic profile offers the most benign stress distributions. When$n = \infty$, the model can be degenerated to a perfectly plastic case.

4 Conclusions

The analytical solutions on the thermo-elastic and thermo-elastoplastic stress solutions for functionally graded materials using averaging technique of composites are studied in this work The proposed relations for the stress distributions within a generic metal-FGM-ceramic system can predict accurately complex stress distributions induced by thermal loading. Compared with the elastic results, the stresses in the Ni rich regions show a considerable reduction By choosing an appropriate FGM compositional gradation, the stress distribution within the system can be controlled so that undersirable stresses at critical locations are minimized or avoided. The analytical elastoplastic solutions presented here may be accounted for in many potential FGM composites design and can provide a simple, yet accurate tool for the prediction of thermally induced stresses in an FGM layer sandwiched between two homogeneous materials.

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