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Influence of Stress Singularities on Scaling of Fracture of Metal-Composite Hybrid Structures

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Abstract: It has been recently shown that the nominal structural strength of metal-composite structures depends on the structure size, and such dependence is strongly influenced by the stress singularities. Nevertheless, previous studies only focused on structures that exhibit very strong stress singularities, which are close to the crack-like stress singularity. In the actual engineering designs, due to the mismatch of material properties and complex structural geometries, many metal-composite structures may contain stress singularities that are much weaker than the crack-like stress singularity. This paper presents a numerical study on the size dependence of scaling of fracture of metal-composite hybrid structures for a wide range of stress singularities. The numerical examples include a series of metal-composite hybrid beams with a V-notch under three-point bending with different notch angles, which lead to various magnitudes of stress singularities. By assuming that the bimaterial interface is weaker than both metal and composite, we use a mixed-mode cohesive element model to simulate the fracture behavior of these hybrid beams. It is shown that the resulting size effect curves strongly depend on the magnitude of stress singularities. The simulation results agree well with a recently developed energetic-statistical scaling model.

Keywords: Size effect, quasibrittle fracture, weakest link model, cohesive fracture

1 Introduction

Metal-composite hybrid structures are designed to combine the advantages of both materials, such as the high stiffness of the metals and the light weight and excellent corrosion resistance of the composites. These bimaterial hybrid structures have been used in many engineering designs. Recent applications include the metal-composite joints for large ship hull and modern aircraft wings and fuselages [Bar-

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soum (2003)]. Understanding the fracture behavior of bimaterial structures is a critical aspect of design. Due to the complex structural geometries and material mismatch, bimaterial structures often exhibit stress concentrations at the bimaterial corner. Substantial efforts have been devoted to analyzing the singular stress and strain fields at the bimaterial corner, e.g. [Bogy (1971); Rice (1988); Hutchinson and Suo (1992); Desmorat and Leckie (1998); Qian and Akisanya (1998); Liu and Fleck (1999); Jr. (1993); Labossiere, Duun, and Cunningham (2002)]. These analyses were largely derived from the linear elastic fracture mechanics (LEFM), which is only applicable to perfectly brittle structures. For metal-composite hybrid structures, both adhesives and composite materials usually have a gradual strain-softening behavior, which leads to a transitional failure mechanism lying between the perfectly brittle and plastic failure modes, often termed as quasibrittle failure. Therefore, the metal-composite hybrid structures can be considered as quasibrittle structures. It has been well demonstrated that the failure of quasibrittle structures is subjected to a scale effect [Bažant and Planas (1998); Bažant (2004, 2005)], i.e. the nominal structural strength varies with the structure size. Therefore, it is expected that the metal-composite hybrid structures would also exhibit a size-dependent failure behavior. Exploring such a size effect is critical for extrapolating small-scale laboratory tests to full-scale design.

The scale effect on the strength of metal-composite hybrid structures has recently been studied analytically, numerically and experimentally [Yu, Bažant, Bayldon, Le, Caner, Ng, Waas, and Daniel (2010); Le, Bažant, and Yu (2010); Yu, Z. P. Bažant, and Le (2013)]. The structures that were considered in these studies exhibit strong stress singularities (very close to the"$-1/2$" stress singularity), and therefore the structural failure must be caused by the damage at the bimaterial corner. By applying the LEFM for large-size structures and asymptotic matching, it has been shown that for two-dimensional problem the size effect on the nominal structural strength can be expressed as:

$$\sigma_N = \sigma_s \left(1 + D/D_0\right)^{\kappa} \tag{1}$$

where σ_N = nominal strength = $c_n P_{\max}/bD$, $P_{\max}$ = load capacity of the structure, D = characteristic size of the structure to be scaled, b = size of the structure in the transverse direction, c_n = constant chosen such that σ_N carries some physical meaning, e.g. the maximum elastic stress without considering the stress singularities, D_0 = transitional size, which depends on the structural geometry and the size of the fracture process zone (FPZ), κ = order of dominant stress singularity, and σ_s = nominal strength of the structure at the small-size limit. Eq. 1 implies that, as the stress singularities vanish, the size effect on the nominal strength would disappear. Nevertheless, it has been well known that for quasibrittle structures without stress singularities the nominal strength is also strongly size dependent [Bažant and Li (1995); Bažant (2004, 2005)], which can be explained by either the distributed cracking theory [Bažant and Li (1995)] or the finite weakest link theory [Bažant and Pang (2007); Bažant, Le, and Bazant (2009); Le, Bažant, and Bazant (2011)]. Clearly Eq. 1 does not describe the scaling of nominal strength for the case where the stress singularities are weak or vanishing. In a recent study [Le (2011)], a general scaling equation was developed to bridge the cases of strong and zero singularities by combining the finite weakest link model and fracture mechanics of bimaterial corner. Nevertheless, no numerical and experimental studies have been carried out to investigate the effect of stress singularities on the scaling of strength of metal-composite hybrid structures.

This study present a numerical study on the scale effect on the strength of metal-composite hybrid beams with a V-notch under three-point bending, where the interface is weaker than both base materials. By varying the notch angle, we are able to investigate the influence of stress singularities on the scaling of strength of these beams. This paper is planned as follows: Section 2 reviews a recently developed general scale effect equation for the strength of bimaterial quasibrittle structures [Le (2011)], Section 3 describes the details of the numerical simulation, and Section 4 discusses the simulation results.

2 Review of theoretical formulation

Consider a general bimaterial structure with a weakly bonded interface, as shown in Fig. 1. Here we limit our attention to structures of positive geometry, which is defined such that the peak load is reached once the FPZ is fully developed.

The material mismatch and geometry of the bimaterial corner could cause a singular stress field. For metal-composite hybrid structures, where the composite materials are usually orthotropic, the stress singularities at the bimaterial corner can be solved by the complex potential method [Stroh (1958); Lekhnitskii (1963); Desmorat and Leckie (1998); Le, Bažant, and Yu (2010)]. Here we focus on structures with a power-law singular stress field, i.e. $\sigma \propto r^{\lambda}$ (r = radial distance from the notch tip, λ = order of stress singularity), which applies to most bimaterial corners. Since the order of stress singularities is weaker than $-1/2$, the energy release rate at the notch tip is zero. Therefore, the direct use of fracture energy would not yield a failure criterion. It has been shown that a general fracture criterion can be formulated by using the equivalent LEFM framework [Le, Bažant, and Yu (2010); Le (2011)]. As a brief review, let us consider a general case where the stress field at the bimaterial corner is governed by two distinct real stress singularities, κ_1, κ_2.

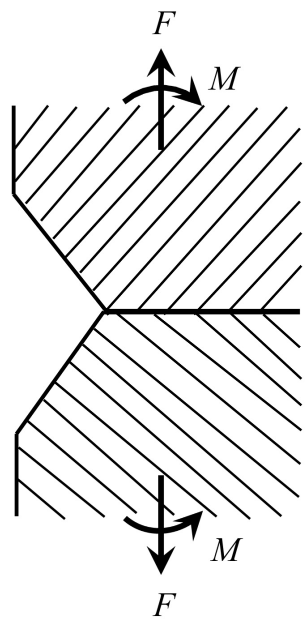

Figure 1: Geometry of bimaterial structures

The corresponding stress intensity factors are:

$$H_k = \sigma D^{-\kappa_k} h_k \tag{2}$$

where σ = nominal stress = P/bD, P = the applied load, and $h_k(k=1,2)$ = dimensionless stress intensity factors determined by the geometry of the structure.

As the structure reaches its peak load, a FPZ, which contains numerous microcracks, will form at the tip of the bimaterial notch. Within the framework of the equivalent LEFM, we can approximate this FPZ by an equivalent interfacial crack [Grenestedt and Hallstrom (1997); Liu and Fleck (1999); Le, Bažant, and Yu (2010); Le (2011)]. At the large size limit, this equivalent crack is fully enclosed by the singular stress zone at the notch tip and therefore the energy release rate $\mathscr{G}$ at the tip of the equivalent crack can be calculated from the stress intensity factors of the bimaterial notch, i.e. $\mathscr{G} = F(H_1, H_2)$, [Liu and Fleck (1999); Le, Bažant, and Yu (2010); Le (2011)]. By considering that the peak load is reached once the energy release rate reaches a critical value G_f, we reach a general expression for the nominal structural strength [Le (2011)]:

$$\sigma_N = \frac{\sqrt{EG_f/l_c}}{[A_1(D/l_c)^{-2\kappa_1} + A_2(D/l_c)^{-2\kappa_2} + A_3(D/l_c)^{-\kappa_1-\kappa_2}]^{1/2}} \tag{3}$$

where E = Young's modulus of steel, l_c = length of the equivalent crack, which is about half of the FPZ length, and $A_k(k=1-3)$ = geometrical constants, which can be obtained by an elastic analysis [Le (2011)]. At the small-size limit, the entire interfacial ligament behaves as a crack filled by a plastic glue, and therefore the size effect vanishes. An approximate scaling equation has been proposed to bridge the small- and large-size limits:

$$\sigma_N = \sigma_s \left\{1 + \left[(D/D_1)^{-2\kappa_1} + (D/D_2)^{-2\kappa_2} + (D/D_3)^{-\kappa_1-\kappa_2}\right]^{\gamma}\right\}^{-1/2\gamma} \tag{4}$$

where γ = positive constant, $D_k(k=1-3)$ can directly be related to constants A_k and the FPZ size l_c by matching Eq. 3 with the large-size asymptote of Eq. 4. We often refer Eq. 4 to as energetic (deterministic) scaling, since it is derived from an energetic perspective and the random material properties do not contribute to this size dependence.

For some material mismatch and corner geometries, it is possible that only one real stress singularity dominates or a pair of complex conjugate stress singularities prevails. In such cases, Eq. 4 reduces to:

$$\sigma_N = \sigma_s \left[1 + (D/D_1)^{-2\gamma\kappa}\right]^{-1/2\gamma} \tag{5}$$

where κ = the dominant real stress singularity or the real part of the complex stress singularities. Recent studies on scaling of fracture of metal-composite joints, which consist of complex stress singularities, have shown that Eq. 5 agrees well with both numerical and experimental investigations [Yu, Bažant, Bayldon, Le, Caner, Ng, Waas, and Daniel (2010); Le, Bažant, and Yu (2010)]. It should be emphasized here that Eqs. 4 and 5 are only applicable to structures where the magnitude of stress singularities is sufficiently strong, since the entire analysis is based on the fracture of the bimaterial corner.

If the stress singularities at the bimaterial corner are very weak, then there is no guarantee that the fracture would initiate and propagate from the corner. To account for this, we would need to consider the statistics of random strength of the bimaterial interface. Note that here we do not consider the potential failure of composites and metals since a weak interface is assumed. Since we consider the structures of positive geometry, the failure statistics of the structure can be calculated by using the finite weakest link model [Bažant and Pang (2007); Bažant, Le, and Bazant (2009); Le, Bažant, and Bazant (2011)], which implies that the structure reaches its peak load once one representative volume element (RVE) along the interface fails. In the meantime, we note that, at the bimaterial corner, a singular stress field still exists though the singularity is weak. Therefore, if the crack initiates from the corner, we need to consider the aforementioned scaling mechanism. To describe the probabilistic failure of the entire bimaterial interface, we must incorporate the energetic scaling mechanism of the corner tip fracture into the finite weakest link model. To this end, we propose to separate the bimaterial interface into two regions: 1) the region within the singular stress zone, and 2) the region outside the singular stress zone. The failure probability of the first region can be calculated as:

$$P_{f,V_I}(\sigma_N) = 1-\prod_{i=1}^{N_1}\{1-P_1[\mu(D)\sigma_N s(x_i)]\} \tag{6}$$

where

$$\mu(D) = \left\{1+\left[(D/D_1)^{-2\kappa_1}+(D/D_2)^{-2\kappa_2}+(D/D_3)^{-\kappa_1-\kappa_2}\right]^{\gamma}\right\}^{1/2\gamma} \tag{7}$$

where the parameters in scaling term $\mu(D)$ follow the same definition as those in Eq. 4, N_1 = number of RVEs in the singular stress zone, $s(x_i)$ = dimensionless stress field such that $\sigma_N s(x_i)$ = maximum principal stress at the center of ith RVE, and $P_1(x)$ = cumulative distribution function (cdf) of strength of one RVE, which can be derived from atomistic fracture mechanics and a multiscale statistical model [Bažant, Le, and Bazant (2009); Le, Bažant, and Bazant (2011)].

For the remaining part of the interface, the failure probability can directly be calculated from the elastic stress:

$$P_{f,V_{II}}(\sigma_N) = 1-\prod_{i=1}^{N_2}\{1-P_1[\sigma_N s(x_i)]\} \tag{8}$$

Therefore, the failure probability of the entire structure can be calculated as:

$$P_f(\sigma_N) = 1-[1-P_{f,V_I}(\sigma_N)]\,[1-P_{f,V_{II}}(\sigma_N)] \tag{9}$$

By considering geometrically similar specimens of different sizes, we can obtain a size effect on the mean structural strength. Though a closed form solution is next to impossible, an approximate scaling equation has been proposed [Le (2011)]:

$$\begin{aligned}\bar{\sigma}_N = \sigma_0\Bigg\{ & C_1[\mu^m(D)\Psi_1+\Psi_2]^{-r/m}\left(\frac{D+l_s}{l_0}\right)^{-r/m}\exp[-(\kappa/\kappa_1)^2] \\ & +\frac{\mu^{-r}(D)D_b}{\exp[-(\kappa/\kappa_2)^2]D+l_p}\Bigg\}^{1/r}\end{aligned} \tag{10}$$

where σ_0 = reference stress, m = Weibull modulus of strength distribution of the interface, l_0 = size of the RVE along the interface, κ = dominant stress singularity for the case where there are two distinct real stress singularities, or the real part of the complex stress singularities, $\Psi_1 = \int_{V_I} \langle s(x) \rangle^m \mathrm{d}V(x)$, $\Psi_2 = \int_{V_{II}} \langle s(x) \rangle^m \mathrm{d}V(x)$ and $C_1, r, \kappa_1, \kappa_2, l_s, l_p, D_b$ = constants. Constants l_s and l_p are introduced to regularize the functional behavior as $D \to 0$. Note that, as the stress singularities get stronger, the failure of the entire structure is dominated by the failure of the RVE at the bimaterial corner. Therefore, all the statistical scaling terms have to vanish. Here two exponential functions are used to approximate such a transition.

It should be pointed out that for small and intermediate-size structures with weak stress singularities the size effect derived from this finite weakest link model with the use of elastic stresses is expected to agree well with the prediction by nonlinear deterministic calculations. This is because the mean size effect behavior for small- and intermediate-size structures is mainly caused by the stress redistribution, which can be well predicted by the nonlinear deterministic calculation. Meanwhile, this mechanism can also be captured by the finite weakest link model, where the multiscale transition model used for the formulation of the cdf of RVE strength could statistically represent the damage localization and load redistribution mechanisms at different scales (albeit only the elastic stresses are used), see detailed discussion in [Le, Bažant, and Bazant (2011); Le, Elias, and Bažant (2012)]. For large-size structures, the zone of stress redistribution is negligible compared to the structure size and the size effect is mainly caused by randomness of material strength, which cannot be captured by the deterministic calculation.

3 Numerical simulation

Eq. 10 clearly indicates that the scaling of strength of bimaterial structures depends on the magnitude of stress singularities. To verify such dependence, we perform a numerical study on a series of metal-composite hybrid beams with a V-notch under three-point bending (Fig. 2). The steel material is considered to be isotropic with a Young modulus E = 200 GPa and Poisson's ratio $\nu = 0.3$. The composite material is considered to be unidirectional Carbon/Epoxy composite with the following properties: $E_1 = 147$ GPa, $E_2 = E_3 = 10.3$ GPa, $G_{12} = G_{13} = 7.0$ GPa, $G_{23} = 3.7$ GPa, $\nu_{12} = \nu_{13} = 0.27$, and $\nu_{23} = 0.54$ (G denotes the shear modulus).

Here we consider four different notch angles, i.e. $\theta = 0°, 120°, 135°$ and $170°$. For each notch angle, geometrically similar specimens of a wide size range, i.e. 1:2:4:8:16:32 for θ=0°, 120°, and 135°, and 1:2:4:8:16:32:64:100 for θ=170° are simulated. Based on the complex potential method [Stroh (1958); Lekhnitskii (1963); Desmorat and Leckie (1998); Le, Bažant, and Bazant (2011)], the orders of stress singularities are found to be: $\kappa_{1,2} = -0.5 \pm 0.081i$ for $\theta = 0°$ ($i = \sqrt{-1}$), $\kappa_{1,2} = -0.396, -0.177$ for $\theta = 120°$, $\kappa_{1,2} = -0.363, -0.098$ for $\theta = 135°$ and $\kappa_{1,2} = -0.136, -0.01$ for $\theta = 170°$. It can be seen that for the case of $\theta = 0°$ the beam has a pair of complex conjugate stress singularities, which is well-known for

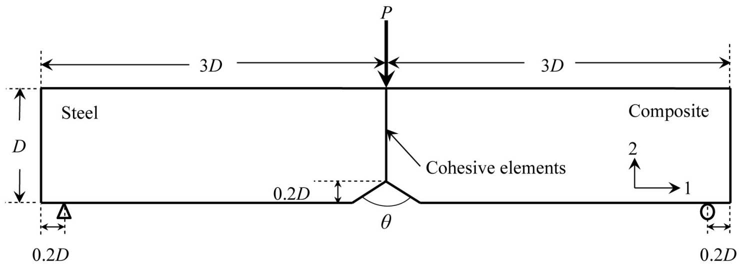

Figure 2: Metal-composite hybrid beam under three-point bending

a bimaterial crack. For other notch angles, the beam exhibits two real stress singularities, where one is much stronger than the other one. Furthermore, we see that, as the notch gets wider, the stress singularities become weaker. Clearly here we have a sufficiently large range of stress singularities to study the dependence of scaling law on the magnitude of the stress singularities.

By assuming that the bimaterial interface is much weaker than steel and composite, we can consider that fracture will always initiate and propagate along the bimaterial interface. This allows us to use mixed-mode cohesive elements to model the interface whereas the steel and composite materials are treated linear elastic. For the bimaterial interface, we consider that the adhesive layer is very thin and it does not vary with the beam size. Consequently, in the present simulation, we use a finite-thickness cohesive element (thickness $t = 1$ mm) with its formulation provided ABAQUS [ABAQUS (2011)]. Before reaching its strength, the cohesive layer is assumed to be linear elastic, where the traction-separation law can be written as:

$$T = \begin{bmatrix} T_n \\ T_l \\ T_m \end{bmatrix} = \begin{bmatrix} K_{nn} & K_{nm} & K_{nl} \\ K_{ln} & K_{ll} & K_{lm} \\ K_{mn} & K_{ml} & K_{mm} \end{bmatrix} \begin{bmatrix} \delta_n \\ \delta_m \\ \delta_l \end{bmatrix} = K\delta \tag{11}$$

where T = traction vector, K = stiffness tensor, δ = relative displacement vector, T_n = normal force, and T_l and T_m are the two orthogonal components of shear force T_s. The normal and shear tractions are considered to be uncoupled in the elastic regime, and so $K_{ij} = 0$ $(i \neq j)$. When the stress in the cohesive layer reaches the strength criterion, the interfacial crack will initiate and propagate which, in general, is subjected a combined normal and shear loading. Therefore, a mixed-mode fracture criterion for the cohesive layer is needed.

The mixed-mode damage initiation criterion is assumed to follow a quadratic form:

$$\frac{\langle T_n \rangle^2}{{f_n}^2} + \frac{T_l^2 + T_m^2}{{f_s}^2} = 1 \tag{12}$$

Here f_n, f_s = tensile and shear strengths, respectively. The Maclaulay bracket, defined as $\langle x \rangle = \max(x,0)$, is used here to ensure that the normal pressure would not contribute to the damage initiation. The damage evolution is formulated in an effective traction-displacement space, where the effective displacement is defined as $\bar{\delta} = \sqrt{\delta_n^2 + \delta_m^2 + \delta_n^2}$ [Camanho and Davila (2002); ABAQUS (2011)]. For the sake of simplicity, a linear softening behavior is adopted here to describe cohesive debonding after damage initiation. The total energy G^c, represented by the area under the softening curve of the effective stress-displacement space, follows an energetic mixed-mode criterion [ABAQUS (2011)]:

$$\left(\frac{G_I}{G_{If}}\right)^2 + \left(\frac{G_{II}}{G_{IIf}}\right)^2 = 1 \tag{13}$$

where G_{If}, G_{IIf} = fracture energy corresponding to Mode I and Mode II, respectively; G_{If}, G_{IIf} are the mode-I and II energy dissipations; and $G_c = G_I + G_{II}$. The following values are used for $f_n, f_s, G_{If}, G_{IIf}$: $f_n = 24$ MPa, $f_s = 12$ MPa, $G_n^c = 0.73$ KN/m, and $G_s^c = 1.15$ KN/m, which is similar to what was used in a recent study [Yu, Z. P. Bažant, and Le (2013)]. The cohesive laws in pure normal and shear modes are shown in Fig. 3.

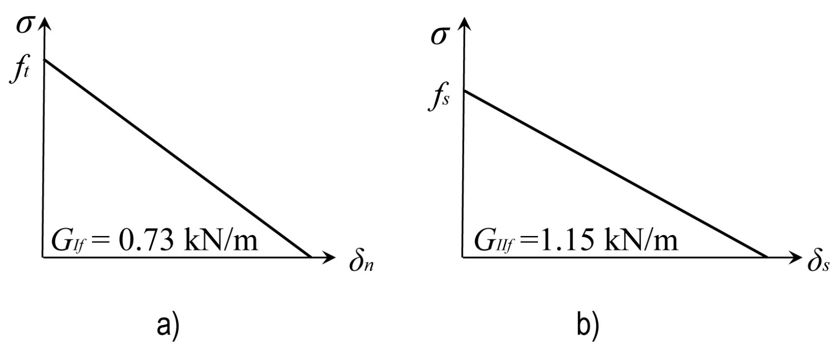

Figure 3: Mode-I and II cohesive laws

The present numerical study is performed in a deterministic framework. As mentioned earlier, deterministic simulations can correctly capture the size effect except for the large-size asymptote for structures with weak stress singularities. For the purpose of comparison with the deterministic calculation, we can consider the Weibull modulus in the aforementioned analytical formulation to be infinity so that the influence of the Weibull statistics on the large-size asymptote vanishes.

4 Results and discussion

Fig. 4 presents the simulated nominal stress-relative displacement curves for all the specimens, where the nominal stress is defined as $\sigma = P/bD$ (P = applied loading, D = beam depth, and b = beam width, which is chosen to be 1) and the relative displacement is calculated by normalizing the load-point displacement with respect to the beam depth D. It can be seen that as the beam size increases the post-peak part of the load-deflection curve becomes steeper, which indicates a more brittle failure behavior. For $D \geq 800$ mm, the load-deflection curve exhibit a sudden vertical drop, which implies a possible snap-back behavior. Since we use displacement controlled loading in the simulation, the snap-back behavior cannot be captured by the present simulation. Such a size-dependent failure behavior is a typical feature of quasibrittle fracture, which leads to the size effect on the nominal strength.

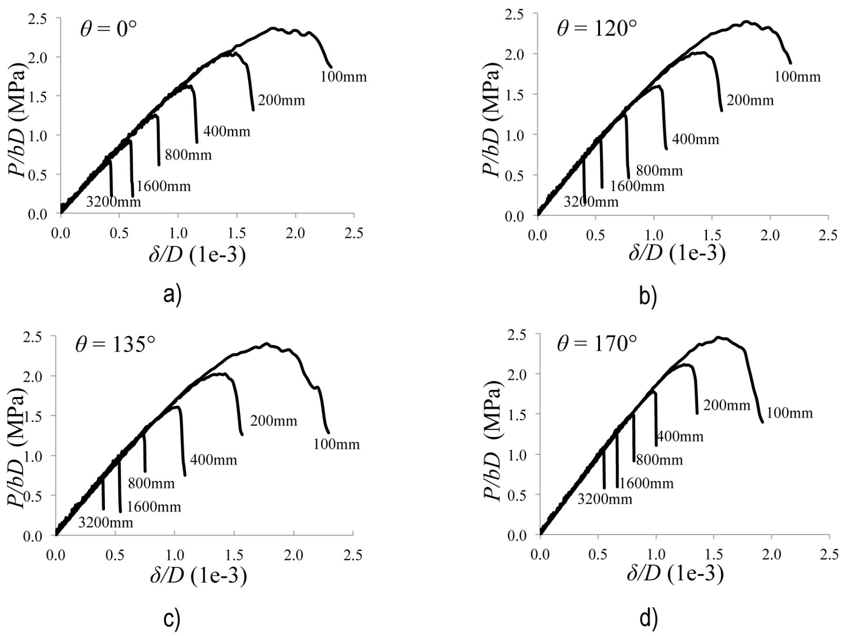

Figure 4: Simulated nominal stress-relative displacement curves

Denote the nominal strength of hybrid beams as $\sigma_N = P_{\max}/bD$. Fig. 5 shows the simulated size effect curves for the hybrid beams with the four different notch angles. It is clear that the size effect is strongly influenced by the magnitude of the stress singularities. When the stress singularities are sufficiently strong, the size effect is very similar to the classical Type-2 size effect [Bažant (1984); Bažant and Planas (1998); Bažant (2004, 2005)], which applies to quasibrittle structures with a large pre-existing crack formed prior to the peak load. As the stress singularity becomes weaker, the size effect is close to the Type-1 size effect [Bažant and Novák (2000); Bažant, Le, and Bazant (2009); Le, Bažant, and Bazant (2011)], which applies to quasibrittle structures with a smooth boundary. This qualitatively confirms the analytical model, which shows the transition from the Type-2 kind of size effect to the Type-1 kind as the stress singularities get weaker.

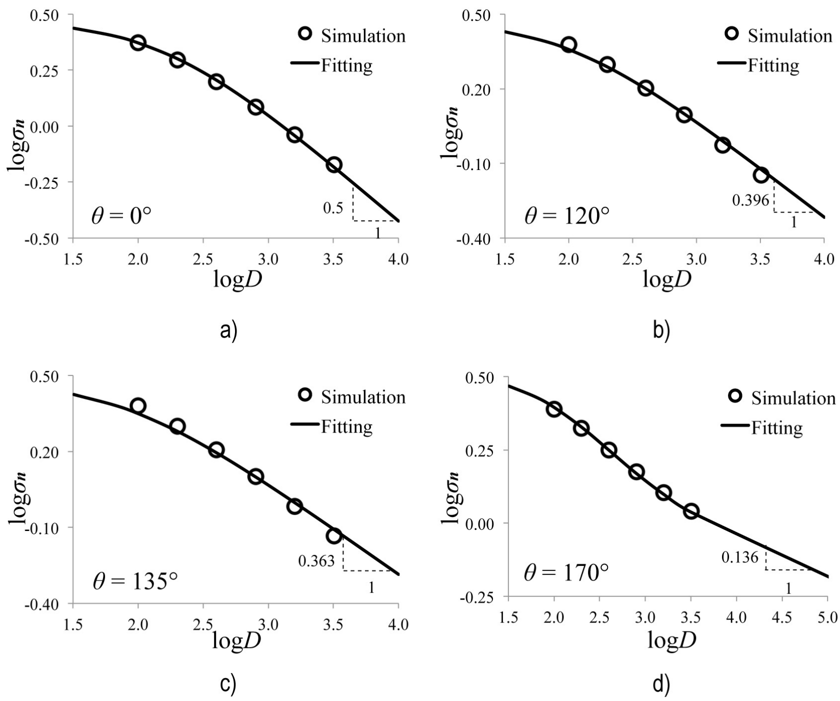

Figure 5: Simulated size effect curves and the optimum fits of Eq. 14

Now we use Eq. 10 to fit the simulated size effect curves. As mentioned earlier, here we consider that the Weibull modulus, which influences the large-size asymptote of the scaling behavior for structures with weak stress singularities, approaches infinity. Therefore, Eq. 10 becomes:

$$\bar{\sigma}_N = \sigma_0 \left\{ C_1 \mu^{-r}(D) \Psi_1^{-r/m} \exp[-(\kappa/\kappa_1)^2] + \frac{\mu^{-r}(D) D_b}{\exp[-(\kappa/\kappa_2)^2] D + l_p} \right\}^{1/r} \tag{14}$$

For beams with a sharp notch ($\theta = 0°$), the notch-tip stress field is governed by a pair of complex conjugate stress singularities, where the real part of the stress singularities is equal to $-1/2$. In this case, the energetic scaling term $\mu(D)$ simply reads: $\mu(D) = [1+(D/D_0)^{\gamma}]^{-1/2\gamma}$, according to Eq. 4. For the other notch angles, the singular stress field is governed by two distinct real stress singularities, and the difference in the magnitudes of these two singularities is large. Therefore, we may consider that the scaling of fracture of the material element at the notch tip is governed by the strongest stress singularity, and the energetic scaling term $\mu(D)$ becomes: $\mu(D) = [1+(D/D_0)^{-2\gamma\kappa_1}]^{-1/2\gamma}$.

For the optimum fitting, we used different values of D_0 and γ for different notch angles. As mentioned earlier, D_0 is determined by the structural geometry as well as the FPZ size. Under general mixed-mode fracture, the FPZ size would depend on the mode mixity. However, little information is available about such dependence. Therefore, here we just consider D_0 as a fitting parameter. Since D_b, l_p and r only governs the scaling behavior of structures with weak stress singularities, which is a relatively narrow range (e.g. $-0.2 \leq \kappa \leq 0$), we may assume that these parameters are independent of the notch angles. Since the Weibull modulus is considered to be infinity, $\Psi^{1/m}$ is equal to the maximum elastic principal stress in the singular stress zone. Based on the weakest link model, this maximum elastic principal stress must be evaluated at the center of the RVE at the notch tip. C_1 and σ_0 are left as two fitting constants.

Fig. 5 shows that the simulated size effect curves can be well fitted by Eq. 14. It is clear that, by ignoring the Weibull statistics (i.e. $m \to \infty$), the proposed size effect equation predicts a power law at the large-size asymptote, and the power-law exponent is equal to the dominant stress singularity for all cases. If we introduce randomness in cohesive behavior of the interface for the simulation, the Weibull statistics would prevail at the large-size limit and the power-law exponent would be equal to $\kappa - 1/m$. The good agreement between the simulation results and Eq. 14 indicates that the general dependence of the size effect on the magnitude of stress singularities can be explained by the energetic-statistical scaling model. From Fig. 5, it is also observed that, for beams with strong stress singularities (i.e. $\theta = 0°, 120°, 135°$), the power-law large-size asymptote (i.e. $\sigma_N \propto D^{\kappa}$) has not been reached even for the largest beam (e.g. $D = 3200$ mm), which implies that the LEFM is insufficient. This implies that for most metal-composite hybrid structures the failure would be quasibrittle and we should use nonlinear fracture mechanics for the failure analysis.

5 Concluding remarks

By using a simple mixed-mode cohesive crack model, we investigate the scaling of fracture of a series of bimaterial hybrid beams with a V-notch under three-point bending. It is shown that the size effect on the structural strength strongly depends on the magnitude of the stress singularities. This can be explained by the fact that, as the stress singularities are strong, the fracture will always initiate from the bimaterial corner and the corresponding size effect is energetic (deterministic) whereas for structures with weak stress singularities the location of crack initiation becomes uncertain and a combined energetic-statistical size effect would prevail.

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