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Generalized Rayleigh Wave Dispersion Analysis in a Pre-stressed Elastic Stratified Half-space with Imperfectly Bonded Interfaces

M. Negin[1], S.D. Akbarov[2,3] and M.E. Erguven[1]

Abstract: Within the framework of the piecewise homogeneous body model the influence of the shear-spring type imperfect contact conditions on the dispersion relation of the generalized Rayleigh waves in the system consisting of the initially stressed covering layer and initially stressed half plane is investigated. The second version of the small initial deformation theory of the three-dimensional linearized theory of elastic waves in initially stressed bodies is applied and the elasticity relations of the materials of the constituents are described by the Murnaghan potential. The magnitude of the imperfectness of the contact conditions is estimated through the shear-spring type parameter. Consequently, the influence of the imperfectness of the contact conditions on the generalized Rayleigh wave propagation velocity is studied through the influence of the values of this parameter. Numerical results on the action of the imperfectness of the contact conditions and the influence of the initial stresses in the constituents on the wave dispersion curves are presented and discussed. In particular, it is established that the magnitude of action of the imperfectness of the contact conditions under the influence of the initial stresses on the wave propagation velocity cannot be limited with corresponding ones obtained in the case where the contact between the constituents is complete and in the case where this contact is full slipping one. The possible application of the obtained results on the geophysical and geotechnical engineering is also discussed.

Keywords: Generalized Rayleigh wave, shear-spring type imperfect contact, initial stresses, Murnaghan potential.

[1] Istanbul Technical University, Faculty of Civil Engineering, Department of Earthquake Engineering, 34469 Maslak, Istanbul, Turkey. Email: mnegin@itu.edu.tr
[2] Yildiz Technical University, Faculty of Mechanical Engineering, Department of Mechanical Engineering, Yildiz Campus, 34349 Besiktas, Istanbul, Turkey. Email: akbarov@yildiz.edu.tr
[3] Institute of Mathematics and Mechanics of National Academy of Sciences of Azerbaijan, 37041Baku, Azerbaijan.

1 Introduction

The theory of elastic surface waves in layered half-spaces, though it is an old topic in classical sense, it has found highly important scientific and engineering applications through the last couple of decades. Fields of applications are vast but some notable areas might be acoustic, smart materials, metrology, Earth sciences, subsurface explorations, non-destructive testing and damage detection. Indeed, variety of mechanical, material and structural properties, presence of damages and/or cracks, different external loading, etc. make the study of these wave processes still an active and interesting field of research these days.

There are two types the most important investigations in this regard the first of which relates to study of the effect of imperfect bonding between the covering layer and half-space on a surface wave propagation and its characteristics. But the other one is the study of influence of initial stresses which exist in the constituents of the stratified half-space on this wave.

The investigations of the first type problems are motivated by very high sensitivity of the wave propagation characteristics to interface defects such as weak-bonding between the constituents which can be caused by interface damages or chemical actions and etc. The results of these investigations can be successfully applied for determination of various defects between the covering layer and half-space.

But the investigations of the second type problems are motivated by the non-destructive determination of mostly the residual (or initial) stresses in the elements of constructions. Currently the results of these investigations are successfully employed for determination of the mentioned residual stresses. At the same time, the foregoing investigations are also propounded the fundamental questions of the dynamics of the non-homogeneous deformable solid bodies.

It is evident from the foregoing discussions that the investigations of the generalized Rayleigh wave dispersion in a pre-stressed elastic stratified half-plane with imperfectly bonded interfaces, which is the topic of the present paper, lies at the junction of the above-noted two type problems.

Note that propagation of the elastic waves in pre-stressed bodies is studied by utilizing of the Three-dimensional Linearized Theory of Elastic Waves in Initially Stressed Bodies (TLTEWISB). The relations and equations of the TLTEWISB are obtained from the exact relations and equations of the non-linear theory of elastodynamics by linearization with respect to small dynamical perturbations. The general questions of the TLTEWISB have been elaborated in many investigations such as in works by Biot (1965), Truestell (1961), Eringen and Suhubi (1975), Guz (2004) and others. It should be noted that there are some versions of the TLTEWISB which were detailed in the monograph by Guz (2004). These versions of the TLTEWISB are distinguished from each other with respect to the magnitude of the initial strains. The version of the TLTEWISB developed for high-elastic materials, according to which the initial strains in the bodies are determined within the scope of the non-linear theory of elasticity without any restrictions on the magnitude of the initial strains, is called the large (or finite) initial deformation version. The version of the TLTEWISB, according to which, an initial stress-strain state in bodies is determined within the scope of the geometrical nonlinear theory of elasticity and changes to the elementary areas and volumes as a result of the initial deformation are not taken into account, is called the first version of the small initial deformation theory of the TLTEWISB. The second version of the small initial deformation theory of the TLTEWISB is the version, according to which, an initial stress-strain state in bodies is determined within the scope of the classical linear theory of elasticity.

The review of the investigations carried out before the year 2007 on the wave propagation in pre-stressed bodies has been made in the papers by Guz (2002), Guz (2005) and Akbarov (2007). The detail consideration of the results of these investigations was made in the monographs by Biot (1965), Eringen and Suhubi (1975), Guz (2004). The review of the more resent related investigations can be found in papers by Akbarov (2012), Akbarov and Ipek (2012), Akbarov; Agasiyev and Zamanov (2011) and etc. However, in considerable part of these works the interface between layers was assumed to be bonded perfectly, which it is not the real case in many applications.

Two classical boundary conditions, that are, perfect bonded interfaces and full slipping ones idealize real physical contact between two layers. In perfect contact condition also known as welded interfaces all the stress and displacement components are continuous across the interface, whereas, in the case of full slipping conditions also known as non-welded interfaces there is a discontinuity in the shear component of the displacement [Rokhlin and Wang (1991)]. An actual interface conditions between two layers is much more complicated in mathematical modeling viewpoint and different investigators spent significant efforts to describe the real physical conditions by different mechanical models. To summarize some, Martin (1992) has reviewed imperfect interface models and formulated the problem mathematically. Pecorari (2001) has investigated the scattering problem of a Rayleigh wave by surface-breaking cracks with partial contact interfaces. Leungvichcharoen and Wijeyewickrema (2003) has discussed the effect of an imperfect interface on harmonic extensional wave propagation in a pre-stressed, symmetric layered composite by employing shear spring type resistance model to simulate the imperfect interface. Zhou; Lu and Chen (2012) also have tried to simulate the imperfect interface conditions by using linear spring model to study bulk wave propagation in laminated piezomagnetic and piezoelectric plates with initial stresses. Kumara and Singh (2009) have considered the propagation of plane waves at an imperfectly bonded interface of two orthotropic generalized thermoelastic rotating half-spaces with different elastic and thermal properties. Liu; Wang and Wang (2010) have analyzed SH surface waves in a piezoelectric elastic layer and an elastic half-space structure with imperfect bonding. Similar model was used by Huang and Li (2010) to study the propagation of shear waves along a weak interface of two dissimilar magnetoelectric or magnetoelectroelastic materials. Reflection and transmission problem of plane waves between piezoelectric and piezomagnetic media with imperfectly bonded interfaces has also been considered by Pang and Liu (2011). Akbarov and Ipek (2010, 2012) have studied the influence of the imperfectness of the interface conditions on the dispersion of the axisymmetric longitudinal waves in the pre-strained compound cylinder under the shear spring type model of the contact condition between layers. Kepceler (2010) has also carried out investigations of a similar type for the initially stressed bi-material compounded circular cylinder.

To the best of the authors' knowledge, up to now there has not been made any investigation related to the study of the influence of imperfectness of the contact conditions on the dispersion characteristics of the generalized Rayleigh wave not only in initially stressed stratified half-space, but also in the stratified half-spaces which has not any initial stresses. Akbarov and Ozisik (2003) have studied the influence of the third order elastic constants on the velocity of the generalized Rayleigh wave propagation in a pre-stressed stratified half-plane, but they also considered only the perfect contact conditions in their analysis. So in the present work, within the framework of the second version of the small initial deformation theory of the TL-TEWISB we attempt to study the effects of the imperfect interface conditions on the generalized Rayleigh wave propagation in a pre-stressed stratified half-plane. Piecewise homogeneous body model were applied and elasticity relations for materials of the constituents were described through the Murnaghan potential. In the classical sense (i.e. in the cases where the initial stresses in the constituents are absent), the investigations carried out in the present paper can be considered as a development of the paper by Tolstoy and Usdin (1953) (in which the generalized Rayleigh waves were studied for the perfectly bounded stratified half-plane) for the concrete selected pair of materials under imperfect contact between the layer and half-plane. At the same time, the investigations carried out in the present paper can be considered as a development of the paper by Akbarov and Ozisik (2003) also for the case where the contact between the constituents of the stratified half-plane is imperfect. Consequently, the goal of the investigations is a study of the role of imperfectness of the contact conditions on the dispersion characteristics of the pre-stressed bi-material non-linear elastic systems.

2 Formulation of the problem

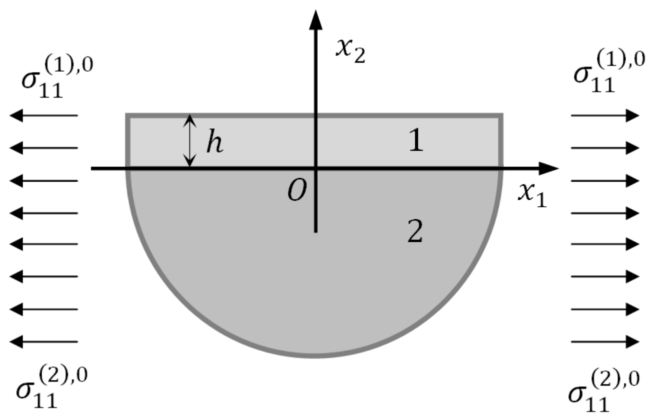


Figure 1: The geometry of the considered stratified half-plane.

The geometry of the problem is shown in Fig. 1. It is assumed that the half-plane covered by the layer with thickness h. In the natural state we determine the positions of the points by the Lagrangian coordinates which coincide with the Cartesian system of coordinates $Ox_1x_2x_3$. The layer and the half-plane occupy the regions $\{-\infty < x_1 < +\infty, 0 \leq x_2 \leq h, -\infty < x_3 < +\infty\}$ and $\{-\infty < x_1 < +\infty, -\infty \leq x_2 \leq 0, -\infty < x_3 < +\infty\}$, respectively. Note that the following notation will be used through the formulations: the values related to the layer and half-plane are denoted by upper indices (1) and (2) respectively and the values related to the initial (or residual stresses) are denoted by upper indices (m), 0 where m = 1, 2. We consider the case where initial stresses in the layer and half-plane are determined as follows:

$$\sigma_{11}^{(m),0} = \text{const}_m \neq 0, \quad m = 1,2, \quad \sigma_{ij}^{(m),0} = 0, \quad \text{for} \quad i = j \neq 1. \tag{1}$$

All investigations in the present paper are made in the framework of the second version of the small initial deformation theory of the TLTEWISB in the plane strain state in the Ox_1x_2 plane.

According to Guz (2004), the equations of motion for the considered case are written as:

$$\frac{\partial \sigma_{11}^{(m)}}{\partial x_1} + \frac{\partial \sigma_{12}^{(m)}}{\partial x_2} + \sigma_{11}^{(m),0} \frac{\partial^2 u_1^{(m)}}{\partial x_1^2} = \rho^{(m)} \frac{\partial^2 u_1^{(m)}}{\partial t^2},$$
$$\frac{\partial \sigma_{12}^{(m)}}{\partial x_1} + \frac{\partial \sigma_{22}^{(m)}}{\partial x_2} + \sigma_{11}^{(m),0} \frac{\partial^2 u_2^{(m)}}{\partial x_1^2} = \rho^{(m)} \frac{\partial^2 u_2^{(m)}}{\partial t^2}. \tag{2}$$

In (2) the conventional notation is used. We assume that the following boundary conditions on the free face plane of the covering layer satisfy:

$$\sigma_{12}^{(1)}\Big|_{x_2=h}=0, \qquad \sigma_{22}^{(1)}\Big|_{x_2=h}=0. \tag{3}$$

Now we consider the formulation of the imperfect contact conditions on the interface plane between the covering layer and half-plane. It should be noted that, in general, the imperfectness of the contact conditions is identified by discontinuities of the displacements and forces across the mentioned interface. A review of the mathematical modeling of the various types of incomplete contact conditions for elastodynamics problems has been detailed in a paper by Martin (1992). It follows from this paper that for most models the discontinuity of the displacement $\mathbf{u}^+$ and force $\mathbf{f}^+$ vectors on one side of the interface are assumed to be linearly related to the displacement $\mathbf{u}^-$ and force $\mathbf{f}^-$ vectors on the other side of the interface. This statement, as in the paper by Rokhlin and Wang (1991), can be presented as follows:

$$[\mathbf{f}]=\mathbf{C}\mathbf{u}^-+\mathbf{D}\mathbf{f}^-, \quad [\mathbf{u}]=\mathbf{G}\mathbf{u}^-+\mathbf{F}\mathbf{f}^-, \tag{4}$$

where $\mathbf{C}$, $\mathbf{D}$, $\mathbf{G}$ and $\mathbf{F}$ are three-dimensional (3×3) matrices and the square brackets indicate a jump in the corresponding quantity across the interface. Consequently, if the interface is at $x_2=0$, then $[\mathbf{u}]=\mathbf{u}|_{x_2=0^+}-\mathbf{u}|_{x_2=0^-}$, $[\mathbf{f}]=\mathbf{f}|_{x_2=0^+}-\mathbf{f}|_{x_2=0^-}$. It follows from (4) that we can write incomplete contact conditions for various particular cases by selection of the matrices $\mathbf{C}$, $\mathbf{D}$, $\mathbf{G}$ and $\mathbf{F}$. One such selection was made in the paper by Jones and Whitter (1967), according to which, it was assumed that $\mathbf{C}=\mathbf{D}=\mathbf{G}=\mathbf{0}$. In this case the following can be obtained from (4):

$$[\mathbf{f}]=\mathbf{0}, \quad [\mathbf{u}]=\mathbf{F}\mathbf{f}^-, \tag{5}$$

where $\mathbf{F}$ is a constant diagonal matrix. The model (5) simplifies significantly the solution procedure of the corresponding problems and is adequate in many real cases. Therefore, this model (i.e. the model (5)) is called a shear-spring type resistance model and has been used in many investigations carried out within the framework of classical elastodynamics by Jones and Whitter (1967), Berger; Martin and McCaffery (2000), and within the framework of the TLTEWISB by Kepceler (2010) and by Akbarov and Ipek (2010, 2012). According to this statement, we also use the model (5) for the mathematical formulation of the imperfectness of the contact conditions and these conditions are written as follows:

$$\sigma_{i2}^{(1)}|_{x_2=0}=\sigma_{i2}^{(2)}|_{x_2=0}\ ,\ i=1,2, \quad u_2^{(1)}|_{x_2=0}=u_2^{(2)}|_{x_2=0},$$

$$u_1^{(1)}|_{x_2=0}-u_1^{(2)}|_{x_2=0}=F\frac{h}{\mu^{(1)}}\sigma_{12}^{(1)}|_{x_2=0}, F>0 \tag{6}$$

where F is the non-dimensional shear spring parameter and $0 \leq F \leq \infty$. Note that the case where $F = 0$ means that the displacement component across the interface is continuous and therefore the half-space and covering layer are perfectly bonded together or to say that they are in welded contact condition. At the other extreme, $F = \infty$ implies that the half-space and covering layer are completely unbounded together and the full slipping condition is satisfied. Thus, any other finite positive values of F expresses different imperfect interface conditions in the problem.

Moreover, we assume that the following decay conditions are satisfied:

$$\sigma_{ij}^{(2)}\Big|_{x_2 \to -\infty} \to 0, \qquad u_i^{(2)}\Big|_{x_2 \to -\infty} \to 0, \quad i = 1,2. \tag{7}$$

As stated above, we assume that the constitutive relations of the materials of the constituents are given by the Murnaghan potential which is given as follows Guz (2004):

$$\Phi^{(m)} = \frac{1}{2}\lambda^{(m)}\left(A_1^{(m)}\right)^2 + \mu^{(m)}A_2^{(m)} + \frac{a^{(m)}}{3}\left(A_1^{(m)}\right)^3 + b^{(m)}A_1^{(m)}A_2^{(m)} + \frac{c^{(m)}}{3}A_3^{(m)}, \tag{8}$$

where $\lambda^{(m)}$ and $\mu^{(m)}$ are Lame's and $a^{(m)}$, $b^{(m)}$ and $c^{(m)}$ are the third order elasticity constants. Here $A_1^{(m)}$, $A_2^{(m)}$ and $A_3^{(m)}$ are the first, second and the third algebraic invariants of Green's strain tensor respectively. For the case under consideration, the expressions of these invariants are:

$$A_1^{(m)} = \varepsilon_{11}^{(m)} + \varepsilon_{22}^{(m)}, \quad A_2^{(m)} = \left(\varepsilon_{11}^{(m)}\right)^2 + 2\left(\varepsilon_{12}^{(m)}\right)^2 + \left(\varepsilon_{22}^{(m)}\right)^2,$$

$$A_3^{(m)} = \left(\varepsilon_{11}^{(m)}\right)^3 + 3\left(\varepsilon_{12}^{(m)}\right)^2\left(\varepsilon_{11}^{(m)} + \varepsilon_{22}^{(m)}\right) + \left(\varepsilon_{22}^{(m)}\right)^3, \tag{9}$$

where

$$\varepsilon_{ij}^{(m)} = \frac{1}{2}\left(\frac{\partial u_i^{(m)}}{\partial x_j} + \frac{\partial u_j^{(m)}}{\partial x_i}\right). \tag{10}$$

According to Guz (2004), linearized constitutive relations for the layer and half-plane materials are obtained in the following form:

$$\sigma_{11}^{(m)} = A_{11}^{(m)}\varepsilon_{11}^{(m)} + A_{12}^{(m)}\varepsilon_{22}^{(m)}, \quad \sigma_{22}^{(m)} = A_{12}^{(m)}\varepsilon_{11}^{(m)} + A_{22}^{(m)}\varepsilon_{22}^{(m)}, \quad \sigma_{12}^{(m)} = 2\mu_{12}^{(m)}\varepsilon_{12}^{(m)}, \tag{11}$$

where

$$\begin{aligned} A_{11}^{(m)} =& \lambda^{(m)} + 2\mu^{(m)} + \frac{\sigma_{11}^{(m),0}}{\mu^{(m)}}\left(2b^{(m)} + c^{(m)}\right) \\ &+ \frac{2\sigma_{11}^{(m),0}}{3K_0^{(m)}}\left[\left(a^{(m)} + b^{(m)}\right) - \left(2b^{(m)} + c^{(m)}\right)\frac{\lambda^{(m)}}{2\mu^{(m)}}\right], \end{aligned}$$

$$A_{22}^{(m)} = \lambda^{(m)} + 2\mu^{(m)} + \frac{2\sigma_{11}^{(m),0}}{3K_0^{(m)}}\left[\left(a^{(m)}+b^{(m)}\right) - \left(2b^{(m)}+c^{(m)}\right)\frac{\lambda^{(m)}}{2\mu^{(m)}}\right],$$

$$A_{12}^{(m)} = \lambda^{(m)} + \frac{b^{(m)}}{\mu^{(m)}}\sigma_{11}^{(m),0} + \frac{2\sigma_{11}^{(m),0}}{3K_0^{(m)}}\left[a^{(m)} - b^{(m)}\frac{\lambda^{(m)}}{\mu^{(m)}}\right],$$

$$\mu_{12}^{(m)} = \mu^{(m)} + \frac{b^{(m)}}{3K_0^{(m)}}\sigma_{11}^{(m),0} + \frac{c^{(m)}\sigma_{11}^{(m),0}}{4\mu^{(m)}}\left[\frac{\lambda^{(m)}+2\mu^{(m)}}{3K_0^{(m)}}\right],$$

$$K_0^{(m)} = \lambda^{(m)} + \frac{2\mu^{(m)}}{3}. \tag{12}$$

This completes the formulation of the problem and in the case where $\sigma_{11}^{(1),0} = \sigma_{11}^{(2),0} = 0$ this formulation transforms to the corresponding one made within the scope of the classical linear theory of elastodynamics.

3 Solution procedure and obtaining the dispersion relation

Each displacements component of the considered system are represent as follows:

$$u_1^{(m)} = \phi_1^{(m)}(x_2)\sin(kx_1-\omega t), \quad u_2^{(m)} = \phi_2^{(m)}(x_2)\cos(kx_1-\omega t). \tag{13}$$

This way we obtain the following equations for the $\phi_1^{(m)}(x_2)$ and $\phi_2^{(m)}(x_2)$ from the Eqs. (1), (2), (11)–(13).

$$\begin{cases} \frac{d^2\phi_1^{(m)}}{d(kx_2)^2} + b_{21}^{(m)}\phi_1^{(m)} + c_{21}^{(m)}\frac{d\phi_2^{(m)}}{d(kx_2)} = 0 \\ \frac{d^2\phi_2^{(m)}}{d(kx_2)^2} + b_{22}^{(m)}\phi_2^{(m)} + c_{22}^{(m)}\frac{d\phi_1^{(m)}}{d(kx_2)} = 0 \end{cases}, \tag{14}$$

where

$$b_{21}^{(m)} = -\frac{A_{11}^{(m)}}{\mu_{12}^{(m)}} - \frac{\sigma_{11}^{(m),0}}{\mu_{12}^{(m)}} + \frac{\rho^{(m)}\omega^2}{\mu_{12}^{(m)}k^2}, \quad c_{21}^{(m)} = \frac{-A_{12}^{(m)} - \mu_{12}^{(m)}}{\mu_{12}^{(m)}},$$

$$b_{22}^{(m)} = -\frac{\mu_{12}^{(m)}}{A_{22}^{(m)}} - \frac{\sigma_{11}^{(m),0}}{A_{22}^{(m)}} + \frac{\rho^{(m)}\omega^2}{A_{22}^{(m)}k^2}, \quad c_{22}^{(m)} = \frac{\mu_{12}^{(m)} + A_{12}^{(m)}}{A_{22}^{(m)}}. \tag{15}$$

We can solve the system (14) using linear operator method as follows:

$$\begin{cases} \left(D^2 + b_{21}^{(m)}\right)\phi_1^{(m)} + c_{21}^{(m)}D\phi_2^{(m)} = 0 \\ c_{22}^{(m)}D\phi_1^{(m)} + \left(D^2 + b_{22}^{(m)}\right)\phi_2^{(m)} = 0 \end{cases}, \tag{16}$$

or in matrix form as:

$$\begin{bmatrix} D^2+b_{21}^{(m)} & c_{21}^{(m)}D \\ c_{22}^{(m)}D & D^2+b_{22}^{(m)} \end{bmatrix} \begin{bmatrix} \phi_1^{(m)} \\ \phi_2^{(m)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{17}$$

where D is the differentiation operator: $D = d/d\,(kx_2)$. This homogenous system (17) has non-trivial solution only when the determinant of operational matrix be zero, that is:

$$\left(D^2+b_{21}^{(m)}\right)\left(D^2+b_{22}^{(m)}\right) - c_{21}^{(m)}c_{22}^{(m)}D^2 = 0,$$

or

$$D^4 + B_2^{(m)}D^2 + C_2^{(m)} = 0, \tag{18}$$

where

$$B_2^{(m)} = b_{22}^{(m)} + b_{21}^{(m)} - c_{21}^{(m)}c_{22}^{(m)}, \quad C_2^{(m)} = b_{21}^{(m)}b_{22}^{(m)}. \tag{19}$$

Therefore we derive the following differential equation for $\phi_2^{(m)}(x_2)$:

$$\left(D^4 + B_2^{(m)}D^2 + C_2^{(m)}\right)\phi_2^{(m)}(x_2) = 0. \tag{20}$$

We determine the solution to the Eq. (20) as follows:

$$\begin{aligned} \phi_2^{(1)}(x_2) =& Z_1^{(1)}\exp\left(R_1^{(1)}kx_2\right) + Z_2^{(1)}\exp\left(-R_1^{(1)}kx_2\right) \\ & + Z_3^{(1)}\exp\left(R_2^{(1)}kx_2\right) + Z_4^{(1)}\exp\left(-R_2^{(1)}kx_2\right), \end{aligned}$$

$$\phi_2^{(2)}(x_2) = Z_1^{(2)}\exp\left(R_1^{(2)}kx_2\right) + Z_3^{(2)}\exp\left(R_2^{(2)}kx_2\right), \tag{21}$$

where

$$R_1^{(m)} = \sqrt{-\frac{B_2^{(m)}}{2} + \sqrt{\frac{\left(B_2^{(m)}\right)^2}{4} - C_2^{(m)}}}, \quad R_2^{(m)} = \sqrt{-\frac{B_2^{(m)}}{2} - \sqrt{\frac{\left(B_2^{(m)}\right)^2}{4} - C_2^{(m)}}}. \tag{22}$$

In a similar way using (22) we can also determine the function $\phi_1^{(m)}(x_2)$ from Eq. (16). Finally, we obtain the dispersion equation considering the conditions (3)–(7).

This dispersion equation after some mathematical manipulations can be expressed formally as follows:

$$\det\left\|\alpha_{ij}\left(c,kh,F,\sigma_{11}^{(1),0},\sigma_{11}^{(2),0},a^{(1)},b^{(1)},c^{(1)},a^{(2)},b^{(2)},c^{(2)}\right)\right\|=0, \tag{23}$$

where $i;j=1,2,...,6$ and

$$c=\frac{\omega}{k},\quad c_1^{(m)}=\sqrt{\frac{\lambda^{(m)}+2\mu^{(m)}}{\rho^{(m)}}},\quad c_2^{(m)}=\sqrt{\frac{\mu^{(m)}}{\rho^{(m)}}}. \tag{24}$$

The explicit expressions of the α_{ij} in the dispersion equation (23) are presented in Appendix A by formulae A1.

This completes the solution method of the problem under consideration.

4 Numerical results and discussion

We will assume that,

$$\text{Re}\,R_1^{(1)}=\text{Re}R_2^{(1)}=0,\quad R_1^{(2)}>0,\quad R_2^{(2)}>0. \tag{25}$$

In this case, the solution (21) corresponds to such a wave propagation in the layered half-plane that the layer undergoes an oscillatory motion in the Ox_2 direction propagating in the Ox_1 direction with velocity c. The disturbances in the layer decay exponentially with depth in the half-plane and therefore the wave can be considered as a generalized Rayleigh wave confined to the pre-stressed covered layer [Tolstoy and Usdin (1953)].

First we consider the numerical results for the case where $\sigma_{11}^{(1),0}=\sigma_{11}^{(2),0}=0$, i.e. for the case where initial stresses in the constituents are absent. We recall that this case for the Poisson material has been discussed by Tolstoy and Usdin (1953) and also discussed in the monograph by Eringen and Suhubi (1975). It was established by Tolstoy and Usdin (1953) that the dispersion equation (23) has infinitely many modes unlike ordinary Rayleigh waves, which can propagate only in one mode. Moreover, the dispersion curves related to each mode has two branches which were denoted by M_{1n} and M_{2n} respectively for the $n-th$ mode. For the first M_{1n} branch the displacement of the layer circumscribes the ellipse similar to the ordinary Rayleigh waves, but for the second M_{2n} branch leads to an opposite type of motion.

As noted above, numerical results are given by Tolstoy and Usdin (1953) for the Poisson materials, i.e. for the case where $\lambda^{(1)}=\mu^{(1)}$, $\lambda^{(2)}=\mu^{(2)}$ (it is assumed that $\upsilon^{(1)}=\upsilon^{(2)}=0.25$, where $\upsilon^{(m)}$ is Poisson's coefficient), under $c_1^{(1)}/c_2^{(1)}=$

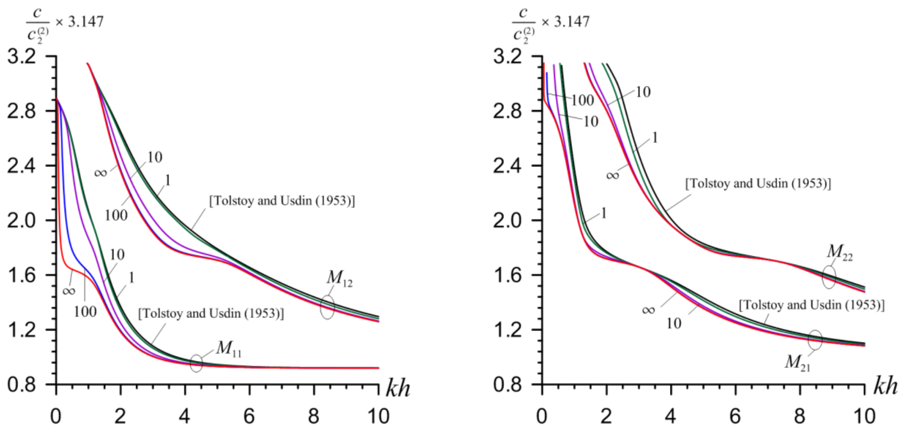
$c_1^{(2)}/c_2^{(2)} = \sqrt{3}$, $c_2^{(2)}/c_2^{(1)} = 3.147$, $\rho^{(2)}/\rho^{(1)} = 1.374$ (where $\rho^{(m)}$ is the density of the $m-th$ material). After programming of the Eq. (23), the above mentioned case (Poisson materials) was considered first in our numerical studies. Fig. 2 shows the dependencies (dispersion curves) between c'' and kh for different values of shear spring parameter F. In this figure the graphs denoted by $M_{11}$and M_{12} (Fig. 2a) correspond respectively to the first and second branch of the first mode, whereas the graphs denoted by M_{21} and M_{22} (Fig. 2b) correspond to the first and second branch of the second mode. It should be noted that the graphs denoted by M_{11}, M_{12}, M_{21} and M_{22}, and obtained for the case where $F = 0$(i.e. for the case where the contact between the constituents is perfect) coincide with the corresponding ones given in the paper by Tolstoy and Usdin (1953). This situation gives a certain guarantee on the correctness of the algorithm-programs constructed for the numerical solution to the dispersion equation (23).

We also, consider similar results obtained for some pairs of the real materials. Values of the mechanical constants of these materials are given in Tab. 1 (i.e. the values of the mechanical constants which enter the expression (8) of the Murnaghan potential). We select four pairs from these materials. For the *I*, *II*, *III* and *IV* pairs, the material of the covering layer we take as *bronze*, *brass* 59-1, *brass* 62 and *acrylic plastic* respectively, but for all the pairs the material of the half-plane we take as *steel*. The graphs related to the foregoing pairs are given in Figs. 3 (for the *I* pair), 4 (for the pairs *II* and *III*) and 5 (for the pair *IV*). Note that the results given in these figures and obtained for the case where $F = 0$ coincide with corresponding ones obtained in the paper by Akbarov and Ozisik (2003).

Note that, according to the expressions (11) and (12), as under calculation of the foregoing results it is assumed that $\sigma_{11}^{(1),0} = \sigma_{11}^{(2),0} = 0$, therefore the third order of elastic constants of the selected materials do not influence on these results.

Table 1: The values of elastic constants of the selected materials [Guz (2004)].

Materials	ρ (g/cm^3)	$\lambda \times 10^{-4}$ (MPa)	$\mu \times 10^{-4}$ (MPa)	$a \times 10^{-5}$ (MPa)	$b \times 10^{-5}$ (MPa)	$c \times 10^{-5}$ (MPa)
Steel 3	7.795	9.26	7.75	-2.35	-2.75	-4.90
Bronze	7.20	8.16	3.84	1.20	-3.10	4.80
Brass 59–1	7.20	9.49	4.47	-0.70	2.70	-3.40
Brass 62	7.20	9.49	4.47	-2.80	-2.10	-3.20
Plexiglass (Lucite)	1.16	0.404	0.19	2.68×10^{-3}	-3.12×10^{-2}	-6.77×10^{-2}



(a) (b)

Figure 2: Dispersion curves for Poisson material [Tolstoy and Usdin (1953)]: First and second branches of the first (a) and the second (b) modes.

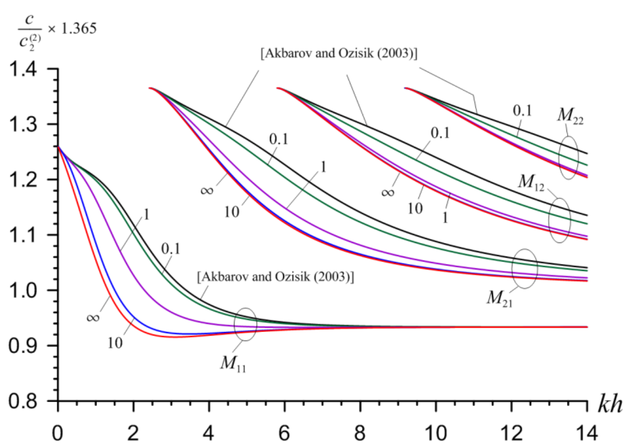


Figure 3: Dispersion curves for the *I* pair of materials.

Thus the dispersion curves obtained for the pairs *II* and *III* coincide because, according to the data given in Tab. 1, the constants ρ, λ and μ are the same for *brass* 59-1 and *brass* 62 (Fig 4). Moreover note that, for clarity of the illustration the first and second modes of the graphs obtained for the *IV* pair are given separately in Figs. 5a and 5b respectively.

Now we analyze the foregoing numerical results which are obtained within the

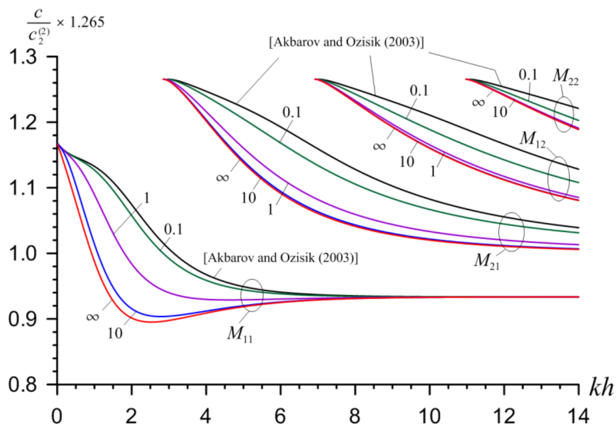


Figure 4: Dispersion curves for the *II* and *III* pair of materials.

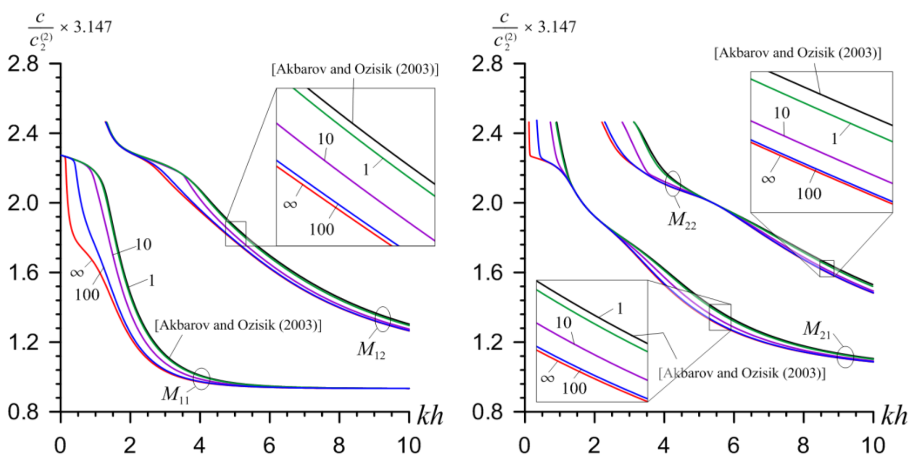


Figure 5: Dispersion curves for the *IV* pair of materials: The first and the second branches of the first (a) and the second (b) modes.

framework of the restriction (25). According to this restriction, it must be $c\big/c_2^{(2)} < 1$ and $c\big/c_1^{(1)} > 1$, i.e. the near-surface waves propagated in the system under consideration is subsonic in the half-plane, but it is supersonic in the covering layer. Thus, it follows from Figs. 2, 3, 4 and 5 that the dimensionless wavenumber kh has cut off values for the second branch of the first mode, the first and second branches of the second mode. We denote these cut of values through $(kh)_{cf21}^{P}$,

$(kh)_{cf12}^{P}$, $(kh)_{cf22}^{P}$for the pair of materials selected by Tolstoy and Usdin (1953) (Fig. 2), through $(kh)_{cf21}^{I}$' $(kh)_{cf12}^{I}$, $(kh)_{cf22}^{I}$ for the *I* pair of materials (Fig. 3), through $(kh)_{cf21}^{II}$' $(kh)_{cf12}^{II}$, $(kh)_{cf22}^{II}$ for the *II* pair of materials (Fig. 4) and through $(kh)_{cf21}^{IV}$' $(kh)_{cf12}^{IV}$, $(kh)_{cf22}^{IV}$ for the *IV* pair of materials (Fig. 5). Moreover, we introduce the notation c_{11}^{P}, c_{21}^{P} (c_{12}^{P}, c_{22}^{P}) for the pair of materials selected by Tolstoy and Usdin (1953), c_{11}^{I}, c_{21}^{I} (c_{12}^{I}, c_{22}^{I}) for the *I* pair of materials; c_{11}^{II}, c_{21}^{II} (c_{12}^{II}, c_{22}^{II}) for the *II* pair of materials and c_{11}^{IV}, c_{21}^{IV} (c_{12}^{IV}, c_{22}^{IV}) for the *IV* pair of materials for indication of the wave propagation velocity related to the first (second) branches of the first and the second modes.

According to the foregoing results, we can conclude that the imperfectness between the constituents causes to decrease of the wave propagation velocity of the all above-noted branches and modes. In this cases values of the velocity decrease monotonically with the shear-spring parameter F. This conclusion can be explained with the fact that the shear wave propagation velocity $c_2 = \sqrt{\mu/\rho}$ of the half-plane material (i.e. of the steel) is greater than that of the covering layer materials for the all considered pairs. Therefore the complete contact of the covering layer with the half-plane made of the steel makes the system under consideration more suitable ones for high wave propagation velocity.

We analyze the influence of the imperfectness of the contact conditions, i.e. the influence of the shear-spring type parameter Fon the low and high wavenumber limit values of the wave propagation velocities of the fırst branch of the fırst mode. The foregoing results show that the low wavenumber limit values of the wave propagation velocity related to the first branches of the first mode obtained for the all pairs of materials denoted by c_{11L}^{P}, c_{11L}^{I}, c_{11L}^{II} and c_{11L}^{IV}, and determined from the relations:

$$c_{11}^{P} \to c_{11L}^{P}, \quad c_{11}^{I} \to c_{11L}^{I}, \quad c_{11}^{II} \to c_{11L}^{II}, \quad c_{11}^{IV} \to c_{11L}^{IV} \quad \text{as} \quad kh \to 0, \tag{26}$$

do not depend on the shear-spring type parameter F. At the same time, the foregoing results show that the wave propagation velocities related to the second branch of the first mode and the first and second branches of the second mode approach to the $c_2^{(2)}$ as the dimensionless wavenumber kh approach to its corresponding cut off values, i.e.

$$\begin{aligned}
&c_{12}^{P} \to c_2^{(2)} \quad \text{as} \quad kh \to (kh)_{cf12}^{P}, \quad c_{12}^{I} \to c_2^{(2)} \quad \text{as} \quad kh \to (kh)_{cf12}^{I}, \\
&c_{12}^{II} \to c_2^{(2)} \quad \text{as} \quad kh \to (kh)_{cf12}^{II}, \quad c_{12}^{IV} \to c_2^{(2)} \quad \text{as} \quad kh \to (kh)_{cf12}^{IV}, \\
&c_{2n}^{P} \to c_2^{(2)} \quad \text{as} \quad kh \to (kh)_{cf2n}^{P}, \quad c_{2n}^{I} \to c_2^{(2)} \quad \text{as} \quad kh \to (kh)_{cf2n}^{I}, \\
&c_{2n}^{II} \to c_2^{(2)} \quad \text{as} \quad kh \to (kh)_{cf2n}^{II}, \quad c_{2n}^{IV} \to c_2^{(2)} \quad \text{as} \quad kh \to (kh)_{cf2n}^{IV},
\end{aligned} \tag{27}$$

where $n = 1,2$. In this cases the cut of values $(kh)^{P}_{cf12}$, $(kh)^{I}_{cf12}$, $(kh)^{I}_{cf2n}$ and $(kh)^{II}_{cf2n}$ do not depend on the parameter F, but cut off values $(kh)^{II}_{cf12}$, $(kh)^{IV}_{cf12}$, $(kh)^{P}_{cf2n}$ and $(kh)^{IV}_{cf2n}$ depend significantly on F and an increase in the values of the parameter F causes to decrease in the values of the $(kh)^{II}_{cf12}$, $(kh)^{IV}_{cf12}$, $(kh)^{P}_{cf2n}$ and $(kh)^{IV}_{cf2n}$.

According to the well-known physical considerations, for the wave propagation velocity related to the first branch of the first mode of each pair of materials the following high wavenumber limit relation:

$$c \to \min\left(c_R^{(1)}, c_S\right) \quad \text{as} \quad kh \to \infty \tag{28}$$

must satisfy, where $c_R^{(1)}$ is a velocity of the Rayleigh wave in the covering layer material, c_S is a velocity of the Stonoley wave for the corresponding pair of materials. It is known that the Stoneley waves exist for a few pairs of materials only and do not exist for the pairs of materials selected in the present investigation. This conclusion follows from nature of the problem under consideration. Consequently, the high wavenumber limit value of the wave propagation velocity related to the first branch of the first mode of each pair of materials is $c_R^{(1)}$. This conclusion is illustrated in Fig. 6 for the I pair of materials.

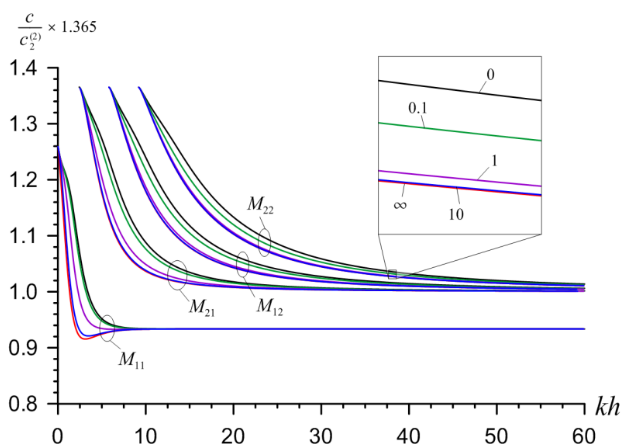


Figure 6: Asymptotic behavior of dispersion curves for the I pair materials as $kh \to \infty$.

Moreover, the results illustrated in Fig. 6 and other numerical results which are not given here show that the second branch of the first mode, the first and second branches of the second mode have the same high wavenumber limit value and this limit value is $c_2^{(1)}$. It follows from the foregoing discussions that, the high wavenumber limit values of the wave propagation velocities do not depend also on the parameter F. This and foregoing conclusions in the qualitative sense agree with corresponding ones obtained in works by Berger; Martin and McCaffery (2000), Kepceler (2010), Akbarov and Ipek (2010, 2012). Finally, we note the following consideration which follows from the results illustrated in Figs. 3 and 4. In the cases where $F \geq 10$ the dispersion curves obtained for the I and II pairs of materials have a point $kh = (kh)_*$ ($0 < (kh)_* < \infty$) at which:

$$\frac{dc}{d(kh)} = 0. \tag{29}$$

It is known that (see, Achenbach; Keshava and Hermann (1967), Akbarov and Ilhan (2008, 2009), Akbarov and Salmanova (2009) and others listed therein) velocities related to the case (29) is taken as a critical velocity for a moving load acting on the free face plane of the covering layer and under this velocity of the moving load a resonance type behavior of the system takes place. Moreover, at $kh = (kh)_*$ for which the relation (29) satisfies the group velocity becomes equal to the corresponding phase velocity and namely the velocity of the moving load which is equal to the group velocity is also called a critical velocity (see, Dieterman and Metrikine (1997)).

According to investigations carried out by Achenbach; Keshava and Hermann (1967), Akbarov and Ilhan (2008, 2009), Akbarov and Salmanova (2009) and other ones listed therein, under perfect contact between the constituents the foregoing type situation, i.e. the existence of the critical velocity of the mentioned moving load takes place in the cases where $c_2^{(1)}$ (shear wave velocity in the covering layer material) is greater than $c_2^{(2)}$ (shear wave velocity in the half-plane material) only. However, in the present case, i.e. in the case where the contact between the covering layer and half-plane is imperfect, the critical velocity may arise also in the cases where $c_2^{(1)} < c_2^{(2)}$. Consequently, the shear-spring type imperfectness between the constituents can acts on the dispersion curves and, in general, on the dynamics of the system under consideration not only quantitatively, but also qualitatively.

Now we analyze the numerical results related to the influence of the initial stresses in the constituents on the wave propagation velocity. For estimation of the magnitude of the initial stresses we introduce the parameters:

$$\psi^{(1)} = \sigma_{11}^{(1),0}/\mu^{(1)}, \quad \psi^{(2)} = \sigma_{11}^{(2),0}/\mu^{(2)}. \tag{30}$$

Here we will present the results only for cases:

$$\begin{aligned} &\text{Case 1. } \psi^{(1)} > 0, \quad \psi^{(2)} = 0; \qquad \text{Case 2. } \psi^{(1)} = 0, \quad \psi^{(2)} < 0; \\ &\text{Case 3. } \psi^{(1)} > 0, \quad \psi^{(2)} < 0; \qquad \text{Case 4. } \psi^{(1)} > 0, \quad \psi^{(2)} > 0. \end{aligned} \tag{31}$$

Moreover, we introduce the notation:

$$\eta = \frac{c|_{\psi^{(1)}\neq 0;\, or\, \psi^{(2)}\neq 0} - c|_{\psi^{(1)}=0;\ \psi^{(2)}=0}}{c|_{\psi^{(1)}=0;\ \psi^{(2)}=0}}, \tag{32}$$

for estimation of the influence of the initial stresses in the constituents, i.e. the influence of the parameters $\psi^{(1)}$ and $\psi^{(2)}$ on the wave propagation velocity.

Thus, through the graphs of the dependencies between η (32) and kh constructed for various values of the parameters F, $\psi^{(1)}$ and $\psi^{(2)}$ we analyze the effect of the imperfectness of the contact conditions between the covering layer and half-plane under the influence of the initial stresses in the constituents on the wave propagation velocity in the cases noted in (31). For the *I* and *III* pairs of materials in Case 1 these graphs are given in Figs. 7 and 8 respectively. Note that in these figures the graphs indicated by letters *a* and *c* (*b* and *d*) correspond to the first and second branches of the first (second) mode. Moreover, note that the results given in Figs. 7 and 8 and obtained in the case where $F = 0$ coincide with corresponding ones obtained in the paper by Akbarov and Ozisik (2003). The graphs show that for the *I*pair of the materials in Case 1 the initial stretching stress in the covering layer causes to increase the wave propagation velocity and in this case the values of $c / c_2^{(2)}$ increase monotonically with $\psi^{(1)}$. Also, the graphs show that the wave propagation velocity related to the first branch of the first mode and to the second branch of the second mode increase monotonically with the parameter F. Consequently, the imperfectness of the contact conditions causes to increase the influence of the initial stress in the covering layer on the wave propagation velocity related to the first branch of the first mode and to the second branch of the second mode of the *I* pair of materials. However, the character of the effect of the imperfectness of the contact conditions, i.e. of the parameter F on the influence of the initial stress in the covering layer on the wave propagation velocity related to the second branch of the first mode and to the first branch of the second mode depends on the values of the dimensionless wavenumber *kh*. So, it follows from the Figs. 7b and 7c that, before (after) a certain value of the *kh*, the imperfectness of the contact conditions causes to increase (decrease), the wave propagation velocity.

Fig. 8 shows that as a result of the initial stretching stress in the covering layer the wave propagation velocity related to the *III* pair of materials decreases. This conclusion was also noted in the paper by Akbarov and Ozisik (2003). In this case the imperfectness of the contact conditions, in general, causes to increase the wave propagation velocity related to the first branches of the first and the second modes. However, the imperfectness of the contact conditions before (after) a certain value of the *kh* causes to decrease (increase) the wave propagation velocity related to the second branches of the first and the second modes. At the same time, it should be noted that the influence of the parameter F on the graphs between η and kh which are shown in Fig. 8, has a complicate character. For instance, in the cases where $0 < kh < 3.0$ the imperfectness of the contact conditions can cause to change the character of the influence of the initial stress in the covering layer on the wave propagation velocity related to the first branch of the first mode (Fig. 8a). Moreover, the influence of the parameter F on the values η related to the first branch of the second mode (Fig. 8b) is non-monotonic.

Now we analyze the effect of the imperfectness of the contact conditions on the influence of the initial compressional stress in the half-plane on the wave propagation velocity to the *II* (or *III*) and *IV* pairs of materials in Case 2. The graphs of the dependencies between η and kh related to the *II* (or *III*) and *IV* pairs of materials and constructed for various values of the parameters F and $\psi^{(2)}$ are given in Figs. 9 and 10, respectively. Note that the graphs constructed in the case where $F = 0$ coincide with corresponding ones obtained in the paper by

Akbarov and Ozisik (2003). It follows from the graphs given in Figs. 9 and 10 that as a result of the initial compression of the half-plane the wave propagation velocity related to the *II* (or *III*) and *IV* pairs of materials in Case 2 increase monotonically with the absolute values of the parameter $\psi^{(2)}$. In this case before (after) a certain value of the kh, the influence of the parameter F causes to increase (decrease) the wave propagation velocity related to the first branch of the first mode for the *II* pair of materials. At the same time, as a result of the influence of the parameter F the wave propagation velocities related to the second branch of the first mode, the first and second branches of the second mode of the *II* pair of materials decrease. The magnitude of this decreasing depends significantly on the values of the dimensionless wavenumber kh.

Analyses of the graphs given in Figs. 10a, 10b and 10c show that the wave propagation velocity related to the first and second branches of the first mode and to the first branch of the second mode of the *IV* pair of materials decrease with the parameter F. However, the character of the influence of the parameter F on the wave propagation velocity related to the second branch of the second mode has a complicate character. This complication is also caused by decreasing of the cut off values of the kh with the parameter F. The similar situation also takes place for the first branch of the second mode. Consequently, the influence of the imperfectness of the contact conditions on the wave propagation velocities related to the first and second branch of the second mode has not only quantitative, but also qualitative character. Moreover, the graphs given in Figs. 10b and 10d show that the results obtained for the first and second branch of the second mode for various values of the parameter F cannot be limited with the corresponding ones obtained in the cases where $F = 0$ (complete contact) and $F = \infty$ (full slipping). Note that this conclusion rises again

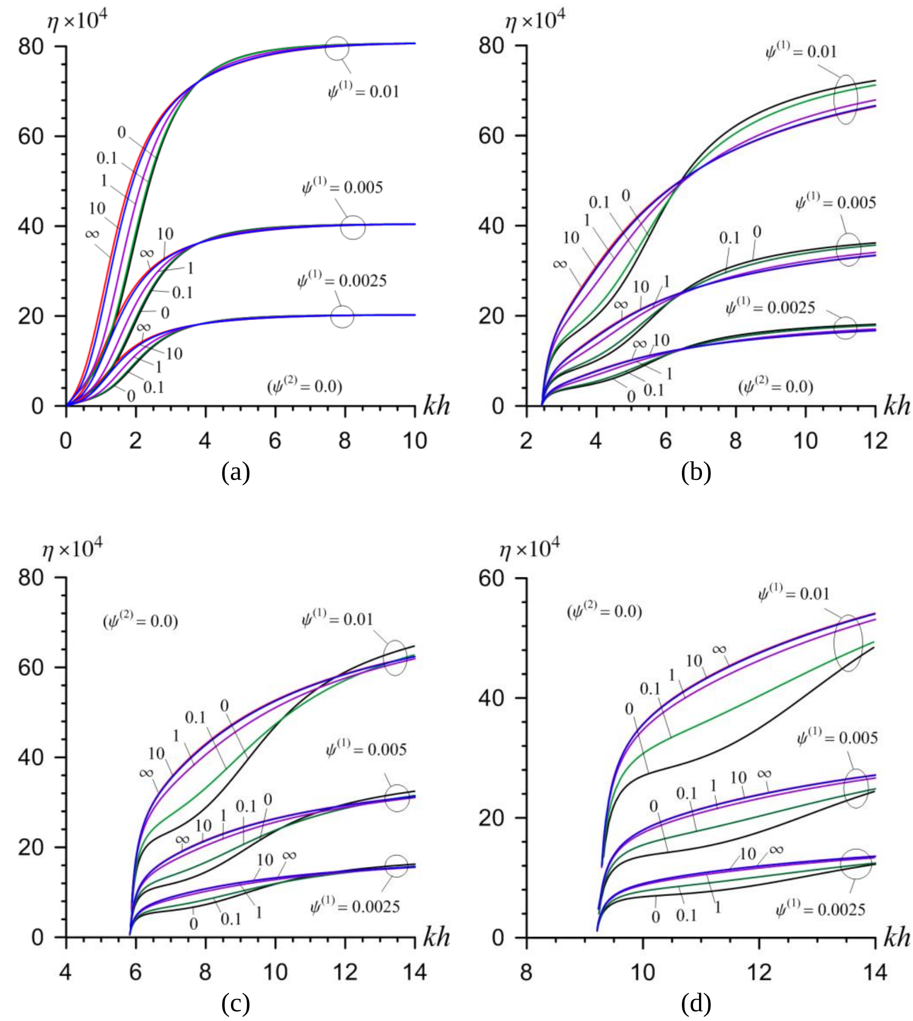


Figure 7: The influence of the imperfect bonding conditions and initial stresses to the dispersion of the generalized Rayleigh wave for the *I* pair of materials in Case 1: First (a) and second (c) branches of the first mode; First (b) and second (d) branches of the second mode.

the significance of the investigations carried out in the present paper.

Now we consider the results obtained in Case 3 for the *II* pair of materials. These results are given in Fig. 11 in the case where $\psi^{(2)} = -0.01$ for various values of the parameter $\psi^{(1)}(> 0)$. Note that the corresponding results obtained in the case where $\psi^{(2)} = 0.0$ are given in Fig. 7. Consequently, it can be conclude from the

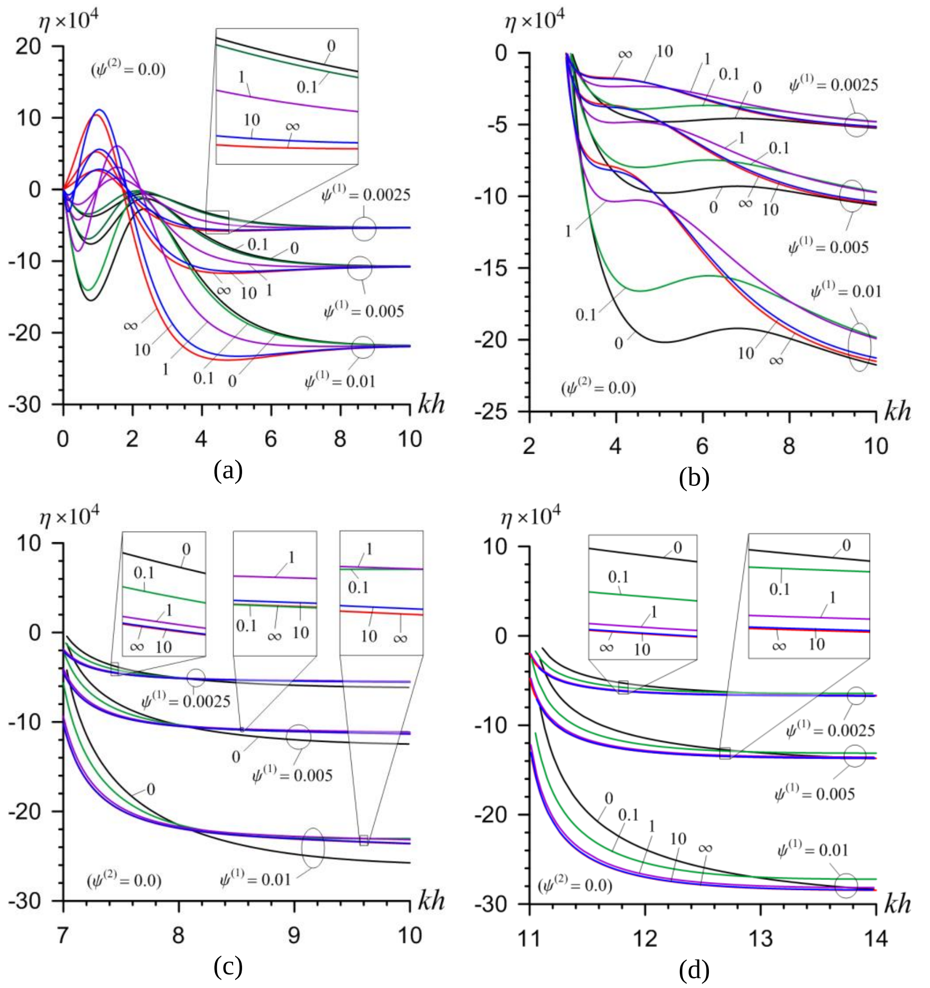


Figure 8: The influence of the imperfect bonding conditions and initial stresses to the dispersion of the generalized Rayleigh wave for the *III* pair of materials in Case 1: First (a) and second (c) branches of the first mode; First (b) and second (d) branches of the second mode.

comparison of the results given in Fig. 11 with the corresponding ones given in Fig. 7, that how the initial compression of the half-plane acts on the influence of the parameter F on the wave propagation velocities under initial stretching of the covering layer. First, this comparison shows that the initial compression of the half-plane causes to considerable increase the wave propagation velocity with respect to the wave propagation velocity obtained in the case where the initial compression

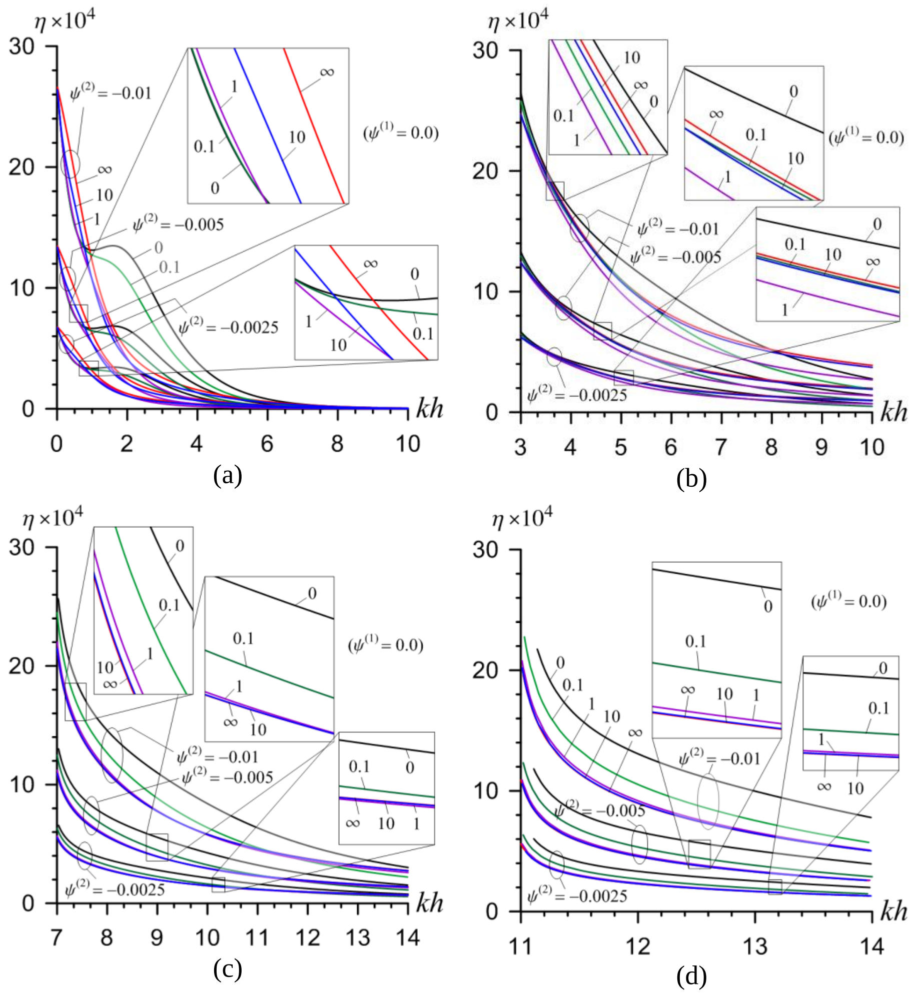


Figure 9: The influence of the imperfect bonding conditions and initial stresses to the dispersion of the generalized Rayleigh wave for the *II* and *III* pair materials in Case 2: First (a) and second (c) branches of the first mode; First (b) and second (d) branches of the second mode.

in the half-plane is absent. Moreover, this comparison shows that the influence of the parameter F on the wave propagation velocity in the latter case is more complicated than the influence of that on the wave propagation velocity obtained in the case where $\psi^{(2)} = 0$. At the same time, the analyses of the results given in Fig. 11 shows that the graphs obtained for various values of the parameter F cannot be limited with corresponding ones obtained in the cases where $F = 0$and $F = \infty$.

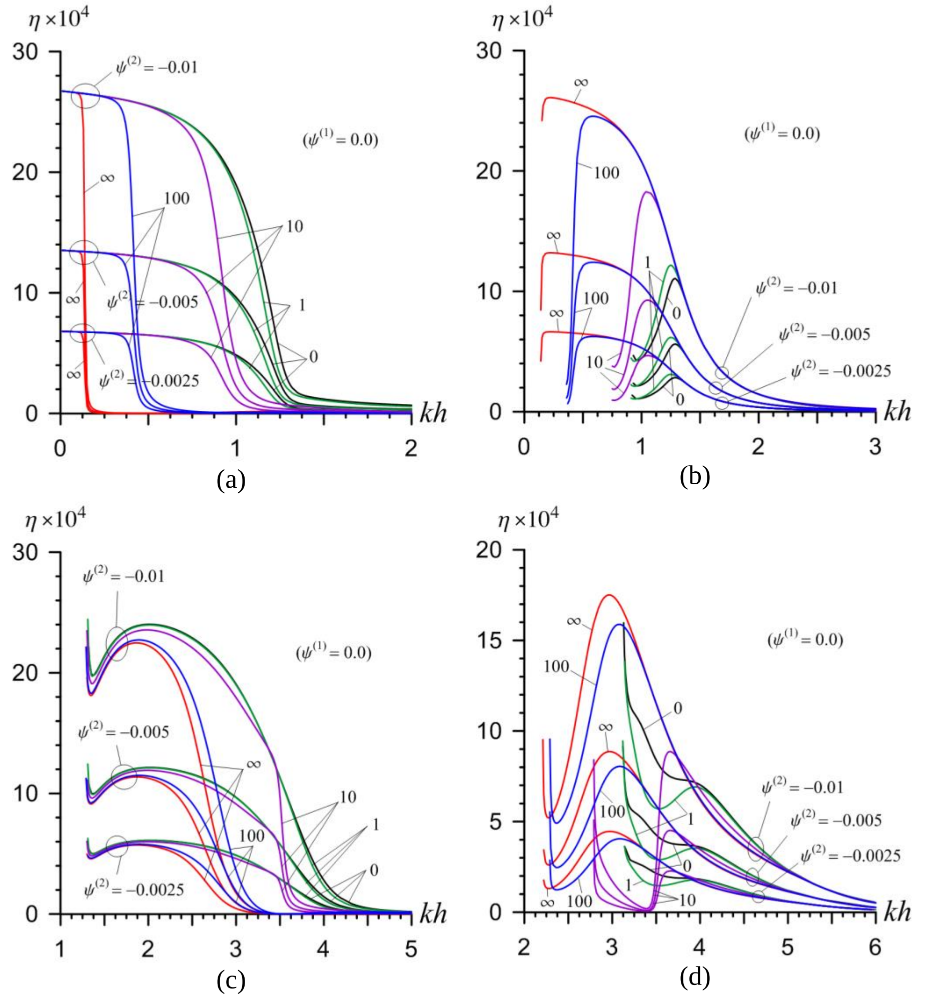


Figure 10: The influence of the imperfect bonding conditions and initial stresses to the dispersion of the generalized Rayleigh wave for the *IV* pair of materials in Case 2: First (a) and second (c) branches of the first mode; First (b) and second (d) branches of the second mode.

Finally, we consider the graphs given in Fig. 12 which show the dependence between the η and *kh* for the *III* pair of materials in Case 4, i.e. in the case where the covering layer and half-plane are initially stretched simultaneously and $\psi^{(1)} = \psi^{(2)}$. Note that these graphs can be taken as generalization of the graphs given in Fig. 8 for the case where the initial stretching exists not only in the covering layer, but also

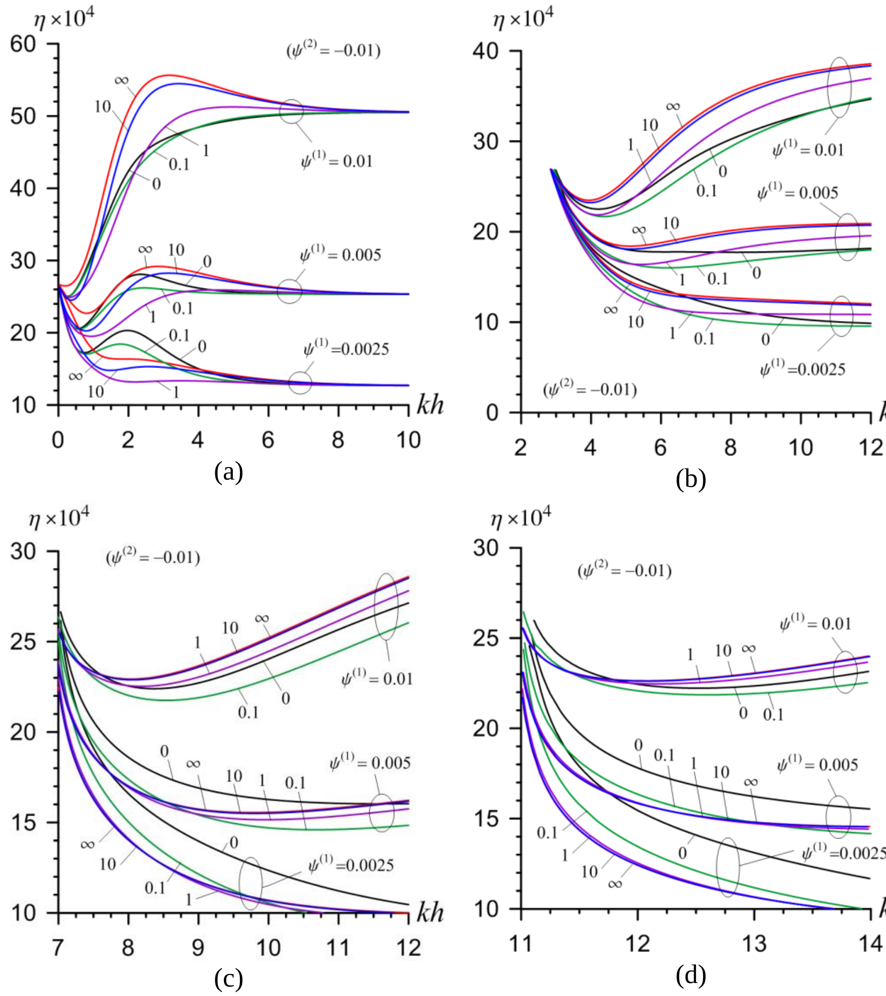


Figure 11: The influence of the imperfect bonding conditions and initial stresses to the dispersion of the generalized Rayleigh wave for the *II* pair of materials in Case 3: First (a) and second (c) branches of the first mode; First (b) and second (d) branches of the second mode.

in the half-plane. Consequently, through the comparison of the graphs given in Fig. 12 with the corresponding ones given in Fig. 8 we can conclude on the action of the initial stretching of the half-plane under the influence of the initial stretching of the covering layer on the wave propagation velocity. It follows from this comparison that as a result of the initial stretching of the half plane the influence of the initial stretching of the covering layer on the wave propagation velocity related to the *III*

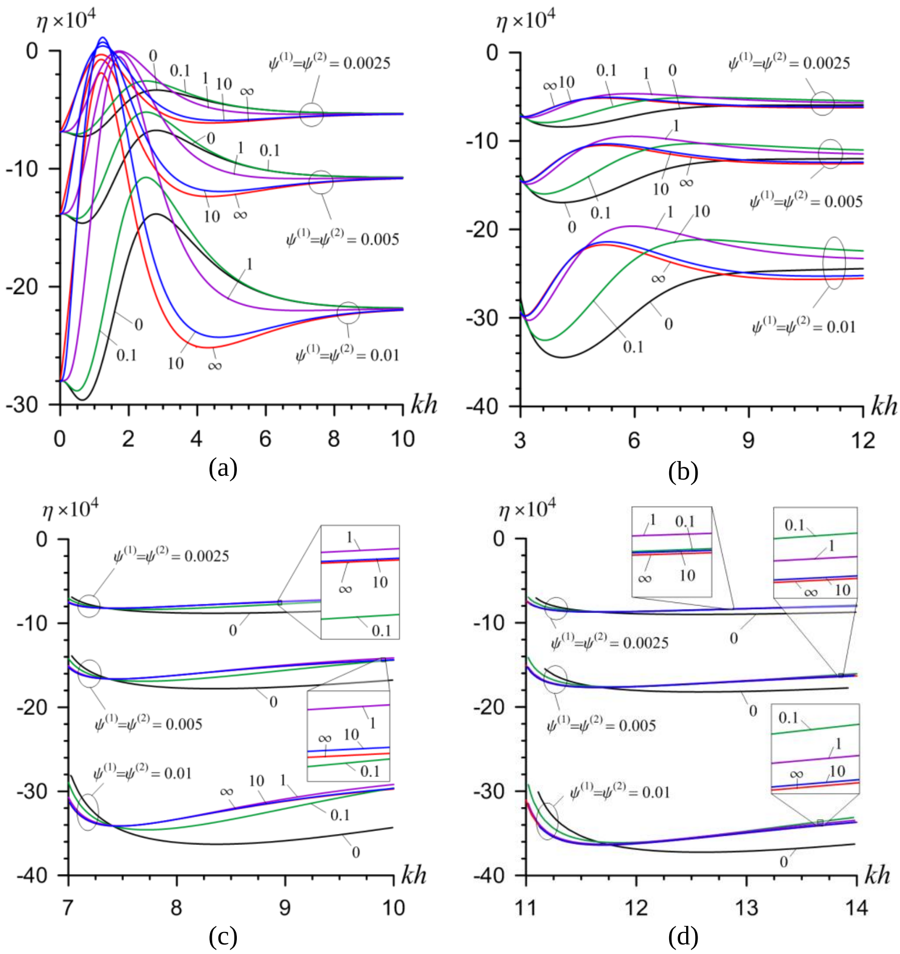


Figure 12: The influence of the imperfect bonding conditions and initial stresses to the dispersion of the generalized Rayleigh wave for the *III* pair of materials in Case 4: First (a) and second (c) branches of the first mode; First (b) and second (d) branches of the second mode.

pair of materials increase significantly and in this case the initial stretching of the covering layer causes to decrease of the wave propagation velocity. The analyses of the graphs given in Fig. 12 show that the character of the influence of the parameter F on the behavior of these graphs is similar with the character of this influence on the graphs given in Fig. 8.

This completes the analyses of the numerical results related to the four pairs of materials shown in Tab. 1. Note that these results are theoretical ones. The experimental studies on the generalized Rayleigh waves for the IV pair of materials was made in a paper by Lu; Zhang and Wang (2006). These studies was carried out within the complete contact condition between the Plexiglass (Lucite) covering layer and Steel half-plane and the dispersive characteristics of Rayleigh waves are investigated experimentally. Under these studies the thickness of the covering layer is taken $h = 5mm$ (Fig. 1) and the experimental results were compared with the theoretical results obtained for the first branches of the first and second modes of the dispersion curves, i.e. the experimental results are compared with the theoretical results given in Fig. 5a. However, in the paper by Lu; Zhang and Wang (2006) the mentioned dispersion curves are given as graphs of dependencies between the phase velocity c and frequency ω. For clarity, in Fig. 13 the dispersion curves given in Fig. 5a are reconstructed as a dependencies between the phase velocity c and frequency ω. Note that, namely, the curves obtained in the case where $F = 0$ and given in Fig. 13 were used in the paper by Lu; Zhang and Wang (2006) for verification of the experimental results and this verification illustrate a very good agreement between the theoretical and corresponding experimental results. Consequently, the experimental method used in the paper by Lu; Zhang and Wang (2006) can also be employed for verification of the theoretical results obtained for the cases where $F > 0$, i.e. for verification of the imperfectness degree of the contact between covering layer and substrate. Note that the experimental methods based on the measurement of the Rayleigh waves for determination of the bonded defects in fiber-layered composites were developed in papers by Zurn and Mantell (2001) and Castaings; Hosten and Francois (2004) and others listed therein. Also, in the papers by Zurn and Mantell (2001) and Castaings; Hosten and Francois (2004) it was established that the mentioned bonded defects causes to decrease the generalized Rayleigh wave propagation velocity. Consequently, the theoretical results obtained in the present paper and related to the influence of the imperfectness of the contact conditions on the generalized Rayleigh wave propagation velocity is validated with the experimental results obtained in the papers by Zurn and Mantell (2001) and Castaings; Hosten and Francois (2004) in the qualitative sense.

It is known that the experimental measurement of the generalized Rayleigh wave propagation is successfully used in the non-destructive determination of the structural parameters and residual stresses in the elements of constructions. It should be noted that under these determinations alongside with experimental data the theoretical results similar with the results presented in the present paper are also used. For example, in a paper by Lakestani; Coste and Denis (1995) generalized Rayleigh waves of various frequencies were generated using a broadband pulse and their velocities were measured as a function of the frequency and compared to the theoret-

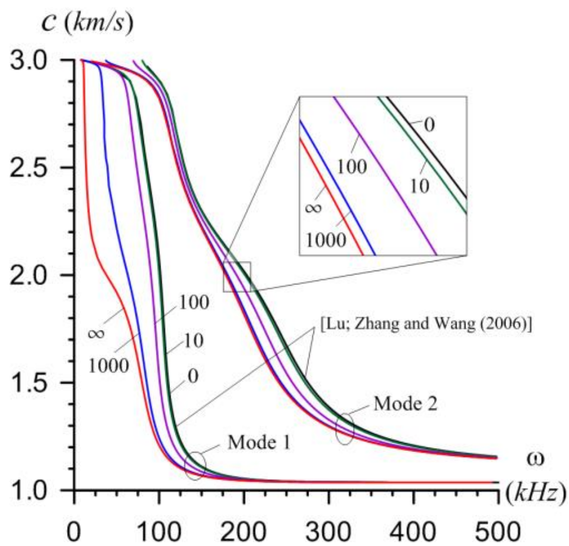


Figure 13: Dispersion curves for steel half-space covered by Lucite.

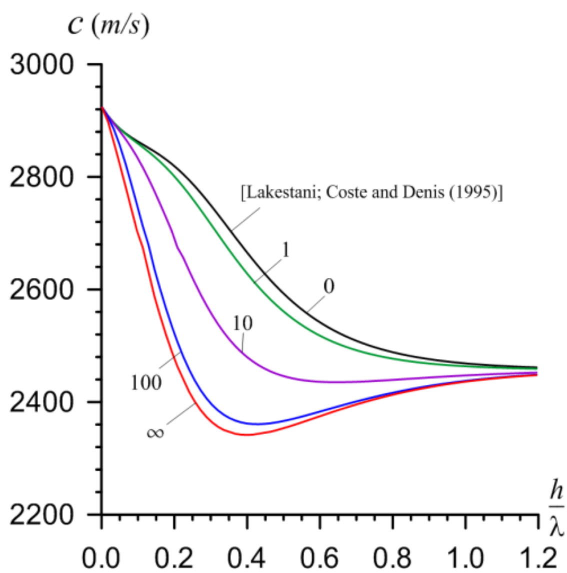


Figure 14: Dispersion curves for AISI 316L stainless steel coated with vacuum plasma sprayed NiCoCrAIY.

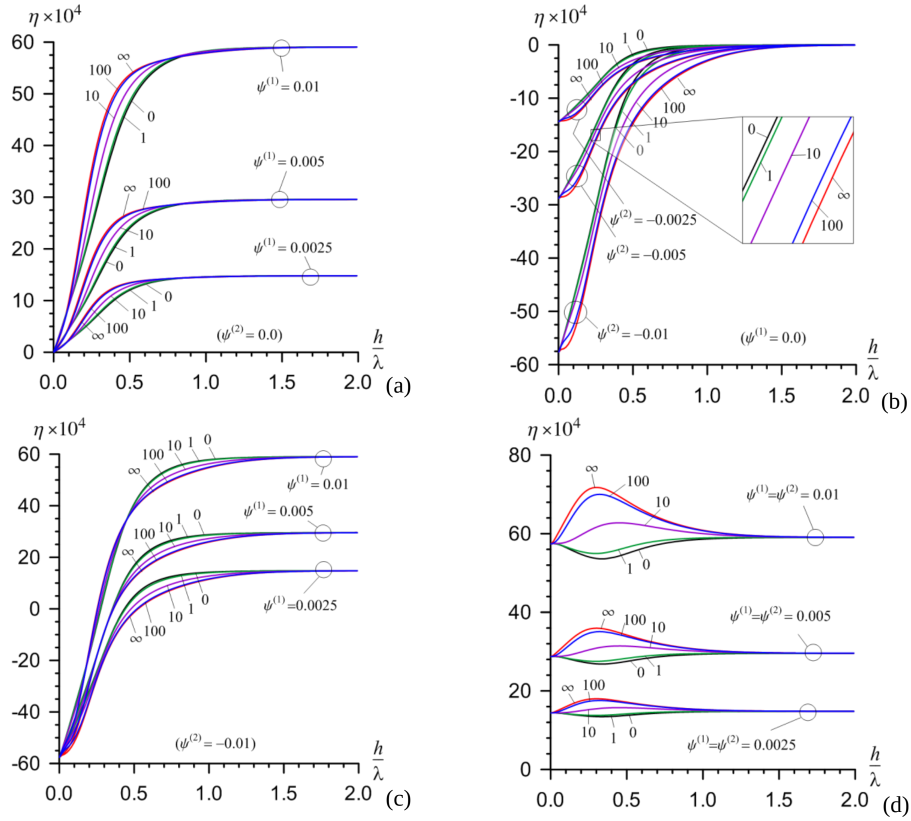


Figure 15: The influence of the imperfect bonding conditions and initial stresses to the dispersion of the generalized Rayleigh wave for AISI 316L steel coated with (VPS) NiCoCrAIY for the first branch of the first mode: (a) Case 1; (b) Case 2; (c) Case 3 and (d) Case 4.

ical dispersion curve of the specimen. Experiments were carried out on AISI 316L specimens coated with vacuum plasma sprayed NiCoCrAlY of various thickness (190-330 μm). Fig. 14 shows the dispersion curves, i.e. the dependence between the generalized Rayleigh wave propagation velocity c and the ratio h/λ, where h is a thickness of the coating and λ is a wave length, obtained for the mentioned pair of materials under various values of the shear-spring type parameter F. Note that the dispersion curve constructed under $F=0$ and shown in Fig. 14 is used in the paper by Lakestani; Coste and Denis (1995) as the theoretical results, according to which, using the experimental data the thickness of the coating is determined. Consequently, the dispersion curves obtained in the cases where $F>0$ and shown

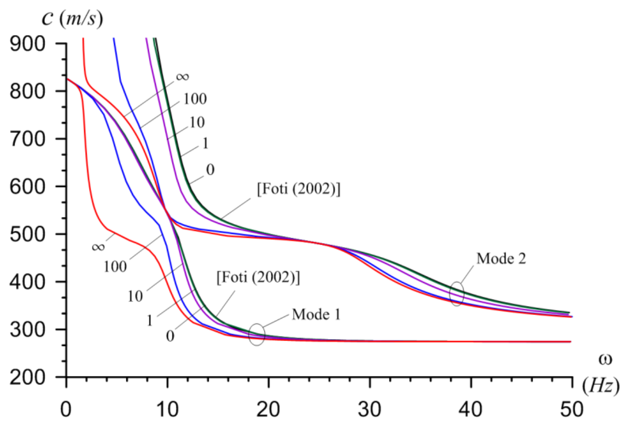


Figure 16: Dispersion curves related to surface waves in the soil which is modeled as a covering layer + half-plane [Foti (2002)].

in Fig. 14 can also successfully be used for determination of the imperfectness between the coating and substrate material. Moreover, the results given in Fig. 15 which illustrate the influence of the initial stresses in the constituents on the wave propagation velocity of the first branch of the first mode, i.e. on the parameter η (32) in Case 1 (Fig. 15a), Case 2 (Fig. 15b), Case 3 (Fig. 15c) and Case 4 (Fig. 15d) which are determined by the expression (31), can also be used as theoretical ones for determination of the quantities of the considered type initial stresses in the coating and substrate material used under the experimental investigations carried out in the paper by Lakestani; Coste and Denis (1995).

The other application field of the generalized Rayleigh wave measurement methods is the geophysical and geotechnical engineering. This method in these engineering fields is employed for determination of the soil stiffness profile. This profile is constructed with an inversion process starting from the experimentally determined dispersion behavior of the Rayleigh waves. After determination of the mentioned stiffness profile, the corresponding theoretical dispersion curves are also calculated for validation of the experimentally determined dispersion curves. Consequently, the dispersion curves constructed within the scope of the assumptions used in the present paper can also be used in the geophysical and geotechnical engineering under determination of the soil stiffness profile. As an example, we consider the case which was considered in a paper by Foti (2002), according to which the soil is modeled as covering layer + half-plane. The thickness of the covering layer is $h = 10m$, the densities of the covering layer and half-plane materials are equal to each other and is $1800 kg/m^3$, shear and bulk waves velocities in the covering

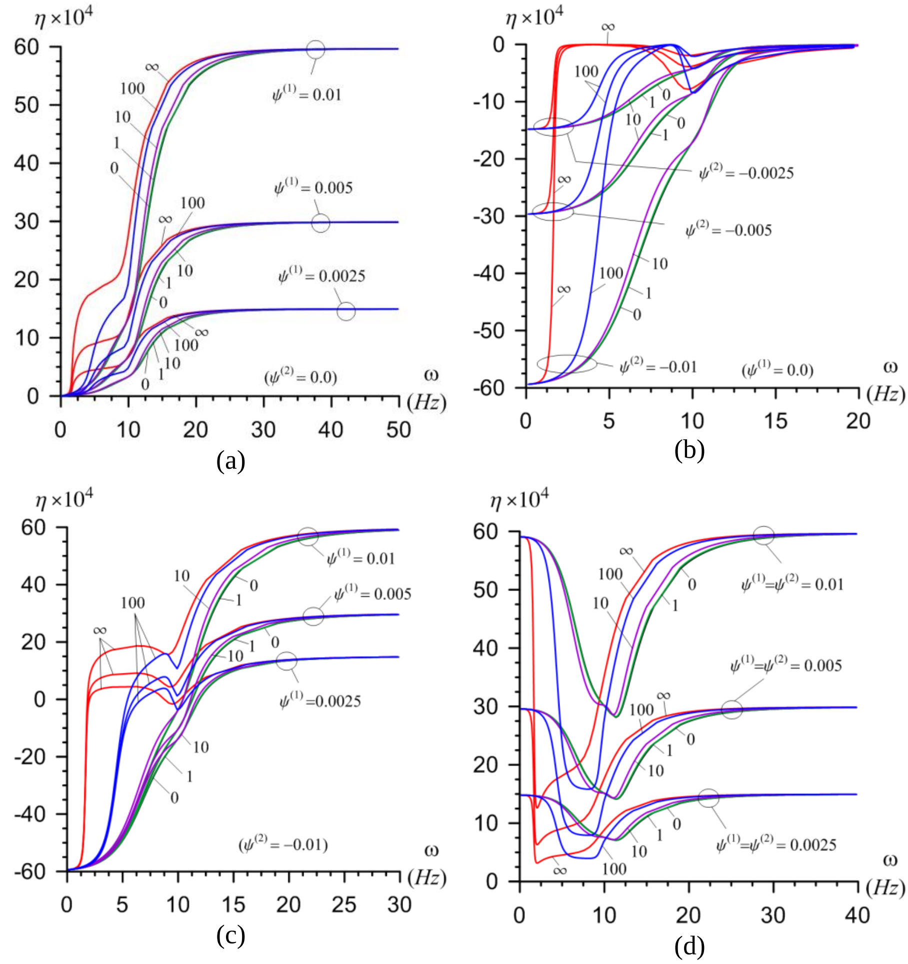


Figure 17: The influence of the imperfect bonding conditions and initial stresses to the surface wave dispersion in the soil which is modeled as a covering layer + half -plane [Foti (2002)]. The first branch of the first mode: (a) Case 1; (b) Case 2; (c) Case 3 and (d) Case 4.

layer (half-plane) material are $300m/s$ $(900m/s)$ and $500m/s$. Fig. 16 shows the graphs of dependencies between the phase velocity and frequency and these graphs relate to the first branches of the first and the second modes of the dispersion curves constructed for the above mentioned case. Note that, namely the graphs constructed in the case where $F = 0$ and shown in Fig. 16 were used in the paper by Foti (2002) for validation of the experimentally constructed dispersion curves.

Consequently the other results show in Fig. 16 and obtained in the case where $F > 0$ can also be used in the corresponding cases related to the geophysical and geotechnical engineering. Moreover, the results given in Fig. 17, which shows the influence of the initial stresses in the soil constituents under consideration in Case 1 (Fig. 17a), Case 2 (Fig.17b), Case 3 (Fig. 17c) and Case 4 (Fig. 17d), allows to determine the magnitude of the initial stresses in the soil layers using the experimentally constructed dispersion curves by employing the method described by Foti (2002). Note that the graphs constructed in Fig. 17 relate to the first branch of the first mode.

5 Conclusion

Thus, in the present paper within the framework of the piecewise homogeneous body model with the use of the second version of the small initial deformation theory of the three-dimensional linearized theory of elastic waves in initially stressed bodies the influence of the shear-spring type imperfect contact conditions on the dispersion relation of the generalized Rayleigh waves in the system consisting of the initially stressed covering layer and initially stressed half plane has been investigated. The elasticity relations of the materials of the constituents are described by the Murnaghan potential. The magnitude of the mentioned imperfectness of the contact conditions on the wave propagation velocity has been estimated through the shear-spring type parameter F(6), where $0 \leq F \leq \infty$ and the case $F = 0$ ($F = \infty$) corresponds to the complete (full slipping) contact between the constituents. Consequently, the influence of the imperfectness of the contact conditions on the generalized Rayleigh wave propagation velocity has been studied through the influence of the parameter F on this velocity.

The numerical results are obtained and discussed for the pair of the Poisson materials by Tolstoy and Usdin (1953), and four pairs of materials composed from the materials the values of the mechanic constants of which are given in Tab. 1. From these discussions the following main conclusions are derived:

- The imperfectness of the contact conditions cause to decrease of the wave propagation velocity of the generalized Rayleigh waves.
- The dispersion curve constructed for each value of the parameter F is limited with corresponding ones obtained at $F = 0$ (upper limit) and $F = \infty$(lower limit).
- The low wavenumber and high wavenumber limit values of the wave propagation velocity do not depend on the imperfectness of the contact conditions. However, the cut off values of the dimensionless wavenumber kh of the first

and second branches of the second modes for the pair of materials used by Tolstoy and Usdin (1953) and for the *IV* pair of materials depend significantly on the parameter F.

- In the case where the complete contact conditions are satisfied between the constituents the wave propagation velocity decrease monotonically with the dimensionless wavenumber kh, however in the case where there exists the shear-spring type imperfect conditions between the constituents the dependence between the wave propagation velocity and the dimensionless wavenumber kh may become non-monotonic for some pair of materials. Consequently, the imperfectness of the contact conditions acts on the dispersion curves and, in general, on the dynamics of the system under consideration not only quantitatively, but also qualitatively.

Note that the foregoing conclusions are made for the case where the initial stresses in the constituents are absent. In the paper the numerical results related to the action of the initial stresses in the constituents under the influence of the imperfectness parameter F on the wave propagation velocity are also presented and discussed for four pairs of materials (Tab. 1). Throughout these discussions the magnitude of the initial stresses is estimated by the parameters $\psi^{(1)}$ and $\psi^{(2)}$ (30), and four cases indicated in (31) with respect to the signs of the $\psi^{(1)}$ and $\psi^{(2)}$are considered, but the change in the values of the wave propagation velocity is estimated through the parameter η (32). We can make the following main conclusions related to the action of the parameter F on the influence of the initial stresses on the wave propagation velocity:

- The imperfectness of the contact conditions causes to increase the influence of the initial stress in the covering layer on the wave propagation velocity related to the first branch of the first mode and to the second branch of the second mode of the *I* pair of materials. However, the character of the effect of the imperfectness of the contact conditions, i.e. of the parameter F on the influence of the initial stress in the covering layer on the wave propagation velocity related to the second branch of the first mode and to the first branch of the second mode depends on the values of the dimensionless wavenumber kh.

- As a result of the initial compression of the half-plane the wave propagation velocity related to the *II* (or *III*) and *IV* pairs of materials in Case 2 increase monotonically with the absolute values of the parameter $\psi^{(2)}$. In this case before (after) a certain value of the kh, the influence of the parameter F causes to increase (decrease) the wave propagation velocity related to the first branch of the first mode of the *II* pair of materials. At the same time, as a result of the influence of the parameter F the wave propagation velocities related to the second branch of the first mode, the first and second branches of the second mode of the *II* pair of materials decrease. The magnitude of this decreasing depends significantly on the values of the dimensionless wavenumber kh.

- In general, the graphs of the dependence between the parameters η and kh, i.e. the influence of the initial stresses on the wave propagation velocity obtained for each value of F cannot be limited with the corresponding ones obtained at $F = 0$ (complete contact) and $F = \infty$ (full slipping). This conclusion rises again the significance of the investigations carried out in the present paper.

- Numerical results obtained for the *IV* pair of materials under complete contact conditions are validated with the corresponding experimental ones which were detailed in the paper by Lu; Zhang and Wang (2006).

- The character of the influence of the imperfectness of the contact between the covering layer and half plane on the generalized Rayleigh wave propagation velocity in the qualitative sense is validated with the experimental ones given by Zurn and Mantell (2001) and Castaings; Hosten and Francois (2004).

- Dispersion curves for the pair of materials considered in the paper by Lakestani; Coste and Denis (1995) are also obtained and the possible application of the numerical results under determination of the structural parameters and residual stresses in the coated materials is proposed.

- The possible application of the numerical results which are similar to the obtained ones and relate to the determination of the soil structure and stiffness in the geophysical and geotechnical engineering is also discussed and corresponding numerical results are presented for the case considered in the paper by Foti (2002).

Many other details of the results obtained for the initially stressed cases are discussed in the text of the paper.

Appendix A:

The expressions of the components $\alpha_{ij} \quad (i; j = 1,2,3,4,5,6)$ in (23) are:

$$\alpha_{11} = -\frac{\mathrm{R}_1^{(1)}}{\mathrm{c}_{22}^{(1)}} - \frac{\mathrm{b}_{22}^{(1)}}{\mathrm{R}_1^{(1)}\mathrm{c}_{22}^{(1)}}, \quad \alpha_{12} = \frac{\mathrm{R}_1^{(1)}}{\mathrm{c}_{22}^{(1)}} + \frac{\mathrm{b}_{22}^{(1)}}{\mathrm{R}_1^{(1)}\mathrm{c}_{22}^{(1)}},$$

$$\alpha_{13}=-\frac{R_2^{(1)}}{c_{22}^{(1)}}-\frac{b_{22}^{(1)}}{R_2^{(1)}c_{22}^{(1)}},\quad \alpha_{14}=\frac{R_2^{(1)}}{c_{22}^{(1)}}+\frac{b_{22}^{(1)}}{R_2^{(1)}c_{22}^{(1)}},$$

$$\alpha_{15}=\frac{R_1^{(2)}}{c_{22}^{(2)}}+\frac{b_{22}^{(2)}}{R_1^{(2)}c_{22}^{(2)}}+\frac{\mu_{12}^{(2)}Fkh}{\mu^{(2)}}\left(\frac{\left(R_1^{(2)}\right)^2+b_{22}^{(2)}}{c_{22}^{(2)}}+1\right),$$

$$\alpha_{16}=\frac{R_2^{(2)}}{c_{22}^{(2)}}+\frac{b_{22}^{(2)}}{R_2^{(2)}c_{22}^{(2)}}+\frac{\mu_{12}^{(2)}Fkh}{\mu^{(2)}}\left(\frac{\left(R_2^{(2)}\right)^2+b_{22}^{(2)}}{c_{22}^{(2)}}+1\right),$$

$$\alpha_{21}=1,\quad \alpha_{22}=1,\quad \alpha_{23}=1,\quad \alpha_{24}=1,\quad \alpha_{25}=-1,\quad \alpha_{26}=-1.$$

$$\alpha_{31}=-\mu_{12}^{(1)}\left(\frac{\left(R_1^{(1)}\right)^2+b_{22}^{(1)}}{c_{22}^{(1)}}+1\right),\quad \alpha_{32}=-\mu_{12}^{(1)}\left(\frac{\left(R_1^{(1)}\right)^2+b_{22}^{(1)}}{c_{22}^{(1)}}+1\right),$$

$$\alpha_{33}=-\mu_{12}^{(1)}\left(\frac{\left(R_2^{(1)}\right)^2+b_{22}^{(1)}}{c_{22}^{(1)}}+1\right),\quad \alpha_{34}=-\mu_{12}^{(1)}\left(\frac{\left(R_2^{(1)}\right)^2+b_{22}^{(1)}}{c_{22}^{(1)}}+1\right),$$

$$\alpha_{35}=\mu_{12}^{(2)}\left(\frac{\left(R_1^{(2)}\right)^2+b_{22}^{(2)}}{c_{22}^{(2)}}+1\right),\quad \alpha_{36}=\mu_{12}^{(2)}\left(\frac{\left(R_2^{(2)}\right)^2+b_{22}^{(2)}}{c_{22}^{(2)}}+1\right),$$

$$\alpha_{41}=A_{22}^{(1)}R_1^{(1)}-A_{12}^{(1)}\left(\frac{R_1^{(1)}}{c_{22}^{(1)}}+\frac{b_{22}^{(1)}}{R_1^{(1)}c_{22}^{(1)}}\right),$$

$$\alpha_{42}=-A_{22}^{(1)}R_1^{(1)}+A_{12}^{(1)}\left(\frac{R_1^{(1)}}{c_{22}^{(1)}}+\frac{b_{22}^{(1)}}{R_1^{(1)}c_{22}^{(1)}}\right),$$

$$\alpha_{43}=A_{22}^{(1)}R_2^{(1)}-A_{12}^{(1)}\left(\frac{R_2^{(1)}}{c_{22}^{(1)}}+\frac{b_{22}^{(1)}}{R_2^{(1)}c_{22}^{(1)}}\right),$$

$$\alpha_{44}=-A_{22}^{(1)}R_2^{(1)}+A_{12}^{(1)}\left(\frac{R_2^{(1)}}{c_{22}^{(1)}}+\frac{b_{22}^{(1)}}{R_2^{(1)}c_{22}^{(1)}}\right),$$

$$\alpha_{45}=-A_{22}^{(2)}R_1^{(2)}+A_{12}^{(2)}\left(\frac{R_1^{(2)}}{c_{22}^{(2)}}+\frac{b_{22}^{(2)}}{R_1^{(2)}c_{22}^{(2)}}\right),$$

$$\alpha_{46}=-\mathrm{A}_{22}^{(2)}\mathrm{R}_2^{(2)}+\mathrm{A}_{12}^{(2)}\left(\frac{\mathrm{R}_2^{(2)}}{\mathrm{c}_{22}^{(2)}}+\frac{\mathrm{b}_{22}^{(2)}}{\mathrm{R}_2^{(2)}\mathrm{c}_{22}^{(2)}}\right),$$

$$\alpha_{51}=-e^{\mathrm{R}_1^{(1)}kh}\left(1+\frac{\left(\mathrm{R}_1^{(1)}\right)^2}{\mathrm{c}_{22}^{(1)}}+\frac{\mathrm{b}_{22}^{(1)}}{\mathrm{c}_{22}^{(1)}}\right),\quad \alpha_{52}=-e^{-\mathrm{R}_1^{(1)}kh}\left(1+\frac{\left(\mathrm{R}_1^{(1)}\right)^2}{\mathrm{c}_{22}^{(1)}}+\frac{\mathrm{b}_{22}^{(1)}}{\mathrm{c}_{22}^{(1)}}\right),$$

$$\alpha_{53}=-e^{\mathrm{R}_2^{(1)}kh}\left(1+\frac{\left(\mathrm{R}_2^{(1)}\right)^2}{\mathrm{c}_{22}^{(1)}}+\frac{\mathrm{b}_{22}^{(1)}}{\mathrm{c}_{22}^{(1)}}\right),\quad \alpha_{54}=-e^{-\mathrm{R}_2^{(1)}kh}\left(1+\frac{\left(\mathrm{R}_2^{(1)}\right)^2}{\mathrm{c}_{22}^{(1)}}+\frac{\mathrm{b}_{22}^{(1)}}{\mathrm{c}_{22}^{(1)}}\right),$$

$$\alpha_{55}=0,\quad \alpha_{56}=0,$$

$$\alpha_{61}=e^{\mathrm{R}_1^{(1)}kh}\left(A_{22}^{(1)}\mathrm{R}_1^{(1)}-A_{12}^{(1)}\left(\frac{\mathrm{R}_1^{(1)}}{\mathrm{c}_{22}^{(1)}}+\frac{\mathrm{b}_{22}^{(1)}}{\mathrm{R}_1^{(1)}\mathrm{c}_{22}^{(1)}}\right)\right),$$

$$\alpha_{62}=e^{-\mathrm{R}_1^{(1)}kh}\left(A_{12}^{(1)}\left(\frac{\mathrm{R}_1^{(1)}}{\mathrm{c}_{22}^{(1)}}+\frac{\mathrm{b}_{22}^{(1)}}{\mathrm{R}_1^{(1)}\mathrm{c}_{22}^{(1)}}\right)-A_{22}^{(1)}\mathrm{R}_1^{(1)}\right),$$

$$\alpha_{63}=e^{\mathrm{R}_2^{(1)}kh}\left(A_{22}^{(1)}\mathrm{R}_2^{(1)}-A_{12}^{(1)}\left(\frac{\mathrm{R}_2^{(1)}}{\mathrm{c}_{22}^{(1)}}+\frac{\mathrm{b}_{22}^{(1)}}{\mathrm{R}_2^{(1)}\times\mathrm{c}_{22}^{(1)}}\right)\right),$$

$$\alpha_{64}=e^{-\mathrm{R}_2^{(1)}kh}\left(A_{12}^{(1)}\left(\frac{\mathrm{R}_2^{(1)}}{\mathrm{c}_{22}^{(1)}}+\frac{\mathrm{b}_{22}^{(1)}}{\mathrm{R}_2^{(1)}\mathrm{c}_{22}^{(1)}}\right)-A_{22}^{(1)}\mathrm{R}_2^{(1)}\right),\quad \alpha_{65}=0,\quad \alpha_{66}=0.$$

References

Achenbach, J. D.; Keshava, S. P.; Hermann, G. (1967): Moving load on a plate resting on an elastic half-space. *Journal of Applied Mechanics*, vol. 34, no. 4, pp. 910-914.

Akbarov, S. D.; Ozisik, M. (2003): The influence of the third order elastic constants to the generalized Rayleigh wave dispersion in a pre-stressed stratified half-plane. *International Journal of Engineering Sciences*, vol. 41, pp. 2047–2061.

Akbarov, S. D. (2007): Recent investigations on the dynamical problems of the elastic body with initial (residual) stresses (review). *International Applied Mechanics*, vol. 43, no. 12, pp. 3–27.

Akbarov, S. D.; Ilhan, N. (2008): Dynamics of a system comprising an orthotropic layer and pre-stressed orthotropic half-plane under the action of a moving load. *International Journal of Solids and Structures*, vol. 45, pp. 4222-4235.

Akbarov, S. D.; Ilhan, N. (2009): Dynamics of a system comprising an orthotropic layer and pre-stressed orthotropic half-plane under the action of an oscillating moving load. *International Journal of Solids and Structures*, vol. 46, pp. 3873-3881.

Akbarov, S. D.; Salmanova, K. A. (2009): On the dynamics of a finite pre-strained bi-layered slab resting on a rigid foundation under the action of an oscillating moving load. *Journal of Sound and Vibration*, vol. 327, pp. 454-472.

Akbarov, S. D.; Ipek, C. (2010): The influence of the imperfectness of the interface conditions on the dispersion of the axisymmetric longitudinal waves in the pre-strained compound cylinder. *CMES: Computer Modeling in Engineering and Sciences*, vol. 70, no. 2, pp. 93-121.

Akbarov, S. D.; Agasiyev, E.R.; Zamanov, A. D. (2011): Wave propagation in a pre-strained compressible elastic sandwich plate. *European Journal of Mechanics A/Solids*, vol. 30, pp. 409-422.

Akbarov, S. D. (2012): The influence of third order elastic constants on axisymmetric wave propagation velocity in the two-layered pre-stressed hollow cylinder. *CMC: Computers, Materials &Continua*, vol. 32, no.1, pp. 29–60.

Akbarov, S. D.; Ipek, C. (2012): Dispersion of axisymmetric longitudinal waves in the pre-strained imperfectly bonded bi-layered hollow cylinder. *CMC: Computers, Materials &Continua*, vol. 30, no. 2, pp. 99–144.

Berger, J. R.; Martin, P. A.; McCaffery, S. J. (2000): Time-harmonic torsional waves in a composite cylinder with an imperfect interface. *Journal of the Acoustical Society of America*, vol. 107, no. 3. pp. 1161-1167.

Biot, M. A. (1965): *Mechanics of incremental deformations: Theory of elasticity and viscoelasticity of initially stressed solids and fluids, including thermodynamic foundations and applications to finite strain*. John Wiley & Sons.

Castaings, M.; Hosten, B.; Francois, D. (2004): The sensitivity of surface guided modes to the bond quality between a concrete block and a composite plate. *Ultrasonics*, vol. 42, pp. 1067-1071.

Dieterman, H. A.; Metrikine, A. (1997): Critical velocities of a harmonic load moving uniformly along an elastic layer. *Journal of Applied Mechanics*: *ASME*, vol. 64, pp. 596-600.

Eringen, A. C.; Suhubi, E. S. (1975): *Elastodynamics, Volume I, Finite Motions*, Academic Press.

Eringen, A. C.; Suhubi, E. S. (1975): *Volume II, Linear Theory*, Academic Press.

Foti, S. (2002): Numerical and experimental comparison between two-station and multistation methods for spectral analysis of surface waves. *Rivista Italiana di Geotecnica*, vol.1, pp. 11-22.

Guz, A. N. (2002): Elastic waves in bodies with initial (residual) stresses. *International Applied Mechanics*, vol. 38, no. 1, pp. 23-59.

Guz, A. N. (2004): *Elastic waves in Bodies with Initial (Residual) Stresses*, "A.S.K", Kiev, (in Russian).

Guz, A. N. (2005): On foundations of the ultrasonic non-destructive method of determination of stresses in near-the-surface layers of solid bodies. *CMES: Computer Modeling in Engineering and Sciences*, vol. 8, no. 3, pp. 217-230.

Huang, Y.; Li, X. F. (2010): Shear waves guided by the imperfect interface of two magnetoelectric materials. *Ultrasonics*, vol. 50, pp. 750–757.

Jones, J. P.; Whitter, J. S. (1967): Waves at a flexibly bonded interface. *Journal of Applied Mechanics,* vol. 34, pp. 905-909.

Kepceler, T. (2010): Torsional wave dispersion relations in a pre-stressed bi-material compounded cylinder with an imperfect interface. *Applied Mathematical Modeling*, vol. 34, pp. 4058–4073.

Kumara R.; Singh, M. (2009): Effect of rotation and imperfection on reflection and transmission of plane waves in an isotropic generalized thermoelastic media. *Journal of Sound and Vibration*, vol. 324, pp. 773–797.

Lakestani, F.; Coste, J. F.; Denis, R. (1995): Application of ultrasonic Rayleigh waves to thickness measurement of metallic coatings. *NDT&T International*, vol. 28, no. 3, pp. 171-178.

Leungvichcharoen, S.; Wijeyewickrema, Anil C. (2003): Dispersion effects of extensional waves in pre-stressed imperfectly bonded incompressible elastic layered composites. *Wave Motion*, vol. 38, pp. 311–325.

Liu, J.; Wang, Y.; Wang, B. (2010): Propagation of Shear Horizontal Surface Waves in a Layered Piezoelectric Half-Space With an Imperfect Interface. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 57, no. 8, pp. 1875-1879.

Lu, L. Y.; Zhang, B. X.; Wang, C. H. (2006): Experimental and inversion studies on Rayleigh wave considering higher modes. *Chines Journal of Geophysics*, vol. 49, no. 4, pp. 974-985.

Martin, P. A. (1992): Boundary integral equations for the scattering of elastic waves by elastic inclusions with thin interface layers. *Journal of Nondestructive Evaluation*, vol. 11, pp. 167–174.

Pang, Y.; Liu, J. X. (2011): Reflection and transmission of plane waves at an imperfectly bonded interface between piezoelectric and piezomagnetic media. *European Journal of Mechanics A/Solids*, vol. 30, pp. 731-740.

Pecorari, C. (2001): Scattering of a Rayleigh wave by a surface-breaking crack with faces in partial contact. *Wave Motion*, vol. 33, pp. 259–270.

Rokhlin, S. I.; Wang, Y. J. (1991): Analysis of boundary conditions for elastic wave interaction with an interface between two solids. *Journal of the Acoustical Society of America*, vol. 89, pp. 503–515.

Tolstoy, I.; Usdin, E. (1953): Dispersive Properties of Stratified Elastic and Liquid Media: A Ray Theory. *Geophysics*, vol. 18, pp. 844–870.

Truestell, C. (1961): General and exact theory of waves in finite elastic strain. *Archive for Rational Mechanics and Analyses*, vol. 8, no. 1, pp. 263–296.

Zhou, Y. Y.; Lü, C. F.; Chen, W. Q. (2012): Bulk wave propagation in layered piezomagnetic/piezoelectric plates with initial stresses or interface imperfections. *Composite Structures*, vol. 94, pp. 2736–2745.

Zurn, B.; Mantell, S. C. (2001): Nondestructive evaluation of laminated composites using Rayleigh waves. *Journal of Composite Materials*, vol. 35, no. 12, pp. 1026-1044.