 CMC, vol.45, no.2, pp.107-125, 2015

Structural Continuous Dependence in Micropolar Porous Bodies

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Abstract: Our study is dedicated to mixed initial boundary value problem for porous micropolar bodies. We prove that the solution of this problem depends continuously on coefficients which couple the micropolar deformation equations with the equations that model the evolution of voids. The evaluation of this dependence is made by using an appropriate measure.

Keywords: micropolar bodies, voids, continuous dependence, uncoupled systems, convergence.

1 Introduction

The theory of materials with voids or vacuous pores is the simplest extension of the classical theory of elasticity and was first proposed by Nunziato and Cowin in the paper Nunziato and Cowin (1979). In this theory the authors introduce an additional degree of freedom in order to develope the mechanical behavior of a body in which the matrix material is elastic and interstices are voids of material. It is worth to recall that porous materials have applications in many fields of engineering such as petroleum industry, material science, biology and so on. The intended applications of the theory are in geological materials like rocks and soil and in manufactured porous materials. The linear theory of elastic materials with voids was developed by Cowin and Nunziato in Cowin and Nunziato (1983). Here the uniqueness and weak stability of solutions are also derived. Then, the problem of bodies with voids was approached in a large number of studies, of which we mention only some of the most recent. For example, Iesan (2011) considered the problem of Almansi for porous Cosserat elastic solids. The non-linear deformations of porous elastic solids were approached in Iesan and Quintanilla (2013).

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Also, in the paper Mora and Waas (2007) the micropolar and Lamè constants for a circular cell polycarbonate honeycomb are calculated by using a finite element representation of the honeycomb microstructure. The known smeared crack approach is revisited in the paper Heinrich and Waas (2013) to describe post-peak softening in laminated composite materials.

The paper Abbas and Kumar (2014) is a study of the plane problem in initially stressed generalized thermoelastic half-space with voids. In the paper Mahmoud and Abd-Alla (2014) the equations of elastodynamic problems of the orthotropic hollow sphere in terms of displacement are solved The minimum principle for dipolar materials with strec is considered in the paper Marin (2009). For the same category of materials, Marin and Stan (2013) give some weak solutions, while the paper Marin et al. (2013 a) presents some results obtained with the help of Lagrange Identity. A study of temporal behaviour of solutions in Thermoelasticity of porous micropolar bodies is given in Marin and Florea (2014) and in Marin et al. (2013 b). Some considerations regarding the localization in time of solutions for thermoelastic micropolar materials with voids can be found in the paper Marin et. al (2014). Also, the study Sharma and Marin (2014) is dedicated to micropolar thermoelastic solids.

Continuous dependence of solutions with regards to the coefficients of the equations that govern the deformation of a body is more important than continuous dependence the initial data and on boundary conditions. This is because, in the mathematical modeling of the continuum, there may be errors or disturbances due the idealization of the model. Also, the continuous dependence of solutions with regards to the coefficients of the equations is important in obtaining some numerical approximations of the solutions of the models. Continuous dependence is important both in terms of pure mathematics and in terms of practical applications. Therefore, many studies published in recent years are devoted to this topic. We recall only the fundamental paper of Knops and Payne (1988) and also the papers Chirita and Ciarletta (2011), Franchi and Straughan (1996), Iovane and Passarela (2004), Green and Naghdi (1993).

The paper is structured as follows. First, we formulate the mixed intial-boundary value problem in the context of micropolar porous bodies. Then we will prove some preliminaries identities which we will use in order to derive the continous dependence theorems. In the last part of our paper we deduce the convergence of the solution of our mixed problem, when the coupling coefficients tend to zero, by using an appropriate measure, which is specified in advance.

2 Basic equations

An anisotropic elastic material is considered. Assume a body of this type that occupies a properly regular region B of the three-dimensional Euclidian space R^3 bounded by a piecewise smooth surface ∂B and we denote the closure of B by $\bar{B}$. The boundary ∂B is smooth enough to apply the divergence theorem.

We use a fixed system of rectangular Cartesian axes $Ox_i, (i = 1,2,3)$ and adopt Cartesian tensor notation. A superposed dot stands for the material time derivative while a comma followed by a subscript denotes partial derivatives with respect to the spatial coordinates. Einstein summation convention on repeated indices is used. Also, the spatial argument and the time argument of a function will be omitted when there is no likelihood of confusion.

The motion of a micropolar porous body is described by the independent variables $u_i(x,t)$, the displacement vector field, $\varphi_i(x,t)$, the microrotation vector field and ϕ, the change in volum fraction.

We consider the mixed problem associated with the theory of elasticity of micropolar bodies with voids on the time interval I. In the absence of supply terms, it is known that the basic equations on $B \times I$ are, [see, for instance Iesan (2011)]

$$t_{ij,j} = \rho \ddot{u}_i,$$

$$m_{ij,j} + \varepsilon_{ijk} t_{jk} = I_{ij} \ddot{\varphi}_j, \tag{1}$$

$$h_{i,i} + g = \rho \kappa \ddot{\phi}, \tag{2}$$

Here, the equations (1) are the motion equations and (2) is the balance of the equilibrated forces.

Next, we restricte our considerations only to the case where the materials have a center of symmetry. Consequently, the constitutive tensors of odd order must vanish and therefore the constitutive equations become

$$t_{ij} = C_{ijmn} \varepsilon_{mn} + B_{ijmn} \gamma_{mn} + a_{ij} \phi,$$

$$m_{ij} = B_{mnij} \varepsilon_{mn} + C_{ijmn} \gamma_{mn} + b_{ij} \phi,$$

$$h_i = A_{ij} \phi_{,j}, \tag{3}$$

$$g = -a_{ij} \varepsilon_{ij} - b_{ij} \gamma_{ij} - \xi \phi - \tau \dot{\phi},$$

where the strain tensors ε_{ij} and γ_{ij} are defined by means of the kinetic relations

$$\varepsilon_{ij} = u_{j,\,i} + \varepsilon_{jik} \varphi_k, \quad \gamma_{ij} = \varphi_{j,\,i}. \tag{4}$$

In the above equations we used the following notations: ρ-the constant mass density; $I_{ij} = I_{ji}$-the coefficients of microinertia; κ-the equilibrated inertia; u_i-the components of displacement vector; φ_i-the components of microrotation vector; ϕ-the volume distribution function which in the reference state is ϕ_0; $\varepsilon_{ij}, \gamma_{ij}$-kinematic characteristics of the strain; t_{ij}-the components of the stress tensor; m_{ij}-the components of the couple stress tensor; h_i-the components of the equlibrated stress vector; g-the intrinsic equilibrated force; A_{ijmn}, B_{ijmn}, C_{ijmn}, A_{ij}, a_{ij}, b_{ij}, τ and ξ from the constitutive equations are prescribed characteristic functions of the material, and they obey to the symmetry relations

$$A_{ijmn} = A_{mnij}, \quad C_{ijmn} = C_{mnij}, \quad A_{ij} = A_{ji}. \tag{5}$$

As time interval I we take $I = (0,\ T)$ and consider the mixed boundary-final problem P defined by the system of equations (1)-(4), the initial conditions in $\bar{B}$

$$\begin{aligned}
&u_i(x,0) = u_i^0(x), \quad \dot{u}_i(x,0) = u_i^1(x),\\
&\varphi_i(x,0) = \varphi_i^0(x), \quad \dot{\varphi}_i(x,0) = \varphi_i^1(x),\\
&\phi(x,0) = \phi^0(x), \quad \dot{\phi}(x,0) = \phi^1(x),
\end{aligned} \tag{6}$$

and the following boundary conditions

$$\begin{aligned}
&u_i(x,\ t) = \tilde{u}_i(x,\ t) \ \text{ on } \ \partial B \times (0,\ T),\\
&\varphi_i(x,\ t) = \tilde{\varphi}_i(x,\ t) \ \text{ on } \ \partial B \times (0,\ T),\\
&\phi(x,\ t) = \tilde{\phi}(x,\ t) \ \text{ on } \ \partial B \times (0,\ T).
\end{aligned} \tag{7}$$

Introducing the constitutive relations (3) into equations (1) and (2), we obtain the following system of equations

$$\begin{aligned}
&\rho \ddot{u}_i = \left(A_{ijmn}\varepsilon_{mn} + B_{ijmn}\gamma_{mn} + a_{ij}\phi\right)_{,j},\\
&I_{ij}\ddot{\varphi}_j = \left(B_{mnij}\varepsilon_{mn} + C_{ijmn}\gamma_{mn} + b_{ij}\phi\right)_{,j} + \varepsilon_{ijk}\left(A_{jkmn}\varepsilon_{mn} + B_{jkmn}\gamma_{mn} + a_{jk}\phi\right),\\
&\rho\kappa\ddot{\phi} = \left(A_{ij}\phi_{,i}\right)_{,j} - a_{ij}\varepsilon_{ij} - b_{ij}\gamma_{ij} - \xi\phi - \tau\dot{\phi}
\end{aligned} \tag{8}$$

By a solution of the mixed initial-boundary value problem of the theory of thermoelasticity of micropolar bodies with voids in the cylinder $\Omega_0 = B \times (0,\ T)$ we mean an ordered array (u_i, φ_i, ϕ) which satisfies the system of equations (8) for all $(x,t) \in \Omega_0$, the boundary conditions (7) and the initial conditions (6).

In our subsequent analysis we need to require the following restrictions

$$\rho > 0, \quad \kappa > 0, \quad \tau \geq 0 \tag{9}$$

Also, we need to impose the positivity of constitutive tensors from the above relations

$$
\begin{aligned}
&A_{ijmn}\eta_{ij}\eta_{mn} \geq a_1|\eta|, \quad a_1 > 0 \\
&B_{ijmn}\eta_{ij}\eta_{mn} \geq b_1|\eta|, \quad b_1 > 0 \qquad (10)\\
&C_{ijmn}\eta_{ij}\eta_{mn} \geq c_1|\eta|, \quad c_1 > 0
\end{aligned}
$$

$$A_{ij}\xi_i\xi_j \geq a_2|\xi|, \quad a_2 > 0 \qquad (11)$$

These assumptions are in agreement with the usual restrictions imposed in the mechanics of continua. We can couple these restrictions with the assumption that the Helmholtz free energy is a positive definite quadratic form and the best of their interpretation finds its place in the theory of mechanical stability:

$$
\begin{aligned}
\Psi = &\frac{1}{2}A_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + \frac{1}{2}C_{ijmn}\gamma_{ij}\gamma_{mn} + \frac{1}{2}A_{ij}\phi_{,i}\phi_{,j} + \\
&+\frac{1}{2}\xi\phi^2 + B_{ijmn}\varepsilon_{ij}\gamma_{mn} + a_{ij}\varepsilon_{ij}\phi + b_{ij}\gamma_{ij}\phi \geq \qquad (12)\\
&\geq \frac{a_0}{2}\left(|\nabla u|^2 + |\nabla v|^2 + |\phi|^2\right) + \frac{a_2}{2}|\nabla\phi|^2, \; a_0 > 0.
\end{aligned}
$$

In the above relations a_0, a_1 and a_2 are appropiate constants. Also, u and v are the vector notations, namely $u = (u_i)$, $v = (\varphi_i)$.

The functions, together with their domains of definition, used in the above equations of motion, in initial conditions and boundary conditions are supposed to be as smooth as required.

3 Preliminary identities

Before tackling the mixed initial-boundary value problem of the theory of thermoelasticity of micropolar bodies with voids we will construct certain auxiliary problems and will prove some bounds for the solutions of these auxiliary problems. The identities that we will obtain are commonly called Rellich identities.

Using an idea suggested in the paper Knops and Payne (1988), we will consider the following formal boundary value problem

$$
\begin{aligned}
&\left(A_{ij}\,H_{,j}\right)_{,i} = 0, \text{ in } B \\
&H = q, \text{ on } \partial B. \qquad (13)
\end{aligned}
$$

We assume that the functions H and q (as well as the surface ∂B) satisfy the regularity conditions required by the theorem of existence (see Fichera (1972)). Therefore, based on this theorem, we ensure the existence of a solution of the problem (13).

Theorem 1. *If H is a solution of the problem (13), then the quantities*

$$\int_B A_{ij}H_{,i}H_{,j}\, dV,\;\; H^2,\;\; \oint_{\partial B}\left(\frac{\partial H}{\partial n}\right)^2 dA$$

admit some bounds in terms of the data function q.

Proof. Starting from the obvious equality

$$\int_B x_k H_{,k}(A_{ij}H_{,j})_{,i} dx = 0.$$

we can derive, without much difficulty (integrating by parts), the following sequence of relations

$$\begin{aligned}
0 &= -\int_B x_{k,i}H_{,k}A_{ij}H_{,j}\, dV - \int_B x_k H_{,ki}A_{ij}H_{,j}\, dV + \oint_{\partial B} n_i x_k H_{,k}A_{ij}H_{,j}\, dA \\
&= -\int_B A_{ij}H_{,i}H_{,j}\, dV - \frac{1}{2}\int_B x_k A_{ij}(H_{,i}H_{,j})_{,k}\, dV + \oint_{\partial B} n_i x_k H_{,k}A_{ij}H_{,j}\, dA \\
&= \frac{1}{2}\int_B A_{ij}H_{,i}H_{,j}\, dV + \frac{1}{2}\int_B x_k A_{ij,k}H_{,i}H_{,j}\, dV + \oint_{\partial B} n_i x_k H_{,k}A_{ij}H_{,j}\, dA \\
&- \frac{1}{2}\oint_{\partial B} n_i x_k A_{ij}H_{,k}H_{,j}\, dA
\end{aligned} \tag{14}$$

where n_i represent the components of the unit normal to the surface ∂B. If we denote by s_i the components of the tangential vector to the surface ∂B, we can write the derivative $H_{,i}$ in the form

$$H_{,i} = n_i \frac{\partial H}{\partial n} + s_i \nabla_s H$$

where $\partial/\partial n$ denotes the derivative in the direction of the normal and ∇_s is the tangential derivative.

If we denote by $a^{\alpha\beta}$ the coefficients of the first fundamental form of the surface ∂B, then the tangential derivative can be written in the form

$$\nabla_s H = x_{i;\alpha} a^{\alpha\beta} H_{;\beta}$$

where the notation $f_{;\alpha}$ represents the differentiation of function f with respect to surface variables t^{α}.

In what follows we suppose that the surface ∂B is star shaped with respect to origin and, furthermore, we have

$$x_k n_k \geq h_0,\; |x_k s_k| \leq \delta_0 \text{ on } \partial B.$$

Also, we need the following limitation of the gradients of A_{ij}

$$\left|x_k A_{ij,k}\right| \leq a_s < \mu\, a_2, \quad 0 < \mu < 1 \tag{15}$$

In the above relations h_0, δ_0, a_s and μ are constants.
If we take into account these conditions, (14) can be rewritten in the form

$$\begin{aligned}
&\frac{1-\mu}{2}\int_B A_{ij}H_{,i}H_{,j}\,dV + \frac{h_0 a_2}{2}\oint_{\partial B}\left(\frac{\partial H}{\partial n}\right)^2 dA \\
&\leq \oint_{\partial B}\left(\frac{1}{2}x_k n_k s_i s_j - n_i s_j s_k x_k\right)A_{ij}|\nabla_s q|^2\,dA \\
&+\frac{\delta_0\alpha}{2}\oint_{\partial B}A_{ij}n_i n_j\left(\frac{\partial H}{\partial n}\right)^2 dA + \frac{\delta_0\alpha}{2\alpha}\oint_{\partial B}A_{ij}n_i n_j|\nabla_s q|^2 dA
\end{aligned} \tag{16}$$

where α will be conveniently chosen.
Thus, if we choose α of the form

$$\alpha = \frac{h_0 a_2}{2\delta_0 a_3}$$

then

$$h_0 a_2 > \delta_0 \alpha a_3$$

where we used the notation

$$a_3 = \max_{\partial B}\left|\, n_i n_j A_{ij}\,\right|$$

With these considerations, we can write the inequality (16) in the form

$$\begin{aligned}
&\frac{1-\mu}{2}\int_B A_{ij}H_{,i}H_{,j}\,dV + \frac{h_0 a_2}{4}\oint_{\partial B}\left(\frac{\partial H}{\partial n}\right)^2 dA \\
&\leq \oint_{\partial B}\left[\left(\frac{1}{2}x_k n_k s_i s_j - n_i s_j s_k x_k\right) + \frac{\delta_0^2 a_3}{h_0 a_2}n_i n_j\right]A_{ij}|\nabla_s q|^2\,dA
\end{aligned} \tag{17}$$

On the other hand, with the help of the Poincare inequality we are lead to

$$\lambda_1\int_B H^2\,dV \leq \int_B H_{,i}H_{,j}\,dV + C_1\oint_{\partial B}H^2\,dA \tag{18}$$

Inequalities (17) and (18) give us the desired bounds for quantities $\int_B A_{ij}H_{,i}H_{,j}\,dV$, $\|\phi\|^2$ and $\oint_{\partial B}\left(\frac{\partial\varphi}{\partial n}\right)^2 dA$, these bounds being expressed with the help of the boundary function q. The proof of Theorem 1 is concluded.

Our next considerations are analog to those of Theorem 1 but for a vector version of the inequality (17).

By analogy with (13), taking into account the geometric equations (4), we consider (U_i, F_i) a solution of the boundary value problem

$$\left[A_{ijmn}\left(U_{j,i}+\varepsilon_{jik}F_k\right)+B_{ijmn}F_{n,m}\right]_{,j}=0, \text{ in } B$$
$$\left[B_{mnij}\left(U_{n,m}+\varepsilon_{nmk}F_k\right)+C_{ijmn}F_{n,m}\right]_{,j}+$$
$$+\varepsilon_{ijk}\left[A_{jkmn}\left(U_{n,m}+\varepsilon_{nmk}F_k\right)+B_{jkmn}F_{n,m}\right]=0, \text{ in } B \tag{19}$$
$$U_i=\dot{g}_i,\ F_i=\dot{f}_i, \text{ on } \partial B.$$

We assume that the functions (U_i, F_i), (g_i, f_i), as well as the surface ∂B, satisfy the regularity conditions required by the theorem of existence of Fichera (see the paper Fichera (1972). Based on this theorem, we ensure the existence of a solution of the problem (19).

Theorem 2. *If (U_i, F_i) is a solution of the problem (19), then the quantities*

$$\int_B\left[A_{ijmn}\left(U_{j,i}+\varepsilon_{jik}F_k\right)\left(U_{n,m}+\varepsilon_{nmk}F_k\right)+2B_{ijmn}\left(U_{j,i}+\varepsilon_{jik}F_k\right)F_{n,m}+\right.$$
$$\left.+C_{ijmn}F_{j,i}F_{n,m}\right]dV,\ \|U\|^2,\ \|F\|^2,\ \oint_{\partial B}\left(\frac{\partial U_i}{\partial n}\right)\left(\frac{\partial U_i}{\partial n}\right)dA,\ \oint_{\partial B}\left(\frac{\partial F_i}{\partial n}\right)\left(\frac{\partial F_i}{\partial n}\right)dA$$

admit some bounds in terms of the data functions g_i and f_i

Proof. To obtain an a priori bounds for

$$\int_B\left[A_{ijmn}\left(U_{j,i}+\varepsilon_{jik}F_k\right)\left(U_{n,m}+\varepsilon_{nmk}F_k\right)+2B_{ijmn}\left(U_{j,i}+\varepsilon_{jik}F_k\right)F_{n,m}+C_{ijmn}F_{j,i}F_{n,m}\right]dV$$
$$\oint_{\partial B}\left(\frac{\partial U_i}{\partial n}\right)\left(\frac{\partial U_i}{\partial n}\right)dA,$$
$$\oint_{\partial B}\left(\frac{\partial U_i}{\partial n}\right)\left(\frac{\partial U_i}{\partial n}\right)dA,$$

we can use similar assessments with those in the proof of Theorem 1.

To obtain an a priori bounds for $\|U\|^2$, $\|F\|^2$ we can use the Poincare inequality, and thus we find that

$$\begin{aligned}\lambda_1\int_B U_iU_i\,dV &\le \int_B U_{i,j}U_{i,j}dV+C_1\oint_{\partial B}\dot{g}_i\dot{g}_i\,dA\\ \lambda_2\int_B F_iF_i\,dV &\le \int_B F_{i,j}F_{i,j}dV+C_2\oint_{\partial B}\dot{f}_i\dot{f}_i\,dA\end{aligned} \tag{20}$$

and these inequalities give a bound for PUP^2 and PFP^2. W

Now, we proceed to derive some estimates for a solution of equations (8). For this purpose, we use the bounds found in Theorem 1 and in Theorem 2 for the solutions of the auxiliary problems (13) and (19).

First, we write equations (8) in the form

$$
\begin{aligned}
&-\rho\ddot{u}_i+\left(A_{ijmn}\varepsilon_{mn}+B_{ijmn}\gamma_{mn}+a_{ij}\phi\right)_{,j}=0\\
&-I_{ij}\ddot{\varphi}_j+\left(B_{mnij}\varepsilon_{mn}+C_{ijmn}\gamma_{mn}+b_{ij}\phi\right)_{,j}+\\
&+\varepsilon_{ijk}(A_{jkmn}\varepsilon_{mn}+B_{jkmn}\gamma_{mn}+a_{jk}\phi)=0, \qquad (21)\\
&-\rho\kappa\ddot{\phi}-a_{ij}\varepsilon_{ij}-b_{ij}\gamma_{ij}-\xi\phi-\tau\dot{\phi}+\left(A_{ij}\phi_{,j}\right)_{,i}=0
\end{aligned}
$$

the system of equations (21) being defined on the cylinder $B\times(0,T)$

We assume that a solution $(u_i,\ \varphi_i,\ \phi)$ of the system (21) satisfies the initial conditions (6) and boundary conditions (7).

Theorem 3. *If $(u_i,\ \varphi_i,\ \phi)$ is a solution of the system (21) which satisfies the initial conditions (6) and boundary conditions (7), then the quantities*

$$\int_0^t\|\nabla v\|^2ds,\ \int_0^t\|\nabla\varphi\|^2ds$$

admit some bounds in terms of the data functions u_i^0, $u_i^1\,\varphi_i^0$, φ_i^1, ϕ^0 and ϕ^1, with $v=(u_i,\ \varphi_i)$.

Proof. Let us denote by H the solution of equation $(13)_1$ which satisfies the boundary condition $H=\dot{h}$ on ∂B. If we take into account equation $(21)_3$ we obtain easily the identity

$$\int_0^t\left(\left(A_{ij}\varphi_{,j}\right)_{,i}-a_{ij}\varepsilon_{ij}-b_{ij}\gamma_{ij}-\xi\varphi-\tau\dot{\varphi}-\rho\kappa\ddot{\varphi},\ H-\dot{\varphi}\right)ds=0. \qquad (22)$$

We integrate (22) by parts with respect to $s\in[0,t]$ and with respect to the spatial variable on domain B and thus we are lead to the identity

$$
\begin{aligned}
&\frac{1}{2}\int_B\xi\varphi^2(t)\,dV+\frac{1}{2}\int_B\rho\kappa\dot{\varphi}^2(t)\,dV+\int_0^t\int_B\tau\dot{\varphi}^2(s)\,dV\,ds\\
&+\frac{1}{2}\int_B A_{ij}\varphi_{,i}(t)\varphi_{,j}(t)\,dV+\int_0^t\int_B(a_{ij}\varepsilon_{ij}+b_{ij}\gamma_{ij})\,\dot{\varphi}(s)\,dV\,ds\\
&=\frac{1}{2}\int_B A_{ij}\varphi^0_{,i}\varphi^0_{,j}\,dV+\int_0^t\oint_{\partial B}n_in_jA_{ij}h\frac{\partial H}{\partial n}\,dA\,ds \qquad (23)
\end{aligned}
$$

$$+\int_0^t \oint_{\partial B} n_j s_i A_{ij} h \nabla_s \dot{h}\, dA\, ds + \frac{1}{2}\left(\xi \varphi^0,\ \varphi^0\right) - \left(\rho\kappa\varphi^1, H(0)\right)$$
$$+\frac{1}{2}\left(\rho\kappa\varphi^1, \varphi^1\right) + \int_0^t \left(a_{ij}\varepsilon_{ij} + b_{ij}\gamma_{ij},\ H\right)\, ds + \int_0^t \left(\xi\varphi,\ H\right)\, ds$$
$$+\left(\rho\kappa\dot{\varphi},\ H\right) - \int_0^t \left(\rho\kappa\dot{\varphi}, \dot{H}\right) ds + \int_0^t \left(\tau\dot{\varphi}, H\right) ds$$

For the last five terms on the right-hand side of relation (23) we will use the arithmetic-geometric mean inequality in the form

$$< a,b > \le \frac{1}{2}\left(\mu \|a\|^2 + \frac{1}{\mu}\|b\|^2\right),\ \mu > 0. \tag{24}$$

For this reason we use the notation

$$a_M = \max_{\bar{B}} \left|a_{ij}\right|,\ b_M = \max_{\bar{B}} \left|b_{ij}\right|,\ \rho_M = \max_{\bar{B}} |\rho|,$$

$$\kappa_M = \max_{\bar{B}} |\kappa|,\ \xi_M = \max_{\bar{B}} |\xi|,\ \tau_M = \max_{\bar{B}} |\tau|$$

Using this notation, we can conveniently choose the constants μ_1, μ_2, μ_3, μ_4, μ_5 and so by applying the arithmetic-geometric mean inequality (24), from relation (23) we can deduce

$$\begin{aligned}
&\frac{1}{2}\int_B \xi\varphi^2(t)\, dV + \frac{1}{2}\int_B \rho\kappa\dot{\varphi}^2(t)\, dV + \int_0^t \int_B \tau\dot{\varphi}^2(s)\, dV\, ds \\
&+\frac{1}{2}\int_B A_{ij}\varphi_{,i}(t)\varphi_{,j}(t)\, dV + \int_0^t \int_B \left(a_{ij}\varepsilon_{ij} + b_{ij}\gamma_{ij}\right)\dot{\varphi}(s)\, dV\, ds \\
&\le \frac{1}{2}\int_B A_{ij}\varphi^0_{,i}\varphi^0_{,j}\, dV + \int_0^t \oint_{\partial B} n_i n_j A_{ij} h \frac{\partial H}{\partial n}\, dA\, ds \\
&+\int_0^t \oint_{\partial B} n_j s_i A_{ij} h \nabla_s \dot{h}\, dA\, ds + \frac{1}{2}\left(\xi\varphi^0,\ \varphi^0\right) - \left(\rho\kappa\varphi^1, H(0)\right) \\
&+\frac{1}{2}\left(\rho\kappa\varphi^1, \varphi^1\right) + \frac{\mu_1}{2}\int_0^t \left\|\varepsilon_{ij}\right\|^2 ds + \frac{a_M^2}{2\mu_1}\int_0^t \|H\|^2 ds \\
&+\frac{\mu_2}{2}\int_0^t \left\|\gamma_{ij}\right\|^2 ds + \frac{b_M^2}{2\mu_2}\int_0^t \|H\|^2 ds + \frac{\mu_3}{2}\int_0^t \|\varphi\|^2 ds \\
&+\frac{\xi_M^2}{2\mu_3}\int_0^t \|H\|^2 ds + \frac{\mu_4}{2}\left(\rho\kappa\dot{\varphi}, \dot{\varphi}\right) + \frac{\rho_M\kappa_M}{2\mu_4}\|H\|^2 \\
&+\frac{\mu_5}{2}\int_0^t \|\dot{\varphi}\|^2 ds + \frac{\left(\rho_M\kappa_M\right)^2}{2\mu_5}\int_0^t \left\|\dot{H}\right\|^2 ds + \frac{\mu_6}{2}\int_0^t \|\dot{\varphi}\|^2 ds
\end{aligned} \tag{25}$$

$$+\frac{\tau_M^2}{2\mu_6}\int_0^t \|H\|^2 ds$$

Terms relating to ϕ, $\dot{\phi}$ and $\phi_{,i}$ from the right-hand side of relation (25) may be bounded using the left-hand side, while the terms relating to H from the right-hand side of relation (25) may be bounded by using the estimations found by using Theorem 1 and Theorem 2.

Analog considerations with the above one will now be made relative to the first two equations of system (21)

So, we consider the equation $(21)_1$ and if we take into account equation $(19)_1$ we find the identity

$$\int_0^t \Big(\left[A_{ijmn}\left(U_{n,m}+\varepsilon_{nmk}F_k\right)+B_{ijmn}F_{n,m}\right]_{,j}+\left(a_{ij}\varphi\right)_{,j}-\rho\ddot{u}_i, U_i-\dot{u}_i \Big)\, ds = 0 \tag{26}$$

Finally, if we consider the equation $(21)_2$ and if we take into account equation $(19)_2$ we find the identity

$$\int_0^t \Big(\left[B_{mnij}\left(U_{n,m}+\varepsilon_{nmk}F_k\right)+C_{ijmn}F_{n,m}\right]_{,j}+\left(b_{ij}\varphi\right)_{,j}-I_{ij}\ddot{\phi}_j, F_i-\dot{\phi}_i \Big)\, ds = 0 \tag{27}$$

Adding together the relations (26) and (27), term by term, then integrating by parts in space and in time and taking into account the geometric equations (4) we get

$$\begin{aligned}
&\frac{1}{2}\int_0^t \frac{d}{ds}\left[\int_B \left(\rho\dot{u}_i\dot{u}_i+I_{ij}\dot{\phi}_i\dot{\phi}_j+A_{ijmn}\varepsilon_{ij}\varepsilon_{mn}+2B_{ijmn}\varepsilon_{ij}\gamma_{mn}+C_{ijmn}\gamma_{ij}\gamma_{mn}\right)dV\right]ds \\
&+\int_0^t\int_B \left(a_{ij}\dot{\varepsilon}_{ij}+b_{ij}\dot{\gamma}_{ij}\right)\varphi\, dV ds = \int_0^t \oint_{\partial B} n_m\left(A_{ijmn}g_nU_{i,j}+B_{ijmn}f_nF_{i,j}\right)dAds \\
&+\int_0^t \oint_{\partial B} n_m\left(B_{ijmn}g_nU_{i,j}+C_{ijmn}f_nF_{i,j}\right)dAds-\int_B\left(\rho U_i(0)u_i^1+I_{ij}F_i(0)\phi_j^1\right)dV \\
&+\int_0^t\int_B \left(a_{ij}U_{i,j}+b_{ij}F_{i,j}\right)\varphi\, dVds+\int_B\left(\rho U_i\dot{u}_i+I_{ij}F_i\dot{\phi}_j\right)dV \\
&-\int_0^t\int_B\left(\rho\dot{U}_i\dot{u}_i+I_{ij}\dot{F}_i\dot{\phi}_j\right)dVds
\end{aligned} \tag{28}$$

For the last three integrals on the right-hand side of relation (28) we will use the arithmetic-geometric mean inequality in the form (24). Thus we can conveniently choose the positive constants α_1, α_2, ..., α_6 so that

$$\frac{1}{2}\int_B \left(\rho\dot{u}_i\dot{u}_i+I_{ij}\dot{\phi}_i\dot{\phi}_j+A_{ijmn}\varepsilon_{ij}\varepsilon_{mn}+2B_{ijmn}\varepsilon_{ij}\gamma_{mn}+C_{ijmn}\gamma_{ij}\gamma_{mn}\right)dV$$

$$
\begin{aligned}
&+\int_0^t\int_B \left(a_{ij}\dot{\varepsilon}_{ij}+b_{ij}\dot{\gamma}_{ij}\right)\varphi\, dVds \leq \frac{1}{2}\int_B \left(\rho \dot{u}_i^1\dot{u}_i^1+I_{ij}\dot{\phi}_i^1\dot{\phi}_j^1\right)dV\\
&+\frac{1}{2}\int_B \left(A_{ijmn}\varepsilon_{ij}^0\varepsilon_{mn}^0+2B_{ijmn}\varepsilon_{ij}^0\gamma_{mn}^0+C_{ijmn}\gamma_{ij}^0\gamma_{mn}^0\right)dV\\
&+\int_0^t\oint_{\partial B} n_m\left(A_{ijmn}+B_{ijmn}\right)g_n\left(n_j\frac{\partial U_i}{\partial n}+s_j\nabla_s\dot{g}_i\right)dAds\\
&+\int_0^t\oint_{\partial B} n_m\left(B_{mnij}+C_{ijmn}\right)f_n\left(n_j\frac{\partial F_i}{\partial n}+s_j\nabla_s\dot{f}_i\right)dAds \qquad (29)\\
&-\int_B \left(\rho U_i(0)u_i^1(0)+I_{ij}F_i(0)\phi_j^1(0)\right)dV+\frac{\alpha_1}{2}\int_0^t\|\varphi\|^2ds+\frac{a_M^2}{2\alpha_1}\int_0^t\left\|U_{i,\,j}\right\|^2ds\\
&+\frac{\alpha_2}{2}\int_0^t\|\varphi\|^2ds+\frac{b_M^2}{2\alpha_2}\int_0^t\left\|F_{i,\,j}\right\|^2ds+\frac{\alpha_3}{2}\int_B\rho\dot{u}_i\dot{u}_idV+\frac{1}{2\alpha_3}\int_B\rho U_iU_idV\\
&+\frac{\alpha_4}{2}\int_B I_{ij}\dot{\phi}_i\dot{\phi}_jdV+\frac{1}{2\alpha_4}\int_B I_{ij}F_iF_jdV+\frac{\alpha_5}{2}\int_0^t\int_B\rho\dot{u}_i\dot{u}_idVds\\
&+\frac{1}{2\alpha_5}\int_0^t\int_B\rho\dot{U}_i\dot{U}_idVds+\frac{\alpha_6}{2}\int_0^t\int_B I_{ij}\dot{\phi}_i\dot{\phi}_jdV+\frac{1}{2\alpha_6}\int_0^t\int_B I_{ij}\dot{F}_i\dot{F}_jdV
\end{aligned}
$$

where $\varepsilon_{ij}^0=u_{j,\,i}^0+\varepsilon_{jik}\varphi_k^0,\ \gamma_{ij}^0=\varphi_{j,\,i}^0$.

By adding the relations (25) and (29) we are lead to

$$
\begin{aligned}
&\frac{1}{2}\int_B\left(\rho\dot{u}_i\dot{u}_i+I_{ij}\dot{\phi}_i\dot{\phi}_j+A_{ijmn}\varepsilon_{ij}\varepsilon_{mn}+2B_{ijmn}\varepsilon_{ij}\gamma_{mn}+C_{ijmn}\gamma_{ij}\gamma_{mn}\right)dV\\
&+\frac{1}{2}(\xi\varphi,\ \varphi)+\frac{1}{2}(\rho\kappa\dot{\varphi},\ \dot{\varphi})+\int_0^t(\tau\dot{\varphi},\ \dot{\varphi})ds+\frac{1}{2}\int_B A_{ij}\varphi_{,\,i}\varphi_{,\,j}\,dV\\
&+\int_B\left(a_{ij}\varepsilon_{ij}+b_{ij}\gamma_{ij}\right)\varphi\,dV\leq\frac{\mu_1}{2}\int_0^t\left\|\varepsilon_{ij}\right\|^2ds+\frac{\mu_2}{2}\int_0^t\left\|\gamma_{ij}\right\|^2ds \qquad (30)\\
&+\frac{\mu_3+\alpha_1+\alpha_2}{2}\int_0^t\|\varphi\|^2ds+\frac{\mu_4}{2}(\rho\kappa\dot{\varphi},\dot{\varphi})+\frac{\mu_5+\mu_6}{2}\int_0^t\|\dot{\varphi}\|^2ds\\
&+\frac{\alpha_3}{2}\int_B\left(\rho\dot{u}_i\dot{u}_i+I_{ij}\dot{\phi}_i\dot{\phi}_j\right)dV+\frac{\alpha_5}{2}\int_0^t\int_B\left(\rho\dot{u}_i\dot{u}_i+I_{ij}\dot{\phi}_i\dot{\phi}_j\right)dVds+R
\end{aligned}
$$

Here we have denoted by R the other terms resulting from the gathered formulas (25) and (29). The remaining R contains either terms that involve direct data either terms that can be estimated in terms of data based on Theorem 1 and Theorem 2, generically denoted by M. With these considerations, we can deduce that the left-hand side of inequality (30) is a positive definite measure. Also, if we use as energy measure the function E, given by

$$
E(t)=\frac{1}{2}\int_B\left[\rho\dot{u}_i(t)\dot{u}_i(t)+I_{ij}\dot{\phi}_i(t)\dot{\phi}_j(t)+\rho\kappa\dot{\varphi}^2(t)\right]dV
$$

$$+\frac{1}{2}\int_B [A_{ijmn}\varepsilon_{ij}(t)\varepsilon_{mn}(t)+2B_{ijmn}\varepsilon_{ij}(t)\gamma_{mn}(t)+C_{ijmn}\gamma_{ij}(t)\gamma_{mn}(t)]\,dV$$

$$+\int_B [a_{ij}\varepsilon_{ij}(t)+b_{ij}\gamma_{ij}(t)]\,\varphi(t)dV+\frac{1}{2}\left(\xi\varphi(t),\varphi(t)\right)+\frac{1}{2}\int_B A_{ij}\varphi_{,i}(t)\varphi_{,j}(t)dV \quad (31)$$

then from (30) we can deduce the following inequality

$$E(t)\le K\int_0^t E(s)ds+M, \quad (32)$$

in which the positive constant K is conveniently chosen.

Now we multiply both members of inequality (32) by e^{-Kt} and after the resulting inequality is integrated over $[0,t]$ we are led to the result

$$\int_0^t E(s)ds\le\frac{e^{KT}}{K}M. \quad (33)$$

It is clear that this inequality provides a priori estimates for quantities

$$\int_0^t \|\nabla v\|^2ds,\ \int_0^t \|\nabla\varphi\|^2ds$$

which concludes the proof of Theorem 3.

4 Main result

In this section we will prove the continuous dependence of solutions of the mixed initial-boundary value problem P_1 consisting of equations (8), the initial conditions (6) and the boundary conditions (7) with respect to coupling coefficients a_{ij} and b_{ij}

To this end we consider a solution (u_i,φ_i,ϕ) of our problem P_1 and (v_i,ψ_i,χ) a solution of problem P_2 which is similar to the problem P_1 namely, it has the same initial data, the same boundary data, but different coupling coefficients. The coupling coefficients of problem P_1 are a_{ij}, b_{ij} and of problem P_2 are α_{ij}, β_{ij}. We assume that the other characteristics coefficients of the material are the same for both problems P_1 and P_2, namely A_{ijmn}, B_{ijmn}, C_{ijmn}, A_{ij}, ρ, κ, τ and ξ

Let us denote by (w_i,ϖ_i,θ) the difference between the two solutions and by c_{ij}, d_{ij} the difference between the coupling coefficients, that is

$$w_i=u_i-v_i,\quad \varpi_i=\varphi_i-\psi_i,\quad \theta=\phi-\chi,$$

$$c_{ij}=a_{ij}-\alpha_{ij},\quad d_{ij}=b_{ij}-\beta_{ij} \quad (34)$$

Due to linearity, the difference (w_i,ϖ_i,θ) satisfies a system of equation similar to that of (8):

$$-\rho\ddot{w}_i+\left(A_{ijmn}\bar{\varepsilon}_{mn}+B_{ijmn}\bar{\gamma}_{mn}\right)_{,j}+\left(\alpha_{ij}\theta\right)_{,j}=-\left(c_{ij}\phi\right)_{,j}$$

$$-I_{ij}\ddot{\psi}_j+(B_{mnij}\bar{\varepsilon}_{mn}+C_{ijmn}\bar{\gamma}_{mn})_{,\,j}+(\beta_{ij}\theta)_{,j}+$$
$$+\varepsilon_{ijk}(A_{jkmn}\bar{\varepsilon}_{mn}+B_{jkmn}\bar{\gamma}_{mn}+\alpha_{jk}\theta)=-(d_{ij}\phi)_{,\,j}, \tag{35}$$

$$-\rho\kappa\ddot{\theta}-\alpha_{ij}\bar{\varepsilon}_{ij}-\beta_{ij}\bar{\gamma}_{ij}-\xi\theta-\tau\dot{\theta}+(A_{ij}\theta_{,\,j})_{,\,i}=-c_{ij}\varepsilon_{ij}-d_{ij}\gamma_{ij} \tag{36}$$

where we used the notations

$$\bar{\varepsilon}_{ij}=w_{j,\,i}+\varepsilon_{jik}\varpi_k, \quad \bar{\gamma}_{ij}=\varpi_{j,\,i}$$

Equations (35)-(36) are satisfied on the cylinder $B\times(0,T)$

Also, due to linearity, the difference (w_i,ϖ_i,θ) satisfies the initial conditions in their homogeneous form

$$w_i(x,0)=0,\ \dot{w}_i(x,0)=0,\ \varpi_i(x,0)=0,$$

$$\dot{\varpi}_i(x,0)=0,\ \theta(x,0)=0,\ \dot{\theta}(x,0)=0 \tag{37}$$

and, also, the boundary conditions in their homogeneous form

$$w_i(x,t)=0,\ \varpi_i(x,t)=0,\ \theta(x,t)=0,\ (x,t)\in\partial B\times(0,T) \tag{38}$$

The main result regarding the continuous dependence with respect to coupling coefficients is proved in the following theorem.

Theorem 4. *The solutions of the mixed initial-boundary value problem consists of equations (8), the initial conditions (6) and the boundary conditions (7) depend continuously with respect to coupling coefficients a_{ij} and b_{ij}*

Proof. We multiply equation $(35)_1$ by $\dot{w}_i$ and then integrate over B to find

$$\frac{d}{dt}\frac{1}{2}\left[\int_B\rho\dot{w}_i\dot{w}_idV+\int_B(A_{ijmn}\bar{\varepsilon}_{mn}+B_{ijmn}\bar{\gamma}_{mn})\,w_{i,\,j}dV\right]+$$
$$+\int_B\alpha_{ij}\,\theta\,\dot{w}_{i,\,j}dV=\int_B(c_{ij}\varphi)_{,\,j}\,\dot{w}_idV \tag{39}$$

Similarly, we multiply equation $(35)_2$ by $\dot{\varpi}_i$ and then integrate over B to find

$$\frac{d}{dt}\frac{1}{2}\left[\int_B I_{ij}\dot{\varpi}_i\dot{\varpi}_jdV+\int_B(B_{mnij}\bar{\varepsilon}_{mn}+C_{ijmn}\bar{\gamma}_{mn})\,\varpi_{i,\,j}\,dV+\right.$$
$$\left.+\int_B\varepsilon_{ijk}\left(A_{jkmn}\bar{\varepsilon}_{mn}+B_{jkmn}\bar{\gamma}_{mn}+\alpha_{jk}\,\theta\right)\dot{\varpi}_i\,dV\right]+ \tag{40}$$
$$+\int_B\beta_{ij}\,\theta\,\dot{\varpi}_{i,\,j}dV=\int_B(d_{ij}\varphi)_{,\,j}\,\dot{\varpi}_i\,dV$$

Now, we add equations (39) and (40) and take into account the geometric equations (6) to see that

$$\frac{d}{dt}\frac{1}{2}\left[\int_B (\rho \dot{w}_i \dot{w}_i + I_{ij}\dot{\varpi}_i\dot{\varpi}_j)\, dV + \right.$$

$$\left. + \int_B (A_{ijmn}\bar{\varepsilon}_{ij}\bar{\varepsilon}_{mn} + 2B_{ijmn}\bar{\varepsilon}_{ij}\bar{\gamma}_{mn} + C_{ijmn}\bar{\gamma}_{ij}\bar{\gamma}_{mn})\, dV\right] + \tag{41}$$

$$+ \int_B \left(\alpha_{ij}\dot{\bar{\varepsilon}}_{ij} + \beta_{ij}\dot{\bar{\gamma}}_{ij}\right) \theta dV = \int_B \left[(c_{ij}\varphi)_{,j}\dot{w}_i + (d_{ij}\varphi)_{,j}\, \dot{\varpi}_i\right] dV$$

We will multiply now equation (36) by $\dot{\theta}$ and then integrate over B so that we find

$$\frac{d}{dt}\frac{1}{2}\left[\left(\rho\kappa\dot{\theta}, \dot{\theta}\right) + \int_B A_{ij}\theta_{,i}\theta_{,j} dV + (\xi\theta, \theta)\right] + \left(\tau\dot{\theta}, \dot{\theta}\right) +$$

$$+ \int_B (\alpha_{ij}\bar{\varepsilon}_{ij} + \beta_{ij}\bar{\gamma}_{ij})\, \dot{\theta} dV = -\int_B (c_{ij}\varepsilon_{ij} + d_{ij}\gamma_{ij})\, \dot{\theta} dV \tag{42}$$

If we add the equations (41) and (42), term by term, we deduce that

$$\frac{d}{dt}\left\{\frac{1}{2}\left[\int_B (\rho \dot{w}_i \dot{w}_i + I_{ij}\dot{\varpi}_i\dot{\varpi}_j)\, dV + (\xi\theta, \theta) + \left(\rho\kappa\dot{\theta}, \dot{\theta}\right) + \right.\right.$$

$$+ \int_B (A_{ijmn}\bar{\varepsilon}_{ij}\bar{\varepsilon}_{mn} + 2B_{ijmn}\bar{\varepsilon}_{ij}\bar{\gamma}_{mn} + C_{ijmn}\bar{\gamma}_{ij}\bar{\gamma}_{mn})\, dV + \tag{43}$$

$$\left.\left. + \int_B A_{ij}\theta_{,i}\theta_{,j} dV\right] + \int_B (\alpha_{ij}\bar{\varepsilon}_{ij} + \beta_{ij}\bar{\gamma}_{ij})\, \theta dV\right\} + \left(\tau\dot{\theta}, \dot{\theta}\right) =$$

$$= \int_B \left[(c_{ij}\varphi)_{,j}\dot{w}_i + (d_{ij}\varphi)_{,j}\dot{\varpi}_i\right] dV - \int_B (c_{ij}\varepsilon_{ij} + d_{ij}\gamma_{ij})\, \dot{\theta} dV$$

We will use the notation

$$c^2 = c_{ij}\, c_{ij},\ c^{*2} = c_{ij,j}\, c_{ik,k},\ d^2 = d_{ij}\, d_{ij},$$

$$d^{*2} = d_{ij,j}\, d_{ik,k},\ \rho_m = \min_B \rho,\ \kappa_m = \min_B \kappa,\ I_m = \min_B I_{ij}$$

and we apply the Schwarz's inequality in relation (43) and taking into account the hypotheses (10) we find that

$$\frac{d}{dt}\left\{\frac{1}{2}\left[\int_B (\rho \dot{w}_i \dot{w}_i + I_{ij}\dot{\varpi}_i\dot{\varpi}_j)\, dV + (\xi\theta, \theta) + \left(\rho\kappa\dot{\theta}, \dot{\theta}\right) + \right.\right.$$

$$+ \int_B (A_{ijmn}\bar{\varepsilon}_{ij}\bar{\varepsilon}_{mn} + 2B_{ijmn}\bar{\varepsilon}_{ij}\bar{\gamma}_{mn} + C_{ijmn}\bar{\gamma}_{ij}\bar{\gamma}_{mn})\, dV +$$

$$+\int_B A_{ij}\theta_{,i}\theta_{,j}dV\Big]+\int_B(\alpha_{ij}\bar{\varepsilon}_{ij}+\beta_{ij}\bar{\gamma}_{ij})\,\theta dV\Big\}+\left(\tau\dot{\theta},\dot{\theta}\right)\leq \tag{44}$$

$$\leq\frac{1}{2\rho_m}\int_B\rho\dot{w}_i\dot{w}_i dV+\frac{1}{2I_m}\int_B I_{ij}\dot{\varpi}_i\dot{\varpi}_j dV+\frac{1}{2}\int_B\left(c^2+d^2\right)\varphi_{,k}\varphi_{,k}dV$$

$$+\int_B\left(\frac{1}{2a_1}A_{ijmn}\varepsilon_{ij}\varepsilon_{mn}+\frac{1}{b_1}B_{ijmn}\varepsilon_{ij}\gamma_{mn}+\frac{1}{2c_1}C_{ijmn}\gamma_{ij}\gamma_{mn}\right)dV$$

$$+\frac{1}{2}\int_B\left(c^{*2}+d^{*2}\right)\varphi^2dV+\frac{1}{2\kappa_m\rho_m}\left\|\dot{\theta}\right\|$$

Now we introduce the measure M by

$$M=\frac{1}{2}\Big[\int_B(\rho\dot{w}_i\dot{w}_i+I_{ij}\dot{\varpi}_i\dot{\varpi}_j)\,dV+(\xi\theta,\theta)+\left(\rho\kappa\dot{\theta},\dot{\theta}\right)+$$

$$+\int_B(A_{ijmn}\bar{\varepsilon}_{ij}\bar{\varepsilon}_{mn}+2B_{ijmn}\bar{\varepsilon}_{ij}\bar{\gamma}_{mn}+C_{ijmn}\bar{\gamma}_{ij}\bar{\gamma}_{mn})\,dV+ \tag{45}$$

$$+\int_B A_{ij}\theta_{,i}\theta_{,j}dV\Big]+\int_B(\alpha_{ij}\bar{\varepsilon}_{ij}+\beta_{ij}\bar{\gamma}_{ij})\,\theta dV$$

With the help of notations

$$c_M^2=\max_{\bar{B}}\left\{c^2+d^2\right\},\ c_M^{*2}=\max_{\bar{B}}\left\{c^{*2}+d^{*2}\right\},\ C=\max_{\bar{B}}\left\{\frac{2}{\rho_m},\frac{1}{\rho_m\kappa_m}\right\}$$

from (45) we deduce that

$$\frac{dM}{dt}\leq CM+c_M^2\Big[\int_B\left(\frac{1}{2a_1}A_{ijmn}\varepsilon_{ij}\varepsilon_{mn}+\frac{1}{b_1}B_{ijmn}\varepsilon_{ij}\gamma_{mn}+\frac{1}{2c_1}C_{ijmn}\gamma_{ij}\gamma_{mn}\right)dV$$

$$+\frac{1}{2}\int_B\varphi_{,k}\varphi_{,k}dV\Big]+\frac{1}{2}c_M^{*2}\|\varphi\|^2$$

After we integrate this inequality over $[0,t]$, we are led to

$$M(t)\leq C\int_0^t M(s)ds+$$

$$+c_M^2\Big[\int_0^t\int_B\left(\frac{1}{2a_1}A_{ijmn}\varepsilon_{ij}\varepsilon_{mn}+\frac{1}{b_1}B_{ijmn}\varepsilon_{ij}\gamma_{mn}+\frac{1}{2c_1}C_{ijmn}\gamma_{ij}\gamma_{mn}\right)dVds \tag{46}$$

$$+\frac{1}{2}\int_0^t\int_B\varphi_{,k}\varphi_{,k}dVds\Big]+\frac{1}{2}c_M^{*2}\int_0^t\|\varphi\|^2ds$$

Using the *a priori* estimate (33) we deduce that the last term of inequality (46) is bounded. Then we can write, formally, inequality (46) in the following form

$$M(t) \leq C \int_0^t M(s)ds + c_M^2 D_1 + c_M^{*2} D_2 \tag{47}$$

Here D_1 and D_1 are some terms that depend on data.

After integration over the interval $[0,t]$, inequality (47) leads to

$$\int_0^t M(s)ds \leq \left(c_M^2 D_1 + c_M^{*2} D_2\right) \frac{1}{C} \left(e^{CT} - 1\right) \tag{48}$$

Inequality (47) proves that the solution of the mixed initial-boundary value problem consists of the system of equations (8), the initial conditions (6) and the boundary conditions (7) which depend continuously on coupling coefficients a_{ij} and b_{ij}.

The evaluation of this dependence is made by means of measure $\int_t^\int 0$.

On the other hand, if we substitute the integral $\int_t^\int 0$ from (48) in (47), we obtain the following inequality

$$M(t) \leq \left(c_M^2 D_1 + c_M^{*2} D_2\right) e^{CT}$$

This inequality proves that the solution depends continuously on coupling coefficients c_{ij} and d_{ij} from (34). This time the evaluation of this dependence is made by means of measure $M(t)$. With this, the proof of Theorem 4 is complete.

5 Conclusions

In the first part of the study, we attach to our mixed problem certain auxiliary problems and prove some bounds for solutions of these problems. The second part of the study is devoted to obtain *a priori* estimates for the gradient of displacement (u_i) and microrotation (φ_i) and the gradient of the volume distribution function ϕ.

In the main result of the study we prove that the solution of this problem depends continuously on coefficients which couple the micropolar deformation equations the equations that model the evolution of voids. The evaluation of this dependence is made by using an appropriate measure.

Acknowledgement: The authors are deeply grateful to the reviewers, whose very pertinent suggestions have helped improve the paper.

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