 CMC, vol.45, no.3, pp.145-162, 2015

Flexoelectricity in Solid Dielectrics: From Theory to Applications

Jianfeng Lu[1], Xu Liang[1,2] and Shuling Hu[1,2]

Abstract: Flexoelectricity phenomenologically describes the universal electromechanical coupling effect between electric polarization and strain gradient, and electric field gradient and elastic strain. In contrast to piezoelectricity which is invalid in materials with inversion symmetry, flexoelectricity exists, commonly, in all solid dielectrics. In this paper, a summary of the research on flexoelectricity is presented to illustrate the development of this topic. Flexoelectricity still have many open questions and unresolved issues in the developing field, although it has attracted a surge of attention recently. Here we review the theoretical investigations and experimental studies on flexoelectricity, and the aim of the current paper is to look into the potential applications of this electromechanical coupling effect.

Keywords: Flexoelectricity, Strain gradient, Electric field gradient, Electromechanical coupling.

1 Introduction

The development of nanotechnology, such as high performance electronics, integrated circuit, microelectromechanical systems and nanoelectromechanical systems, has the deepest effect on our daily life [Craighead (2000); Ekinci and Roukes (2005)]. The conversion between mechanical energy and electrical energy has attracted a surge of attention, such as field effect transistors [Nishi (1978); Javey *et al.* (2003)], self-powered nanogenerators [Wang (2008); Xu *et al.* (2010); Fan *et al.* (2012)], sensors and actuators [Park and Gao (2006)]. A novel application is proposed to harvest the mechanical energy in the ambient based on the classical piezoelectricity [Sodano *et al.* (2004); Hong and Moon (2005); Friswell and Adhikari (2010)]. However, piezoelectric effect is commonly allowed in noncentrosymmetric media. The presence of non-uniform strain field such as strain

[1] State Key Laboratory for Strength and Vibration of Mechanical Structures, School of Aerospace, Xi'an Jiaotong University, Xi'an 710049, P. R. China.
[2] Corresponding Authors. E-mail: xul594@gmail.com; slhu@mail.xjtu.edu.cn

gradient can locally break the inversion symmetry and induces electric polarization in solid dielectrics, which has been termed as flexoelectric effect. Conversely, mechanical stress can be generated by an electric field gradient [Tagantsev (1987); Tagantsev (1991); Ma (2010); Lee and Noh (2012); Nguyen *et al.* (2013)]. Flexoelectricity phenomenologically describes the coupling between polarization and strain gradient, and electric field gradient and stress. In contrast to piezoelectricity which is invalid in materials with inversion symmetry, flexoelectricity exists in all solid dielectrics, even in soft membranes [Petrov (2002); Deng *et al.* (2014)] and biological tissues [Fu (2010)]. Flexoelectricity also manifests as a size-dependent electromechanical coupling effect due to the including of strain gradient and electric field gradient. Moreover, flexoelectricity hold the promising applications in nanoelectronics where strong strain gradients often be presented [Majdoub *et al.* (2009a); Fu *et al.* (2011); Lee *et al.* (2012)].

In this paper, a summary of research on flexoelectricity is presented to illustrate the development of such topic. The effect of flexoelectricity on the electromechanical coupling response of nanostructures, the modified electrostatic potential generated in a bent piezoelectric nanowires and piezoelectric semiconductor nanowires has been discussed. Especially, the authors focus on the experimental study on the flexoelectricity in solid materials, the experimental methods and results are discussed in this paper. The aim of this paper is to look into the potential applications of this electromechanical coupling effect in engineering.

2 Fundamental of flexoelectricity

Flexoelectric effect is a fundamental physical property of dielectrics which can be defined as the linear coupling between strain gradient and electric polarization, and linear coupling between stress and electric field gradient. Although flexoelectric effect is a universal electromechanical coupling effect, flexoelectricity has been ignored for a long time. Recently, it was realized that the flexoelectric effect may explain various physical phenomena in solids, such as the intrinsic “dead-layer” in ferroelectric capacitors [Majdoub *et al.* (2009a); Maranganti *et al.* (2009)], the size-dependent electromechanical coupling response of nanostructures [Liang and Shen (2013); Yan and Jiang (2013a); Yan and Jiang (2013b); Liang *et al.* (2014)], the rotation of electric polarization in ferroelectrics [Catalan *et al.* (2011)]. By introducing the flexoelectricity, Liu *et al.* [Liu *et al.* (2012)] analytically solved the electrostatic potential generated in a bent piezoelectric nanowire and Xu [Xu *et al.* (2013)] discussed the interaction between flexoelectric effect and semiconductor properties.

The fundamental physical formulation for the theory of flexoelectricity can be found in many literatures, Hu and Shen [Hu and Shen (2009)] developed a theory for nano-dielectrics with electric field gradient effect, surface and electrostatic force, Shen and Hu [Shen and Hu (2010)] developed a theory for solid dielectrics with flexoelectric effect, surface effect and electrostatic force. These works provided the fundamental physical and mathematical description of the flexoelectricity. Based on these theories, the effect of flexoelectric can be expressed as [Hu and Shen (2009); Liang *et al.* (2014)]:

$$\begin{cases} \sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \mu_{klij}\dfrac{\partial E_k}{\partial x_l} \\ P_k = \varepsilon_0\chi_{kl}E_l + e_{kij}\varepsilon_{ij} + \mu_{klij}\dfrac{\partial \varepsilon_{ij}}{\partial x_l} \end{cases} \tag{1}$$

where c_{ij} is the elastic modulus, e_{kij} is the piezoelectric constants, ε_0 is the dielectric constant of vacuum, χ_{ij} is the relative susceptibility and μ_{ijkl} is the flexoelectric coefficients. ε_{ij} and E_k are the strain and electric field, σ_{ij} and P_k are the Cauchy stress and electric polarization, respectively. The third terms in the right hand of Eq. (1) describe the direct and converse flexoelectric effect.

It is worth mentioning that in the case of small gradients (such as mechanical bending), Eq. (1) is suitable, and however, in the case of strong gradients the following expressions are suggested [Shen and Hu (2010); Yudin and Tagantsev (2013)]:

$$\begin{cases} \sigma_{ij} = c_{ijkl}\varepsilon_{kl} + d_{kij}P_k + e_{klij}\dfrac{\partial P_k}{\partial x_l} \\ E_k = \left(\varepsilon_0\chi_{kl}\right)^{-1}P_l + d_{kij}\varepsilon_{ij} + f_{klij}\dfrac{\partial \varepsilon_{ij}}{\partial x_l} \end{cases} \tag{2}$$

where e_{klij} and f_{klij} are the converse and direct flexocoupling coefficients, respectively. Eq. (1) and Eq. (2) give the completely full coupled description of flexoelectricity. Based on these phenomenological descriptions, a series of theoretical works have been done to investigate the flexoelectric effect in solid dielectrics, i.e., Yang [Yang and Shen (2014)] solved the embedded inclusion problem by the generalized Green's function method, in which the flexoelectricity is taken into consideration.

Although there are some review papers on such topic [Maranganti *et al.* (2006); Majdoub *et al.* (2008a); Yudin and Tagantsev (2013); Zubko *et al.* (2013)], flexoelectricity still have many open questions in the developing field. Especially, review on experimental studies of flexoelectricity has not been done so far, that is the focus of this paper.

Flexoelectric effect has been discovered in the middle twentieth century, however, it has been ignored for a long time by the researchers because this effect is quite small at macroscopic level. With the development of new techniques and nanotechnology, flexoelectricity has attracted an increasing amount of attention. Typically, flexoelectricity has been found in presence of strong electromechanical coupling in nano scaled materials and structures. In this section, we give a briefly summary of the development of flexoelectricity.

Kogan (1964) developed the phenomenological description for electric polarization due to strain gradient in solid crystals while Meyer (1969) discussed the contribution of electric quadrupole to flexoelectricity. Indenbom (1981) suggested the flexoelectricity for such phenomenon as was discussed in liquid crystals. In the 1980s, Tagantsev (1985,1986) gave a more extensively study on the flexoelectric effect, and systematically studied four contributions to this effect, i.e. the bulk static flexoelectric effect, the bulk dynamic flexoelectric effect, the surface flexoelectric effect, and the surface piezoelectric effect. Based on the lattice dynamics theory, an explicit expression for the flexoelectric coefficients is [Tagantsev (1986); Fu *et al.* (2006)]:

$$\mu_{ijkl} = \chi_{ij}\gamma_{kl}\frac{e}{a} \tag{3}$$

whereχ is the dielectric susceptibility, γ is the material parameter constant, e is the electron charge and a the lattice parameter.

Inspired by the Tagantsev's theory and lattice dynamics theory's prediction, there spring up numerous investigation on flexoelectricity. Marvan et al (1994) proposed the parallel chains of harmonic oscillator model combined with surface force rather than strain gradient to understand the physical reason of flexoelectric effect. Klic *et al.* (2004) used the potential double-well model to derive the formulation of flexoelectric coefficient which is compatible with Tagantsev's expression. Maranganti (2006) developed the fundamental solutions for spherical and cylindrical inclusion problems from the framework of flexoelectricity. After that, Majdoub (2008b,2009b) employed molecular dynamics to interpret the flexoelectric effect, and investigated the size-dependent piezoelectric and elastic behavior by combining atomistic and theoretical approaches. Deng (2014) developed a nonlinear theoretical framework for flexoelectricity in soft material, and proposed a concept of designing soft piezoelectric composite without using piezoelectric materials.

Variational principle has been regarded as the bases of the computational for electromechanical coupling problems for a long time. Hu and Shen (2009, 2010), Shen and Hu (2010) proposed a variational principle based on electric enthalpy for nano-sized dielectrics concerning the effects of flexoelectricity, surface and electrostatic force. This works provide the physical fundamentals and computational method for flexoelectricity. Based on this work, the size-dependent piezoelectricity and elasticity due to strain gradient-electric field coupling has been studied based on a modified Bernoulli-Euler beam model [Liang and Shen (2013)], the effect of flexoelectricity on the electrostatic potential in bent ZnO and piezoelectric semiconductive nanowire has been investigated and discussed [Liu *et al.* (2012); Xu *et al.* (2013)]. The flexoelectric effect on elastic wave propagating in periodically layered nanostructure has also been performed using the transfer matrix method [Liu *et al.* (2014)]. There are a series of theoretical works considering the flexoelectric effect in nanoscale dielectrics, however, the flexoelectric coefficients have not be experimentally measured. The difficulties in measuring flexoelectric coefficients of dielectrics are in measuring tiny electric signals generated in bulk dielectrics or the need of new detection techniques for nano scaled dielectrics.

3 Development of experiments on flexoelectricity

3.1 Experimental measurement of flexoelectric coefficients of ferroelectrics

Although lattice dynamics predict a much small magnitudes of the flexoelectric coefficients, theoretical analysis have shown that flexoelectricity plays an important role in enhancing the electromechanical coupling effect, especially in where strong strain gradients is presented [Maranganti *et al.* (2006); Majdoub *et al.* (2008a); Majdoub *et al.* (2008b); Majdoub *et al.* (2009a)]. To understand the flexoelectricity better, it is very necessary to measure the flexoelectric coefficients of dielectrics, typically for dielectrics with high dielectric constants (high dielectric susceptibility) as suggested by the lattice dynamic theory. For cubic crystals, there are only three independent non-zero components of the flexoelectric coefficients [Ma and Cross (2001b); Ma (2007); Shu *et al.* (2011)]. By stretching or compressing a truncated pyramid specimen, the flexoelectric coefficient μ_{11} has been measured [Fu *et al.* (2006)]. By bending a cantilever beam specimen, the flexoelectric coefficient μ_{12} for a series of un-poled ferroelectrics has been measured [Ma and Cross (2001b, a); Ma and Cross (2002); Ma and Cross (2005, 2006)]. Four point bending method is also employed to measure the flexoelectric coefficient μ_{12} [Ma and Cross (2003)]. In these works, giant flexoelectric coefficients which are 4-5 order larger than the predictions of lattice dynamics have been observed.

In the last decades, experiments on a series of ferroelectrics have been performed inspired by the intrinsic property of flexoelectricity. Cross *et al.* studied the flexoelectric effect in various perovskite ceramics, such as ferroelectric and paraelectric Barium Titanate [Ma and Cross (2006)], Barium Strontium Titanate (BST) [Ma and Cross (2002)], Lead Magnesium Niobate (PMN) [Ma and Cross (2001b)], Lead ZirconateTitanate (PZT) [Ma and Cross (2003)]. In their analysis of experimental, the quasi-static or low frequency dynamic techniques as well as four point bending configuration were employed to measure the flexoelectric coefficients. It is found that the flexoelectric coefficient can come up to $100\mu C/m$, 4-5 orders larger than the lattice dynamic predictions ($\sim 10^{-10}$ C/m). The temperature dependence of flexoelectric coefficients has also been investigated in perovskite ceramics, and it is found that flexoelectric coefficient approaches its peak at the phase transition point [Ma and Cross (2006)].

By bending beam methods, the flexoelectric coefficient μ_{12} has been measured for various ceramics. Figure 1 illustrated the bending method for measuring flexoelectric coefficient. The wire connects to electrodes on the surface of the specimen for current detection, and the displacement of the specimen is monitored. The electric charge can be calculated from the measured electric current in the external electrical circuit by $P_i = i/2\pi f A$, where i is the measured electrical current, f is the driving frequency of the applied load and A is the area of the electrodes on the top and bottom surface [Cross (2006)]. The flexoelectric effect of the specimen can be simplified as

$$P_3 = \mu_{12}\frac{\partial \varepsilon_{11}}{\partial x_3} \tag{4}$$

where μ_{12} is the transverse flexoelectric coefficient.

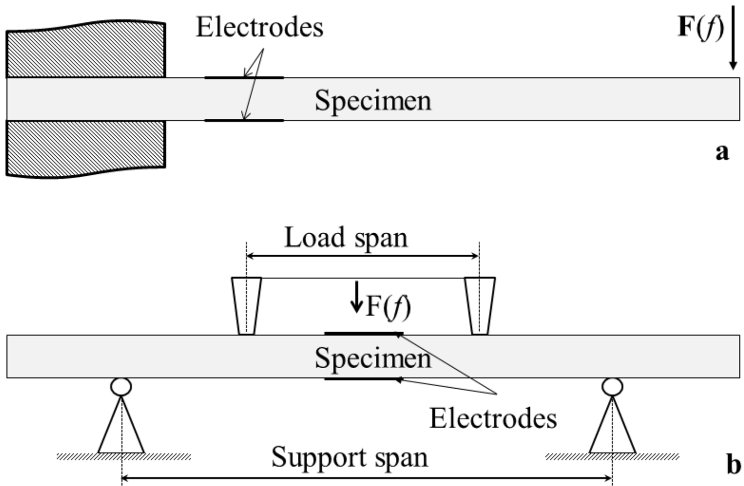

Figure 1: Schematic for experiments measurement of flexoelectric coefficients by bending mothed. a: cantilever bending method; b: four point bending method.

By stretching or compressing truncated pyramid specimens, the flexoelectric coefficient μ_{11} for various ceramics has also been measured. Figure 2 gives the schematic for the experiments set up. This special geometrical shape of the specimen was designed to generate strain gradient when elastic stress is applied. The average strain gradient in the truncated pyramid can be calculated from

$$\frac{\partial \varepsilon_{11}}{\partial x_1} = \frac{\Delta \varepsilon_{11}}{\Delta x_1} = \frac{\varepsilon_{11}^{u} - \varepsilon_{11}^{l}}{h} \tag{5}$$

The electric charge can be calculated from the measured electric current in the external electrical circuit by $P_i = i/2\pi f A$, where i is the measured electrical current, f is the driving frequency of the applied load and A is the area of the electrodes on the top and bottom surface [Cross (2006)]. The definition of the direct flexoelectric effect holds

$$P_1 = \mu_{11} \frac{\partial \varepsilon_{11}}{\partial x_1} \tag{6}$$

where μ_{11} is longitudinal flexoelectric coefficient.

After measured the electric current and calculated the average strain gradient, the flexoelectric coefficient can be calculated

$$\mu_{11} = \frac{P_1}{\left(\partial \varepsilon_{11}/\partial x_1\right)_{average}}$$

and effective piezoelectric stress constant [Cross (2006)] can be defined from the experiments as

$$d_{33} = \mu_{11} \frac{a_2^2 - a_1^2}{a_1^2 c_{11} h} \tag{7}$$

Eq. (6) indicates that flexoelectric effect can perform as piezoelectric effect, however, the effective piezoelectric stress constant related to the geometric parameters of the specimen.

The flexoelectric coefficients and the material parameters for different ceramics are listed in Table 1. These works proved the flexoelectric effect by experiments, in addition it is found that the flexoelectric coefficient for high-K ceramics are 4-5 orders larger than the prediction by the lattice dynamic theory. It is also found that the flexoelectric coefficients in ceramics have been enhanced by the high dielectric susceptibility, which agrees well with the predictions of lattice dynamic theory.

Inspired by Cross's works and the lattice dynamic prediction, ferroelectric composites with high dielectric susceptibility were fabricated. Giant flexoelectric coefficients in these composites are observed [Li *et al.* (2013); Shu *et al.* (2013); Kwon

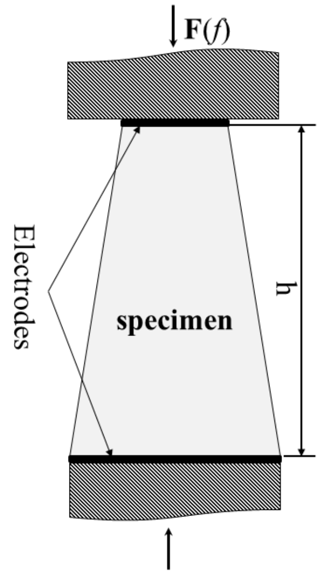

Figure 2: Schematic for measuring flexoelectric coefficient μ_{11} by compressing a truncated pyramid specimen.

Table 1: flexoelectric coefficient of various materials at room temperature (24˚C).

Material	Flexoelectric coefficient	Material parameter (γ)	Relative dielectric susceptibility (χ/ε_0)
PMN ($PbMg_{1/3}NB_{2/3}O_3$) [Ma and Cross (2001a)]	$\mu_{12} = 4 \times 10^{-6} C/m$	0.65	$\approx 13,000$
BST [Ma and Cross (2002)]	$\mu_{12} = 100 \times 10^{-6} C/m$	9.3	$\approx 20,000$
PZT [Ma and Cross (2003)]	$\mu_{12} = 2 \times 10^{-6} C/m$	0.57	$\approx 2,200$
BT [Ma and Cross (2006)]	$\mu_{12} = 5 \times 10^{-6} C/m$	11.4	$\approx 10,000$
BST [Zhu *et al.* (2006)]	$\mu_{11} = 120 \times 10^{-6} C/m$	9.3	$\approx 20,000$

et al. (2014); Li *et al.* (2014); Shu *et al.* (2014a); Shu *et al.* (2014b)]. Although there are many attempts on measuring the flexoelectric coefficient of ferroelectrics, no works are made on measuring the flexoelectric μ_{44} of ceramics.

3.2 Measurement of flexoelectric effect in polyvinylidene fluoride films (PVDF)

Besides ferroelectrics, flexoelectric coefficients in some thermoplastic polymers such as PVDF have been measured. Fu *et al.* [Fu *et al.* (2006); Fu *et al.* (2007); Baskaran *et al.* (2011a); Baskaran *et al.* (2011b); Baskaran *et al.* (2011c); Baskaran *et al.* (2012); He *et al.* (2012)] observed giant flexoelectric effect in polyvinylidene fluoride (PVDF) films. Different shapes of no stretched and poled PVDF films were measured via lock-in detection setup to verify the flexoelectric effect [Baskaran *et al.* (2011a)].

The polarization in the film includes the residual piezoelectricity effect and the flexoelectric effect. The generated electric polarization in PVDF films can be written as:

$$P_1 = d^r_{11} E \bar{S}_{tra} + \mu_{11} \nabla S_{tra} \tag{8}$$

where d^r_{11} represents the effective coefficient of the residual piezoelectricity due to the residual electric polarization, microstructural effects such as defects, cracks and might be the interaction between the α-phase and the amorphous phase in the film [Baskaran *et al.* (2011a)]. E is the Young's modulus, $\bar{S}_{tra}$ is the average strain and ∇S_{tra} is the average strain gradient. The flexoelectric coefficient can be derived as:

$$\mu_{11} = (P_1 - d_{33} E \bar{S}_{tra}) / \nabla S_{tra} \tag{9}$$

Theoretically, the flexoelectric effect in polymers such as PVDF is similar to that in liquid crystals. Therefore the flexoelectric effect in polymers is more complicated than that in solid crystals. However, the mechanism of flexoelectric effect in polymers has not been adequately understood so far.

4 Development of numerical methods of flexoelectricity

Strain gradient and electric field gradient are included in the theory of flexoelectricity. Analytical solutions for the electromechanical coupled problems with flexoelectricity can be obtained for simple models such as beams, plates and so on. For the case where the shapes and boundary conditions are complex, the numerical methods are needed and urgent.

At the atomic level, Hong (2013) used the first-principles to calculate the flexoelectric coefficient for cubic insulating materials. Mbarki (2014) used the molecular dynamics (MD) approach with specially tailored interatomic force-field to verify flexoelectric effect of BST/STO and its temperature dependence. Atomic and MD simulations, however, are expensive and restricted by the hardware conditions.

At the macroscopic level, numerical methods can be used to solve the complicated electromechanical coupling problems with flexoelectricity. Classical finite element methods cannot solve the higher order theories which including the gradients of strain and electric field. The mixed finite element methods or the meshless methods might be the appropriate methods to solve the electromechanical coupling problems with flexoelectricity. There are also some attempts on solving such electromechanical coupling problems. Arias *et al.* (2014, 2015) introduced the smooth meshfree basis function to deal with the higher-order partial differential equations which could be convenient when handle the general geometries and boundary conditions. Darrall *et al.* (2015) provided the variational formulation and used the mixed finite element method to solve the size-dependent problem. Several examples were bringing out to illustrate the size-dependent characteristics. Some other researchers also conducted numerical study on flexoelectricity [Fang *et al.* (2013); Yurkov (2015)].

5 Potential applications of flexoelectricity

There are also some applications based on the flexoelectric effect, such as curvature detection by flexoelectric sensors [Kwon *et al.* (2013); Yan *et al.* (2013a); Yan *et al.* (2013b)] and flexoelectric actuators [Hu *et al.* (2011)]. Among these structural health monitoring (SHM) in mechanical, civil, shipbuilding, transportation and aircraft structures may be the key point. The system defects such as cracks could cause a catastrophic failure. The present detection technology involves time consuming, expensive and low accuracy, so the researchers and enterprise are always hunting for the high efficiency with low cost structure health monitoring systems. Strain gradient distribution changes abruptly in the vicinity of a crack due to the stress concentration. Strain gradient in the vicinity of a crack can be measured based on flexoelectric effect, and precautionary measures can be carried out based on the estimation of loading parameter to avoid accident. A novel technique has been proposed [Huang *et al.* (2012); Kwon *et al.* (2013); Yan *et al.* (2013b); Huang *et al.* (2014a, b)] for structural health monitoring and crack detection based on the flexoelectric effect. The strain gradient sensors were attached in the neighboring of crack and hole with varied tension stress, the charge generated by flexoelectric effect was measured to predict the position of crack. In the centrosymmetric crystals, the flexoelectricity can be written as[Huang *et al.* (2014a, b)]:

$$P_i = \mu_{11}\frac{\partial \varepsilon_{ii}}{\partial x_i} + \mu_{12}\left(\frac{\partial \varepsilon_{jj}}{\partial x_i} + \frac{\partial \varepsilon_{kk}}{\partial x_i}\right) + \mu_{44}\left(\frac{\partial \varepsilon_{ji}}{\partial x_j} + \frac{\partial \varepsilon_{ki}}{\partial x_k}\right) \tag{10}$$

Another novel application for flexoelectric effect is to fabricate piezoelectric composite but without any piezoelectric constituents. Cross [Fousek *et al.* (1999)] analyzed the piezoelectric response of 0-3 composite made of non-piezoelectric constituent. Then they presented a flexure mode multilayer composite in which giant piezoelectric effect was observed [Chu *et al.* (2009)]. Zhu *et al.* (2006) devised the pyramid array structure based on the enhanced flexoelectric effect.

The flexoelectric coefficients of ceramics are affected by the grain size, temperature and loading frequency. Systematic investigations are needed to analyze these factors. The flexoelectric effect in polymers is more complex, the mechanism has not been fully understood so far. There is still a long way to go from theory to engineering applications, in view of the difficulties in theoretical and experimental works.

6 Conclusion

As a universal electromechanical coupling effect, flexoelectricity attracted an increasing of attention. Flexoelectricity phenomenologically describes the coupling between electric polarization and strain gradient, and electric field gradient and stress. Flexoelectricity plays an important role in determining the electro-elastic response of nanoscaled structures. In the last decades, a lot of experimental works have been done to measure the flexoelectric coefficients of non-poled ferroelectrics and thermoplastic polymers. The experimental methods and experimental results are summarized and discussed in this paper. The potential applications such as flexoelectric sensors, actuators, structural health monitoring and crack detection have also been briefly summarized.

Acknowledgement: The support from NSFC (Grants No. 11372238) is appreciated.

References

Abdollahi, A.; Millán, D.; Peco, C.; Arroyo, M.; Arias, I. (2015): Revisiting pyramid compression to quantify flexoelectricity: A three-dimensional simulation study. *Physical Review B,* vol. 91, no. 10, 104103.

Abdollahi, A.; Peco, C.; Millán, D.; Arroyo, M.; Arias, I. (2014): Computational evaluation of the flexoelectric effect in dielectric solids. *Journal of Applied Physics,* vol. 116, no. 9, 093502.

Baskaran, S.; He, X.; Chen, Q.; Fu, J. Y. (2011a): Experimental studies on the direct flexoelectric effect in alpha-phase polyvinylidene fluoride films. *Applied Physics Letters,* vol. 98, no. 24, 2901.

Baskaran, S.; He, X.; Fu, J. Y. (2011b): Gradient scaling phenomenon of piezoelectricity in non-piezoelectric polyvinylidene fluoride films. *Materials Science*.

Baskaran, S.; He, X.; Wang, Y.; Fu, J. Y. (2012): Strain gradient induced electric polarization in α-phase polyvinylidene fluoride films under bending conditions. *Journal of Applied Physics,* vol. 111, no. 1, 014109.

Baskaran, S.; Ramachandran, N.; He, X.; Thiruvannamalai, S.; Lee, H. J.; Heo, H.; Chen, Q.; Fu, J. Y. (2011c): Giant flexoelectricity in polyvinylidene fluoride films. *Physics Letters A,* vol. 375, no. 20, pp. 2082-2084.

Catalan, G.; Lubk, A.; Vlooswijk, A.; Snoeck, E.; Magen, C.; Janssens, A.; Rispens, G.; Rijnders, G.; Blank, D.; Noheda, B. (2011): Flexoelectric rotation of polarization in ferroelectric thin films. *Nature materials,* vol. 10, no. 12, pp. 963-967.

Chu, B.; Zhu, W.; Li, N.; Eric Cross, L. (2009): Flexure mode flexoelectric piezoelectric composites. *Journal of Applied Physics,* vol. 106, no. 10, 4109.

Craighead, H. G. (2000): Nanoelectromechanical systems. *Science,* vol. 290, no. 5496, pp. 1532-1535.

Cross, L. E. (2006): Flexoelectric effects: Charge separation in insulating solids subjected to elastic strain gradients. *Journal of Materials Science,* vol. 41, no. 1, pp. 53-63.

Darrall, B. T.; Hadjesfandiari, A. R.; Dargush, G. F. (2015): Size-dependent piezoelectricity: A 2D finite element formulation for electric field-mean curvature coupling in dielectrics. *European Journal of Mechanics-A/Solids,* vol. 49, pp. 308-320.

Deng, Q.; Liu, L.; Sharma, P. (2014): Flexoelectricity in soft materials and biological membranes. *Journal of the Mechanics and Physics of Solids,* vol. 62, pp. 209-227.

Ekinci, K.; Roukes, M. (2005): Nanoelectromechanical systems. *Review of scientific instruments,* vol. 76, no. 6, 061101.

Fan, F.-R.; Lin, L.; Zhu, G.; Wu, W.; Zhang, R.; Wang, Z. L. (2012): Transparent triboelectric nanogenerators and self-powered pressure sensors based on micropatterned plastic films. *Nano letters,* vol. 12, no. 6, pp. 3109-3114.

Fang, D.; Li, F.; Liu, B.; Zhang, Y.; Hong, J.; Guo, X. (2013): Advances in Developing Electromechanically Coupled Computational Methods for Piezoelectrics/Ferroelectrics at Multiscale. *Applied Mechanics Reviews,* vol. 65, no. 6, 060802.

Fousek, J.; Cross, L.; Litvin, D. (1999): Possible piezoelectric composites based on the flexoelectric effect. *Materials Letters,* vol. 39, no. 5, pp. 287-291.

Friswell, M. I.; Adhikari, S. (2010): Sensor shape design for piezoelectric cantilever beams to harvest vibration energy. *Journal of Applied Physics,* vol. 108, no. 1, 014901.

Fu, J. (2010): *Experimental studies of the direct flexoelectric effect in bone materials.* Paper presented at the APS Meeting Abstracts18013.

Fu, J. Y.; Liu, P. Y.; Cheng, J.; Bhalla, A. S.; Guo, R. (2007): Optical measurement of the converse piezoelectric d 33 coefficients of bulk and microtubular zinc oxide crystals. *Applied physics letters,* vol. 90, no. 21, 212907-212907-212903.

Fu, J. Y.; Zhu, W.; Li, N.; Cross, L. E. (2006): Experimental studies of the converse flexoelectric effect induced by inhomogeneous electric field in a barium strontium titanate composition. *Journal of Applied Physics,* vol. 100, no. 2, 024112.

Fu, Q.; Zhang, Z. Y.; Kou, L.; Wu, P.; Han, X.; Zhu, X.; Gao, J.; Xu, J.; Zhao, Q.; Guo, W. (2011): Linear strain-gradient effect on the energy bandgap in bent CdS nanowires. *Nano Research,* vol. 4, no. 3, pp. 308-314.

He, X.; Baskaran, S.; Fu, J. Y. (2012): *On the flexoelectricity in Polyvinylidene fluoride films.* Paper presented at the MRS Proceedingsmrsf11-1403-v1417-1440.

Hong, J.; Vanderbilt, D. (2013): First-principles theory and calculation of flexoelectricity. *Physical Review B,* vol. 88, no. 17, 174107.

Hong, Y. K.; Moon, K. S. (2005): *Single crystal piezoelectric transducers to harvest vibration energy.* Paper presented at the Optomechatronic Technologies 200560480E-60480E-60487.

Hu, S.; Li, H.; Tzou, H. (2011): *Static nano-control of cantilever beams using the inverse flexoelectric effect.* Paper presented at the ASME 2011 international mechanical engineering congress and exposition463-470.

Hu, S.; Shen, S. (2009): Electric field gradient theory with surface effect for nano-dielectrics. *Computers, Materials & Continua (CMC),* vol. 13, no. 1, pp. 63.

Hu, S.; Shen, S. (2010): Variational principles and governing equations in nano-dielectrics with the flexoelectric effect. *Science China Physics, Mechanics and Astronomy,* vol. 53, no. 8, pp. 1497-1504.

Huang, W.; Yan, X.; Kwon, S. R.; Zhang, S.; Yuan, F.-G.; Jiang, X. (2012): Flexoelectric strain gradient detection using Ba0. 64Sr0. 36TiO3 for sensing. *Applied Physics Letters,* vol. 101, no. 25, 252903.

Huang, W.; Yang, S.; Zhang, N.; Yuan, F.-G.; Jiang, X. (2014a): *Cracks monitoring and characterization using Ba0.64Sr0.36TiO3 flexoelectric strain gradient sensors*, 906119-906119-906119.

Huang, W.; Yang, S.; Zhang, N.; Yuan, F.-G.; Jiang, X. (2014b): Direct Measurement of Opening Mode Stress Intensity Factors Using Flexoelectric Strain Gradient Sensors. *Experimental Mechanics*, vol. 55, no. 2, pp. 313-320.

Indenbom, V.; Loginov, E.; Osipov, M. (1981): Flexoelectric effect and crystal-structure. *Kristallografiya,* vol. 26, no. 6, pp. 1157-1162.

Javey, A.; Guo, J.; Wang, Q.; Lundstrom, M.; Dai, H. (2003): Ballistic carbon nanotube field-effect transistors. *Nature,* vol. 424, no. 6949, pp. 654-657.

Klíč, A.; Marvan, M. (2004): Theoretical study of the flexoelectric effect based on a simple model of ferroelectric material. *Integrated Ferroelectrics,* vol. 63, no. 1, pp. 155-159.

Kogan, S. M. (1964): Piezoelectric effect during inhomogeneous deformation and acoustic scattering of carriers in crystals. *Soviet Physics-Solid State,* vol. 5, no. 10, pp. 2069-2070.

Kwon, S.; Huang, W.; Zhang, S.; Yuan, F.; Jiang, X. (2013): Flexoelectric sensing using a multilayered barium strontium titanate structure. *Smart Materials and Structures,* vol. 22, no. 11, 115017.

Kwon, S. R.; Huang, W.; Shu, L.; Yuan, F.-G.; Maria, J.-P.; Jiang, X. (2014): Flexoelectricity in barium strontium titanate thin film. *Applied Physics Letters,* vol. 105, no. 14, 142904.

Lee, D.; Noh, T. W. (2012): Giant flexoelectric effect through interfacial strain relaxation. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences,* vol. 370, no. 1977, pp. 4944-4957.

Lee, D.; Yang, S. M.; Yoon, J.-G.; Noh, T. W. (2012): Flexoelectric Rectification of Charge Transport in Strain-Graded Dielectrics. *Nano letters,* vol. 12, no. 12, pp. 6436-6440.

Li, Y.; Shu, L.; Huang, W.; Jiang, X.; Wang, H. (2014): Giant flexoelectricity in Ba0. 6Sr0. 4TiO3/Ni0. 8Zn0. 2Fe2O4 composite. *Applied Physics Letters,* vol. 105, no. 16, 162906.

Li, Y.; Shu, L.; Zhou, Y.; Guo, J.; Xiang, F.; He, L.; Wang, H. (2013): Enhanced flexoelectric effect in a non-ferroelectric composite. *Applied Physics Letters,* vol. 103, no. 14, 142909.

Liang, X.; Hu, S.; Shen, S. (2014): Effects of surface and flexoelectricity on a piezoelectric nanobeam. *Smart Materials and Structures,* vol. 23, no. 3, 035020.

Liu, C.; Hu, S.; Shen, S. (2012): Effect of flexoelectricity on electrostatic potential in a bent piezoelectric nanowire. *Smart Materials and Structures,* vol. 21, no. 11, 115024.

Liu, C.; Hu, S.; Shen, S. (2014): Effect of Flexoelectricity on Band Structures of One-Dimensional Phononic Crystals. *Journal of Applied Mechanics,* vol. 81, no.

5, 051007.

Ma, W. (2007): Flexoelectricity: strain gradient effects in ferroelectrics. *Physica Scripta,* 2007(T129), 180.

Ma, W. (2010): Flexoelectric charge separation and size dependent piezoelectricity in dielectric solids. *physica status solidi (b),* vol. 247, no. 1, pp. 213-218.

Ma, W.; Cross, L. E. (2001a): Large flexoelectric polarization in ceramic lead magnesium niobate. *Applied Physics Letters,* vol. 79, no. 26, pp. 4420-4422.

Ma, W.; Cross, L. E. (2001b): Observation of the flexoelectric effect in relaxor Pb (Mg1/3Nb2/3) O3 ceramics. *Applied Physics Letters,* vol. 78, no. 19, pp. 2920-2921.

Ma, W.; Cross, L. E. (2005): Flexoelectric effect in ceramic lead zirconate titanate. *Applied Physics Letters,* vol. 86, no. 7, 072905.

Ma, W.; Cross, L. E. (2006): Flexoelectricity of barium titanate. *Applied Physics Letters,* vol. 88, no. 23, 232902-232902-232903.

Ma, W.; Eric Cross, L. (2002): Flexoelectric polarization of barium strontium titanate in the paraelectric state. *Applied Physics Letters,* vol. 81, no. 18, pp. 3440-3442.

Ma, W.; Eric Cross, L. (2003): Strain-gradient-induced electric polarization in lead zirconate titanate ceramics. *Applied Physics Letters,* vol. 82, no. 19, pp. 3293-3295.

Majdoub, M.; Maranganti, R.; Sharma, P. (2009a): Understanding the origins of the intrinsic dead layer effect in nanocapacitors. *Physical Review B,* vol. 79, no. 11, 115412.

Majdoub, M.; Sharma, P.; Cagin, T. (2008a): Enhanced size-dependent piezoelectricity and elasticity in nanostructures due to the flexoelectric effect. *Physical Review B,* vol. 77, no. 12, 125424.

Majdoub, M.; Sharma, P.; Cagin, T. (2009b): Erratum: Enhanced size-dependent piezoelectricity and elasticity in nanostructures due to the flexoelectric effect [Phys. Rev. B 77, 125424 (2008)]. *Physical Review B,* vol. 79, no. 11, 119904.

Majdoub, M.; Sharma, P.; Çağin, T. (2008b): Dramatic enhancement in energy harvesting for a narrow range of dimensions in piezoelectric nanostructures. *Physical Review B,* vol. 78, no. 12, 121407.

Maranganti, R.; Majdoub, M.; Sharma, P. (2009): *Flexoelectricity in nanostructures and ramifications for the dead-layer effect in nanocapacitors and "giant" piezoelectricity.* Paper presented at the APS March Meeting Abstracts1193.

Maranganti, R.; Sharma, N.; Sharma, P. (2006): Electromechanical coupling in nonpiezoelectric materials due to nanoscale nonlocal size effects: Green's function solutions and embedded inclusions. *Physical Review B,* vol. 74, no. 1, 014110.

Marvan, M.; Janus, V.; Havranek, A. (1994): *Electric polarization induced by strain gradient.* Paper presented at the Electrets, 1994.(ISE 8), 8th International Symposium on623-627.

Mbarki, R.; Haskins, J.; Kinaci, A.; Cagin, T. (2014): Temperature dependence of flexoelectricity in BaTiO 3 and SrTiO 3 perovskite nanostructures. *Physics Letters A,* vol. 378, no. 30, pp. 2181-2183.

Meyer, R. B. (1969): Piezoelectric effects in liquid crystals. *Physical Review Letters,* vol. 22, no. 18, 918.

Nguyen, T. D.; Mao, S.; Yeh, Y. W.; Purohit, P. K.; McAlpine, M. C. (2013): Nanoscale flexoelectricity. *Advanced Materials,* vol. 25, no. 7, pp. 946-974.

Nishi, Y. (1978): Field effect transistors): Google Patents.

Park, S.; Gao, X. (2006): Bernoulli–Euler beam model based on a modified couple stress theory,*Journal of Micromechanics and Microengineering,* 16(11), 2355.

Petrov, A. G. (2002): Flexoelectricity of model and living membranes. *Biochimica et Biophysica Acta (BBA)-Biomembranes,* vol. 1561, no. 1, pp. 1-25.

Shen, S.; Hu, S. (2010): A theory of flexoelectricity with surface effect for elastic dielectrics. *Journal of the Mechanics and Physics of Solids,* vol. 58, no. 5, pp. 665-677.

Shu, L.; Huang, W.; Kwon, S. R.; Wang, Z.; Li, F.; Wei, X.; Zhang, S.; Lanagan, M.; Yao, X.; Jiang, X. (2014a): Converse flexoelectric coefficient f1212 in bulk Ba0. 67Sr0. 33TiO3. *Applied Physics Letters,* vol. 104, no. 23, 232902.

Shu, L.; Li, F.; Huang, W.; Wei, X.; Yao, X.; Jiang, X. (2014b): Relationship between direct and converse flexoelectric coefficients. *Journal of Applied Physics,* vol. 116, no. 14, 144105.

Shu, L.; Wei, X.; Jin, L.; Li, Y.; Wang, H.; Yao, X. (2013): Enhanced direct flexoelectricity in paraelectric phase of Ba (Ti0. 87Sn0. 13) O3 ceramics. *Applied Physics Letters,* vol. 102, no. 15, 152904.

Shu, L.; Wei, X.; Pang, T.; Yao, X.; Wang, C. (2011): Symmetry of flexoelectric coefficients in crystalline medium. *Journal of Applied Physics,* vol. 110, no. 10, 104106.

Sodano, H. A.; Inman, D. J.; Park, G. (2004): A review of power harvesting from vibration using piezoelectric materials. *Shock and Vibration Digest,* vol. 36, no. 3, pp. 197-206.

Tagantsev, A. (1985): Theory of flexoelectric effect in crystals. *Zhurnal Eksperi-*

mental'noi i Teoreticheskoi Fiziki, vol. 88, no. 6, pp. 2108-2122.

Tagantsev, A. (1986): Piezoelectricity and flexoelectricity in crystalline dielectrics. *Physical Review B,* vol. 34, no. 8, 5883.

Tagantsev, A. (1987): Pyroelectric, piezoelectric, flexoelectric, and thermal polarization effects in ionic crystals. *Physics-Uspekhi,* vol. 30, no. 7, pp. 588-603.

Tagantsev, A. K. (1991): Electric polarization in crystals and its response to thermal and elastic perturbations. *Phase Transitions: A Multinational Journal,* vol. 35, no. 3-4, pp. 119-203.

Wang, Z. L. (2008): Towards Self-Powered Nanosystems: From Nanogenerators to Nanopiezotronics. *Advanced Functional Materials,* vol. 18, no. 22, pp. 3553-3567.

Xu, L.; SHEN, S. (2013): Size-dependent piezoelectricity and elasticity due to the electric field-strain gradient coupling and strain gradient elasticity. *International Journal of Applied Mechanics,* vol. 5, no. 02.

Xu, S.; Qin, Y.; Xu, C.; Wei, Y.; Yang, R.; Wang, Z. L. (2010): Self-powered nanowire devices. *Nature nanotechnology,* vol. 5, no. 5, pp. 366-373.

Xu, Y.; Hu, S.; Shen, S. (2013): Electrostatic potential in a bent flexoelectric semiconductive nanowire. *CMES–Computer Modeling in Engineering & Sciences,* vol. 91, no. 5, pp. 397-408.

Yan, X.; Huang, W.; Kwon, S.; Yang, S.; Jiang, X.; Yuan, F. (2013a): *Design of a curvature sensor using a flexoelectric material.* Paper presented at the SPIE Smart Structures and Materials+ Nondestructive Evaluation and Health Monitoring86920N-86920N-86910.

Yan, X.; Huang, W.; Kwon, S. R.; Yang, S.; Jiang, X.; Yuan, F.-G. (2013b): A sensor for the direct measurement of curvature based on flexoelectricity. *Smart Materials and Structures,* vol. 22, no. 8, 085016.

Yan, Z.; Jiang, L. (2013a): Flexoelectric effect on the electroelastic responses of bending piezoelectric nanobeams. *Journal of Applied Physics,* vol. 113, no. 19, 194102.

Yan, Z.; Jiang, L. (2013b): Size-dependent bending and vibration behaviour of piezoelectric nanobeams due to flexoelectricity. *Journal of Physics D: Applied Physics,* vol. 46, no. 35, 355502.

Yang, S.; Shen, S. (2014): Anti-plane Circular Nano-inclusion Problem with Electric Field Gradient and Strain Gradient Effects. *CMC: Computers, Materials & Continua,* vol. 40, no. 3, pp. 219-239.

Yudin, P.; Tagantsev, A. (2013): Fundamentals of flexoelectricity in solids. *Nanotechnology,* vol. 24, no. 43, 432001.

Yurkov, A. (2015): Calculation of flexoelectric deformations of finite-size bodies. *Physics of the Solid State,* vol. 57, no. 3, pp. 460-466.

Zhu, W.; Fu, J. Y.; Li, N.; Cross, L. (2006): Piezoelectric composite based on the enhanced flexoelectric effects. *Applied physics letters,* vol. 89, no. 19, 192904-192904-192903.

Zubko, P.; Catalan, G.; Tagantsev, A. K. (2013): Flexoelectric effect in solids. *Annual Review of Materials Research,* vol. 43, pp. 387-421.