 CMC, vol.48, no.3, pp.147-161, 2015

Investigating Creep Performance and Predicting Rupture Time for Rotating FGM Disc under Different Thermal Gradients

K. Khanna[1,2], V.K. Gupta[3], S.P. Nigam[1]

Abstract: A mathematical model is developed to describe the steady state creep in a rotating Al-SiC$_p$ disc having a non-linear thickness profile and distribution of SiC particles along the radial direction. The model is used to investigate the effect of imposing three different kinds of radial temperature profiles viz. linear, parabolic and exponential with fixed values of inner and outer surface temperatures, on the creep stresses and strain rates. It is noticed that by increasing the temperature exponent (n_T), the radial stress (over the entire radius) and tangential stress (near the inner radius) increase in the disc. However, the tangential stress decreases near the outer radius. The radial and tangential strain rates in the functionally graded (FG) disc reduce significantly with the increase in exponent n_T. Besides reduction in the magnitude, the distribution of strain rates also become relatively more uniform throughout with the increase in n_T. It is concluded that FG disc operating under exponential temperature profile performs better. It is also revealed that amongst several FG discs operating under radial thermal gradients, with different values of temperature exponent (n_T) but having the same average and fixed outer surface temperature, the FGM disc with lower value of n_T exhibits the maximum creep life.

Keywords: Creep; Rotating disc; Functionally graded material; Rupture time; Thermal gradient.

1 Introduction

Rotating disc is a common component in many engineering applications like steam and gas turbine rotors, turbo generators, flywheels, gears, centrifugal compressors, ship propellers, automotive brakes, disc grinders, disc cutters, internal combustion

[1] Mechanical Engineering Dept., Thapar University, Patiala-147004, India
[2] Corresponding author (Email: kishore.khanna@thapar.edu)
[3] Mechanical Engineering Dept., Punjabi University, Patiala-147002, India

engines and in aerospace devices [Gupta, Singh, Chandrawat, and Ray (2004a); Hojjati and Hassani (2008); Hassani, Hojjati, Farrahi, and Alashti (2012)]. In some applications, like turbine rotors and disc brakes of racing cars, the disc operates at high speed and is simultaneously subjected to high temperature [Laskaj, Murphy and Houngan (1999); Farshi, Jahed, and Mehrabian (2004)]. Under such severe thermo-mechanical loading, the disc material undergoes creep. Several investigators have performed analysis of stresses and deformations in rotating disc made of functionally graded material (FGM) [Hassani, Hojjati, Farrahi, and Alashti (2012); You, You, Zhang, and Li (2007); Bayat, Saleem, Sahari, Hamouda, and Mahdi (2008); Çallıoğlu, Bektas, and Sayer (2011); Akbari and Ghanbari (2015)]. The studies reveal that a variable thickness disc develops lesser stresses and deformations than a constant thickness disc [Bayat, Saleem, Sahari, Hamouda, and Mahdi (2008)]. Besides disc, the stresses and displacements have also been estimated for FGM structures such as plates, shells and beams by using finite element method (without higher-order theory) [Dong, El-Gizawy, Juhany, and Atluri (2014a,b)].

In some of the earlier investigations concerning analysis of creep in FGM disc, having either constant thickness [Singh and Ray (2001); Gupta, Singh, Chandrawat, and Ray (2004a)] or variable thickness [Deepak, Gupta, and Dham (2010); Nie and Batra (2010)], the disc has been assumed to operate under constant temperature. However, in real life applications, such as turbine rotor and disc brake, the disc operates under a radial thermal gradient [Ali and Mostefa, (2013)]. Gupta, Singh, Chandrawat, and Ray (2005) simulated the effect of imposing radial thermal gradient on the creep behavior of rotating FGM disc having linearly decreasing content of reinforcement (SiC_P) along the disc radius. However, the study assumed disc thickness to be constant. Garg, Salaria, and Gupta (2013) analyzed the effect of imposing radially varying linear thermal gradient on the creep performance of FGM disc with linearly varying thickness profile. The studies reveal that creep response of FGM disc, with either uniform thickness [Gupta, Singh, Chandrawat, and Ray (2005); Kordkheili and Livani (2013)] or linearly variable thickness [Garg, Salaria, and Gupta (2013)], is significantly affected by varying the radial thermal gradient. In our recent study [Khanna, Gupta, and Nigam (2015)], pertaining to the effects of varying disc thickness profile and reinforcement gradient, it is observed that the FGM disc having higher thickness and higher reinforcement gradients exhibits lower strain rates. But, the study assumed the disc to operate at uniform temperature. Some attempts have been made by researchers to investigate the effect of imposing different kinds of radial thermal gradients on elastic stresses and deformation in FGM disc [Afsar and Go (2010); Hassani, Hojjati, Farshi, and Alashti (2011)]. However, to the best of our knowledge no such study is reported that considers the effect of imposing various kinds of radial thermal gradients on the creep behavior of variable thickness rotating FGM disc.

The present study investigates the effect of imposing different kinds of radial thermal gradients, viz. linear, parabolic and exponential, on the creep performance of rotating disc having non-linear thickness profile and non-linear reinforcement gradient. The various kinds of temperature profiles are obtained by varying the value of temperature exponent (n_T) in the equation of radial temperature profile in the disc while keeping the fixed values of temperature at the inner and outer surface of the disc. The effect of varying the value of temperature exponent (n_T) on the creep life of the disc has also been investigated when the discs are subjected to a fixed temperature at the outer surface while maintaining a constant average temperature for all values of n_T, which ultimately reduces the value of temperature at the inner disc radius.

2 Disc geometry and reinforcement profile

The present study assumes a variable thickness disc (Fig. 1), with thickness at the inner and outer radii as h_a (= 49.00 mm) and h_b (= 19.1 mm), respectively. The disc thickness, $h(r)$, at any radius r, is assumed to vary according to,

$$h(r) = h_b \left[\frac{r}{b}\right]^k \tag{1}$$

where a, b and k are the inner disc radius, outer disc radius and disc thickness gradient, respectively.

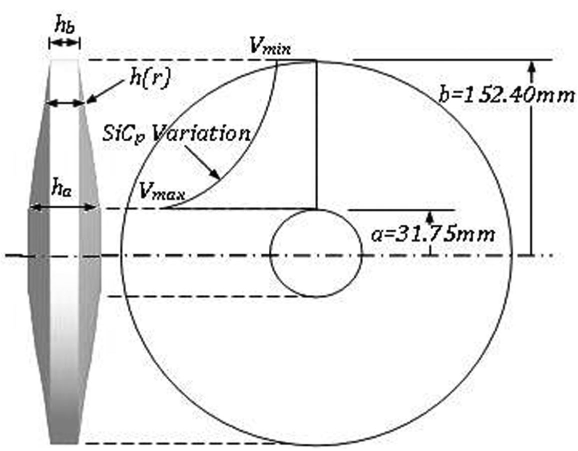

Figure 1: Schematic showing disc geometry and SiC_p distribution.

The variable thickness FGM disc is assumed to rotate at 15000 *rpm* and made of Al-SiC_p, with SiC_p content at the inner and outer radius as V_{max} (= 100%) and V_{min}

(= 5.67%), respectively. The content $V(r)$ of SiC_p at any radius r is assumed to decrease radially from the inner to outer radius according to,

$$V(r) = V_{\min}\left[\frac{r}{b}\right]^m \tag{2}$$

where m is the reinforcement gradient.

The disc geometry and SiC_p distribution in the disc are similar to those taken in earlier work [Khanna, Gupta, and Nigam (2015)]. The values of k and m are taken as -0.6 and -1.8295, respectively since, corresponding to these values the strain rates in the disc were the lowest [Khanna, Gupta, and Nigam (2015)].

3 Creep law

The disc material undergoes steady state creep according to [Gupta, Singh, Chandrawat, and Ray (2004b)],

$$\dot{\bar{\varepsilon}} = [M(r)\{\bar{\sigma} - \sigma_o(r)\}]^n \tag{3}$$

where $\dot{\bar{\varepsilon}}$ is the effective strain rate, $\bar{\sigma}$is the effective stress, $\sigma_0(r)$ is the threshold stress and $M(r)\left[= \frac{1}{E}\left(A' \exp\frac{-Q}{RT}\right)^{1/n}\right]$ is a creep parameter in which the symbols A', n, Q, E, R and T denote the structure dependent parameter, true stress exponent, true activation energy, temperature-dependent Young's modulus, gas constant and operating temperature, respectively.

In this study, the value of exponent n is taken as 5 and the values of parameters $M(r)$ and $\sigma_0(r)$ are estimated from the following regression equations, derived earlier [Deepak, Gupta, and Dham (2010)],

$$M(r) = 0.0288 - \frac{0.0088}{P} - \frac{14.0267}{T(r)} + \frac{0.0322}{V(r)} \tag{4}$$

$$\sigma_0(r) = 22.207 - 0.084P - 0.023T(r) + 1.185V(r) \tag{5}$$

where P is the size of SiC_p (= 1.7 μm) and $T(r)$ is the temperature at any radius r of the disc.

4 Disc temperature profile

As outlined earlier, in applications of rotating disc as turbine rotor and disc brake, the disc is subjected to radial temperature gradient, with the temperature at the outer radius being higher than the inner radius. In case of disc brakes, frictional heat generated due to braking action causes higher temperature at the contact area between the brake pad and disc surface, and the temperature decreases radially towards the inner disc radius. Similarly, in turbine rotors, due to the entrance of steam/gas at the outer radius, a radially decreasing temperature distribution is developed towards the inner radius.

To impose such a radial temperature gradient in the disc, the following equation, as proposed by Hassani, Hojjati, Farshi, and Alashti (2011), in their study on elastic behavior of rotating FGM disc, has been used to estimate temperature, $T(r)$ in the disc,

$$T(r) = (T_o - T_i)\left[\frac{r-a}{b-a}\right]^{n_T} + T_i \tag{6}$$

where T_i (= 623 K) and T_o (= 723 K) are the temperatures at the inner and outer disc radii, respectively and n_T is the temperature exponent. To obtain different kinds of radial temperature profiles in the disc (Fig. 2), the index n_T has been taken as 1, 2 and 10, which refer to linear, parabolic and exponential temperature profiles, respectively, in this study. For comparison, the study also considers an FGM disc operating at uniform temperature of 673 K, which is the mean of temperature at the inner and outer disc radii.

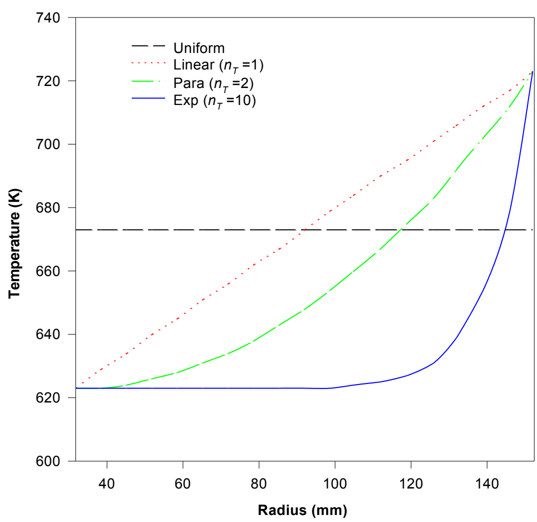

Figure 2: Temperature profiles.

5 Creep modeling

The analysis assumes: (i) disc material to be incompressible, (ii) steady state condition of stresses, (iii) negligible elastic deformations, and (iv) disc under plane stress condition.

The generalized constitutive equations for creep in an isotropic disc, when the reference frame is along the principal directions r, θ and z, are given [Gupta, Singh, Chandrawat, and Ray (2005)] as,

$$\begin{aligned} \dot{\varepsilon}_r &= \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}\left[2\sigma_r(r)-\sigma_\theta(r)\right] \\ \dot{\varepsilon}_\theta &= \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}\left[2\sigma_\theta(r)-\sigma_r(r)\right] \\ \dot{\varepsilon}_z &= \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}}\left[-\sigma_r(r)-\sigma_\theta(r)\right] \end{aligned} \tag{7}$$

where $\dot{\bar{\varepsilon}}$ and $\bar{\sigma}$ are, respectively effective strain rate and effective stress, and $\dot{\varepsilon}$ and σ denote, respectively the strain rate and stress in the disc, with the subscripts r, θ and z indicating radial, tangential and axial components, respectively.

The disc material is assumed to yield according to Tresca's criterion, therefore effective stress ($\bar{\sigma}$) in the disc, under plane stress condition [Khanna, Gupta, and Nigam (2015)], is given by,

$$\bar{\sigma} = \sigma_\theta \qquad (\because \text{ In a disc: } \sigma_\theta \rangle \sigma_r \rangle \sigma_z) \tag{8}$$

The equilibrium equation for a variable thickness rotating FGM disc is given as [Deepak, Gupta, and Dham (2010)],

$$\frac{d}{d_r}\left[rh(r)\sigma_r\right] - h(r)\sigma_\theta + \rho(r)\omega^2 r^2 h(r) = 0 \tag{9}$$

where ω is angular velocity of the disc.

Following previous work [Khanna, Gupta, and Nigam (2015)], the disc is assumed under the following free-free boundary conditions, as applicable for disc fitted on a splined shaft,

$$\sigma_r = 0 \text{ at } r = a \quad \text{and} \quad \sigma_r = 0 \text{ at } r = b.$$

The creep stresses in the disc are estimated by solving equilibrium Eq. (9) alongwith the constitutive Eqs. (7), under the imposed boundary conditions given above. The solution procedure is similar to that described earlier [Khanna, Gupta and Nigam (2015)] and the resulting equations used for the estimation of σ_r and σ_θ are given in the **APPENDIX A.**

6 Results and discussion

The stresses and strain rates in the FGM disc are estimated by following an iterative process. The results are estimated by assuming the disc to operate under four different kinds of radial temperature profiles (Fig. 2).

6.1 Validation

To validate the analysis performed, the value of tangential strain is estimated for a rotating steel disc by following the current analysis scheme. The dimensions of the disc used in this study are similar to those used by Wahl, Sankey, Manjoine and Shoemaker (1954). The analytical results are compared with the available experimental results for steel disc. The creep parameters required for the steel disc are estimated by fitting the experimental values of effective stress and effective strain rate in the steel disc into the creep law given in Eq. (3), which yields the values of parameters M and σ_0 as $2.0408 \times 10^{-4}\text{s}^{-1/5}$/MPa and 37.178 MPa, respectively. The tangential strain estimated by following the current analysis procedure is in good agreement with the experimental values (Fig. 3).

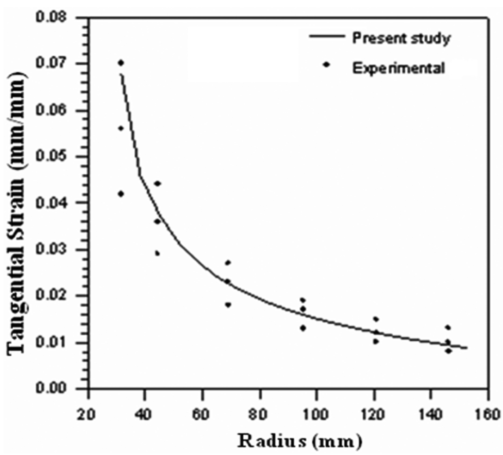

Figure 3: Comparison of theoretical and experimental [Wahl, Sankey, Manjoine, and Shoemaker (1954)] strain in steel disc. (Disc RPM = 15000, Creep duration = 180 hrs, Operating temperature = 810.78 K.

6.2 Effect of temperature exponent on the creep parameters

It is evident from Eqs. (4)–(5) that the creep parameters $M$and σ_0 depend on the operating temperature, therefore in different discs these parameters will vary due to different kinds of radial temperature distributions (Fig. 2). The parameter M increases on moving from the inner to outer radius (Fig. 4). The FGM disc with exponential temperature profile shows the lowest value of M throughout, when compared to FGM disc with linear and parabolic temperature distributions. The variation observed in M is higher in the middle of the disc, although at the inner and outer radii the values of M are same in discs with all the radial temperature variations. The uniform temperature disc exhibits higher M value towards the inner radius but lower M value towards the outer radius, when compared to FGM disc with radially varying temperature. The effect of varying temperature profile in the disc is not significant on the threshold stress (σ_0), Fig. 5, which may be attributed to low value (= 0.023) of the coefficient of $T(r)$ in Eq. (5).

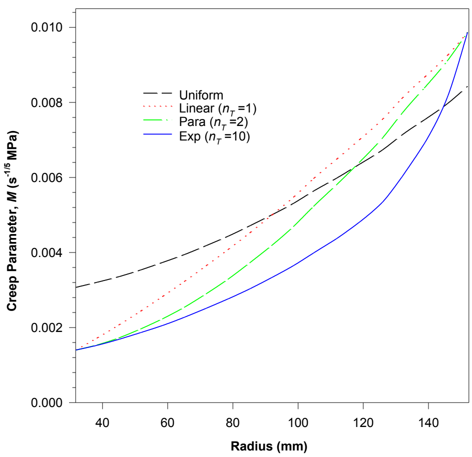

Figure 4: Variation of creep parameter M.

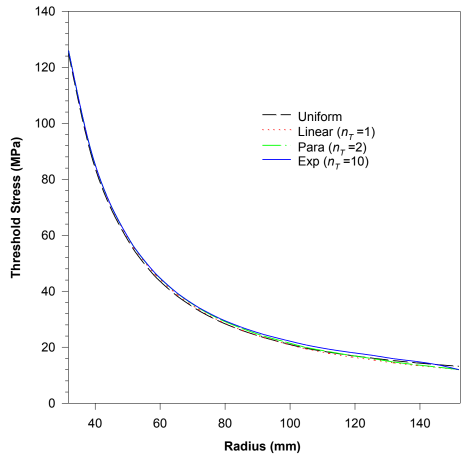

Figure 5: Variation of threshold stress (σ_0).

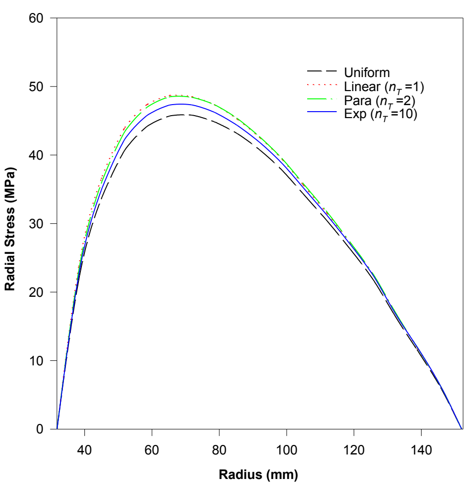

Figure 6: Variation of radial stress in disc.

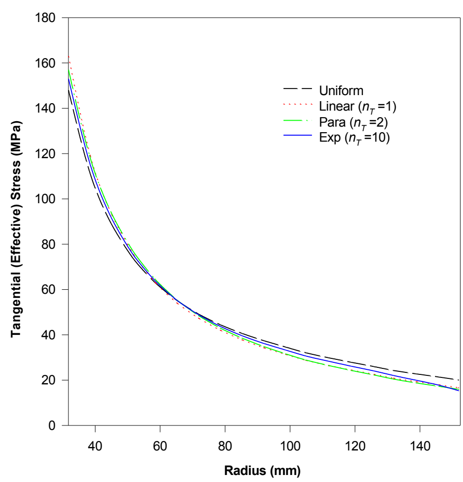

Figure 7: Variation of tangential (effective) stress in disc.

6.3 Effect of temperature exponent on the stresses

As compared to uniform temperature disc, the radial stress in discs with radial temperature distributions is slightly higher throughout (Fig. 6). The maximum variation of 3 MPa at a radius of 65.3 mm is observed in discs with linear temperature profile and uniform temperature. On imposing radial thermal gradient in the disc, the tangential (effective) stress increases towards the inner radius but decreases towards the outer radius (Fig. 7), when compared to uniform temperature disc. The maximum variation of 15 MPa is noticed at the inner disc radius between the discs with linear and uniform temperature profiles.

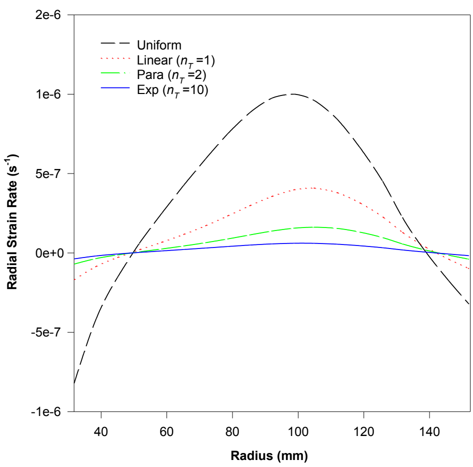

Figure 8: Variation of radial strain rate in the FGM disc.

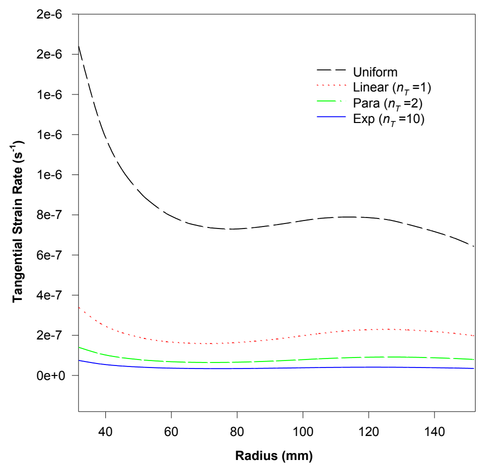

Figure 9: Variation of tangential strain rates in the FGM disc.

6.4 Effect of temperature exponent on the strain rates

The radial strain rate in all the discs is tensile near the ends but shows compressive nature in the middle (Fig. 8). The magnitude of radial strain rate (tensile as well as compressive) is observed to decrease with the increase in temperature exponent (n_T) when compared to those observed in uniform temperature disc. The decrease observed is maximum in the middle region. Besides reduction in magnitude, the strain rate tends to become relatively uniform with the increase in exponent n_T. Similarly, the tangential strain rate in the disc also decreases significantly with the increase in exponent n_T (Fig. 9). The decrease observed is more near the inner radius of the disc. The reduction observed in strain rates near the inner radius of the FGM disc having radially varying temperature is attributed to significant reduction in creep parameter M in these discs (Fig. 4), which dominates over the effect caused by the higher effective stress observed near the inner radius of these discs than those observed in uniform temperature disc (Fig. 7). However, towards the outer radius, inspite of high M value observed in FGM disc having radially varying temperature (Fig. 4), the lower effective stress (Fig. 7) tends to reduce strain rates in these discs, as compared to uniform temperature disc. Thus the FGM disc operating under exponentially varying temperature profile develops lower and relatively uniform distribution of strain rates and will have lesser chances of distortion.

6.5 Effect of temperature exponent on rupture time

In this section, the effect of varying the value of temperature exponent (n_T) has been investigated on the creep rupture time of the FGM discs, when all the discs operate under the same average temperature (T_{avg}) and at a fixed value of outer surface temperature ($T_o = 723$ K). As the superior response of FGM disc is observed for temperature exponent $n_T = 10$, the average temperature ($T_{avg} = 632.09$ K) is calculated for that disc from the following equation, obtained by integrating Eq. (6) between limits a to b.

$$T_{avg} = \frac{(T_o - T_i)}{(n_T + 1)} + T_i \qquad (10)$$

For given values of T_o and T_{avg}, the temperature at the inner surface of the FGM disc (T_i) may be estimated from Eq. (10) for different values of n_T, that is varied in the range 2 to 15. The values T_i, thus obtained, are substituted in Eq. (6) to get the radial variation of temperature in FGM disc for different values of n_T (Fig. 6a). It is important to mention that the value of temperature exponent (n_T) has not been reduced below 2, as it gives negative value of creep parameter M, which is practically impossible. It is observed that for a given average temperature in the disc, the temperature decreases near the inner radius but increases towards the outer radius (Fig. 10). Corresponding to different kinds of temperature profiles in the FGM disc, the radial and tangential strain rates are estimated at the inner and outer disc radii. With the decrease in temperature exponent (n_T) from 15 to 2, both the strain rates are observed to reduce drastically by about 8 orders of magnitude (Fig. 11).

Based on the maximum strain rates (i.e. tangential strain rate), observed at the inner disc radius, the creep life (defined in terms of the creep rupture time) of the FGM disc has been estimated for different values of n_T. The rupture strain for Al-SiC composites is observed in the range of 2 to 4% [Nieh (1984); Orlando and Filho (2004)]. Therefore, the rupture time has been estimated for different values of n_T by taking 2%, 3% and 4% strain as the rupture strain. The rupture time is observed to increase slightly (by few hours) as the value of exponent n_T decreases from 15 to 5 (Fig. 12). On decreasing $n_T$below 5, the rupture time is observed to

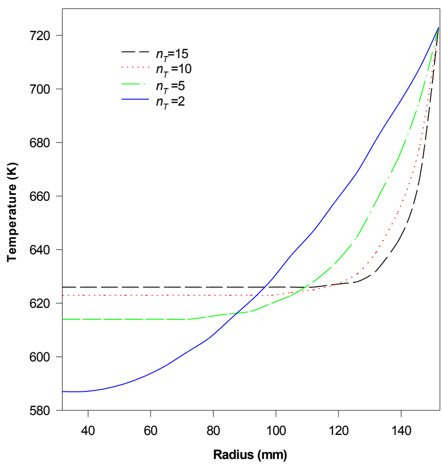

Figure 10: Variation of temperature with n_T ($T_{avg} = 632.09$ K).

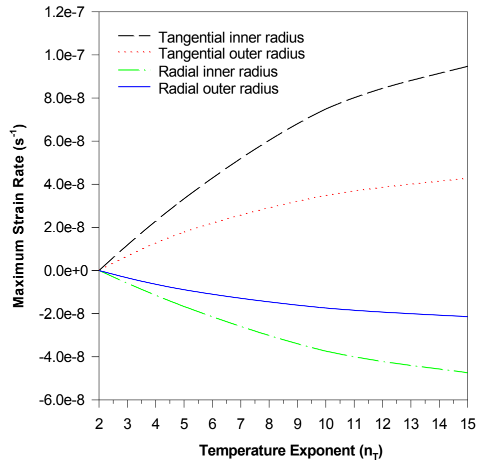

Figure 11: Variation of strain rate with n_T.

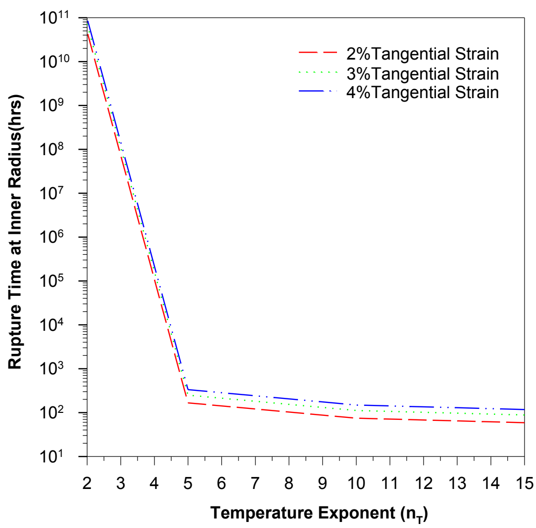

Figure 12: Variation of rupture time with n_T.

increase significantly (by several years). As an example, the rupture time increases by around nine orders of magnitude when the value of n_T decreases from 5 to 2. The increase in rupture time with decreasing the value of exponent n_T is attributed to the reduction in temperature at the inner radius (Fig. 10), which leads to significant reduction in strain rate (Fig. 11). Thus, if one controls the heat transfer along the radial direction of the FGM disc, which is possible by employing suitable thermal barrier coatings, heat sink [Lepeshkin (2012); Padture, Gell, and Jordan (2002)] etc., the creep damage in the FGM disc could be reduced significantly to achieve enhanced creep life.

7 Conclusions

The performance of the Al-SiC$_p$ disc has been investigated by imposing three different kinds of radially decreasing temperature profiles, viz. linear, parabolic and exponential, and compared with a similar FGM disc subjected to uniform temperature. The study indicates that as compared to uniform temperature disc the FGM disc operating under radial temperature profile(s) has higher radial stress throughout and higher tangential stress near the inner radius, with the maximum stress noticed for disc subjected to linear temperature gradient. Though, near the outer radius, the tangential stress is observed to be minimum in FGM disc subjected to radial thermal gradient(s), with the lowest stress noticed for FGM disc subjected to exponential thermal gradient. The radial as well as tangential strain rates in the FGM disc operating under radial thermal gradient(s) are significantly reduced as compared to uniform temperature FGM disc. The maximum reduction in strain rates is observed for FGM disc operating under exponential thermal gradient. Besides reduction in magnitude, the imposition of exponential thermal gradient leads to relatively more uniform distribution of strain rates in the FGM disc. The study also reveals that amongst several FGM discs operating under radial thermal gradients, with different values of temperature exponent (n_T) but having the same average and fixed outer surface temperature, the FGM disc with lower value of n_T exhibits the maximum creep life. By decreasing n_T from 15 to 2, the creep life of the disc could be increased from few hours to several years.

References

Afsar, A. M.; Go, J. (2010): Finite element analysis of thermoelastic field in a rotating FGM circular disc. *Appl. Math. Model.*, vol. 34, pp.3309–3320.

Akbari, M. R.; Ghanbari, J. (2015): Analytical solution of thermo-elastic stresses and deformation of functionally graded rotating hollow discs with radially varying thermo-mechanical properties under internal pressure. *Computers, Materials & Continua*, vol. 45, pp. 187–201.

Ali, B.; Mostefa, B. (2013): Thermomechanical modelling of disc brake contact phenomena. *FME Transactions*, vol. 41, pp. 59–65.

Bayat, M.; Saleem, M.; Sahari, B. B.; Hamouda, A. M. S.; Mahdi, E. (2008): Analysis of functionally graded rotating discs with variable thickness. *Mech. Res. Commun.*, vol. 35, pp. 283–309.

Çallıoğlu, H.; Bektas, N. B.; Sayer, M. (2011): Stress analysis of functionally graded rotating discs: analytical and numerical solutions. *Acta Mech. Sin.*, vol. 27, pp. 950–955.

Deepak, D.; Gupta, V. K.; Dham, A. K. (2010): Creep modeling of functionally graded rotating disc of variable thickness. *J. Mech. Sci. Technol.*, vol. 24, pp. 2221–2232.

Dong, L.; El-Gizawy, A. S.; Juhany, K. A.; Atluri, S. N. (2014a): A simple locking-alleviated 4-node mixed-collocation finite element with over-integration, for homogeneous or functionally-graded or thick-section laminated composite beams. *CMC: Comput Mater Con*, vol. 40, pp. 49–77.

Dong, L.; El-Gizawy, A. S.; Juhany, K. A.; Atluri, S. N. (2014b): A simple locking-alleviated 3D 8-node mixed-collocation C^0 finite element with over-integration, for functionally-graded and laminated thick-section plates and shells, with & without Z-pins. *CMC: Comput Mater Con*, vol. 41, pp. 163–192.

Farshi, B.; Jahed, H.; Mehrabian, A. (2004): Optimum design of inhomogeneous non-uniform rotating discs. *Comput. Struct.*, vol. 82, pp. 773–779.

Garg, M.; Salaria, B. S.; Gupta, V. K. (2013): Effect of thermal gradient on steady state creep in a rotating disc of variable thickness. *Procedia Eng.*, vol. 55, pp. 542–547.

Gupta, V. K.; Singh, S. B.; Chandrawat, H. N.; Ray, S. (2004a) Steady state creep and material parameters in a rotating disc of Al-SiC_p composite. *Eur. J. Mech. A/Solids*, vol. 23, pp. 335–344.

Gupta, V. K.; Singh, S. B.; Chandrawat H. N.; Ray, S. (2004b) Creep behavior of a rotating functionally graded composite disc operating under thermal gradient. *Metall Mater Trans A.*, vol. 35, pp. 1381–1391.

Gupta, V. K.; Singh, S. B.; Chandrawat, H. N.; Ray, S. (2005) Modeling of creep behavior of a rotating disc in the presence of both composition and thermal gradients. *J. Eng. Mater. Technol.*, vol. 127, pp. 97–105.

Hassani, A.; Hojjati, M. H.; Farrahi, G. H.; Alashti, R. A. (2012): Semi-exact solution for thermo-mechanical analysis of functionally graded elastic-strain hardening rotating disks. *Commun. Nonlinear Sci. Numer. Simul*, vol. 17, pp. 3747–3762.

Hassani, A.; Hojjati, M. H.; Farshi, G.; Alashti, R. A. (2011): Semi-exact solutions for thermo-mechanical analysis of functionally graded rotating disks. *Compos. Struct.*, vol. 93, pp. 3239–3251.

Hojjati, M. H.; Hassani, A. (2008): Theoretical and numerical analyses of rotating discs of non-uniform thickness and density. *Int. J. Pressure Vessels Piping*, vol. 85, pp. 694–700.

Khanna, K.; Gupta, V. K.; Nigam, S. P. (2015): Creep analysis of a variable thickness rotating FGM disc using Tresca criterion. *Def. Sci. J.* vol. 65, pp. 163–170.

Kordkheili, S. A. H.; Livani, M. (2013): Thermoelastic creep analysis of a functionally graded various thickness rotating disk with temperature-dependent material properties. *Int. J. Pressure Vessels Piping*, vol. 111–112, pp. 63–74.

Laskaj, M.; Murphy, B.; Houngan, K. (1999): *Improving the efficiency of cooling the front disc brake on a V8 racing car*, A Project Report, Monash University, Australia.

Lepeshkin, A. (2012): Investigations of Thermal Barrier Coatings for Turbine Parts, *in Ceramic Coatings -Applications in Engineering (Edited by Prof. Feng Shi)*, InTech, Croatia.

Nieh, T. G. (1984): Creep rupture of a silicon carbide reinforced aluminum composite. *Metall. Trans. A*, vol. 15A, pp. 139–146.

Nie, G. J.; Batra, R.C. (2010): Stress analysis and material tailoring in isotropic linear thermo elastic incompressible functionally graded rotating disks of variable thickness. *Compos. Struct.*, vol. 92, pp. 720–729.

Orlando, J. A.; Filho, G. (2004): A lower bound to the creep rupture time of pressurised thick cylinders. *Int. J. Mech. Sci.*, vol. 46, pp. 527–539.

Padture, N. P.; Gell, M.; Jordan, E. H. (2002): Thermal Barrier Coatings for Gas-Turbine Engine Applications. *Science*, vol. 296, pp. 280–284.

Singh, S. B.; Ray, S. (2001): Steady-state creep behavior in an isotropic functionally graded material rotating disc of Al-SiC composite *Metall Mater Trans A*, vol. 32, pp. 1679–1685.

Wahl, A. M.; Sankey, G. O.; Manjoine, M. J.; Shoemaker, E. (1954): Creep test of rotating discs at elevated temperature and comparison with theory. *J. Appl. Mech.-T ASME* vol. 21, pp. 225–235.

You, L. H.; You, X. Y.; Zhang, J. J.; Li, J. (2007): On rotating circular discs with varying material properties. *Z. angew. Math. Phys*, vol 58, pp. 1068–1084.

APPENDIX A

$$\sigma_\theta(r) = \frac{\psi_1(r)}{M(r)}\left[\frac{\left[A_0\sigma_\theta(avg) - \int_a^b h(r)\psi_2(r)dr\right]}{\int_a^b \frac{h(r)\psi_1(r)dr}{M(r)}}\right] + \psi_2 \tag{A1}$$

where $\psi_1(r) = \psi(r)^{1/n}$; $\psi_2 = \sigma_0(r)$ and $\psi(r) = \frac{2}{r[2-x(r)]}\exp\int_a^r \frac{\phi(r)}{r}dr$ and

$$\sigma_\theta(avg) = \frac{\omega^2 h_b}{A_0 b^k}\left[\frac{A_\rho}{3+k}\left(b^{3+k} - a^{3+k}\right) + \frac{B_\rho}{3+k+m}\left(b^{3+k+m} - a^{3+k+m}\right)\right]$$

where $A_\rho = \rho_m$, $B_\rho = \frac{(\rho_d - \rho_m)V_{\min}}{b^m(100)}$, $\rho_m(= 2698.9\frac{kg}{m^3})$ is the density of Al and $\rho_d(= 3210\frac{kg}{m^3})$ is the density of SiC_p and $A_0 = \int_a^b h(r)dr$

$$\sigma_r(r) = \frac{1}{rh(r)}\left[\int_a^r h(r)\sigma_\theta dr - \frac{\omega^2 h_b}{b^k}\left\{\frac{A_\rho}{3+k}(r^{3+k} - a^{3+k}) + \frac{B_\rho}{3+k+m}(r^{3+k+m} - a^{3+k+m})\right\}\right] \tag{A2}$$