 CMC, vol.51, no.2, pp.63-79, 2016

Reflection of Plane Waves from Electro-magneto-thermoelastic Half-space with a Dual-Phase-Lag Model

A. M. Abd-Alla[1,2,3], **Mohamed I. A. Othman**[1,4], **S. M. Abo-Dahab**[1,5]

Abstract: The aim of this paper is to study the reflection of plane harmonic waves from a semi-infinite elastic solid under the effect of magnetic field in a vacuum. The expressions for the reflection coefficients, which are the relations of the amplitudes of the reflected waves to the amplitude of the incident waves, are obtained. Similarly, the reflection coefficient ratio variations with the angle of incident under different conditions are shown graphically. Comparisons are made with the results predicted by the dual-phase-lag model and Lord-Shulman theory in the presence and absence of magnetic field.

Keywords: Reflection; Generalized thermoelasticity; Magnetic field; Dual-phase-lag model.

1 Introduction

Biot (1956) introduced the theory of coupled thermoelasticity to overcome the first shortcoming in the classical uncoupled theory of thermoelasticity where it predicts two phenomena not compatible with physical observations. First, the equation of heat conduction of this theory does not contain any elastic terms. Second, the heat equation is of a parabolic type, predicting infinite speeds of propagation for heat waves. The governing equations for the Biot theory are coupled, eliminating the first paradox of the classical theory. However, both theories share the second shortcoming since the heat equation for the coupled theory is also parabolic.

Thermoelasticity theories that predict a finite speed for the propagation of thermal signals have aroused much interest in the last three decades. These theories

[1] Math. Dept., Faculty of Science, Taif University 888, Saudi Arabia
[2] Department of Mathematics, Faculty of Science, Sohag University, Egypt
[3] Corresponding author: Ph.D. Email: mohmrr@yahoo.com
[4] Math. Dept., Faculty of Science, Zagazig University, P.O. Box 44519, Zagazig, Egypt
[5] Math. Dept., Faculty of Science, South Valley University, Qena 83523, Egypt

are known as generalized thermoelasticity theories. The first generalization of the thermo-elasticity theory is due to Lord and Shulman (1967) who introduced the theory of generalized thermoelasticity with one relaxation time by postulating a new law of heat conduction to replace the classical Fourier' law. This law contains the heat flux vector as well as its time derivative. It contains also a new constant that acts as a relaxation time. The heat equation of this theory is of the wave-type, ensuring finite speeds of propagation of heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motion and the constitutive relations remain the same as those for the coupled and the uncoupled theories. This theory was extended by Dhaliwal and Sherief (1980) to general anisotropic media in the presence of heat sources.

A generalization of this inequality was proposed by Green and Laws (1972). Green and Lindsay obtained another version of the constitutive equations in (1972). The theory of thermoelasticity without energy dissipation is another generalized theory and was formulated by Green and Naghdi (1993). It includes the thermal displacement gradient among its independent constitutive variables, and differs from the previous theories in that it does not accommodate dissipation of thermal energy.

Tzou (1995a, 1996) proposed the dual-phase-lag (DPL) model, which describes the interactions between phonons and electrons on the microscopic level as retarding sources causing a delayed response on the macroscopic scale. For macroscopic formulation, it would be convenient to use the (DPL) model for investigation of the micro-structural effect on the behavior of heat transfer. The physical meanings and the applicability of the (DPL) model have been supported by the experimental results [Tzou (1995)]. The dual-phase-lag proposed by Tzou (1995b) is such a modification of the classical thermoelastic model in which the Fourier law is replaced by an approximation to a modified Fourier law with two different time translations: a phase-lag of the heat flux τ_q and a phase-lag of temperature gradient τ_θ. A Taylor series approximation of the modified Fourier law, together with the remaining field equations leads to a complete system of equations describing a dual-phase-lag thermoelastic model. The model transmits thermoelastic disturbance in a wavelike manner if the approximation is linear with respect to τ_q and τ_θ, and $0 \leq \tau_\theta < \tau_q$; or quadratic in τ_q and linear in τ_θ, with $\tau_q > 0$ and $\tau_\theta > 0$. This theory is developed in a rational way to produce a fully consistent theory which is able to incorporate thermal pulse transmission in a very logical manner.

Some researches in the past have investigated different problems of rotating media. In a paper by Schoenberg and Censor (1973), the propagation of plane harmonic waves in a rotating elastic medium without a thermal field has been studied. It was shown there that the rotation causes the elastic medium to be depressive and anisotropic. Chand, Sharma, and Sud (1990) presented an investigation of the distribution of deformation, stresses and magnetic field in a uniformly rotating homogeneous isotropic, thermally and electrically conducting elastic half-space. Clarke and Burdness (1994); Destrade (2004); Othman (2005); Othman (2004) studied the effect of rotation on elastic waves. Sharma and Thakur (2006); Sharma, Walia, and Gupta (2008) discussed the effect of rotation on different types of wave propagating in a thermoelastic medium. Othman and Song (2009) discussed the effect of rotation in a magneto-thermoelastic medium. Abo-Dahab and Mohamed (2010) discussed the influence of magnetic field and hydrostatic initial stress on reflection phenomena of P and SV waves from a generalized thermoelastic solid half-space. Abo-Dahab, Mohamed, and Singh (2011) investigated the rotation and magnetic field effect on the P wave reflection from stress-free surface elastic half-space with voids under one thermal relaxation time. Abo-Dahab (2011) discussed the reflection of P and SV waves from stress-free surface elastic half-space under the influence of magnetic field and hydrostatic initial stress without energy dissipation. Singh and Tomer (2011) studied the effect of rotation on propagation of plane waves in generalized thermoelasticity. Othman and Said (2012) investigated the effect of rotation on the two-dimensional problem of fiber-reinforced thermoelastic with one relaxation time.

In the classical theory of elasticity, the gravity effect is generally neglected. The effect of gravity in the problem of propagation of waves in solids, in particular on an elastic globe, was first studied by Bromwich (1898). Subsequently, an investigation of the effect of gravity was considered by Love (1911) who showed that the velocity of Rayleigh waves is increased to a significant extent by gravitational field when wavelengths are large. De and Sengupta (1974,1976) studied the effect of gravity on the surface waves, on the propagation of waves in an elastic layer. Das, Acharya, and Sengupta (1992) investigated surface waves under the influence of gravity in a non-homogeneous elastic solid medium. Abd-Alla, Yahia, and Abo-Dahab (2003) discussed the reflection of the generalized magneto-thermo-viscoelastic plane waves. Ailawalia and Narah (2009) depicted the effects of rotation and gravity in the generalized thermoelastic medium. Abo-Dahab and Singh (2013) explained rotational and voids effects on the reflection of P waves from stress-free surface of an elastic half-space under a magnetic field, initial stress and without energy dissipation. Allam, Rida, Abo-Dahab, Mohamed, and Kilany (2014) studied (GL) model of reflection of P and SV-waves from the free surface of thermoelastic diffusion solid under influence of the electromagnetic field and initial stress. Abo-Dahab, Abd-Alla, and Gohaly (2014) pointed out the reflection of plane elastic wave problem at a free surface under the initial stress, magnetic field and temperature field. Abo-Dahab and Elsagheer (2014) investigated the reflection of thermoelastic boundary half-space with the magnetic field and rotation. Othman and Lotfy (2013) studied the effect of magnetic field and a rotation of the 2-D problem of a fiber-reinforced thermoelastic under three theories with influence of gravity. Abo-Dahab, Gohaly, and El-Malki (2015) studied the rotation effect on the reflection of plane elastic waves at a free surface under the initial stress, magnetic field and temperature field.

In this paper, the generalized thermoelastic theory is applied to study the reflection of plane wave under the effect of magnetic field on a half-space elastic media near-by a vacuum. The reflection coefficient ratios of various reflected waves with the angle of incidence have been obtained from (DPL) model and LS theory. Also the effects of magnetic field is discussed numerically and illustrated graphically.

2 Formulation of the problem and basic equations

We consider an isotropic, homogeneous, linear, thermally, and electrically conducting thermoelastic half-space $(x \geq 0, -\infty \leq z \leq \infty)$ and x-axis pointing vertically inwards as shown in the geometry of the problem. The surface $(x = 0)$ of the half-space is taken to be traction free and subjected to mechanical and thermal loads. All considered functions are assumed to be bounded as $x \to \infty$. The whole body is at a constant temperature T_0. We consider that the orientation of the primary magnetic field $\boldsymbol{H} = (0, H_0, 0)$ is towards the positive direction of $y-$axis. Due to the application of this magnetic field, there arise in the medium an induced magnetic field $\boldsymbol{h}$ and an induced electric field $\boldsymbol{E}$ as shown in Fig. 1

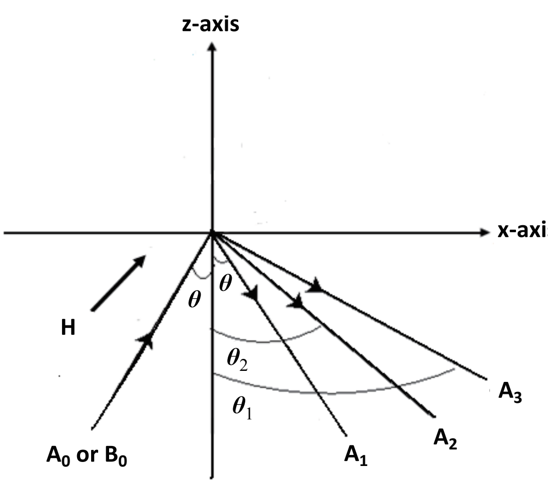

Figure 1: Schematic of the problem.

The variation of the magnetic and electric fields are perfectly conducting slowly moving medium and are given by Maxwell's equations:

$$\text{curl}\ \boldsymbol{h} = \boldsymbol{J} + \varepsilon_0 \dot{\boldsymbol{E}}, \tag{1}$$

$$\text{curl}\ \boldsymbol{E} = -\mu_0 \dot{\boldsymbol{h}}, \tag{2}$$

$$\boldsymbol{E} = -\mu_0 (\dot{\boldsymbol{u}} \times \boldsymbol{H}), \tag{3}$$

$$\text{div}\ \boldsymbol{h} = 0. \tag{4}$$

From the above equations, we can obtain

$$\boldsymbol{E} = \mu_0 H_0 (\dot{w}, 0, -\dot{u}), \tag{5}$$

$$\boldsymbol{h} = (0, -H_0 e, 0), \tag{6}$$

$$\boldsymbol{J} = (-h_{,z} - \varepsilon_0 \mu_0 H_0 \ddot{w}, 0, h_{,x} + \varepsilon_0 \mu_0 H_0 \ddot{u}). \tag{7}$$

The equations of motion have the form

$$\sigma_{ji,j} + F_i = \rho \ddot{u}_i, \;\; i, j = 1, 2, 3. \tag{8}$$

Where F_i is the Lorentz force and is given by:

$$F_i = \mu_0 (\boldsymbol{J} \times \boldsymbol{H})_i. \tag{9}$$

From Eqs. (7) and (9), we obtain

$$\boldsymbol{F} = (F_x, F_y, F_z) = (\mu_0 H_0^2 e_{,x} - \varepsilon_0 \mu_0^2 H_0^2 \ddot{u}, 0, \mu_0 H_0^2 e_{,z} - \varepsilon_0 \mu_0^2 H_0^2 \ddot{w}). \tag{10}$$

The strain-displacement relation

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{11}$$

The constitutive laws

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda e - \gamma T]\delta_{ij}. \tag{12}$$

Substituting Eq. (10) into Eq. (8) we obtain

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ij} + \mu_0 (\boldsymbol{J} \times \boldsymbol{H})_i - \gamma T_{,i} = \rho \ddot{u}_i. \tag{13}$$

The Chandrasekaraiah and Tzou theory (DPL) Othman, Hasona, and Abd-Elaziz (2014) have such a modified of classical thermoelasticity model in which the Fourier law is replaced by an approximation of the equation

$$q_i(x, t + \tau_q) = -K T_{,i}(x, t + \tau_\theta). \tag{14}$$

Where, q_i is the heat flux vector.

The model transmits thermoelastic disturbances in a wave-like-manner if Eq. (5) is approximated by

$$(1+\tau_q\frac{\partial}{\partial t})q_i = -K(1+\tau_\theta\frac{\partial}{\partial t})T_{,i}. \tag{15}$$

Here $0 \le \tau_\theta < \tau_q$, hence, we get the heat conduction equation in the context of (DPL) model in the form

$$K(1+\tau_\theta\frac{\partial}{\partial t})T_{,ii} = (1+\tau_q\frac{\partial}{\partial t})(\rho C_E\frac{\partial T}{\partial t}+\gamma T_0\frac{\partial e}{\partial t}). \tag{16}$$

Moreover, if we put $\tau_\theta = 0$ and $\tau_q = \tau$ (the first relaxation time), then the fundamental equations will be possible for the L-S theory.

Where λ, μ are Lame's constants, T is the temperature distribution, $\gamma = \alpha_t(3\lambda + 2\mu)$, α_t is the coefficient of linear thermal expansion, K is the thermal conductivity, T_0 is the reference temperature, σ_{ij} are the components of the stress tensor, δ_{ij} is the Kronecker delta, ρ, C_E are the density and specific heat respectively, τ_q is the phase-lag of the heat flux and τ_θ is the phase-lag of temperature gradient.

Substituting from Eq. (10) into Eq. (13), we obtain the equations of motion in the form

$$\mu\nabla^2 u + (\lambda+\mu+\mu_0 H_0^2)e_{,x} - \gamma T_{,x} = \rho(1+\frac{\varepsilon_0\mu_0^2H_0^2}{\rho})\ddot{u}, \tag{17}$$

$$\mu\nabla^2 w + (\lambda+\mu+\mu_0 H_0^2)e_{,z} - \gamma T_{,z} = \rho(1+\frac{\varepsilon_0\mu_0^2H_0^2}{\rho})\ddot{w}. \tag{18}$$

$$K(1+\tau_\theta\frac{\partial}{\partial t})\nabla^2 T = \rho C_E(1+\tau_q\frac{\partial}{\partial t})\dot{T} + \gamma T_0(1+\tau_q\frac{\partial}{\partial t})\dot{e}. \tag{19}$$

The constitutive relations can be written as

$$\sigma_{xx} = (\lambda+2\mu)u_{,x} + \lambda w_{,z} - \gamma T, \tag{20}$$

$$\sigma_{yy} = \lambda e - \gamma T, \tag{21}$$

$$\sigma_{zz} = \lambda u_{,x} + (\lambda+2\mu)w_{,z} - \gamma T, \tag{22}$$

$$\sigma_{xz} = \mu(u_{,z}+w_{,x}), \quad \sigma_{xy} = \sigma_{yz} = 0. \tag{23}$$

For simplifications, we shall use the following non-dimensional variables:

$$\{x_i', u_i'\} = \frac{\omega^*}{c_0}\{x_i, u_i\}, \;\; \theta' = \frac{\gamma}{\rho c_0^2}(T-T_0), \;\; \{t', \tau_T', \tau_v', \tau_q'\} = \omega^*\{t, \tau_T, \tau_v, \tau_q\},$$

$$h' = \frac{h}{H_0},\ H' = \frac{H}{H_0},\ (\sigma'_{ij},\ \tau'_{ij}) = \frac{(\sigma_{ij}, \tau_{ij})}{\gamma T_0}, \tag{24}$$
$$\omega^* = \rho C_E c_0^2/K \text{ and } c_0^2 = (\lambda + 2\mu)/\rho \ \ i, j = 1, 2, 3.$$

In terms of non-dimensional quantities defined in Eq. (24), the above governing Eqs. (17)–(19) reduce to (dropping the dashed for convenience)

$$\frac{\mu}{\rho c_0^2}\nabla^2 u + \frac{(\lambda + \mu + \mu_0 H_0^2)}{\rho c_0^2}\frac{\partial e}{\partial x} - \frac{\partial \theta}{\partial x} = (1 + \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho})\ddot{u}, \tag{25}$$

$$\frac{\mu}{\rho c_0^2}\nabla^2 w + \frac{(\lambda + \mu + \mu_0 H_0^2)}{\rho c_0^2}\frac{\partial e}{\partial z} - \frac{\partial \theta}{\partial z} = (1 + \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho})\ddot{w}, \tag{26}$$

$$(1 + \tau_\theta \frac{\partial}{\partial t})\nabla^2 \theta = (1 + \tau_q \frac{\partial}{\partial t})\dot{\theta} + \frac{\gamma^2 T_0}{\rho K \omega^*}(1 + \tau_q \frac{\partial}{\partial t})\dot{e}. \tag{27}$$

3 Solution of the problem

To separate the dilatational and rotational components of strain, we introduce displacement potentials Φ and Ψ defined by the following relations

$$u = \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial z},\ \ v = \frac{\partial \Phi}{\partial z} + \frac{\partial \Psi}{\partial x}. \tag{28}$$

Substituting from Eq. (28) into Eqs. (25) and (26) we get

$$\left(C_1^2 + C_s^2 + R_H^2\right)\nabla^2 \Phi - T = (1 + \varepsilon_T)\ddot{\Phi}, \tag{29}$$
$$C_s^2 \nabla^2 \Psi = (1 + \varepsilon_T)\bar{\Psi}. \tag{30}$$

Where, $C_1^2 = \dfrac{\lambda + \mu}{\rho c_0^2}, C_s^2 = \dfrac{\mu}{\rho c_0^2}, \mathrm{R}_\mathrm{H}^2 = \dfrac{\mu_0 H_0^2}{\rho c_0^2}$ and $\varepsilon_T = \dfrac{\varepsilon_0 \mu_0^2 H_0^2}{\rho}$ are the dilatational and secondary vertically wave velocities, Alfven speed, coupled electromagnetic and $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ is the Laplace operator.
Using Eq. (28) in Eq. (27), we get

$$(1 + \tau_\theta \frac{\partial}{\partial t})\nabla^2 T = (1 + \tau_q \frac{\partial}{\partial t})[\dot{T} + R\nabla^2 \dot{\Phi}]. \tag{31}$$

Where, $R = \dfrac{\gamma^2 T_0}{\rho K \omega^*}$.
We assume now the solution of Eqs. (29)–(31) takes the following form

$$\{\Phi, \Psi, T\} = \left\{\bar{\Phi}, \bar{\Psi}, \bar{T}\right\}\exp[i\xi\,(x\sin\theta + z\cos\theta)) - i\omega t]. \tag{32}$$

Where, $v = \dfrac{\omega}{\xi}$.

Substitute from Eq. (32) into Eqs. (29)–(31), we get

$$[-\xi^2(C_1^2 + C_s^2 + R_H^2) + (1+\varepsilon_T)\omega^2]\bar{\Phi} - \bar{T} = 0, \tag{33}$$

$$[C_s^2\xi^2 - ((1+\varepsilon_T)\omega^2)]\bar{\Psi} = 0, \tag{34}$$

$$\left[i\omega\xi^2 t_q R\right]\bar{\Phi} + \left[\xi^2 t_\theta - i\omega t_q\right]\bar{T} = 0. \tag{35}$$

where, $t_q = 1 - i\omega\tau_q, t_\theta = 1 - i\omega\tau_\theta$.

Equation (34) indicates that the reflected SV-waves do not affect by the thermal filed, then Eq. (34) has the following solution

$$\Psi = \bar{\Psi}\exp[i\xi_3(x\sin\theta + z\cos\theta) - i\omega t] \tag{36}$$

Where, $\xi_3 = \dfrac{\omega}{\sqrt{C_s^2/(1+\varepsilon_T)}}$.

Equations (33) and (35) have a nontrivial solution if and only if the determinant vanished, so

$$\begin{vmatrix} -\xi^2(C_s^2 + C_1^2 + R_H^2) + (1+\varepsilon_T)\omega^2 & -1 \\ i\omega\xi^2 R t_q & \xi^2 t_\theta - i\omega t_q \end{vmatrix} = 0 \tag{37}$$

This yields an algebraic equation on $\xi^2 = \dfrac{\omega^2}{v^2}$, where, v is the coupled wave velocity.

$$A\xi^4 + B\xi^2 + C = 0. \tag{38}$$

Where,

$$A = -t_\theta(C_s^2 + C_1^2 + R_H^2),$$
$$B = t_\theta\omega^2(1+\varepsilon_T) + i\omega t_q(C_s^2 + C_1^2 + R_H^2) + i\omega R t_q,$$
$$C = -i\omega^3 t_q(1+\varepsilon_T)$$

Eq. (38) indicates that there are two reflected waves; T-wave and p-wave.

4 Solution of the problem

Where, Eq. (38) has two roots in $\xi_i^2, (i = 1, 2)$, there are two coupled waves T-wave and p-wave with two different velocities and SV-wave with ξ_3^2. Assuming that the radiation in vacuum is neglected, when a coupled wave falls on the boundary $z = 0$ from within the thermoelastic medium, it will make an angle θ with the negative direction of the z-axis, and three reflected waves that will make angles θ and θ_i $(i = 2,3)$ with the same direction as shown in Fig. 1.

The displacement potentials Φ, Ψ and T will take the following forms:

$$\Phi = A_0 \exp[i\xi_1(x\sin\theta + z\cos\theta)) - i\omega t] + \sum_{j=1}^{2} A_j \exp[i\xi_j(x\sin\theta_j - z\cos\theta_j)) - i\omega t] \tag{39}$$

$$\Psi = B_0 \exp[i\xi_1 (x\sin\theta + z\cos\theta)) - i\omega t] + B_1 \exp[i\xi_3 (x\sin\theta_3 - z\cos\theta_3)) - i\omega t] \tag{40}$$

$$T = \vartheta_1 A_0 \exp[i\xi_1 (x\sin\theta + z\cos\theta)) - i\omega t] + \sum_{j=1}^{2} \vartheta_j A_j \exp[i\xi_j (x\sin\theta_j - z\cos\theta_j)) - i\omega t] \tag{41}$$

where

$$\vartheta_j = -(C_s^2 + C_1^2 + R_H^2)\xi_i^2 + (1+\varepsilon_T)\omega^2, \ \ j = 1,2 \tag{42}$$

The ratio of the amplitudes of the reflected waves and amplitude of the incident wave, $\frac{A_j}{A_0}$, $j = 1,2$, give the corresponding reflection coefficient ratio. Also, it may be noted that the angle θ, $\theta_j (j = 1,2)$, and the corresponding wave numbers, ξ_j, $j = 1,2,3$, are to be connected by the following relations according to Snell's law as follows

$$\xi_1 \sin\theta = \xi_1 \sin\theta_1 = \xi_2 \sin\theta_2 = \xi_3 \sin\theta_3 \tag{43}$$

5 Boundary conditions

(1) A mechanical boundary condition that the surface of the half-space is traction free

$$\sigma_{xx}(x,0,t) + \tau_{xx}(x,0,t) = \sigma_{xz}(x,0,t) + \tau_{xz}(x,0,t) = 0. \tag{44}$$

Where,

$$\tau_{ij} = \mu_0 \left[\mathrm{H_i h_j + H_j h_i - H_k.h_k \delta_{ij}}\right] \tag{45}$$

Substitute from Eqs. (35)–(37) into Eq. (44), we get

$$\left[\xi_1^2 \left\{\lambda + \mu_0 H_0^2 + 2\mu \sin^2\theta\right\} + \gamma\vartheta_1\right] A_0 + \sum_{j=1}^{2} \left[\xi_j^2 \left\{\lambda + \mu_0 H_0^2 + 2\mu \sin^2\theta_j\right\}\right.$$

$$+\gamma\vartheta_j]A_j+\mu\xi_1^2\sin 2\theta B_0-\mu\xi_3^2\sin 2\theta_3 B_1=0 \tag{46}$$

$$-\xi_1^2\sin 2\theta A_0+\sum_{j=1}^{2}\xi_j^2\sin 2\theta_j A_j+\xi_1^2\cos 2\theta B_0+\xi_3^2\cos 2\theta_3 B_1=0 \tag{47}$$

(2) Assuming that the boundary $z=0$ is thermally insulated. This means that the following relation will be

$$\frac{\partial T}{\partial z}=0 \text{ on } z=0 \tag{48}$$

Substitute from Eq. (39), we obtain

$$\xi_1\vartheta_1\cos\theta A_0-\sum_{j=1}^{3}[\xi_j\vartheta_j\cos\theta_j]A_j=0 \tag{49}$$

From Eqs. (46), (47) and (49) we can put them in the following algebraic equation

$$\sum_{j=1}^{3}a_{ij}Z_j=b_i, \tag{50}$$

Now we consider the incidence of p-wave or SV-wave as follows:

(i) For the incidence of p-wave: $B_0=0, \theta_1=\theta$ and

$$Z_{1,2}=\frac{A_{1,2}}{A_0}, Z_3=\frac{B_1}{A_0}$$

$$a_{1j}=\xi_j^2\left\{\lambda+\mu_0H_0^2+2\mu\sin^2\theta_j\right\}+\gamma\vartheta_j,\quad a_{13}=-\mu\xi_3^2\sin 2\theta_3,$$

$$a_{2j}=\xi_j^2\sin 2\theta_j,\quad a_{23}=\xi_3^2\cos 2\theta_3,$$

$$a_{3j}=\xi_j\vartheta_j\cos\theta_j,\quad a_{33}=0,$$

$$b_1=a_{13},\ b_2=a_{21},\ b_3=a_{31}.$$

(ii) For the incidence of SV-wave: $A_0=0, \theta_3=\theta$ and

$$Z_{1,2}=\frac{A_{1,2}}{B_0}, Z_3=\frac{B_1}{B_0}$$

$$a_{1j}=\xi_j^2\left\{\lambda+\mu_0H_0^2+2\mu\sin^2\theta_j\right\}+\gamma\vartheta_j,\quad a_{13}=-\mu\xi_3^2\sin 2\theta_3,$$

$$a_{2j}=\xi_j^2\sin 2\theta_j,\quad a_{23}=\xi_3^2\cos 2\theta_3,$$

$$a_{3j}=\xi_j\vartheta_j\cos\theta_j,\quad a_{33}=0,$$

$$b_1=-\mu\xi_1^2\sin 2\theta,\ b_2=\xi_1^2\cos 2\theta,\ b_3=0.$$

6 Numerical results and discussion

To illustrate the theoretical results obtained in the preceding section, to compare these in the context of the (DPL) model, and to study the effect of rotation and gravity on wave propagation, we now present some numerical results. For this purpose, Crust is taken as the thermoelastic material for which we take the following values of the different physical constants

$$\lambda = \mu = 3 \times 10^{10}\ \mathrm{N \cdot m^{-2}},\ K = 3\mathrm{w} \cdot \mathrm{m}^{-1} \cdot k^{-1},\ T_0 = 300\ \mathrm{K},\ g = 9.8,$$
$$\gamma = 1.6 \times 10^{11} k^{-1},\ \rho = 2900\ \mathrm{kg \cdot m^{-3}},\ C_E = 1100\ \mathrm{J \cdot kg^{-1}} \cdot k^{-1}.$$

The computations were carried out for:

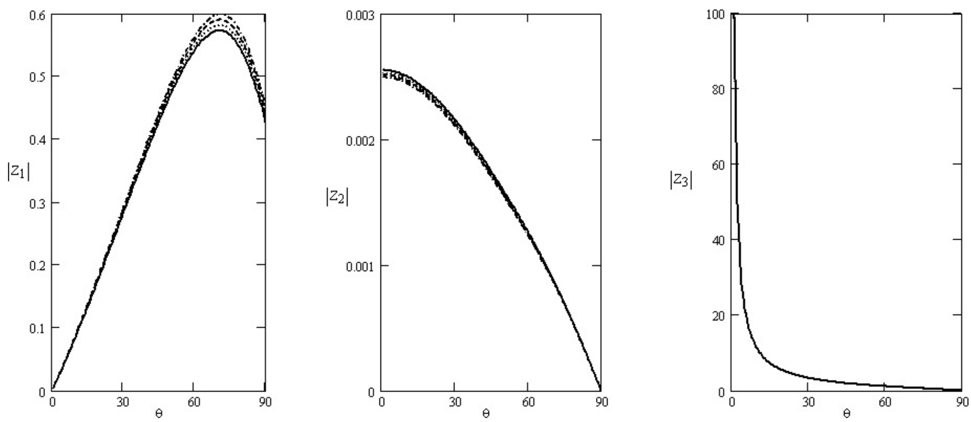

Figure 2: Variation of the magnitude of amplitude ratios $Z_i (i = 1,2,3)$ with respect to the angle of incidence θ with variation of $\tau_\theta = 1$ ——, 3 ······, 5 — —, 7 — · — for p-wave incidence.

Fig. 2 displays the variation of the magnitude of amplitude ratios $|Z_i|, (i = 1,2,3)$ with respect to the angle of incident θ for *p-wave* for different values of the phase-lag of temperature gradient τ_θ. It is observed that the magnitude of amplitude ratios $|Z_1|$ increases with increasing of the phase-lag of temperature gradient τ_θ and it has very large value at $\theta = 60$ and vanishes at $\theta = 0$, while it increases and decreases with increasing of the angle of incident, the magnitude of amplitude ratios $|Z_2|$ and $|Z_3|$ decreases with increasing of the angle of incident until vanish at $\theta = 90$, while the magnitude of amplitude ratios $|Z_2|$ decreases with increasing of the phase-lag of temperature gradient, as well there is no effect of the phase-lag of temperature gradient on the $|Z_3|$. Fig. 3 shows the variation of the magnitude of amplitude ratios $|Z_i|, (i = 1,2,3)$ with respect to the angle of incident θ for *p-wave* for different values of the magnetic field H. It is observed that the magnitude of amplitude ratios $|Z_1|$ increases with increasing of the magnetic field and it has very large value at $\theta = 60$ and vanishes at $\theta = 0$, while it increases and decreases with increasing of the angle of incident, the magnitude of amplitude ratios $|Z_2|$ and $|Z_3|$ decreases with increasing of the angle of incident until vanish at $\theta = 90$, while the magnitude of amplitude ratios $|Z_2|$ increases with increasing of the magnetic field, as well the magnitude of amplitude ratios $|Z_3|$ decreases with increasing of magnetic field.

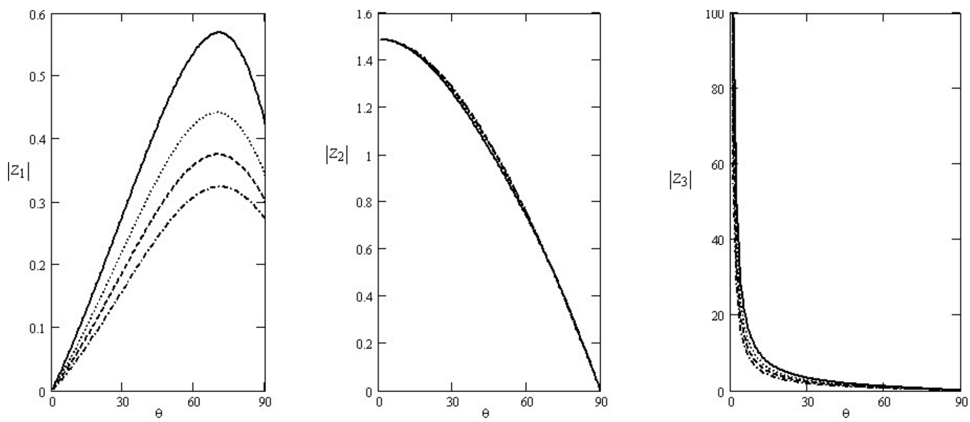

Figure 3: Variation of the magnitude of amplitude ratios $Z_i (i = 1,2,3)$ with respect to the angle of incidence θ with variation of $H = 0.1$ ——, 0.5 ······, 0.7 — —, 0.9 — · — for *p-wave* incidence.

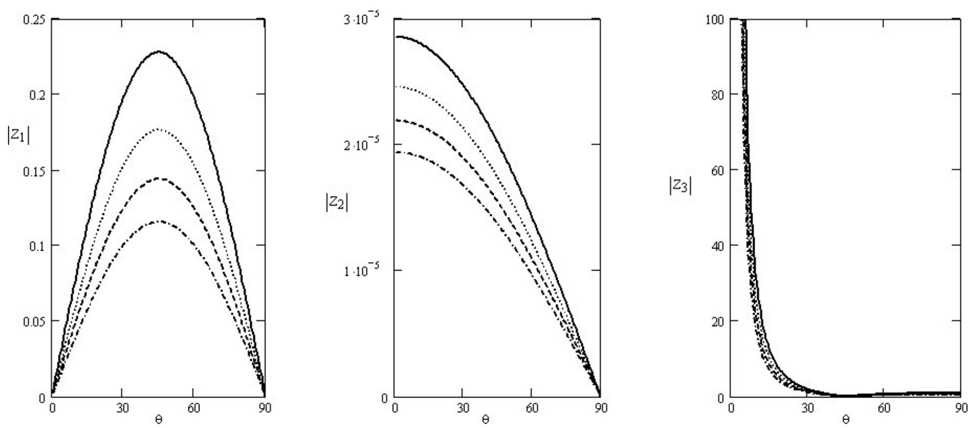

Figure 4: Variation of the magnitude of amplitude ratios $Z_i (i = 1,2,3)$ with respect to the angle of incidence θ with variation of $\tau_\theta = 1$ ——, 3 ······, 5 — —, 7 — · — for *SV-wave* incidence.

Fig. 4 shows the variation of the magnitude of amplitude ratios $|Z_i|, (i = 1,2,3)$ with respect to the angle of incident θ for *SV-wave* for different values of the phase-lag of temperature gradient τ_θ. It is observed that the magnitude of amplitude ratios $|Z_1|$ decreases with increasing of the phase-lag of temperature gradient τ_θ and it has very large value at $\theta = 60$ and it vanishes at $\theta = 0,90$, which it has an oscillatory behavior of thermoelastic half-space in the whole range of the angle of incident θ, the magnitude of amplitude ratios $|Z_2|$ and $|Z_3|$ decreases with increasing of the angle of incident until vanish at $\theta = 90$, while it decreases with increasing of the phase-lag of temperature gradient. Fig. 5 shows the variation of the magnitude of amplitude ratios $|Z_i|, (i = 1,2,3)$ with respect to the angle of incident θ for *SV-wave* for different values of the magnetic field H. It is observed that the magnitude of amplitude ratios $|Z_1|$ decreases with increasing of the magnetic field and it has very large value at $\theta = 60$ and it vanishes at $\theta = 0,90$, which it has an oscillatory behavior for thermoelastic half-space in the whole range of the angle of incident θ, as well the magnitude of amplitude ratios $|Z_2|$ and $|Z_3|$ decreases with increasing of the angle of incident until vanish at $\theta = 90$, while the magnitude of amplitude ratios $|Z_2|$ increases with increasing of the magnetic field, as well there is no effect of the magnetic field on the $|Z_3|$.

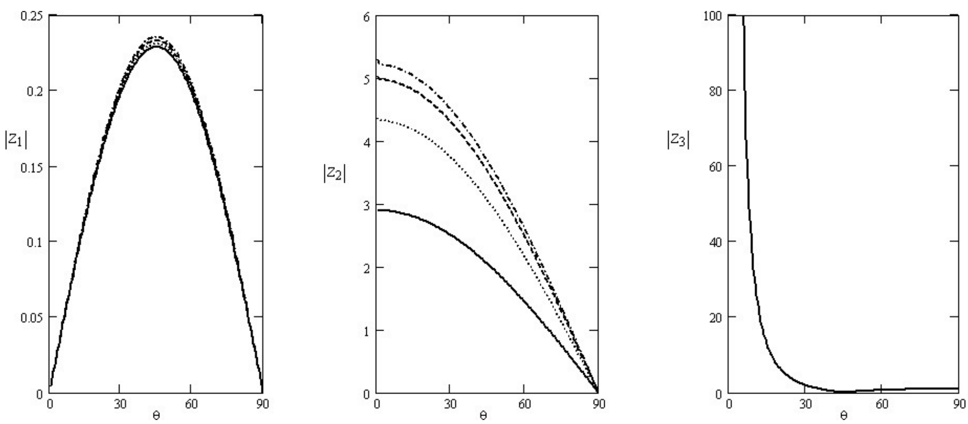

Figure 5: Variation of the magnitude of amplitude ratios $Z_i (i = 1,2,3)$ with respect to the angle of incidence θ with variation of $H = 0.1$ ——, 0.5 ······, 0.7 — —, 0.9 — · — for *SV-wave* incidence.

7 Conclusion

According to the above results, we can conclude that:

1. The magnitude of amplitude ratios depends on the angle of incidence, the

phase-lag of temperature gradient τ_θ and magnetic field, the nature of this dependence is different for different magnitude of amplitude ratios.

2. The phase-lag of temperature gradient τ_θ and magnetic field play a significant role and the two effects have the inverse trend for the magnitude of amplitude ratios.

3. The phase-lag of temperature gradient and magnetic field have a strong effect on the magnitude of amplitude ratios. It is observed that the magnitude of amplitude ratios, changes their values in the phase-lag of temperature gradient and magnetic field. Hence, the phase-lag of temperature gradient and magnetic field affect on the magnitude of amplitude ratios phenomena significantly.

4. The results presented in this paper will be very helpful for researchers concerning with material science, designers of new materials, low-temperature physicists, as well as for those working on the development of a theory of hyperbolic propagation of the magnitude of amplitude ratios of thermoelastic waves. Study of the phenomenon of the phase-lag of temperature gradient and magnetic field are also used to improve the conditions of oil extractions.

References

Abd-Alla, A. N.; Yahia, A. A.; Abo-Dahab, S.M. (2003): On the reflection of the generalized magneto-thermo-viscoelastic plane waves. *Chaos, Solitons & Fractals*, vol. 16, no. 2, pp. 211–231.

Abo-Dahab, S. M. (2011): Reflection of P and SV waves from stress-free surface elastic half-space under influence of magnetic field and hydrostatic initial stress without energy dissipation. *J. Vib. & Control*, vol. 17, no. 14, pp. 2213–2221.

Abo-Dahab, S. M.; Abd-Alla, A. M.; Gohaly, A. (2014): On reflection of plane elastic waves problem at a free surface under initial stress, magnetic field and temperature field. *J. Comput. Theor. Nanosci.*, vol. 11, no. 10, pp. 2171–2184.

Abo-Dahab, S. M.; Elsagheer, M. (2014): On the reflection of thermoelastic boundary half space with the magnetic field and rotation. *J. Comput. & Theor. Nanosci.*, vol. 11, no. 11, pp. 2370–2378.

Abo-Dahab, S. M.; Gohaly, A.; El-Malki, F. (2015): Rotation effect of reflection of plane elastic waves at a free surface under initial stress, magnetic field and temperature field. *J. Comput. & Theor. Nanosci.*, vol. 12, no. 2, pp. 244–256.

Abo-Dahab, S. M.; Mohamed, R. A. (2010): Influence of magnetic field and hydrostatic initial stress on reflection phenomena of P and SV waves from a generalized thermoelastic solid half-space. *J. Vib. & Control*, vol. 16, pp. 685–699.

Abo-Dahab, S. M.; Mohamed, R. A.; Singh, B. (2011): Rotation and magnetic field effects on P wave reflection from stress-free surface elastic half-space with voids under one thermal relaxation time. *J. Vib. & Control*, vol. 17, no. 12, pp. 1827–1839.

Abo-Dahab, S. M.; Singh, B. (2013): Rotational and voids effects on the reflection of P waves from stress-free surface of an elastic half-space under magnetic field, initial stress and without energy dissipation. *Appl. Math. Mod.*, vol. 37, pp. 8999–9011.

Ailawalia, P.; Narah, N. S. (2009): Effect of rotation in generalized thermoelastic solid under the influence of gravity with an overlying infinite thermoelastic fluid. *Appl. Math. Mech.*, vol. 30, pp. 1505–1518.

Allam, M. N. M.; Rida, S. Z.; Abo-Dahab, S. M.; Mohamed, R. A.; Kilany, A. A. (2014): GL model on reflection of P and SV-waves from the free surface of thermoelastic diffusion solid under influence of the electromagnetic field and initial stress. *J. Thermal Stresses*, vol. 37, pp. 471–487.

Biot, M. A. (1956): Thermoelasticity and irreversible thermodynamics. *J. Appl. Phys.*, vol. 27, pp. 240–253.

Bromwich, T. J. J. A. (1898): On the influence of gravity on elastic waves and in particular on the vibrations of an elastic globe. *Proc. London Math. Soc.*, vol. 30, pp. 98–120.

Chand, D.; Sharma, J. N.; Sud, S. P. (1990): Transient generalized magneto thermo-elastic waves in a rotating half space. *Int. J. Eng. Sci.*, vol. 28, pp. 547–556.

Clarke, N. S.; Burdness, J. J. (1994); Rayleigh waves on rotating surface. *ASME J. Appl. Mech.*, vol. 61, pp. 724–726.

Das, S. C.; Acharya, D. P.; Sengupta, P. R. (1992): Surface waves in an inhomogeneous elastic medium under the influence of gravity. *Rev. Roum. Des. Sci. Tech.*, vol. 37, pp. 539–551.

De, S. N.; Sengurta, P. R. (1974): Influence of gravity on wave propagation in an elastic layer. *J. Acoust. Soc. Am.*, vol. 55, pp. 919–921.

De, S. N.; Sengupta; P. R. (1976): Surface waves under the influence of gravity. *Gerlands Beitr. Geophys.* (Leipzig), vol. 85, pp. 311–318.

Destrade, M. (2004): Surface waves in rotating rhombic crystal. *Proceeding of the Royal Society of London.* Series A, vol. 460, pp. 653–665.

Dhaliwal, R.; Sherief, H. H. (1980): Generalized thermoelasticity for anisotropic media. *Q. J. Appl. Math.*, vol. 33, pp. 1–8.

Green, A. E.; Laws, N. (1972): On the entropy production inequality. *Arch. Rat. Mech. Anal.*, vol. 45, pp. 47–53.

Green, A. E.; Lindsay, K. A. (1972): Thermoelasticity. *J. Elasticity*, vol. 2, pp. 1–7.

Green, A. E.; Naghdi, P. M. (1993): Thermoelasticity without energy dissipation. *J. Elasticity*, vol. 31, pp. 189–208.

Lord, H. W.; Shulman, Y. (1967): A generalized dynamical theory of thermoelasticity. *J. Mech. Phys. Sol.*, vol. 15, pp. 299–309.

Love, A. E. H. (1911): *Some problems of Geodynamics*. New York, Dover.

Othman, M. I. A. (2004): Effect of rotation on plane waves in generalized thermoelasticity with two relaxation times. *Int. J. Sol. & Str.*, vol. 41, pp. 2939–2956.

Othman, M. I. A. (2005): Effect of rotation in case of 2-D problems of the generalized thermoelasticity with thermal relaxation. *Mech. & Mechanical Eng.*, vol. 9, pp. 115–130.

Othman, M. I. A.; Song, Y. Q. (2009): The effect of rotation on 2-D thermal shock problems for a generalized magneto-thermoelasticity half space under three theories. *Mult. Model Math. & Str.*, vol. 5, no. 1, pp. 43–58.

Othman, M. I. A.; Said, S. M. (2012): The effect of rotation on two dimensional problem of a fiber-reinforced thermoelastic with one relaxation time. *Int. J. Thermophysics*, vol. 33, pp. 160–171.

Othman, M. I. A.; Lotfy, Kh. (2013): The effect of magnetic field and rotation of the 2-D problem of a fiber-reinforced thermoelastic under the theories with influence of gravity. *Mech. Mater.*, vol. 60, pp. 129–143.

Othman, M. I. A.; Hasona, W. M.; Abd-Elaziz, E. M. (2014): Effect of rotation on micropolar generalized thermoelasticity with two temperature using a dual-phase-lag model. *Can. J. Phys.*, vol. 92, no. 2, pp. 149–158.

Schoenberg, M.; Censor, D. (1973): Elastic waves in rotating media. *Q. J. Appl. Maths.*, vol. 31, pp. 115–125.

Singh, J.; Tomar, S. K. (2011): Plane waves in a rotating generalized thermoelastic solid with voids. *Int. J. Eng. Sci. and Tech.*, vol. 3, pp. 34–41.

Sharma, J. N.; Thakur, D. (2006): Effect of rotation on Rayleigh-Lamb waves in magneto-thermoelastic media. *J. Sound & Vibration*, vol. 296, pp. 871–887.

Sharma, J. N.; Walia, V.; Gupta, S. K. (2008): Effect of rotation and thermal relaxation on Rayleigh waves in piezo-thermoelastic half space. *Int. J. Mech.*, vol. 50, pp. 433–444.

Tzou, D. Y. (1995a): A unified approach for heat conduction from macro-to micro-scales. *J. Heat Transfer*, vol. 117, pp. 8–16.

Tzou, D. Y (1995b): Experimental support for the lagging behavior in heat propagation. *J. Thermophysics and Heat Transfer*, vol. 9, pp. 686–693.

Tzou, D. Y. (1996): *Macro-to micro-scale heat transfer: The lagging behavior*. 1st Edition, Taylor & Francis, Washington.