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Prediction of Compressive Strength of Various SCC Mixes Using Relevance Vector Machine

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Abstract: This paper discusses the applicability of relevance vector machine (RVM) based regression to predict the compressive strength of various self compacting concrete (SCC) mixes. Compressive strength data various SCC mixes has been consolidated by considering the effect of water cement ratio, water binder ratio and steel fibres. Relevance vector machine (RVM) is a machine learning technique that uses Bayesian inference to obtain parsimonious solutions for regression and classification. The RVM has an identical functional form to the support vector machine, but provides probabilistic classification and regression. RVM is based on a Bayesian formulation of a linear model with an appropriate prior that results in a sparse representation. Compressive strength model has been developed by using MATLAB software for training and prediction. About 75% of the data has been used for development of model and 30% of the data is used for validation. The predicted compressive strength for SCC mixes is found to be in very good agreement with those of the corresponding experimental observations available in the literature.

Keywords: Relevance Vector Machine, Self-compacting concrete, Compressive strength, Variance.

1 Introduction

Concrete has been one of the most commonly used construction materials in the world. One of the major problems civil engineers face today is concerned with preservation, maintenance and retrofitting of structures. It is well know that the self compacting concrete is developed in view of free flow of concrete without segregation where reinforcement is congested. The Self compacting concrete (SCC) is a concrete which has the ability to flow by its own weight and achieve good compaction with no external vibration. In addition, SCC is found to have resistance to segregation and bleeding because of its cohesive properties [Okamura and Ouchi (2003)]. The raw material selection is an important aspect of the mix design process for SCC, since it influences significantly the stability as well as the cost of the mix, which are two primarily elements in the successful use of SCC.

There is no standard method for SCC mix design, but many educational institutions, precast and contracting companies and admixture ready-mix have developed their own mix proportioning methods for SCC mix design. Mix designs generally employ volume based

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procedure as one of the key parameters for design. It may be due to filling of voids between the aggregate particles. Some studies were reported in the literature on the use of ingredients in optimized way [European Project Group (2005)]. However, any SCC mix must satisfy the criteria on filling ability, passing ability and segregation resistance. The base for SCC mix design is the general method developed by the University of Tokyo and since then, many attempts were made to modify this method to suit local conditions or specific requirements [Hodws, Sheinn, Ng et al. (2001)]. Broadly, there are some thumb rules, rational methods and EFNARC guidelines for proportioning and design of SCC mix [European Project Group 2005; Collepardi (2006); Okamura and Ozawa (1995)].

In view of difficulty in conducting experiments several times and to reduce time and effort, some times, analytical models to predict the required data will be very much useful. There are several advanced statistical models such as Artificial Neural Network, Gaussian regression process, least squares support vector machine, relevance vector machine, extreme learning machine and multivariate adaptive regression splines to predict the response of the structural components or concrete mixes [Yuvaraj, Murthy, Iyer et al. (2013a); Yuvaraj, Murthy, Iyer et al. (2013b); Yuvaraj, Murthy, Iyer et al. (2014a); Yuvaraj, Murthy, Iyer et al. (2014b); Shantaram, Shah, Samui et al. (2014); Shah, Shah, Samui et al. (2014); Dutta, Murthy, Kim et al. (2017); Kaur and Kaur (2017)]. In the present investigation, it is proposed to employ relevance vector machine to predict the compressive strength of various SCC mixes.

Tipping [Tipping (2001)] proposed a model, namely, relevance vector machine (RVM) which has additional advantages than the base model of support vector machine (SVM). In SVM, the target function minimises a measure of error on the training set and simultaneously maximises the 'margin' between the two classes (in the feature space implicitly defined by the kernel). In order to avoid over fitting, this is an effective mechanism [Tipping (2001)]. Though there are good predictions of SVM, it was found that there are several limitations and demerits [Tipping (2000); Caesarendra, Widodo and Yang (2009)]. RVM is a special case of a sparse kernel model, which consists of a Bayesian treatment of a generalized linear model of identical functional form as in the case of support vector machine (SVM). RVM differs from SVM in the case of solution, which is based on probabilistic interpretation of its output [Wei, Yang, Nishikawa et al. (2005)]. RVM evades the complexity by producing simple models that have both a structure and a parameterization process together in relation to the data type. RVM is a probabilistic based approach, introduces a prior over the model weights governed by a set of hyperparameters associated with each weight, whose most probable values are iteratively estimated from the data. The important feature of RVM is that it requires less kernel functions. RVM based regression and classifications are popular in many fields [Han, Cluckie, Kang et al. (2002); Wei, Yang, Nishikawa (2005); Das and Samui (2008); Widodo, Kim, Son et al. (2009); Wang and Duanmu (2009); Liu and Xu (2011); Yuvaraj, Murthy, Iyer et al. (2014b)]. From the above literature, it was found that RVM based models for prediction of data in the field of structural engineering is limited.

In the present study, compressive strength values for SCC mixes are predicted by developing a regression model based on relevance vector machine approach.

2 Compressive strength of various SCC mixes

For various SCC mixes, compressive strength data available in the literature has been compiled and the data is presented in Tab. 1. Compressive strength is compiled against water to binder ratio and water to cement ratio.

Table 1: Compressive strength of various SCC mixes

Reference	Water to binder ratio	w/c ratio	Comp. strength, MPa
Nikbin et al. (2014)	0.23	0.35	75.5
	0.26	0.4	69.2
	0.29	0.45	58.8
	0.32	0.5	54.8
	0.34	0.55	46.0
	0.37	0.6	42.6
	0.39	0.65	35.5
	0.41	0.7	26.0
	0.37	0.47	48.7
	0.31	0.47	52.9
	0.26	0.47	60.6
	0.23	0.47	67.2
	0.48	0.6	37.9
	0.40	0.6	39.7
	0.34	0.6	44.6
	0.30	0.6	45.4
Burak(2007)	0.37	0.6	67.32
	0.33	0.54	59.64
	0.29	0.48	56.16
	0.25	0.42	51.36
Leeman (2005)	0.22	0.37	43.56
	0.33	0.42	61.8
	0.35	0.45	63.1
	0.36	0.46	60.8
	0.40	0.51	52.0
	0.46	0.59	48.7
	0.40	0.53	60.5
	0.43	0.56	51.6

	0.36	0.47	64.0
	0.34	0.44	66.9
Parra (2011)	0.37	0.65	32.58
	0.34	0.55	39.83
	0.34	0.55	48.46
	0.31	0.45	62.67
Dinakar (2008)	0.41	2.71	14.64
	0.34	1.13	34.9
	0.33	1.1	34.83
	0.34	0.68	57.9
	0.34	0.68	50.07
	0.31	0.44	77.08
	0.36	0.51	71.62
	0.29	0.322	86.41
Girish (2007)	0.325	0.487	56.3
	0.33	0.56	47.6
	0.34	0.65	43.8
	0.35	0.78	37.0
	0.356	0.87	31.0
	0.361	0.975	26.5
	0.365	0.5	73.3
	0.371	0.5	66.0
	0.373	0.5	67.3
	0.376	0.5	71.8
	0.377	0.5	65.0
Mounir (2014)	0.4		30.3
Brouwers (2005)	0.34	0.55	51.2
	0.36	0.55	50.7
	0.37	0.55	53.6
Marco (2017)	0.32	0.5	72.22
Rahmat (2012)	0.32	0.32	45
	0.32	0.34	49.5
	0.32	0.36	54
	0.32	0.38	52
	0.32	0.40	47

	0.38	0.38	41.5
	0.38	0.40	45.5
	0.38	0.42	49.5
	0.38	0.45	49.5
	0.38	0.475	45.5
	0.45	0.45	31
	0.45	0.47	33
	0.45	0.50	37
	0.45	0.53	38
	0.45	0.56	35
Subhan (2017)		0.396	44.44
Alireza (2014)	0.48	0.48	47.64
Sherif (2016)	0.36	0.45	72.1
Thiago (2016)	0.34	0.478	43.7
	0.40	0.573	44.4
Valeria (2011)	0.35	0.4	54
Farhad (2013)	0.52	1.3	39.96
Abbas (2013)	0.32	0.49	35.4
Krishnarao and Ravindra (2010)	0.31	0.62	43.51

3 Relevance vector machine

This section provides a brief description about RVM. Full details about model can be found in Tipping [Tipping (2000, 2001)]. RVM is a specialization of a spares Bayesian model which uses the same data dependent kernel basis [Tipping (2001)]. The key feature of RVM is that the inferred predictors are exceedingly sparse in that they contain relatively few ‘‘relevance vectors”, as well as offers a generalized performance.

RVM starts with the concept of linear models, i.e. the function of y(x) to be predicted at some arbitrary point x given a set of (typically noisy) measurements of the function $t=(t_1, y, t_N)$ and with some training points $x=(x_1, y, x_N)$:

$$t_i = y(x_i) + \varepsilon_i \tag{1}$$

where ε_i is the noise component of the measurement with mean 0 and variance σ^2. With a linear model assumption, the unknown function y(x) is a linear combination of some known basis function i.e.

$$y(x) = \sum_{i=1}^{M} w_i \varphi_i(x) \tag{2}$$

where, $w_i=(w_1,\ldots,w_M)$ = a vector consisting of the linear combination weights

$y(x)$ = the output which is a linearly-weighted sum of M, generally nonlinear and fixed basis functions $\varphi_i(x) = (\varphi_1(x), \varphi_2(x), \ldots\ldots\ldots, \varphi_M(x))^T$.

Analysis of functions shown in Eq. (2) is available in Tipping [Tipping (2001)]. During the development of model, the majority of parameters are automatically set to zero in view of good predictions [Tipping (2000, 2001)].

$$t = \Phi w + \varepsilon \tag{3}$$

where, Φ is an NxM design matrix, whose i^{th} column is formed with the values of basis function $\Phi_i(x)$ at all the training points

$\varepsilon_i = (\varepsilon_1,\ldots, \varepsilon_N)$, the noise vector.

As a supervised learning, RVM starts with a set of data input $\{x_n\}_n^N = 1$ and their corresponding target vector $\{t_n\}_n^N = 1$. The basic aim of the 'training' set is to learn a model of the dependency of the target vectors on the inputs to make accurate prediction of t for previously unseen value of x.

For the case of support vector machine (SVM), the prediction is made based on a function of the form

$$y(x) = \sum_{i=1}^{N} w_i K(x, x_i) + w_0 \tag{4}$$

where, $w_i = (w_1, w_2,\ldots, w_N)$ is weight vectors

$K(x,x_i)$ = a kernel function and w_0 is the bias

In the present study, Radial basis kernel function is used and the related equation is given below

$$K(x_i, x) = \exp\left\{-\frac{(x_i - x)^T (x_i - x)}{2\sigma^2}\right\} \tag{5}$$

where, x_i and x are the training and test patterns, respectively.

d = a dimension of the input vector, σ = width of the basis function.

For a given a dataset of input-target pairs $\{x_n, t_n\}_n^N = 1$, it is assumed that $p(t|x)$ is Gaussian $N(t|y(x), \sigma^2)$. The mean of this distribution for a given x was modelled by $y(x)$ as mentioned in Eq. (4). The likelihood of dataset can be expressed as

$$p(t|w, \sigma^2) = (2\pi\sigma^2)^{-N/2} \exp\left\{-\frac{1}{2\sigma^2}\|t - \Phi w\|^2\right\} \tag{6}$$

Where, $t_i = (t_1 \ldots, t_N)^T$, $\omega_i = (\omega_0, \ldots, \omega_N)$ and

$$\Phi^T = \begin{bmatrix} 1 & K(x_1, x_1) & K(x_1, x_2) & \cdots & K(x_1, x_n) \\ 1 & K(x_2, x_1) & K(x_2, x_2) & \cdots & K(x_2, x_n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & K(x_n, x_1) & K(x_n, x_2) & \cdots & K(x_n, x_n) \end{bmatrix}$$

Where, $K(x_i, x_n)$ is the kernel function.

It was mentioned in the literature [Tipping (2001)] that the maximum likelihood estimation of w and σ^2 by using Eq. (6) in general results in overfitting. Tipping [Tipping (2001)] recommended by imposing prior constrains on the parameters w by adding a complexity to the likelihood or error function. This is a priori information that controls the generalization ability of the learning process. Generally, new higher-level parameters are preferred to constrain an explicit zero-mean Gaussian prior probability distribution to the weights

$$p(w|\alpha) = \prod_{i=0}^{N} N\left(w_i | 0, \alpha_i^{-1}\right) \tag{7a}$$

where α is a vector of (N+1) hyperparameters which controls the deviation of weight [Caesarendr (2010)]. By using Bayes' rule, the posterior over all unknowns can be computed, given the defined non-informative prior-distributions. In order to complete the specification of the prior-distribution, one must define hyperpriors over α and noise variance σ^2. These quantities are typical scale parameters and suitable prior are Gamma Distributions [Tipping (2000)]

$$p(\alpha) = \prod_{i=0}^{N} \mathrm{Gamma}(\alpha_i | a, b), \tag{7b}$$

$$p(\beta) = \prod_{i=0}^{N} \mathrm{Gamma}(\beta | c, d) \tag{7c}$$

Where, $\beta = \sigma^{-2}$.

Hence, for α and σ, the distribution is gamma distribution and for w, it is normal distribution and after the prior-distributions, Bayes rule is applied.

$$p\left(w, \alpha, \sigma^2 \middle| t\right) = \frac{p\left(t | w, \alpha, \sigma^2\right) p\left(w, \alpha, \sigma^2\right)}{p(t)} \tag{8a}$$

Then, for a given a new test point (X_*), predictions were performed for the corresponding target (t_*), in terms of the predictive distribution :

$$p(t_* | t) = \int p\left(t_* | w, \alpha, \sigma^2\right) p\left(w, \alpha, \sigma^2 \middle| t\right) dw\, d\alpha\, d\sigma^2 \tag{8b}$$

The solution for the posterior in Eq. 8(a) is difficult due to normalization of integral $p(t) = \int p\left(t | w, \alpha, \sigma^2\right) p\left(w, \alpha, \sigma^2 \middle| t\right) dw\ \ d\alpha\ \ d\sigma^2$.

The preferred solution is decomposition of the posterior as shown in Eq. (9)

$$p\left(w,\alpha,\sigma^2 \middle| t\right) = p\left(w \middle| t,\alpha,\sigma^2\right) p\left(\alpha,\sigma^2 \middle| t\right) \tag{9}$$

It can be noted that one can compute analytically the posterior distribution over the weights because its normalization integral is convolution of gaussians [Tipping (2000)]. Hence, to obtain a solution, Eq. (10) shows the posterior distribution of weights

$$p\left(w \middle| t,\alpha,\sigma^2\right) = \frac{p\left(t \middle| w,\sigma^2\right) p(w,\alpha)}{p(t|\alpha,\sigma^2)} \tag{10}$$

the posterior over the weights is then obtained from Bayes rule

$$p\left(w \middle| t,\alpha,\sigma^2\right) = (2\pi)^{-(N+1)/2} |\Sigma|^{-1/2} \exp\left\{-\frac{1}{2}(w-\mu)^T \Sigma^{-1}(w-\mu)\right\} \tag{11}$$

The analytical solution for Eq. (11) in terms of the posterior covariance and mean are

$$\Sigma = \left(\sigma^{-2}\Phi^T\Phi + A\right)^{-1} \tag{12}$$

$$\mu = \sigma^{-2}\Sigma\Phi^T t \tag{13}$$

where, $A = (\alpha_0, \alpha_{1\ldots}\ \alpha_N)$.

It can be noted that σ^2 is also treated as a hyperparameter, which may be obtained from the data.

Therefore, machine learning process becomes a search for the hyperparameters posterior most probable,

i.e. maximization of $p\left(\alpha,\alpha_{\in_n}^2 \middle| y\right) \alpha p\left(y \middle| \alpha,\alpha_{\in_n}^2\right) p(\alpha) p\left(\alpha_{\in_n}^2\right)$ with respect to a α and σ^2. For uniform hyperpriors, it is necessary to maximize the term $p\left(y \middle| \alpha,\alpha_{\in_n}^2\right)$, which can computed and given by

$$\begin{aligned} p\left(y \middle| \alpha,\alpha_{\in_n}^2\right) &= \int p\left(y \middle| w,\alpha_{\in_n}^2\right) p(w|\alpha) dw \\ &= (2\pi)^{-1/2} \left|\alpha_{\in_n}^2 I + \Phi A^{-1}\Phi^T\right|^{1/2} \\ &\times \exp\left\{-\frac{1}{2} y^T \left(\alpha_{\in_n}^2 I + \Phi A^{-1}\Phi^T\right|\right)^{-1} y\right\} \end{aligned} \tag{14}$$

Tipping [Tipping (2000)] arrived at this approximation and found that this is effective after confirmation with several experiments. Bayesian models of Eq. (18a) refer to the marginal likelihood, and its maximization is known as the type II-maximum likelihood method [Ghosh and Mujumdar (2008)]. Hyperparameter estimation is generally performed with an iterative formula, namely, a gradient ascent on the objective function [Tipping (2000); Ghosh and Mujumdar (2008)]. Predictions for a new data were then made as per integration of the weights to arrive at the marginal likelihood for the hyperparameters. The predictive distribution for a given input vector, {x} can be estimated by using following equation. The predictions were made based on the posterior distribution over the weights, conditioned on the maximized most probable values of α and $\sigma^2_{\in_n}, \alpha_{MP}$ and σ^2_{MP} respectively.

$$p\left(y_*\middle|y,\alpha_{MP},\sigma^2_{MP}\right)=\int p\left(y_*\middle|w,\sigma^2_{MP}\right)p\left(w\middle|y,\alpha_{MP},\middle|\sigma^2_{MP}\right)dw \tag{15}$$

This can readily be computed as

$$p\left(y_*\middle|y,\alpha_{MP},\sigma^2_{MP}\right)=N\left(y_*\middle|t_*,\sigma^2_*\right) \tag{16}$$

$$t_*=\mu^T\Phi(x_*) \tag{17}$$

With $$\sigma^2_*=\sigma^2_{MP}+\Phi(x_*)^T\sum\Phi(x_*) \tag{18}$$

the outcome of the optimization involved in RVM (i.e. max of $p\left(y\middle|\alpha,\sigma^2_{\in_n}\right)$), is that many of α go to infinity such that 'w' will have only a few nonzero weights that can be considered as relevant vectors [Ghosh and Mujumdar (2008)]. The relevant vectors (RVs) can be viewed as counterparts of support vectors (SVs) in SVM. Thus , the developed model contains the benefits of SVM (sparsity and generalization) and in addition, provides estimates of uncertainty bounds in the predictions [Ghosh and Mujumdar (2008)].

4 RVM based analysis

For prediction of the compressive strength, RVM model has been developed. From the experimental studies (Tab. 2), it can be noted that the compressive strength is influenced by the water binder ratio and water cement ratio. These two parameters from the input vector and it can also be noted that the input vector has different quantitative limit as shown in Tab. 2. Hence, a normalization of the data has been performed before presenting the input patterns to statistical machine learning algorithm. Thus, Eq. (29) has been used for the linear normalization of the data to the data values between 0 and 1.

$$x_i^n=\frac{x_i^a-x_i^{\min}}{x_i^{\max}-x_i^{\min}} \tag{19}$$

where, x_i^a and x_i^n are i^{th} components of the input vector before and after normalization, respectively,

$x_i^{\max}$ and $x_i^{\min}$ are the maximum and minimum values of all the components of the input vector before the normalization.

Table 2: Training data set of various SCC mixes

Sl No.	Water to binder ratio	w/c ratio	Comp. strength, MPa
1	0.23	0.35	75.5
2	0.26	0.4	69.2
3	0.29	0.45	58.8
4	0.32	0.5	54.8
5	0.34	0.55	46.0
6	0.34	0.44	66.9
7	0.37	0.65	32.58
8	0.34	0.55	39.83
9	0.34	0.55	48.46
10	0.31	0.45	62.67
11	0.41	2.71	14.64
12	0.34	1.13	34.9
13	0.33	1.1	34.83
14	0.34	0.68	57.9
15	0.34	0.68	50.07
16	0.31	0.44	77.08
17	0.36	0.51	71.62
18	0.29	0.322	86.41
19	0.325	0.487	56.3
20	0.33	0.56	47.6
21	0.34	0.65	43.8
22	0.35	0.78	37.0
23	0.356	0.87	31.0
24	0.361	0.975	26.5
25	0.365	0.5	73.3
26	0.371	0.5	66.0
27	0.373	0.5	67.3
28	0.376	0.5	71.8
29	0.377	0.5	65.0
30	0.4		30.3
31	0.34	0.55	51.2
32	0.36	0.55	50.7

33	0.37	0.55	53.6
34	0.32	0.5	72.22
35	0.32	0.32	45
36	0.32	0.34	49.5
37	0.32	0.36	54
38	0.32	0.38	52
39	0.32	0.40	47
40	0.38	0.38	41.5
41	0.38	0.40	45.5
42	0.38	0.42	49.5
43	0.38	0.45	49.5
44	0.38	0.475	45.5
45	0.45	0.45	31
46	0.45	0.47	33
47	0.45	0.50	37
48	0.45	0.53	38
49	0.45	0.56	35
50		0.396	44.44
51	0.48	0.48	47.64
52	0.36	0.45	72.1
53	0.34	0.478	43.7
54	0.40	0.573	44.4
55	0.35	0.4	54
56	0.52	1.3	39.96
57	0.32	0.49	35.4
58	0.31	0.62	43.51

4.1 Development of RVM model

A Total of 82 data sets were collected from the literature for various SCC mixes. About 70% of data set is used for the development of RVM model and about 30% of the data set is used for testing and verification of the developed model. Testing and verification of the model is done by comparing the experimental compressive strength with the predicted compressive strength by using the RVM model. The key aspect of development of RVM model is the selection of kernel width which was done by using post modelling analysis [Caesarendr, Widodo, Yang et al. (2010)]. Post-modelling analysis of the training and testing R values is related to number of relevance vectors (NRV) involved in the model and their corresponding weights and variation in the kernel width. The value of σ is assumed initially as 0.13 and for the assumed valued of σ, the model is developed. Fig. 1 shows the schematic diagram of RVM model. The developed model gives the NRVs used and their corresponding weights (w_i). The quality of the developed model is assessed based on the coefficient of correlation (R) value which is determined using the Eq. (20).

$$R = \frac{\sum_{i=1}^{n}\left(E_{ai} - \overline{E}_a\right)\left(E_{pi} - \overline{E}_p\right)}{\sqrt{\sum_{i=1}^{n}\left(E_{ai} - \overline{E}_a\right)}\sqrt{\sum_{i=1}^{n}\left(E_{pi} - \overline{E}_p\right)}} \tag{20}$$

where, E_{ai} and E_{pi} are the actual and predicted values, respectively

$\overline{E}_a$ and $\overline{E}_p$ are mean of actual and predicted E values corresponding to n patterns. In each iteration, R value is computed and the model is finalized when the R value is closer to one.

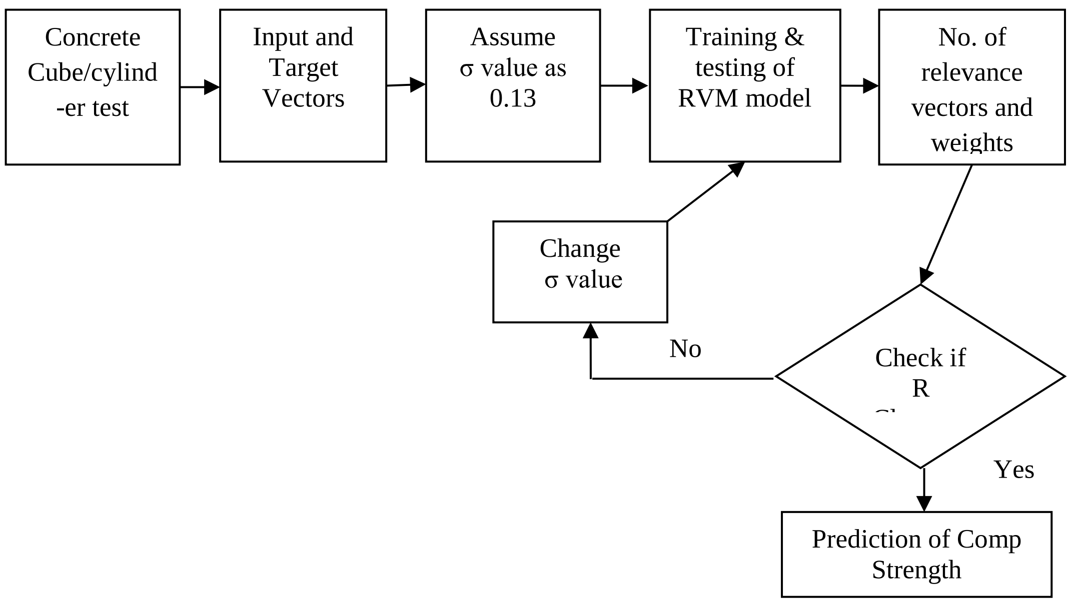

Figure 1: Schematic diagram-development of RVM models

It is observed that the testing R value achieved its maximum at kernel widths shown in Tab. 3 for the corresponding models, involving minimum number of relevance vectors. The training and testing R values obtained for models are presented in Tab. 3.

Table 3: Performance of developed RVM models

Parameters	**Coefficient of correlation (R)**		**width (σ)**	**No. of RVs used out of total 58 dataset**	**No of RVs (% of training data set)**
	Training	**Testing**			
Model I (Comp. Strength)	0.994	0.992	0.13	36	62.06

Tab. 4 shows the weights for RVM model.

Table 4: Values of weights (w_i) for RVM models

i =1,2…58	w_i	i =1,2…58	w_i
1	0	30	0
2	0.0652	31	0
3	0	32	0.0253
4	0	33	0.0252
5	0	34	0.0841
6	0.0342	35	0.0345
7	0	36	0.02785
8	0.1346	37	0
9	0	38	0.06541
10	0.1650	39	0.0982
11	0.1401	40	0
12	0.0565	41	0.0653
13	0.1543	42	0.08761
14	0.0922	43	0
15	0.1065	44	0
16	0.1766	45	0.05415
17	0.1366	46	0
18	0	47	0.60243
19	0.1096	48	0
20	0.0743	49	0
21	0.0789	50	0.1421
22	0	51	0
23	0.0763	52	0.0356
24	0.0542	53	0.0976

25	0	54	0
26	0	55	0
27	0.2431	56	0.5263
28	0.0842	57	0.0904
29	0.0983	58	0.0983

From Eq. (14), (15) and Tab. 4(b) with w_o as zero, the following equation has been obtained from the developed RVM model.

$$y = \text{Comp Str} = \sum_{i=1}^{58} w_i \exp\left\{-\frac{(x_i - x)^T (x_i - x)}{0.034}\right\} \tag{21}$$

The values of weights, w_i for all the training data sets are available in Tab. 4.

Variance for training and testing data set for the developed model are plotted and shown in Fig. 2 and Fig. 3.

Figure 2: Variance of training data set for compressive strength

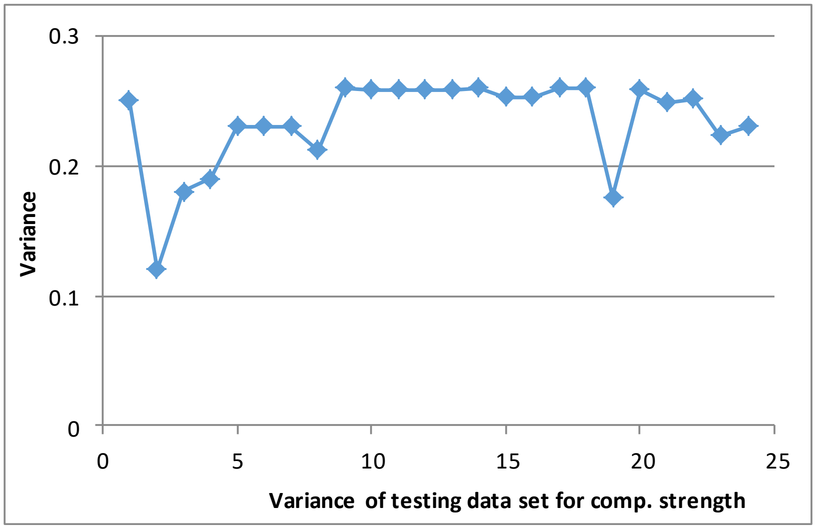

Figure 3: Variance of testing data set for compressive strength

The developed RVM model has been verified with the remaining 24 data sets and the results are shown in Tab. 5.

Table 5: Predicted and experimental compressive strength

Sl No.	Water to binder ratio	Water to cement ratio	Compressive strength, MPa	
			Exptl.	Predicted
1	0.37	0.6	42.6	40.1
2	0.39	0.65	35.5	37.2
3	0.41	0.7	26.0	27.3
4	0.37	0.47	48.7	45.8
5	0.31	0.47	52.9	47.9
6	0.26	0.47	60.6	56.87
7	0.23	0.47	67.2	62.43
8	0.48	0.6	37.9	39.6
9	0.40	0.6	39.7	35.76
10	0.34	0.6	44.6	41.5
11	0.30	0.6	45.4	42.87
12	0.37	0.6	67.32	62.34
13	0.33	0.54	59.64	53.21
14	0.29	0.48	56.16	52.45
15	0.25	0.42	51.36	47.42
16	0.22	0.37	43.56	40.76

17	0.33	0.42	61.8	57.65
18	0.35	0.45	63.1	59.54
19	0.36	0.46	60.8	58.21
20	0.40	0.51	52.0	49.64
21	0.46	0.59	48.7	44.78
22	0.40	0.53	60.5	56.21
23	0.43	0.56	51.6	48.31
24	0.36	0.47	64.0	59.21

The normalised output vector obtained from the RVM model is converted back to original value by using the equation below.

$$x_i^a = x_i^n\left(x_i^{\max} - x_i^{\min}\right) + x_i^{\min} \tag{22}$$

where, x_i^n is the normalized result obtained after the test for the ith component.

x_i^a is the actual result obtained for ith componenet, and $x_i^{\max}$ and $x_i^{\min}$ are the maximum and minimum values of all the components of the corresponding input vector before the normalization.

From Tab. 5, it can be observed that the predicted compressive strength is in very good agreement with the corresponding experimental observations. Fig. 4 shows the comparison plot of predicted and the corresponding experimental compressive strength. From Tab. 4 and Fig. 4, it can be concluded that the developed model is robust and reliable.

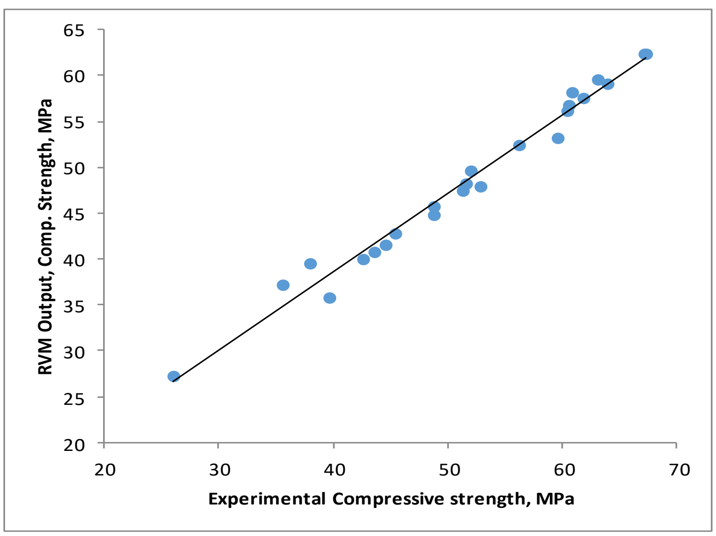

Figure 4: Predicted and experimental compressive strength

5 Summary and conclusions

Relevance vector machine, one of the advanced statistical models was developed to predict a compressive strength for various SCC mixes. The input parameters are water cement ratio and water binder ratio. Compressive strength data available in the literature for various SCC mixes has been consolidated to develop and test the model. MATLAB software has been used for training and prediction. About 75% of the data has been used for development of model and 30% of the data is used validation. The predicted compressive strength for SCC mixes is found to be in very good agreement with those of the corresponding experimental observations available in the literature. The developed equation for prediction of compressive strength can be used for all practical purposes. The R value for the developed model is found to be closer to 1 indicating better predictability of the models. From the overall study, it can be concluded that the developed RVM model is found to be robust and reliable.

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