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Joint Bearing Mechanism of Coal Pillar and Backfilling Body in Roadway Backfilling Mining Technology

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Abstract: In the traditional mining technology, the coal resources trapped beneath surface buildings, railways, and water bodies cannot be mined massively, thereby causing the lower coal recovery and dynamic disasters. In order to solve the aforementioned problems, the roadway backfilling mining technology is developed and the joint bearing mechanism of coal pillar and backfilling body is presented in this paper. The mechanical model of bearing system of coal pillar and backfilling body is established, by analyzing the basic characteristics of overlying strata deformation in roadway backfilling mining technology. According to the Ritz method in energy variation principle, the elastic solution expression of coal pillar deformation is deduced in roadway backfilling mining technology. Based on elastic-viscoelastic correspondence principle, combining with the burgers rheological constitutive model and Laplace transform theory, the viscoelastic solution expression of coal pillar deformation is obtained in roadway backfilling mining technology. By analyzing the compressive mechanical property of backfilling body, the time formula required for coal pillar and backfilling body to play the joint bearing function in roadway backfilling mining technology is obtained. The example analysis indicates that the time is 140 days. The results can be treated as an important basis for theoretical research and process design in roadway backfilling mining technology.

Keywords: Roadway backfilling mining technology, coal pillar, backfilling body, joint bearing mechanism, energy variation principle.

1 Introduction

In order to solve the increasingly serious problem of mining under building, railway and water body at present [Zhou, Guo, Cha et al. (2008); Miao (2012); Cao, Du, Xu et al. (2017)], and reasonably dispose the wastes stacked on the mine like coal gangue and coal ash [Zhang, Zhang, Zhao et al. (2007); Zhang, Miao and Guo (2015); Xin and Ji (2016)], the roadway backfilling mining technology, a kind of safe and high-efficiency coal mining technology, is developed specially and has already obtained good popularizing and application [Qian (2010); Ding, Zhou, Xu et al. (2013); Teng, Wang, Gao et al. (2016)]. In roadway backfilling mining technology, the roadheader is employed for coal cutting in roadway driving work, which conveys the solid backfilling materials, (such as ground

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gangue, coal ash, loess and aeolian sand) to the mined roadway underground after the materials have been processed on the ground [Miao, Zhang and Guo (2010); Xue, Cao, Cai et al. (2017)], for the purpose of mining three-under coal (coal trapped under buildings, waters-bodies and railways) or the corner residual coal resource, controlling roof in the gob, preventing surface subsidence, and disposing solid wastes [Teng, Wang, Gao et al. (2016); Guo, Li and Liu (2017)]. Therefore, the roadway backfilling mining technology can be adopted for mining three-under coal and corner residual coal in certain conditions in small and medium-size coal mines [Ju, Li, Zhang et al. (2014); Cao, Du, Li et al. (2017)].

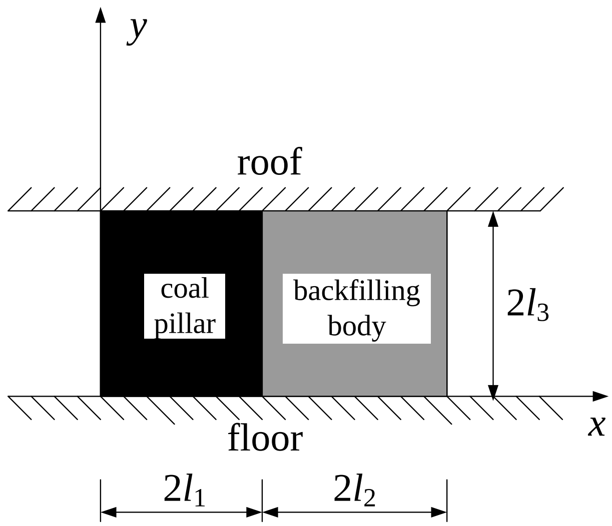
Advances in the roadway backfilling mining technology over the last ten years have seen various research methods and results [Hu, Zhang, Gao et al. (2006); Qian, Miao and Xu (2008); Miao, Huang and Ju (2012); Jia, Qiao and Jiang (2016); Xue, Ranjith, Gao et al. (2017)]. In order to solve the technical problems of mining strip extraction coal-pillar and waste disposal, Zhang et al. [Zhang, Miao, Mao et al. (2007)] proposed the waste substitution extraction by the roadway backfilling mining technology, which determines the layout with two waste filling roadway (the width and height are 4.0 m and 5.0 m) placed in middle of strip extraction coal-pillar (the width is 4.0 m), by reference to the numerical analysis of the stability of substitution extraction coal-pillar and vertical stress distribution of basic roof in strip and substitution extraction. Li et al. [Li, Mao, Bu et al. (2008)] established the mechanical model of waste-filling in roadway and obtained the elastic displacement solution of coal pillar; based on elastic-viscoelastic correspondence principle, viscoelastic displacement solution of coal pillar was found out by importing Maxwell model, therefore, the rule of the compression and the lateral deformation of coal pillar change with time and backfilling material characteristics was achieved. On the basis of strip mining characteristics, Yu et al. [Yu and Wang (2011)] described the exchanging technology of gangue backfill and analyzed the characteristics of quadratic strata movement, according to subject of strata movement and quadratic stability of coal-pillar under three circumstances exchanged by gangue backfill; Besides, the analytic formula of the bearing body of “bearing rock strata and gangue filling support body” was deduced by means of the elastoplastic theory and based on the actual force and ultimate strength of the gangue filling support body. Deng et al. [Deng, Zhang, Zhou et al. (2014)] proposed the longwall-roadway cemented backfilling mining technology to solve the engineering technical problems in extra-thick coal seam, such as low recovery rate and the difficulty in controlling the mining surface subsidence and overlying strata movement and deformation; meanwhile, the layout of filling mining system, main equipment and technics was explained systematically, based on the principle of longwall-roadway cemented backfilling mining technology in extra thick coal seam. Zhou et al. [Zhou, Li, Zhang et al. (2016)] proposed a roadway backfill method during longwall mining, so as to improving coal recovery and preventing the geohazards in room and pillar mining method in Chinese western coal mines; besides, the reasonable ratio of backfill materials for the driving roadway during longwall mining was determined by testing mechanical properties of backfill materials, and the roadway layout and the backfill mining technique were introduced; meanwhile, the distance between roadways and a driving and filling sequence for multiple-roadway driving was designed, based on the effects of the abutment stress from a single roadway driving task. In order to guard against coal pillar instability, mine earthquake, surface subsidence, vegetation deterioration, and other environmental problems in traditional room mining technology, Zhang et al. [Zhang, Sun, Zhou et al. (2016)] proposed the roadway backfill coal mining method and presented the technical principle and key equipment; further, coal pillar stress, plastic zone change, and surface deformation of the roadway backfill coal mining schemes were studied with the FLAC3D numerical simulation software, and a reasonable mining scheme of "mining 7 m and leaving 3 m" was determined. For the sake of dealing with the low coal recovery and environmental problems caused by the traditional coal mining technology in Chinese western ecological fragile coal mining area, Sun et al. [Sun, Zhang, Yin et al. (2017)] proposed the longwall roadway backfill coal mining method and established the mechanical model of coal mining in stope, on the basis of the strata movement characteristics in the roadway backfill coal mining; then, the equation of roof deflection curve with corresponding mechanical analysis and the calculation formulas of coal pillar were deduced.

It can be seen that the current researches mainly focus on the stability of coal pillar and technical practice in roadway backfilling mining technology. Nevertheless, the joint bearing mechanism of coal pillar and backfilling body is the key to success of roadway backfilling mining technology, which should be research systematically to provide an important basis for theoretical research and process design in roadway backfilling mining technology. In this paper, the elastic and viscoelastic solution expression of coal pillar deformation is obtained by establishing the mechanical model of bearing system of coal pillar and backfilling body, on the basis of basic characteristics of overlying strata deformation in roadway backfilling mining technology. Then, the time formula required for coal pillar and backfilling body to play the joint bearing function is obtained and the joint bearing mechanism of coal pillar and backfilling body in roadway backfilling mining technology is achieved, by analyzing the compressive mechanical property of backfilling body.

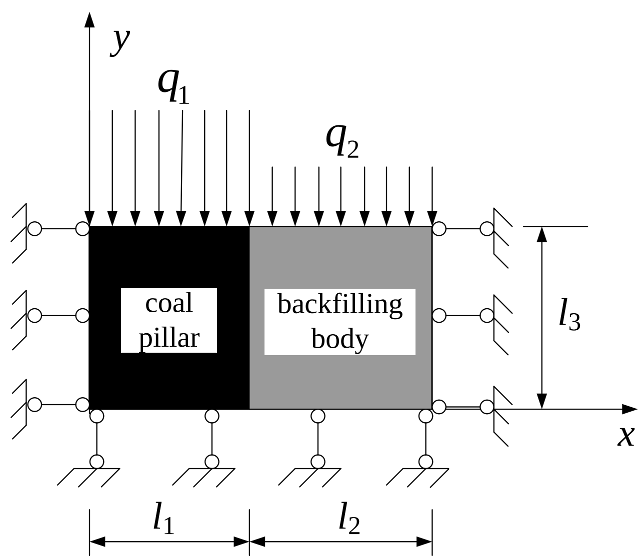
2 Mechanical model of bearing system of roadway backfilling mining

The roadway backfilling mining technology is an effective approach to mining the corner residual coal resource and the quality coal resource under buildings, railways, rivers, which is an important part of coal green mining system. The key equipment of roadway backfilling mining technology includes the bulldozer, continuous miner, belt stage-loader mechanism, and high-speed conveyer (fully hydraulically driven). The high-speed conveyer is entirely mounted on a crawler unit. The basic process in the roadway backfilling mining is as follows: firstly, move out related mining equipment after finishing one cycle of roadway mining with continuous miner; secondly, convey backfilling material to the mined roadway with high speed conveyer under coordination of belt stage-loader mechanism; finally, compact the backfilling materials (such as ground gangue, coal ash, loess and aeolian sand) gradually in mined roadway with bulldozer.

The control process of overlying strata movement in roadway backfilling mining technology can be divided into three stages, namely coal pillar bearing stage, backfilling body dynamic compaction stage and coal pillar and backfilling body combined load bearing stage. In the coal pillar bearing stage, the overburden load is mainly supported by coal pillars on both sides of the roadway; In backfilling body dynamic compaction stage, the backfilling body gradually restores bearing capacity under the compaction action of surrounding rock of roadway, which begins to play a role of bearing upper strata; In combined load bearing stage, the coal pillar and backfilling body play the joint bearing effect on the overlying strata, and achieve long time stability.



(a) Physical model



(b) Mechanical model

Figure 1: Mechanical model of bearing system of roadway backfilling mining

In the roadway backfilling mining technology, collar pillar is a long cylindrical body, and the support conditions do not vary in length direction; besides, the volume force and surface force parallel to the cross section do not vary in length direction. Meanwhile, the displacement vector in coal pillar is parallel to the cross section, and the axial displacement does not occur. Similarly, this is the same to the backfilling body in the roadway backfilling mining. Therefore, the coupling mechanism of coal pillar and backfilling body in roadway backfilling mining can be attributed to a plane strain problem. In order to analyze the mechanical feature of coal pillar and backfilling body in roadway backfilling mining, the assumptions below are given: the coal pillar and backfilling body are both the continuous, homogeneous and isotropic elastic body; the weak surface does not exist in coal pillar and backfilling body; the full thickness mining method is adopted in roadway backfilling mining, thus the cohesive forces (such as the friction, pressure) exists in the interface between coal pillar, backfilling body and roof, floor; the backfilling body occupies the entire roadway.

The physical model of roadway backfilling mining is shown in Fig. 1(a), which is the overburden movement supported by coal pillar and backfilling body. The width of coal pillar is $2l_1$, and the width and height of backfilling roadway is $2l_2$ and $2l_3$, respectively. In consideration of the symmetry of boundary conditions and external loads, the mechanical model is established, by taking a quarter of right-upper coal pillar and a quarter of left-upper coal pillar, which is shown in Fig. 1(b).

According to the symmetry of physical model of roadway backfilling mining, the right boundary of backfilling body and left boundary of coal pillar are both the horizontal simple support constraint; the lower boundary of coal pillar and backfilling body is the vertical simple constraint; the upper boundary condition of coal pillar and backfilling body is the chain constraint in the horizontal direction. Besides, the displacement is continuous in the interface ($x = l_1$) between coal pillar and backfilling body. Therefore, the displacement boundary condition of mechanical model in roadway backfilling mining is obtained,

$$\begin{cases} u\big|_{x=0} = u\big|_{x=l_1+l_2} = u\big|_{y=l_3} = 0 \\ v\big|_{y=0} = 0 \\ u\big|_{x=l_1^-} = u\big|_{x=l_1^+} \\ v\big|_{x=l_1^-} = v\big|_{x=l_1^+} \end{cases} \tag{1}$$

In roadway backfilling mining, the uniform overburden load supported by coal pillar and backfilling body is q_1 and q_2, respectively; meanwhile, the axial stress and shear stress in coal pillar and backfilling body is continuous. Therefore, the stress boundary condition of mechanical model in roadway backfilling mining is obtained,

$$\begin{cases} \sigma_y\big|_{x=0\to l_1,\ y=0} = -q_1 \\ \sigma_y\big|_{x=l_1\to l_1+l_2,\ y=l_2} = -q_2 \\ \sigma_x\big|_{x=l_1^-} = \sigma_x\big|_{x=l_1^+} \\ \tau_{xy}\big|_{x=l_1^-} = \tau_{xy}\big|_{x=l_1^+} \end{cases} \tag{2}$$

In formula (1), u and v represent the horizontal displacement and vertical displacement, respectively; in formula (2), q_1 and q_2 represent the uniform load supported by the coal pillar and backfilling body, respectively.

In roadway backfilling mining, the load weight of overlying strata is regarded as uniform-distributed load $q_0 = \gamma H$; γ is the average weight of overburden, and H is the mining depth of coal seam. The overburden load is supported jointly by coal pillar and backfilling body, namely

$$q_1 l_1 + q_2 l_2 = q_0 (l_1 + l_2) \tag{3}$$

Along the roadway in unit length, the sinking displacement of coal pillar and backfilling body can be expressed as follows:

$$\begin{cases} \Delta l_1 = \dfrac{F_1 l_3}{E_1 A_1} = \dfrac{(q_1 l_1) l_3}{E_1 l_1} = \dfrac{q_1 l_3}{E_1} \\ \Delta l_2 = \dfrac{F_2 l_3}{E_2 A_2} = \dfrac{(q_2 l_2) l_3}{E_2 l_2} = \dfrac{q_2 l_3}{E_2} \end{cases} \tag{4}$$

In formula (4), l_3 represents the height of backfilling roadway, E_1 and E_2 is the elastic modulus of coal pillar and backfilling body, respectively.

In coal pillar and backfilling body combined load bearing stage, the sinking displacement of coal pillar is equal to that of backfilling body, namely $\Delta l_1 = \Delta l_2$, thus

$$q_2 = \frac{E_2}{E_1} q_1 \tag{5}$$

Combining formula (4) and formula (5), the calculative expression of uniform load q_1 and q_2 is obtained,

$$\begin{cases} q_1 = \dfrac{E_1 (l_1 + l_2)}{(E_1 l_1 + E_2 l_2)} q_0 \\ q_2 = \dfrac{E_2 (l_1 + l_2)}{(E_1 l_1 + E_2 l_2)} q_0 \end{cases} \tag{6}$$

The compressive characteristic of backfilling body is the key point in roadway backfilling mining technology. The related research result [Miao, Zhang, and Guo (2010)] indicates that the exponential function rule exists in stress-strain relationship of backfilling body,

$$\sigma = J_1 \exp\left(J_2 \varepsilon\right) \tag{7}$$

In formula (7), J_1 and J_2 are the regression coefficients.

When coal pillar and backfilling body play the joint bearing function in roadway backfilling mining, the stress σ in backfilling body is equal to the uniform load q_2 supported by backfilling body. Combining formula (6) and formula (7), the strain of backfilling body in steady state is obtained,

$$\varepsilon = \frac{1}{J_2} \ln\left[\frac{E_2\left(l_1 + l_2\right) q_0}{\left(E_1 l_1 + E_2 l_2\right) J_1}\right] \tag{8}$$

When the roadway height is l_3, the compaction amount of backfilling body in the steady state is obtained,

$$\Delta l_3 = l_3 \varepsilon = \frac{l_3}{J_2} \ln\left[\frac{E_2\left(l_1 + l_2\right) q_0}{\left(E_1 l_1 + E_2 l_2\right) J_1}\right] \tag{9}$$

3 Elastic analysis of coal pillar deformation

The elastic deformation solution of coal pillar is analyzed by the basic principle of Ritz method, according to the displacement boundary condition of mechanics model, the expressions of displacement field, including some undetermined coefficients, are assumed to satisfy displacement boundary condition; then, the specific values of undetermined coefficients are determined by the displacement variation equation; thus, the elastic solution of displacement field is obtained.

Based on the displacement boundary condition of mechanical model, the displacement field expression is assumed as follows:

$$\begin{cases} u(x, y) = \sin\left(\dfrac{\pi x}{l_1 + l_2}\right)\left(y - l_3\right)^2 \left(A_1 + A_2 x\right) \\ v\left(x, y\right) = By \end{cases} \tag{10}$$

In formula (10), A_1, A_2 and B are the undetermined coefficients.

The deformation potential energy U_1 of coal pillar and deformation potential energy U_2 of backfilling body is expressed as follows:

$$\begin{cases} U_1 = \dfrac{E_1}{2(1+\mu_1)}\displaystyle\int_0^{l_3}\int_0^{l_1}\left[\dfrac{\mu_1}{1-2\mu_1}\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}\right)^2+\left(\dfrac{\partial u}{\partial x}\right)^2+\left(\dfrac{\partial v}{\partial y}\right)^2+\dfrac{1}{2}\left(\dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y}\right)^2\right]dxdy \\ U_2 = \dfrac{E_2}{2(1+\mu_2)}\displaystyle\int_0^{l_3}\int_{l_1}^{l_1+l_2}\left[\dfrac{\mu_2}{1-2\mu_2}\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}\right)^2+\left(\dfrac{\partial u}{\partial x}\right)^2+\left(\dfrac{\partial v}{\partial y}\right)^2+\dfrac{1}{2}\left(\dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y}\right)^2\right]dxdy \end{cases} \tag{11}$$

In formula (11), μ_1 and μ_2 are the Poisson's ratio of coal pillar and backfilling body, respectively.

Substituting formula (10) into formula (11),

$$\begin{cases} \dfrac{\partial U_1}{\partial A_1}=Q_1(E_1,\mu_1,l_1,l_2,l_3)A_1+Q_2(E_1,\mu_1,l_1,l_2,l_3)A_2+Q_3(E_1,\mu_1,l_1,l_2,l_3)B \\ \dfrac{\partial U_1}{\partial A_2}=Q_4(E_1,\mu_1,l_1,l_2,l_3)A_1+Q_5(E_1,\mu_1,l_1,l_2,l_3)A_2+Q_6(E_1,\mu_1,l_1,l_2,l_3)B \\ \dfrac{\partial U_1}{\partial B}=Q_7(E_1,\mu_1,l_1,l_2,l_3)A_1+Q_8(E_1,\mu_1,l_1,l_2,l_3)A_2+Q_9(E_1,\mu_1,l_1,l_2,l_3)B \\ \dfrac{\partial U_2}{\partial A_1}=Q_{10}(E_2,\mu_2,l_1,l_2,l_3)A_1+Q_{11}(E_2,\mu_2,l_1,l_2,l_3)A_2+Q_{12}(E_2,\mu_2,l_1,l_2,l_3)B \\ \dfrac{\partial U_2}{\partial A_2}=Q_{13}(E_2,\mu_2,l_1,l_2,l_3)A_1+Q_{14}(E_2,\mu_2,l_1,l_2,l_3)A_2+Q_{15}(E_2,\mu_2,l_1,l_2,l_3)B \\ \dfrac{\partial U_2}{\partial B}=Q_{16}(E_2,\mu_2,l_1,l_2,l_3)A_1+Q_{17}(E_2,\mu_2,l_1,l_2,l_3)A_2+Q_{18}(E_2,\mu_2,l_1,l_2,l_3)B \end{cases} \tag{12}$$

In formula (12), $Q_i(i=1,2,\cdots,9)$ is the expression which contains variable E_1, μ_1, l_1, l_2 and l_3, while $Q_i(i=10,11,\cdots,18)$ is the expression which contains variable E_2, μ_2, l_1, l_2 and l_3.

The total deformation potential energy of mechanical system is obtained,

$$U=U_1+U_2=\frac{E_1}{2(1+\mu_1)}\int_0^{l_3}\int_0^{l_1}\left[\frac{\mu_1}{1-2\mu_1}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]dxdy$$
$$+\frac{E_2}{2(1+\mu_2)}\int_0^{l_3}\int_{l_1}^{l_1+l_2}\left[\frac{\mu_2}{1-2\mu_2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]dxdy \tag{13}$$

Substituting formula (12) into formula (13),

$$\begin{cases} \dfrac{\partial U}{\partial A_1} = \dfrac{\partial U_1}{\partial A_1} + \dfrac{\partial U_2}{\partial A_1} = m_1 A_1 + m_2 A_2 + m_3 B \\ \dfrac{\partial U}{\partial A_2} = \dfrac{\partial U_1}{\partial A_2} + \dfrac{\partial U_2}{\partial A_2} = m_4 A_1 + m_5 A_2 + m_6 B \\ \dfrac{\partial U}{\partial B} = \dfrac{\partial U_1}{\partial B} + \dfrac{\partial U_2}{\partial B} = m_7 A_1 + m_8 A_2 + m_9 B \end{cases} \tag{14}$$

In formula (14), $m_i (i = 1, 2, \cdots, 9)$ is the expression which contains variable E_1, μ_1, E_2, μ_2, l_1, l_2 and l_3. The specific expression is as follows:

$$\begin{cases} m_1 = Q_1(E_1, \mu_1, l_1, l_2, l_3) + Q_{10}(E_2, \mu_2, l_1, l_2, l_3) \\ m_2 = Q_2(E_1, \mu_1, l_1, l_2, l_3) + Q_{11}(E_2, \mu_2, l_1, l_2, l_3) \\ m_3 = Q_3(E_1, \mu_1, l_1, l_2, l_3) + Q_{12}(E_2, \mu_2, l_1, l_2, l_3) \\ m_4 = Q_4(E_1, \mu_1, l_1, l_2, l_3) + Q_{13}(E_2, \mu_2, l_1, l_2, l_3) \\ m_5 = Q_5(E_1, \mu_1, l_1, l_2, l_3) + Q_{14}(E_2, \mu_2, l_1, l_2, l_3) \\ m_6 = Q_6(E_1, \mu_1, l_1, l_2, l_3) + Q_{15}(E_2, \mu_2, l_1, l_2, l_3) \\ m_7 = Q_7(E_1, \mu_1, l_1, l_2, l_3) + Q_{16}(E_2, \mu_2, l_1, l_2, l_3) \\ m_8 = Q_8(E_1, \mu_1, l_1, l_2, l_3) + Q_{17}(E_2, \mu_2, l_1, l_2, l_3) \\ m_9 = Q_9(E_1, \mu_1, l_1, l_2, l_3) + Q_{18}(E_2, \mu_2, l_1, l_2, l_3) \end{cases} \tag{15}$$

Under the condition of without considering the volume force ($X = 0, Y = 0$), the Ritz equation is shown as follows,

$$\begin{cases} \dfrac{\partial U}{\partial A_1} = \int \bar{X} u_1 dx \\ \dfrac{\partial U}{\partial A_2} = \int \bar{X} u_2 dx \\ \dfrac{\partial U}{\partial B} = \int \bar{Y} v dy \end{cases} \tag{16}$$

In formula (16), $\bar{X}$ and $\bar{Y}$ represent the area force in the boundary. The expression of u_1, u_2 and v is obtained,

$$\begin{cases} u_1 = \left(y - l_3\right)^2 \sin\left(\dfrac{\pi x}{l_1 + l_2}\right) \\ v = y \\ u_2 = \left(y - l_3\right)^2 x \sin\left(\dfrac{\pi x}{l_1 + l_2}\right) \end{cases} \tag{17}$$

According to the stress boundary condition of coal pillar, the relationship is true,

$$\begin{cases} \bar{Y}\Big|_{y=l_3,\ x=0\to l_1} = -q_1 = -\dfrac{E_1\left(l_1 + l_2\right)}{\left(E_1 l_1 + E_2 l_2\right)} q_0 \\ \bar{Y}\Big|_{y=l_3,\ x=l_1\to l_1+l_2} = -q_2 = -\dfrac{E_2\left(l_1 + l_2\right)}{\left(E_1 l_1 + E_2 l_2\right)} q_0 \end{cases} \tag{18}$$

Thus,

$$\begin{cases} \int \bar{X} u_1 dx = 0 \\ \int \bar{X} u_2 dx = 0 \\ \int \bar{Y} v dy = -q_0 (l_1 + l_2) l_3 \end{cases} \tag{19}$$

Substituting the formula (19) into the Ritz equation, namely the formula (16),

$$\begin{cases} m_1 A_1 + m_2 A_2 + m_3 B = 0 \\ m_4 A_1 + m_5 A_2 + m_6 B = 0 \\ m_7 A_1 + m_8 A_2 + m_9 B = -q_0 (l_1 + l_2) l_3 \end{cases} \tag{20}$$

By solving the ternary nonhomogeneous equation system, the undetermined coefficients are expressed as follows:

$$\begin{cases} A_1 = \dfrac{\det\left(\vec{B}_1\right)}{\det\left(\vec{A}\right)} = \dfrac{q_0 l_3 (l_1 + l_2)(m_3 m_5 - m_2 m_6)}{m_1 m_5 m_9 + m_2 m_6 m_7 + m_3 m_4 m_8 - m_1 m_6 m_8 - m_2 m_4 m_9 - m_3 m_5 m_7} \\ A_2 = \dfrac{\det\left(\vec{B}_2\right)}{\det\left(\vec{A}\right)} = \dfrac{q_0 l_3 (l_1 + l_2)(m_1 m_6 - m_3 m_4)}{m_1 m_5 m_9 + m_2 m_6 m_7 + m_3 m_4 m_8 - m_1 m_6 m_8 - m_2 m_4 m_9 - m_3 m_5 m_7} \\ B = \dfrac{\det\left(\vec{B}_3\right)}{\det\left(\vec{A}\right)} = \dfrac{q_0 l_3 (l_1 + l_2)(m_2 m_4 - m_1 m_5)}{m_1 m_5 m_9 + m_2 m_6 m_7 + m_3 m_4 m_8 - m_1 m_6 m_8 - m_2 m_4 m_9 - m_3 m_5 m_7} \end{cases} \tag{21}$$

Therefore, the analytical expressions of displacement field of coal pillar is obtained,

$$\begin{cases} u(x,y)=\sin\left(\dfrac{x\pi}{l_1+l_2}\right)(y-l_3)^2\left(\dfrac{q_0l_3(l_1+l_2)(m_3m_5-m_2m_6)+q_0l_3(l_1+l_2)(m_1m_6-m_3m_4)x}{m_1m_5m_9+m_2m_6m_7+m_3m_4m_8-m_1m_6m_8-m_2m_4m_9-m_3m_5m_7}\right) \\ v(x,y)=\dfrac{q_0l_3(l_1+l_2)(m_2m_4-m_1m_5)y}{m_1m_5m_9+m_2m_6m_7+m_3m_4m_8-m_1m_6m_8-m_2m_4m_9-m_3m_5m_7} \end{cases} \tag{22}$$

4 Viscoelastic analysis of coal pillar deformation

The basic equations of elasticity and viscoelasticity are shown in Tab. 1, which indicates that the viscoelasticity equations can be obtained by doing the Laplace transform on elasticity equations and substituting G into $\bar{Q}/\bar{P}$ in elasticity equations. Therefore, the viscoelastic solution of mechanical model can be transformed by doing the mathematical procession on the elastic solution of mechanical model, and this is the elastic-viscoelastic correspondence principle.

Table 1: Basic equations of elasticity and viscoelasticity

Title	Basic equations of elasticity	Basic equations of viscoelasticity
Balance equation	$\sigma_{ij,i}+X_i=0$	$\bar{\sigma}_{ij,i}+\bar{X}_i=0$
Geometric equation	$\varepsilon_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right)$	$\bar{\varepsilon}_{ij}=\frac{1}{2}\left(\bar{u}_{i,j}+\bar{u}_{j,i}\right)$
Physical equation	$\begin{cases} s_{ij}=2Ge_{ij} \\ \sigma_{ii}=3K\varepsilon_{ii} \end{cases}$	$\begin{cases} \bar{P}\bar{s}_{ij}=2\bar{Q}\bar{e}_{ij} \\ \bar{\sigma}_{ii}=3K\bar{\varepsilon}_{ii} \end{cases}$
Boundary condition	$\begin{cases} \sigma_{ij}n_j=T_i \\ u_i=\varphi_i \end{cases}$	$\begin{cases} \bar{\sigma}_{ij}n_j=\bar{T}_i \\ \bar{u}_i=\bar{\varphi}_i \end{cases}$

Assume that a real function $f(t)$ is defined on an independent variable $t\geq 0$, and the integral $F(s)=\int_0^{+\infty}f(t)\exp(-st)dt$ tends to convergence in a certain range of complex plane, so the function $F(s)$ determined by the integral is denoted as Laplace transform of the real function $f(t)$. Similarly, $f(t)$ is entitled as the inverse Laplace transform of $F(s)$.

Based on the elastic-viscoelastic correspondence principle, Laplace transform can be done on the elastic solution of displacement field expression of coal pillar, namely the formula (10),

$$\begin{cases} \bar{u} = \sin\left(\dfrac{x}{l_1 + l_2}\pi\right)(y - l_3)^2 \left(\dfrac{A_1}{s} + \dfrac{A_2}{s}x\right) \\ \bar{v} = \dfrac{B}{s} y \end{cases} \tag{23}$$

According to the relationship between the elastic modulus E, Poisson's ratio μ and shear modulus G, bulk modulus K, namely

$$\begin{cases} E = \dfrac{9KG}{3K + G} \\ \mu = \dfrac{3K - 2G}{6K + 2G} \end{cases} \tag{24}$$

Thus, the elastic modulus E_1, Poisson's ratio μ_1 in $m_i(i = 1, 2, \cdots, 9)$ expression in formula (15) can be expressed by shear modulus G and bulk modulus K.

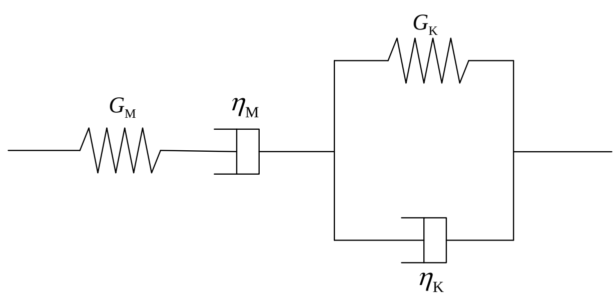


Figure 2: Burgers creep mechanical model

The rheological characteristic of coal pillar illustrated with the burgers creep mechanical model, which is shown in Fig. 2. The three dimensional constitutive equation is as follows,

$$s_{ij} + \left(\frac{\eta_M}{G_M} + \frac{\eta_M + \eta_K}{G_K}\right)\frac{ds_{ij}}{dt} + \frac{\eta_M \eta_K}{G_M G_K}\frac{d^2 s_{ij}}{dt^2} = 2\eta_M \frac{de_{ij}}{dt} + \frac{2\eta_M \eta_K}{G_K}\frac{d^2 e_{ij}}{dt^2} \tag{25}$$

According to the viscoelastic theory, the variables are obtained,

$$\begin{cases} \bar{P} = 1 + \left(\dfrac{\eta_M}{G_M} + \dfrac{\eta_M + \eta_K}{G_K}\right)s + \dfrac{\eta_M \eta_K}{G_M G_K} s^2 \\ \bar{Q} = 2\eta_M s + \dfrac{2\eta_M \eta_K}{G_K} s^2 = 2\eta_M s\left(1 + \dfrac{\eta_K}{G_K} s\right) \end{cases} \tag{26}$$

Based on the elastic-viscoelastic correspondence principle, the viscoelastic solution of displacement field expression of coal pillar can be transformed by doing the inverse Laplace transformation, which is shown in formula (27),

$$\begin{cases} u(x,y,t) = \sin\left(\dfrac{x}{l_1+l_2}\pi\right)\left(y-l_3\right)^2\left[g_1(t)+g_2(t)x\right] \\ v\left(x,y,t\right) = g_3(t)y \end{cases} \tag{27}$$

In formula (27), $g_1(t)$, $g_2(t)$ and $g_3(t)$ are the expressions which only contain the time variable t.

Therefore, when the roadway height is l_3 , the maximum value of vertical creep deformation of coal pillar is $g_3(t)l_3$. Combining with the compaction amount Δl_3 of backfilling body in the steady state in formula (9), the expression of $g_3(t)$ is obtained,

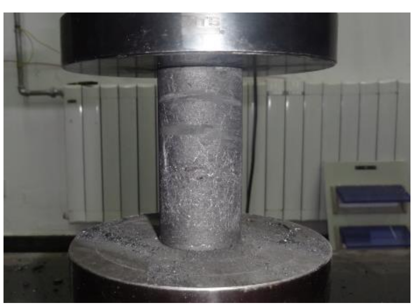
$$g_3(t) = \frac{1}{J_2}\ln\left[\frac{E_2\left(l_1+l_2\right)q_0}{\left(E_1l_1+E_2l_2\right)J_1}\right] \tag{28}$$

By substituting the specific variable values into formula (28), the time required for coal pillar and backfilling body to play the joint bearing function in roadway backfilling mining technology can be obtained.

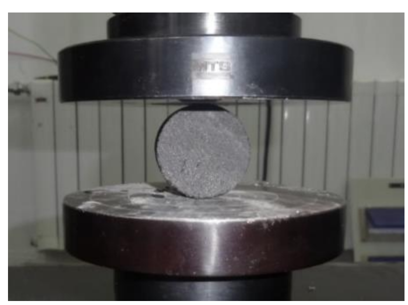
5 Engineer example analysis

5.1 Elastic solution of displacement field of coal pillar

In the engineer practice, the width of coal pillar, the width and height of backfilling roadway is 6m, namely $l_1 = l_2 = l_3 = 3\text{m}$; the elastic modulus and Poisson's ratio of coal pillar is $E_1 = 2.5\text{GPa}$ and $\mu_1 = 0.25$, respectively; the elastic modulus and Poisson's ratio of backfilling body is $E_2 = 0.2\text{GPa}$ and $\mu_2 = 0.3$, respectively. The mechanical parameter values in engineering example are obtained by the uniaxial compression test, the uniaxial tensile test, the corner-mould shear test and the creep test with load rating (Fig. 3), which are carried out with DDL500 electronic universal testing machine and MTS815.02 electro-hydraulic servo testing system (Fig. 4).



(a) the uniaxial compression test



(b) the uniaxial tensile test

(c) the corner-mould shear test (d) the creep test with load rating

Figure 3: The mechanical experiments

(a) the uniaxial compression test (b) the uniaxial tensile test

Figure 4: The equipment of mechanical experiment

According to the calculative expression of uniform load q_1 and q_2 in formula (6), the specific values of q_1 and q_2 are

$$\begin{cases} q_1 = \dfrac{E_1\left(l_1+l_2\right)}{\left(E_1 l_1 + E_2 l_2\right)} q_0 = \dfrac{2.5\times\left(2.5+2.5\right)}{\left(2.5\times 2.5 + 0.2\times 2.5\right)}\times 15 = 27.78\left(\mathrm{MPa}\right) \\ q_2 = \dfrac{E_2\left(l_1+l_2\right)}{\left(E_1 l_1 + E_2 l_2\right)} q_0 = \dfrac{0.2\times\left(2.5+2.5\right)}{\left(2.5\times 2.5 + 0.2\times 2.5\right)}\times 15 = 2.22\left(\mathrm{MPa}\right) \end{cases} \tag{29}$$

The displacement expression of coal pillar is

$$\begin{cases} u(x,y) = \sin\left(\dfrac{\pi x}{6}\right)\left(y-3\right)^2\left(A_1 + A_2 x\right) \\ v\left(x,y\right) = By \end{cases} \tag{30}$$

Thus, the deformation potential energy of coal pillar and backfilling body are

$$\begin{cases} U_1 = \dfrac{E_1}{2(1+\mu_1)}\displaystyle\int_0^3\int_0^3\left[\dfrac{\mu_1}{1-2\mu_1}\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}\right)^2+\left(\dfrac{\partial u}{\partial x}\right)^2+\left(\dfrac{\partial v}{\partial y}\right)^2+\dfrac{1}{2}\left(\dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y}\right)^2\right]dxdy \\ U_2 = \dfrac{E_2}{2(1+\mu_2)}\displaystyle\int_0^3\int_3^6\left[\dfrac{\mu_2}{1-2\mu_2}\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}\right)^2+\left(\dfrac{\partial u}{\partial x}\right)^2+\left(\dfrac{\partial v}{\partial y}\right)^2+\dfrac{1}{2}\left(\dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y}\right)^2\right]dxdy \end{cases} \tag{31}$$

Substituting formula (30) into formula (31),

$$\begin{cases} \dfrac{\partial U_1}{\partial A_1} = -\dfrac{27\left(3\pi^2+40-80\mu_1-3\pi^2\mu_1\right)}{40(1+\mu_1)(2\mu_1-1)}E_1A_1 - \dfrac{9\mu_1}{(1+\mu_1)(2\mu_1-1)}E_1B \\ -\dfrac{27\left(156\pi^2-9\pi^4\mu_1-276\pi^2\mu_1+480-960\mu_1+9\pi^4\right)}{80\pi^2(1+\mu_1)(2\mu_1-1)}E_1A_2 \\ \dfrac{\partial U_1}{\partial A_2} = -\dfrac{27\left(9\pi^4+156\pi^2+480-960\mu_1-276\pi^2\mu_1-9\pi^4\mu_1\right)}{80\pi^2(1+\mu_1)(2\mu_1-1)}E_1A_1 \\ -\dfrac{81\left(3\pi^4+240-480\mu_1+94\pi^2-3\pi^4\mu_1-134\pi^2\mu_1\right)}{40\pi^2(1+\mu_1)(2\mu_1-1)}E_1A_2 - \dfrac{27\mu_1}{(1+\mu_1)(2\mu_1-1)}E_1B \\ \dfrac{\partial U_1}{\partial B} = -\dfrac{9\mu_1}{(1+\mu_1)(2\mu_1-1)}E_1A_1 - \dfrac{27\mu_1}{(1+\mu_1)(2\mu_1-1)}E_1A_2 + \dfrac{9(\mu_1-1)}{(1+\mu_1)(2\mu_1-1)}E_1B \end{cases} \tag{32}$$

$$\begin{cases} \dfrac{\partial U_2}{\partial A_1} = -\dfrac{27\left(3\pi^2+40-80\mu_2-3\pi^2\mu_2\right)}{40(1+\mu_2)(2\mu_2-1)}E_2A_1 + \dfrac{9\mu_2}{(1+\mu_2)(2\mu_2-1)}E_2B \\ -\dfrac{27\left(324\pi^2-27\pi^4\mu_2-684\pi^2\mu_2-480+960\mu_2+27\pi^4\right)}{80\pi^2(1+\mu_2)(2\mu_2-1)}E_2A_2 \\ \dfrac{\partial U_2}{\partial A_2} = -\dfrac{27\left(27\pi^4+324\pi^2-480+960\mu_2-684\pi^2\mu_2-27\pi^4\mu_2\right)}{80\pi^2(1+\mu_2)(2\mu_2-1)}E_2A_1 \\ -\dfrac{81\left(21\pi^4-720+1440\mu_2+262\pi^2-21\pi^4\mu_2-542\pi^2\mu_2\right)}{40\pi^2(1+\mu_2)(2\mu_2-1)}E_2A_2 + \dfrac{27\mu_2}{(1+\mu_2)(2\mu_2-1)}E_2B \\ \dfrac{\partial U_2}{\partial B} = \dfrac{9\mu_2}{(1+\mu_2)(2\mu_2-1)}E_2A_1 + \dfrac{27\mu_2}{(1+\mu_2)(2\mu_2-1)}E_2A_2 + \dfrac{9(\mu_2-1)}{(1+\mu_2)(2\mu_2-1)}E_2B \end{cases} \tag{33}$$

The expression $Q_i(i=1,2,\cdots,9)$ is shown in formula (34), which contains variable E_1 and μ_1, while the expression $Q_i(i=10,11,\cdots,18)$ is shown in formula (35), which contains variable E_2 and μ_2,

$$\begin{cases}
Q_1=-\dfrac{27\left(3\pi^2+40-80\mu_1-3\pi^2\mu_1\right)}{40\left(1+\mu_1\right)\left(2\mu_1-1\right)}E_1 \\
Q_2=-\dfrac{27\left(156\pi^2-9\pi^4\mu_1-276\pi^2\mu_1+480-960\mu_1+9\pi^4\right)}{80\pi^2\left(1+\mu_1\right)\left(2\mu_1-1\right)}E_1 \\
Q_3=-\dfrac{9\mu_1}{\left(1+\mu_1\right)\left(2\mu_1-1\right)}E_1 \\
Q_4=-\dfrac{27\left(9\pi^4+156\pi^2+480-960\mu_1-276\pi^2\mu_1-9\pi^4\mu_1\right)}{80\pi^2\left(1+\mu_1\right)\left(2\mu_1-1\right)}E_1 \\
Q_5=-\dfrac{81\left(3\pi^4+240-480\mu_1+94\pi^2-3\pi^4\mu_1-134\pi^2\mu_1\right)}{40\pi^2\left(1+\mu_1\right)\left(2\mu_1-1\right)}E_1 \\
Q_6=-\dfrac{27\mu_1}{\left(1+\mu_1\right)\left(2\mu_1-1\right)}E_1;\quad Q_7=-\dfrac{9\mu_1}{\left(1+\mu_1\right)\left(2\mu_1-1\right)}E_1 \\
Q_8=-\dfrac{27\mu_1}{\left(1+\mu_1\right)\left(2\mu_1-1\right)}E_1;\quad Q_9=\dfrac{9\left(\mu_1-1\right)}{\left(1+\mu_1\right)\left(2\mu_1-1\right)}E_1
\end{cases} \tag{34}$$

$$\begin{cases} Q_{10} = -\dfrac{27\left(3\pi^2 + 40 - 80\mu_2 - 3\pi^2\mu_2\right)}{40\left(1+\mu_2\right)\left(2\mu_2 - 1\right)} E_2 \\ Q_{11} = -\dfrac{27\left(324\pi^2 - 27\pi^4\mu_2 - 684\pi^2\mu_2 - 480 + 960\mu_2 + 27\pi^4\right)}{80\pi^2\left(1+\mu_2\right)\left(2\mu_2 - 1\right)} E_2 \\ Q_{12} = \dfrac{9\mu_2}{\left(1+\mu_2\right)\left(2\mu_2 - 1\right)} E_2 \\ Q_{13} = -\dfrac{27\left(27\pi^4 + 324\pi^2 - 480 + 960\mu_2 - 684\pi^2\mu_2 - 27\pi^4\mu_2\right)}{80\pi^2\left(1+\mu_2\right)\left(2\mu_2 - 1\right)} E_2 \\ Q_{14} = -\dfrac{81\left(21\pi^4 - 720 + 1440\mu_2 + 262\pi^2 - 21\pi^4\mu_2 - 542\pi^2\mu_2\right)}{40\pi^2\left(1+\mu_2\right)\left(2\mu_2 - 1\right)} E_2 \\ Q_{15} = \dfrac{27\mu_2}{\left(1+\mu_2\right)\left(2\mu_2 - 1\right)} E_2; \quad Q_{16} = \dfrac{9\mu_2}{\left(1+\mu_2\right)\left(2\mu_2 - 1\right)} E_2 \\ Q_{17} = \dfrac{27\mu_2}{\left(1+\mu_2\right)\left(2\mu_2 - 1\right)} E_2; \quad Q_{18} = \dfrac{9\left(\mu_2 - 1\right)}{\left(1+\mu_2\right)\left(2\mu_2 - 1\right)} E_2 \end{cases} \tag{35}$$

Thus, the specific values of $Q_i (i = 1, 2, \cdots, 18)$ and $m_i (i = 1, 2, \cdots, 9)$ can be obtained, which is shown in Tab. 2.

Substituting the related values into formula (20),

$$\begin{cases} 123493A_1 + 277325A_2 + 7962B = 0 \\ 277325A_1 + 936096A_2 + 23885B = 0 \\ 7961A_1 + 23885A_2 + 29423B = -225 \end{cases} \tag{36}$$

Table 2: *Q* and *m* values

coefficient	value	coefficient	value	coefficient	value
Q_1	113958	Q_{10}	9535	m_1	123493
Q_2	240215	Q_{11}	37110	m_2	277325
Q_3	9000	Q_{12}	-1038	m_3	7962
Q_4	240215	Q_{13}	37110	m_4	277325
Q_5	768408	Q_{14}	167688	m_5	936096
Q_6	27000	Q_{15}	-3115	m_6	23885
Q_7	9000	Q_{16}	-1039	m_7	7961

Q_8	27000	Q_{17}	-3115	m_8	23885
Q_9	27000	Q_{18}	2423	m_9	29423

The undetermined coefficients can be solved,

$$\begin{cases} A_1 = \dfrac{186589963575}{1114124726985047} = 0.00017 \\ A_2 = \dfrac{166852947375}{1114124726985047} = 0.00015 \\ B = -\dfrac{8705733233175}{1114124726985047} = -0.00781 \end{cases} \tag{37}$$

Then, the elastic solution of displacement field of coal pillar is obtained,

$$\begin{cases} u(x,y) = \sin\left(\dfrac{\pi x}{6}\right)(y-3)^2(0.00017 + 0.00015x) \\ v(x,y) = -0.00781y \end{cases} \tag{38}$$

The height of coal pillar is 6 m, substitute y=6 into formula (38), so the maximum vertical deformation of coal pillar is approximately 0.045 m, which is reasonable in practice engineer.

5.2 Viscoelastic solution of displacement field of coal pillar

Based on the elastic-viscoelastic correspondence principle, the specific values of creep variable $G_{\mathrm{M}} = 5.1\mathrm{GPa}$, $G_{\mathrm{K}} = 1.1\mathrm{GPa}$, $\eta_{\mathrm{M}} = 1400\mathrm{GPa}\cdot\mathrm{d}$ and $\eta_{\mathrm{K}} = 14\mathrm{GPa}\cdot\mathrm{d}$ are substituting into formula (26),

$$\begin{cases} \bar{P} = 1 + 1560s + 3494s^2 \\ \bar{Q} = 2800s(1+13s) \end{cases} \tag{39}$$

Thus, the shear modulus G in viscoelastic theory is obtained,

$$G = \frac{\bar{Q}}{\bar{P}} = \frac{2800s(1+13s)}{1+1560s+3494s^2} \tag{40}$$

Based on the elastic-viscoelastic correspondence principle, the elastic modulus E and Poisson's ratio μ in viscoelastic theory is shown as follows,

$$\begin{cases} E = \dfrac{9KG}{3K+G} = \dfrac{8400s + 109200s^2}{1 + 2120s + 10774s^2} \\ \mu = \dfrac{3K-2G}{6K+2G} = \dfrac{1 + 440s - 11066s^2}{2 + 4240s + 21548s^2} \end{cases} \tag{41}$$

Substituting formula (41) into formula (34),

$$
\begin{cases}
Q_1 = \dfrac{27(8400+109200s)\left[3\pi^2+\left(11400\pi^2+134400\right)s+\left(97842\pi^2+1747200\right)s^2\right]}{40(1680+21840s)\left(3+4680s+10482s^2\right)} \\
Q_2 = \dfrac{27(105+1365s)\left[\left(156\pi^2+480+9\pi^4\right)\left(2+4240s+21548s^2\right)-\left(9\pi^4+276\pi^2+960\right)\left(1+440s-11066s^2\right)\right]}{\pi^2(1680+21840s)\left(3+4680s+10482s^2\right)} \\
Q_3 = \dfrac{9(8400+109200s)\left(1+440s-11066s^2\right)}{(1680+21840s)\left(3+4680s+10482s^2\right)} \\
Q_4 = \dfrac{27(105+1365s)\left[\left(9\pi^4+156\pi^2+480\right)\left(2+4240s+21548s^2\right)-\left(960+276\pi^2+9\pi^4\right)\left(1+440s-11066s^2\right)\right]}{\pi^2(1680+21840s)\left(3+4680s+10482s^2\right)} \\
Q_5 = \dfrac{81(210+2730s)\left[\left(3\pi^4+94\pi^2+240\right)\left(2+4240s+21548s^2\right)-\left(3\pi^4+134\pi^2+480\right)\left(1+440s-11066s^2\right)\right]}{\pi^2(1680+21840s)\left(3+4680s+10482s^2\right)} \\
Q_6 = \dfrac{27(8400+109200s)\left(1+440s-11066s^2\right)}{(1680+21840s)\left(3+4680s+10482s^2\right)} \\
Q_7 = \dfrac{9(8400+109200s)\left(1+440s-11066s^2\right)}{(1680+21840s)\left(3+4680s+10482s^2\right)} \\
Q_8 = \dfrac{27(8400+109200s)\left(1+440s-11066s^2\right)}{(1680+21840s)\left(3+4680s+10482s^2\right)} \\
Q_9 = \dfrac{9(8400+109200s)\left(1+3800s+32614s^2\right)}{(1680+21840s)\left(3+4680s+10482s^2\right)}
\end{cases}
\tag{42}
$$

Combining formula (15) with formula (21), the analytical expression of undetermined coefficients $A_1(s)$, $A_2(s)$ and $B(s)$ can be obtained; therefore, the formula (23) can be expressed as follows:

$$
\begin{cases}
\bar{u} = \sin\left(\dfrac{\pi x}{6}\right)(y-3)^2\left(\dfrac{A_1(s)}{s}+\dfrac{A_2(s)}{s}x\right) \\
\bar{v} = \dfrac{B(s)}{s}y
\end{cases}
\tag{43}
$$

Based on the elastic-viscoelastic correspondence principle, the viscoelastic solution of displacement field of coal pillar can be transformed, which is shown in formula (44),

$$
\begin{cases}
u(x,y,t) = \sin\left(\dfrac{\pi x}{6}\right)(y-3)^2\left[g_1(t)+g_2(t)x\right] \\
v(x,y,t) = g_3(t)y
\end{cases}
\tag{44}
$$

In formula (44), the specific expression of $g_3(t)$ is

$$g_3(t) = -0.097 - 4.372\times10^{-6}\exp(-0.1129t) + 0.004\exp(-0.0288t) \\ +0.007\exp(-0.0306t) + 0.005\exp(-0.0173t) + 0.059\exp(-0.0055t) \tag{45}$$

When the backfilling body is coal gangue (particle size is 20-25 mm), the specific values of regression coefficients J_1 and J_2 is 0.1442 MPa and 10.31, respectively. Substituting the specific values into formula (28),

$$g_3(t) = \frac{1}{10.31}\ln\left[\frac{0.2\times(3+3)\times15}{(2.5\times3+0.2\times3)\times0.1442}\right]=0.042 \tag{46}$$

Combining formula (45) and formula (46), the time formula required for coal pillar and backfilling body to play the joint bearing function in roadway backfilling mining technology is obtained,

$$0.004\exp(-0.0288t) + 0.007\exp(-0.0306t) + 0.005\exp(-0.0173t) \\ +0.059\exp(-0.0055t) - 4.372\times10^{-6}\exp(-0.1129t)=0.139 \tag{47}$$

By solving the formula (47), the time is $t = 140$. Therefore, the time required for coal pillar and backfilling body to play the joint bearing function in roadway backfilling mining technology is 140 days.

6 Conclusions

(1) The control process of overlying strata movement in roadway backfilling mining technology can be divided into three stages, namely coal pillar bearing stage, backfilling body dynamic compaction stage and coal pillar and backfilling body combined load bearing stage. The coal pillar and backfilling body play the joint bearing effect on the overlying strata and achieve long time stability in combined load bearing stage.

(2) The mechanical model of bearing system of coal pillar and backfilling body is established, on basis of the deformation characteristics of overlying strata in roadway backfilling mining technology. Besides, the elastic solution expression of coal pillar deformation is deduced with the Ritz method in energy variation principle, and the viscoelastic solution expression of coal pillar deformation is obtained, based on elastic-viscoelastic correspondence principle, combining with the burgers rheological constitutive model and Laplace transform theory.

(3) By analyzing the compressive mechanical property of backfilling body, the time formula required for coal pillar and backfilling body to play the joint bearing function in roadway backfilling mining technology is obtained. The engineer example analysis indicates that the time required to play the joint bearing function is 140 days.

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