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Some Topological Indices Computing Results If Archimedean Lattices L(4,6,12)

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Abstract: The introduction of graph-theoretical structure descriptors represents an important step forward in the research of predictive models in chemistry and falls within the lines of the increasing use of mathematical and computational methods in contemporary chemistry. The basis for these models is the study of the quantitative structure-property and structure-activity relationship. In this paper, we investigate Great rhom-bitrihexagonal which is a kind of dodecagon honeycomb net-work covered by quadrangle and hexagon. Many topological indexes of Great rhom-bitrihexagonal have being investigated, such as sum-connectivity index, atom-bond connectivity index, geometric-arithmetic index, fifth, harmonic index, Randić connectivity index, first Zagreb index, second Zagreb index and the corresponding Zagreb polynomials, modified Zagreb index, fourth atom-bond connectivity index, augmented Zagreb index, hyper-Zagreb index, Sankruti index, forgotten topological index, first multiple Zagreb index, second multiple Zagreb index, as well as derived geometric-arithmetic index, Narumi-Katayama index and modified Narumi-Katayama index.

Keywords: Topological index, $L_{4,6,12}$ circumference, graph theory, molecular biology.

1 Introduction

There is an undirected graph without multiple edges and loops that is considered in this paper. We define *G* as a graph, and define *E(G)* and *V(G)* as the edge set and vertex set of G. In additional, we define *d(v)* as the degree of the a vertex *v* and *N(v)* as the set of neighbours of *v* in graph. We use $E_{a,b}$ express the set of edges that the degrees of end vertices *a* and *b*, i.e. $E_{a,b}=\{uv|\ \{a,b\}=\{d(u),d(v)\}$. Let the summation of a vertex *u* be $s_G(u)$, i.e. $s_G(u)=\sum_{v=N(u)} d(v)$ and $E'_{a,b}=\{uv|\{a,b\}=\{s_G(u),s_G(v)\}\}$. Topological Index has been derived for many years, the theory of chemical graph contains both compute graph theory and chemical graph theory. As a newly boundary science between chemistry and computer science, which become more and more concerned [Luo and Zhang (2012)]. It has become a significant branch of mathematical chemistry. In traditional chemistry, the researchers exploit the chemical properties of matter by means of experiment. But in

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molecular chemistry, a molecular structure is viewed as a so-called chemical graph, in which way, vertices is regarded as chemical atom and edge is regarded as chemical bond. As a useful tool of research, chemical graph is applied to reveal the relationships between various physical characteristics and chemical structures, such as biological activity, chemical reactivity [Van, Carter, Grassy et al. (1997)]. In 1947, Harold Wiener first used topological indices on the research of paraffin's boiling points. In his paper, Wiener index is introduced to reveal relationships between the index of their molecular graphs and physicochemical properties of organic compounds [Iranmanesh, Alizadeh and Taherkhani (2008)]. Furthermore, such an index reveals the correlations of physicochemical properties of alkanes, alcohols, amines and their analogous compounds [Ali, Yaser and Bahman (2008)]. There was Randić connectivity index, one of topological indexes, applied to the study of branching properties of alkanes [Randic (1975)].

Definition 1 *The connectivity index (or Randić Index) of a graph G, denoted by $\chi(G)$, was defined as follow:*

$$\chi(\mathrm{G})=\sum_{e=uv\in E(G)}\frac{1}{\sqrt{d(u)d(v)}} \tag{1}$$

There is another Randić index, named the harmonic index, as is first introduced in Fajtlowicz et al. [Fajtlowicz and Waller (1986)].

Definition 2 *The harmonic index H(G) is defined in a graph as follow.*

$$\mathrm{H(G)}=\sum_{v\in V(G)}\frac{1}{d(u)+d(v)} \tag{2}$$

In 1998, Estrada et al. [Estrada, Torres and Rodríguez et al. (1998)] put forward the atom-bond connectivity (ABC) index, in order to improve Randić classic connectivity index.

Definition 3 *Estrada introduced ABC index, also named atom-bond connectivity index, in 1998. And ABC index is defined as*

$$\mathrm{ABC(G)}=\sum_{e=uv\in E(G)}\sqrt{\frac{d(u)+d(v)-2}{d(u)d(v)}} \tag{3}$$

In 1998, Estrada [Estrada (2008)] demonstrated that there is a linear relationship between alkanes' experimental heats of formation and ABC index. Furthermore, in 2008, Estrada built the physical basis for this relationship. A new version of ABC index was introduced by Ghorbani et al. [Ghorbani and Hosseinzadeh (2010)], named (ABC_4) index.

Definition 4 *The new version of atom-bond connectivity* (ABC_4) *index is defined as*

$$\mathrm{ABC_4(G)}=\sum_{e=uv\in E(G)}\sqrt{\frac{s_G(u)+s_G(v)-2}{s_G(u)s_G(v)}} \tag{4}$$

In 2010, Furtula et al. [Furtula and Gutman (2015)] refers a new topological index, named "augmented Zagreb index", and furthermore, the upper and lower bounds of chemical tree can be attained.

Definition 5 *The augmented Zagreb index AZI(G) is defined as*

$$\mathrm{AZI(G)}=\sum_{e=uv\in E(G)}\left(\frac{d(u)d(v)}{d(u)+d(v)-2}\right)^3 \tag{5}$$

In 1972, Gutman et al. [Gutman and Trinajstić (1972)] introduced the first and second Zagreb indiecs of a molecular graph.

Definition 6 *The first Zagreb index $M_1(G)$ [Nikolić, Kovačević, Miličević et al. (2003)] is defined as*

$$M_1(G)=\sum_{v\in V(G)}\left(d(v)\right)^2=\sum_{e=uv\in E(G)}\left(d(u)+d(v)\right) \tag{6}$$

Definition 7 *The second Zagreb index $M_2(G)$ [Nikolić, Kovačević, Miličević et al. (2003)] is defined as*

$$M_2(G)=\sum_{e=uv\in E(G)}\left(d(u)d(v)\right) \tag{7}$$

In 1972, Furtula et al. [Furtula and Gutman (2015)] studied the total π-electron energy's structure-dependency. He found that there was a relationship between the sum of square of the vertex degrees of the molecular graph and the total π-electron energy's structure-dependency. At the same time, he introduced the "forgotten topological index".

Definition 8 *The forgotten topological index F(G) defined by Furtula is shown as follow:*

$$F(G)=\sum_{v\in V(G)}\left(d(v)\right)^3=\sum_{uv\in E(G)}\left(d(u)^2+d(v)^2\right) \tag{8}$$

On the basis of the above Zagreb indices, the first Zagreb polynomial $M_1(G,x)$ and the second Zagreb polynomial $M_2(G,x)$ have been defined [Farahani (2013); Fath-Tabar (2009)].

Definition 9 *The first Zagreb multinomial $M_1(G,x)$ is defined as*

$$M_1(G,x)=\sum_{e=uv\in E(G)}x^{d(u)+d(v)} \tag{9}$$

Definition 10 *The second Zagreb multinomial $M_2(G,x)$ is defined as*

$$M_2(G,x)=\sum_{e=uv\in E(G)}x^{d(u)d(v)} \tag{10}$$

A problem with the Zagreb indices is that their contributing parts give greater weights to inner vertices and edges and smaller weights to outer vertices and edges of a graph. This opposes intuitive reasoning that outer atoms and bonds should have greater weights than inner vertices and bonds because outer vertices and bonds are associated with a larger part of the molecular surface and are consequently expected to make a greater contribution to physical, chemical and biological properties [Miličević and Nikolić (2004)]. So the modified Zagreb index was proposed to correct the problem that former Zagreb M_2^*(G) index contributing more weights to inner bonds and less weights to outer bonds. According to chemists' intuition, this index, on the contrary, outer bonds should have greater weights than inner bonds.

Definition 11 *The modified Zagreb index $M_2^*(G)$ is defined as*

$$M_2^*(G)=\sum_{e=uv\in E(G)}\frac{1}{d(u)d(v)} \tag{11}$$

Zhou et al. [Zhou and Trinajstić (2009)] presents a novel connectivity index for molecular graphs, named sum-connectivity index, gives lower and upper bounds when graph structural invariant.

Definition 12 *The sum-connectivity index is defined [Zhou and Trinajstić (2009)] as*

$$X(G)=\sum_{e=uv\in E(G)}\frac{1}{\sqrt{d(u)+d(v)}} \tag{12}$$

Vukičević proposed a new topological index based on the end-vertex degrees of edges named as geometrical-arithmetic index (GA), and presented its basic features.

Definition 13 *The expression of geometric-arithmetic index is defined as*

$$GA(G)=\sum_{e=uv\in E(G)}\frac{2\sqrt{d(u)d(v)}}{d(u)+d(v)} \tag{13}$$

Graovac et al. [Graovac, Ghorbani and Hosseinzadeh (2011)] lately introdue the fifth geometric-arithmetic topological index.

Definition 14 *The fifth geometric-arithmetic topological index GA_5 is defined as*

$$GA_5(G)=\sum_{e=uv\in E(G)}\frac{2\sqrt{s_G(u)s_G(v)}}{s_G(u)+s_v(v)} \tag{14}$$

In 2013, Shirdel et al. [Shirdel, Rezapour and Sayadi (2013)] defined a new distance-based Zagreb indice named “hyper-zagreb index”.

Definition 15 *The hyper-Zagreb index is defined as*

$$HM(G)=\sum_{e=uv\in E(G)}\left(d(u)+d(v)\right)^2 \tag{15}$$

Based on the many researches recently, Hosamani [Hosamani (2017)] proposed a new topological index, named “Sanskruti index S(G)” of a molecular graph G that can be utilized to estimate the bioactivity of chemical compounds.

Definition 16 *The Sankruti index of a graph G is defined as*

$$S(G)=\sum_{uv\in E(G)}\left(\frac{s_G(u)s_G(v)}{s_G(u)+s_G(v)-2}\right)^3 \tag{16}$$

Based on degrees of vertices in a given molecular graph, Ghorbani et al. [Ghorbani and Azimi (2012)] produced the multiple versions of Zagreb indices. They are named as the first multiple Zagreb index and the second multiple Zagreb index.

Definition 17 *The first multiple Zagreb index $PM_1(G)$ is defined as:*

$$PM_1(G)=\prod_{uv\in E(G)}(d(u)+d(v)) \tag{17}$$

Definition 18 *The second multiple Zagreb index $PM_2(G)$ is defined as:*

$$PM_1(G)=\prod_{uv\in E(G)}(d(u)d(v)) \tag{18}$$

In 1984, Narumi et al. [Narumi and Katayama (1984)] defined "simple topological index" as the product of orders at vertexes of a graph. It can be used on the study of thermodynamic data including boiling points. Recently, more studies on the graph widely introduced "Narumi Katayama index" [Tomovic and Gutman (2001)].

Definition 19 *Narumi et al. [Narumi and Katayama **(1984)**] defined this index as follow:*

$$NK(G)=\prod\nolimits_{u\in V(G)} d(u) \tag{19}$$

Definition 20 *Ghorbani et al. [Ghorbani and Azimi (2012)] defined the modified Narumi-Katayama index, in which each vertex degree d is multiplied d times.*

$$NK^*(G)=\prod\nolimits_{u\in V(G)} d(u)^{d(u)} \tag{20}$$

It can be seen in this paper, the modified Narumi-Katayama index and second multiplicative Zagreb index are the same.

2 Results

The Archimedean lattices are the graphs of vertex transitive, it can be embedded in a plane that each face is a regular polygon [Martinez (1973)].The family of Archimedean lattices contains 11 kinds of 2D lattices, which include the famous honeycomb lattices and square, triangle, kagomé. Based on the sizes of faces incident to a given vertex, the names of the lattices are given. The sizes of face are listed in order, from the smallest to largest. So, in this way, we can nominate the Archimedean lattices as follow (4, 4, 4, 4), abbreviated to (4^4), Kagome is (3, 6, 3, 6) [Codello (2010)] and honeycomb is called (6^3). In this paper, we define an archimedean lattice called (4, 6, 12). And we name this lattice $L_{4,6,12}$ circumference. In geometry, the Great rhombitrihexagonal contains one dodecagon, two hexagons, and three squares on each edge, as is shown in Fig. $L_{4,6,12}$ (*n*) and in Fig. $L_{4,6,12}$ (*n*) molecular structure where *n*=2 circle is shown. Furthermore Fig. $L_{4,6,12}$ (*n*) presents the *n*=3 circle.

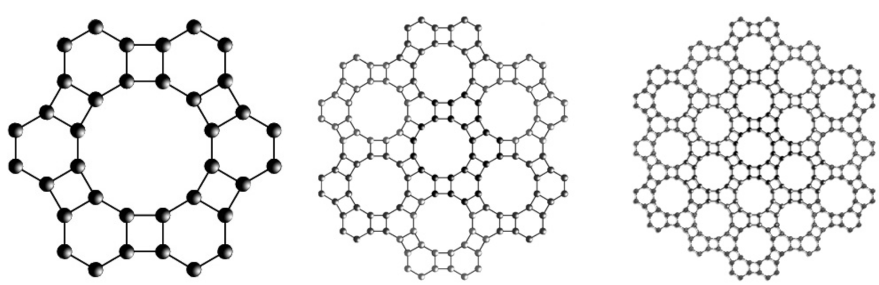

Figure 1: The Archimedean lattice $L_{4,6,12}$(n)

In terms of observing and computing, we can induce the $|V_2|=6n$, $|V_3|=18n^2-6n$. And by means of further calculating, we can infer $E(s,t)$ and $S(s,t)$ corresponding to different s, and t, as is shown in the Tab. 1.

Table 1: Partition the edge set of $L_{4,6,12}(n)$ into $E_{s,t}$

(s,t)	$(3,3)$	$(2,3)$	$(2,2)$
$\|E_{s,t}\|$	$54n^2-24n$	$12n$	$6n$

Table 2: Partition the edge set of $L_{4,6,12}(n)$ into $E'_{s,t}$

(s,t)	(9,9)	(8,9)	(8,8)	(5,8)	(5,5)
$\|E_{s,t}\|$	$54n^2-48n+6$	$24n-12$	6	$12n$	$6n$

Theorem 1 *Let G be and $L_{4,6,12}(n)$ circumference, then $ABC(G)=36n^2-16n+9\sqrt{2}\,n$.*

Proof The result is obtained based on the edge partition given in Tab. 1. By the definition of ABC_4 index, we have

$$
\begin{aligned}
ABC(G) &= \sqrt{\frac{4}{9}}\,|E_{3,3}|+\sqrt{\frac{3}{6}}\,|E_{2,3}|+\sqrt{\frac{2}{4}}\,|E_{2,2}| \\
&= \sqrt{\frac{4}{9}}\,(54n^2-24n)+\sqrt{\frac{3}{6}}\,(12n)+\sqrt{\frac{2}{4}}\,(6n) \qquad (21)\\
&= 36n^2-16n+9\sqrt{2}\,n
\end{aligned}
$$

Theorem 2 *Let G be an $L_{4,6,12}(n)$ circumference, then $ABC_4(G)=\frac{4}{3}(18n^2-16n+2)+\frac{3\sqrt{110}}{5}n+\frac{12\sqrt{2}}{5}n+\sqrt{30}(2n-1)+\frac{3\sqrt{14}}{4}$*

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of Atom-bond Connectivity index, we have

$$
\begin{aligned}
ABC_4(G) &= \sqrt{\frac{16}{81}}\,|E'_{9,9}|+\sqrt{\frac{15}{72}}\,|E'_{8,9}|+\sqrt{\frac{14}{64}}\,|E'_{8,8}|+\sqrt{\frac{11}{40}}\,|E'_{5,8}|+\sqrt{\frac{8}{25}}\,|E'_{5,5}| \\
&= \sqrt{\frac{16}{81}}\,(54n^2-48n+6)+\sqrt{\frac{15}{72}}\,(24n-12)+\sqrt{\frac{14}{64}}\times 6 \qquad (22)\\
&\quad +\sqrt{\frac{11}{40}}\,(12n)+\sqrt{\frac{8}{25}}\,(6n) \\
&= \frac{4}{3}(18n^2-16n+2)+\frac{3\sqrt{110}}{5}n+\frac{12\sqrt{2}}{5}n+\sqrt{30}\,(2n-1)+\frac{3\sqrt{14}}{4}
\end{aligned}
$$

Theorem 3 *Let G be an* $L_{4,6,12}(n)$ *circumference, then* $R(G)= 18n^2-8n+3\sqrt{6}\,n$

Proof Based on the edge partition given in Tab. 2. The result of equation can be obtained. By the definition of Randić connectivity index, we have

$$\boldsymbol{R(G)}=\frac{1}{\sqrt{3\times 3}}\,|E_{3,3}|+\frac{1}{\sqrt{2\times 3}}\,|E_{2,3}|+\frac{1}{\sqrt{2\times 2}}\,|E_{2,2}|$$

$$=\frac{1}{\sqrt{9}}\,(54n^2-24n)+\frac{1}{\sqrt{6}}\,(12n)+\frac{1}{\sqrt{6}}\,(6n) \tag{23}$$

$$=18n^2-8n+3\sqrt{6}\,n$$

Theorem 4 *Let G be an* $L_{4,6,12}(n)$ *circumference, then* $X(G)=\frac{9\sqrt{6}}{9}n^2-\frac{5\sqrt{6}}{2}n+\frac{12\sqrt{5}}{5}n$ **.**

Proof The result is obtained based on the edge partition given in Tab. 2. According to the definition of sum-connectivity index, we have

$$X(G)=\frac{1}{\sqrt{6}}\,|E_{3,3}|+\frac{1}{\sqrt{5}}\,|E_{2,3}|+\frac{1}{\sqrt{4}}\,|E_{2,2}|$$

$$=\frac{1}{\sqrt{6}}\,(54n^2-24n)+\frac{1}{\sqrt{5}}\,(12n)+\frac{1}{\sqrt{4}}\,(6n) \tag{24}$$

$$=\frac{9\sqrt{6}}{9}n^2-\frac{5\sqrt{6}}{2}n+\frac{12\sqrt{5}}{5}n$$

Theorem 5 *Let G be an* $L_{4,6,12}(n)$ *circumference, then* $M_1(G)= 32n^2-60n$**.**

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of First Zagreb index, we have

$$M_1(G)=\mathbf{6}\,|E_{3,3}|+\mathbf{5}|E_{2,3}|+\mathbf{4}|E_{2,2}|$$

$$=\mathbf{6}\,(54n^2-24n)+\mathbf{5}(12n)+\mathbf{4}(6n) \tag{25}$$

$$=324n^2-60n.$$

Theorem 6 *Let G be an* $L_{4,6,12}(n)$ *circumference, then* $M_2G)=486n^2-120n$ **.**

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of Second Zagreb index, we have

$$M_1(G)=\mathbf{9}\,|E_{3,3}|+6|E_{2,3}|+\mathbf{4}|E_{2,2}|$$

$$=\mathbf{9}\,(54n^2-24n)+\mathbf{6}(12n)+\mathbf{4}(6n) \tag{26}$$

$$=486n^2-120n.$$

Theorem 7 *Let G be an* $L_{4,6,12}(n)$ *circumference, then* $M_1(G,x)=(54n^2-24n)x^6+12nx^5+6nx^4$

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of First Zagreb polynomial $M_1(G,x)$ index, we have

$$M_1(G,x)=\sum_{uv\in E(G)} x^{d(u)+d(v)}$$

$$=\sum_{uv\in E_{3,3}}x^{6}+\sum_{uv\in E_{2,3}}x^{5}+\sum_{uv\in E_{2,2}}x^{4} \tag{27}$$

$$=(54n^2-24n)x^6+12nx^5+6nx^4$$

Theorem 8 *Let G be an $L_{4,6,12}(n)$ circumference, then $M_2(G,x)=(54n^2-24n)x^9+12nx^6+6nx^4$*

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of Second Zagreb polynomial $M_2(G,x)$ index, we have

$$M_2(G,x)=\sum_{uv\in E(G)}x^{d(u)d(v)}$$

$$=\sum_{uv\in E_{3,3}}x^{9}+\sum_{uv\in E_{2,3}}x^{6}+\sum_{uv\in E_{2,2}}x^{4} \tag{28}$$

$$=(54n^2-24n)x^9+12nx^6+6nx^4$$

Theorem 9 *Let G be an $L_{4,6,12}(n)$ circumference, then $AZI(G)=\frac{19683}{32}n^2-\frac{1035}{8}n$*

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of Augmented Zagreb index, we have

$$AZI(G)=\left(\frac{3\times 3}{3+3-2}\right)^3|E_{3,3}|+\left(\frac{2\times 3}{2+3-2}\right)^3|E_{2,3}|+\left(\frac{2\times 2}{2+2-2}\right)^3|E_{2,2}|$$

$$=\left(\frac{3\times 3}{3+3-2}\right)^3(54n^2-24n)+\left(\frac{2\times 3}{2+3-2}\right)^3(12n)+\left(\frac{2\times 2}{2+2-2}\right)^3(6n) \tag{29}$$

$$=\frac{19683}{32}n^2-\frac{1035}{8}n.$$

Theorem 10 *Let G be an $L_{4,6,12}(n)$ circumference, then* $\boldsymbol{M_2^*(G)}=6n^2-\frac{5}{6}n$

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of modified Zagreb index, we have

$$\boldsymbol{M_2^*(G)}=\frac{1}{3\times 3}|E_{3,3}|+\frac{1}{2\times 3}|E_{2,3}|+\frac{1}{2\times 2}|E_{2,2}|$$

$$=\frac{1}{3\times 3}(54n^2-24n)+\frac{1}{2\times 3}(12n)+\frac{1}{2\times 2}(6n) \tag{30}$$

$$=6n^2-\frac{5}{6}n.$$

Theorem 11 *Let G be an $L_{4,6,12}(n)$ circumference, then $HM(G)=1944n^2-468n$*

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of hyper-Zagreb index, we have

$$HM(G) = (3+3)^2 |E_{3,3}| + (2+3)^2 |E_{2,3}| + (2+2)^2 |E_{2,2}|$$
$$= (3+3)^2 (54n^2-24n) + (2+3)^2 (12n) + (2+2)^2 (6n) \quad (31)$$
$$=1944n^2-468n.$$

Theorem 12 *Let G be an $L_{4,6,12}(n)$ circumference, then* $GA(G)=54n^2-\frac{24\sqrt{6}}{5}n-18n$

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of Geometric-Arithmetic index, we have

$$GA(G)=\frac{2\times\sqrt{9}}{6}|E_{3,3}|+\frac{2\times\sqrt{6}}{5}|E_{2,3}|+\frac{2\times\sqrt{4}}{4}|E_{2,2}|$$
$$=\frac{2\times\sqrt{9}}{6}(54n^2-24n)+\frac{2\times\sqrt{6}}{5}(12n)+\frac{2\times\sqrt{4}}{4}(6n) \quad (32)$$
$$=54n^2-\frac{24\sqrt{6}}{5}n-18n.$$

Theorem 13 *Let G be an $L_{4,6,12}(n)$ circumference, then* $GA_5(G)=54n^2+\frac{48\sqrt{10}}{13}n-42n+\frac{384\sqrt{2}(2n-1)}{17}+12$

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of GA_5 index, we have

$$GA_5(G)=\frac{2\times\sqrt{81}}{18}|E'_{9,9}|+\frac{2\times\sqrt{72}}{17}|E'_{8,9}|+\frac{2\times\sqrt{64}}{16}|E'_{8,8}|$$
$$+\frac{2\times\sqrt{40}}{13}|E'_{5,8}|+\frac{2\times\sqrt{25}}{10}|E'_{5,5}|$$
$$=\frac{2\times\sqrt{81}}{18}(54n^2-48n+6)+\frac{2\times\sqrt{72}}{17}(24n-12)+\frac{2\times\sqrt{64}}{16}\times 6$$
$$+\frac{2\times\sqrt{40}}{13}(12n)+\frac{2\times\sqrt{25}}{10}(6n) \quad (33)$$
$$=54n^2+\frac{48\sqrt{10}}{13}n-42n+\frac{384\sqrt{2}}{17}(2n-1)+12$$

Theorem 14 *Let G be an $L_{4,6,12}(n)$ circumference, then* $S(G)=\frac{531441}{4096}(54n-48n+6)^2-\frac{258998625}{340736}n+\frac{165888(2n-1)}{125}+\frac{196608}{343}$

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of Sankruti index, we have

$$GA_5(G)=\left(\frac{9\times 9}{9+9-2}\right)^3|E'_{9,9}|+\left(\frac{8\times 9}{8+9-2}\right)^3|E'_{8,9}|+\left(\frac{8\times 8}{8+8-2}\right)^3|E'_{8,8}|$$

$$+\left(\frac{5\times 8}{5+8-2}\right)^3|E'_{5,8}|+\left(\frac{5\times 5}{5+5-2}\right)^3|E'_{5,5}|$$

$$=\left(\frac{9\times 9}{9+9-2}\right)^3(54n^2-48n+6)+\left(\frac{8\times 9}{8+9-2}\right)^3(24n-12)+\left(\frac{8\times 8}{8+8-2}\right)^3\times 6 \qquad (34)$$

$$+\left(\frac{5\times 8}{5+8-2}\right)^3(12n)_+\left(\frac{5\times 5}{5+5-2}\right)^3(6n)$$

$$=\frac{531441}{4096}(54n-48n+6)^2-\frac{258998625}{340736}n+\frac{165888(2n-1)}{125}+\frac{196608}{343}$$

Theorem 15 *Let G be an $L_{4,6,12}(n)$ circumference, then $F(G)=972n^2-228n$.*

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of forgotten index, we have

$$F(G)=\left(3^2+3^2\right)|E_{3,3}|+\left(2^2+3^2\right)|E_{2,3}|$$

$$=\left(3^2+3^2\right)(54n^2-24n)+\left(2^2+3^2\right)(12n)+\left(2^2+2^2\right)(6n) \qquad (35)$$

$$=972n^2-228n.$$

Theorem 16 *Let G be an $L_{4,6,12}(n)$ circumference, then $H(G)=18n^2-\frac{1}{5}n$.*

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of Harmonic index, we have

$$H(G)=\frac{2}{3+3}|E_{3,3}|+\frac{2}{2+3}|E_{2,3}|+\frac{2}{2+2}|E_{2,2}|$$

$$=\frac{2}{3+3}(54n^2-24n)+\frac{2}{2+3}(12n)+\frac{2}{2+2}(6n) \qquad (36)$$

$$=18n^2-\frac{1}{5}n.$$

Theorem 17 *Let G be an $L_{4,6,12}(n)$ circumference, then* $PM_1(G)=6^{54n^2-24n}\times 5^{12n}\times 4^{6n}$

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of first multiple Zagreb index, we have

$$PM_1(G)=\prod_{uv\in E(G)}(d(u)+d(v))$$
$$=6^{|E_{3,3}|}\times 5^{|E_{2,3}|}\times 4^{|E_{2,2}|} \tag{37}$$
$$=6^{54n^2-24n}\times 5^{12n}\times 4^{6n}$$

Theorem 18 *Let G be an $L_{4,6,12}(n)$ circumference, then* $PM_1(G)=9^{54n^2-24n}\times 6^{12n}\times 4^{6n}$

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of first multiple Zagreb index, we have

$$PM_2(G)=\prod_{uv\in E(G)}(d(u)d(v))$$
$$=\mathbf{9}^{|E_{3,3}|}\times 6^{|E_{2,3}|}\times 4^{|E_{2,2}|} \tag{38}$$
$$=\mathbf{9}^{54n^2-24n}\times 6^{12n}\times 4^{6n}$$

Theorem 19 *Let G be an $L_{4,6,12}(n)$ circumference, then* $NK(G)=2^{6n}\times 3^{18n^2-6n}$

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of second multiple Zagreb index, we have

$$NK(G)=\prod_{u\in V(G)}d(u)$$
$$=\mathbf{2}^{|V_2|}\times \mathbf{3}^{|V_3|} \tag{39}$$
$$=2^{6n}\times 3^{18n^2-6n}$$

Theorem 20 *Let G be an $L_{4,6,12}(n)$ circumference, then* $NK^*(G)=4^{6n}\times 27^{18n^2-6n}$

Proof The result is obtained based on the edge partition given in Tab. 2. By the definition of modified Narumi-Katayama index, we have

$$NK^*(G)=\prod_{u\in V(G)}d(u)^{d(u)}$$
$$=\mathbf{4}^{|V_2|}\times 27^{|V_3|} \tag{40}$$
$$=4^{6n}\times 27^{18n^2-6n}$$

3 Conclusions

The topological index is defined for a topological graph which has nothing to do with distances, angles, steric strain, or hindrance. Therefore, without substantial modification, the topological index cannot account for the difference in physical quantities of geometrical isomers, or for the overcrowded effect expected for compounds having vicinal quarter carbon atoms. So these indexes are expected to be dependent on the topological nature of the skeleton of a system like a molecule.

A class of dodecagon honeycomb network which is covered by C_4, C_8 and C_{12} has been investigated here, and formulas for their Randić connectivity index, fourth Atom-Bond Connectivity index, Geometric-Arithmetic index, fifth Geometric-Arithmetic index, Atom-Bond Connectivity index, First Zagreb index, Second Zagreb index and the corresponding Zagreb polynomials, modified Zagreb index, Augmented Zagreb index, hyper-Zagreb index, forgotten topological index, first multiple Zagreb index, second multiple Zagreb, harmonic index, Sankruti index, index, Narumi-Katayama index and modified Narumi-Katayama index have been derived.

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