 CMC, vol.58, no.2, pp.409-420, 2019

Research on the Signal Reconstruction of the Phased Array Structural Health Monitoring Based Using the Basis Pursuit Algorithm

Yajie Sun[1, 2, *], Yanqing Yuan[2], Qi Wang[2], Lihua Wang[3], Enlu Li[2] and Li Qiao[4]

Abstract: The signal processing problem has become increasingly complex and demand high acquisition system, this paper proposes a new method to reconstruct the structure phased array structural health monitoring signal. The method is derived from the compressive sensing theory and the signal is reconstructed by using the basis pursuit algorithm to process the ultrasonic phased array signals. According to the principles of the compressive sensing and signal processing method, non-sparse ultrasonic signals are converted to sparse signals by using sparse transform. The sparse coefficients are obtained by sparse decomposition of the original signal, and then the observation matrix is constructed according to the corresponding sparse coefficients. Finally, the original signal is reconstructed by using basis pursuit algorithm, and error analysis is carried on. Experimental research analysis shows that the signal reconstruction method can reduce the signal complexity and required the space efficiently.

Keywords: Basis pursuit algorithm, compressive sensing, phased array, signal reconstruction.

1 Introduction

Structural Health Monitoring (SHM) technology is an important application of the intelligent material structure practical enginee+96ring [Cao, Thaker, Oseng et al. (2015); Andrea, Pietro, Marco et al. (2016)]. The data acquisition and processing system is an important part of the structural health monitoring. Many scholars internationally have implemented different signal processing methods in structural health monitoring, such as model analysis [Bassoli, Forghieri and Vincenzi (2017)], system identification [Nagarajaiah (2107)], wavelet transform, optimization calculation [Li (2015)] and so on. However, the

[1] Jiangsu Engineering Centre of Network Monitoring, Nanjing University of Information Science and Technology, Nanjing, 210044, China.

[2] School of Computer and Software, Nanjing University of Information Science and Technology, Nanjing, 210044, China.

[3] School of Information and Control, Nanjing University of Information Science and Technology, Nanjing, 210044, China.

[4] Capability Systems Centre, School of Engineering and Information Technology, University of New South Wales, Canberra, 2600, Australia.

[*] Corresponding Author: Yajie Sun. Email: syj@nuist.edu.cn.

majority of research findings are based on the simplified structure and carried out in a controllable laboratory environment. Research has been very successful at the real collected sensor signals which contains various signals. In general, due to the environment noise and the complex properties of composites, the extraction of the damaged characteristic signal is very difficult. In a real-time and on-line monitoring operation a large amount of data can be generated, which increases the complexity of data processing. Thus choosing a suitable method of signal processing is particularly important. Signal processing based on compressive sensing technology has been applied in many disciplines and engineering fields [Jin, Yang, Chen et al. (2014); Liu (2012)]. For example, in the analysis of signal channel bandwidth in wireless communication, the data is sampled in a compressed sampling manner [Haupt, Bajwa, Raz et al. (2010)]. In image processing, Rice University used compressive sensing technology to develop a new “single pixel camera” [Duarte, Davenport, Takhar et al. (2008)]. In the late 1990s, A. Abbate [Abbate, Frenkel and Das (1995)] proposed a new type of signal processing device, and the ultrasonic signal processing is not limited to the analysis of amplitude, phase and frequency domain. It can provide a powerful guarantee for the qualitative and quantitative analysis of the echo signal. This article proposes a method to realize the structural health monitoring of composite materials using ultrasonic phased array technology. The ultrasonic phased array detection technology uses different shapes of multi array elements to generate and receive ultrasonic beam [Sun, Zhang, Qian et al. (2013); Sun and Ji (2015)]. Where, the phased array transducer contains a plurality of piezoelectric elements in a certain way, where each piezoelectric element can independently transmit and receive ultrasonic signals [Liu (2012)]. This method consists of 3 steps: Firstly it proposes the sparse signal using discrete cosine transform; secondly designs a phased array signal acconding to the observation matrix; finally reconstructs the signal based on basis pursuit. The reconstruction uses the compressive sensing theory to process the data acquisition and signal compression at the same time, which greatly reduces the computational complexity and reduces the complexity of the data signal processing and saves the storage space.

The paper is organized as follows: Section 1 introduces the research status of structural health monitoring based on compressive sensing; Section 2 explains the principles of compressive sensing and sparse signal processing methods; Section 3 uses experimental verification signal reconstruction methods to reduce signal complexity and save space complexity degree of effect.

2 Compressive sensing fundamentals

Compressive Sensing (CS), which is derived from the traditional signal sampling theory, is a new sampling and recovery theory for sparse signal [Friedland, Li and Schonfeld (2014)]. The sparse representation of signal, the design of observation matrix and the reconstruction of signal are introduced in Yu et al. [Yu, Li, Wang et al. (2012)]. If the original signal is sparse in the time domain or a certain transformation base, the signal can be collected at low sampling rate, and then achieve high probability reconstruction with compressive sensing technology. Compared to other sampling methods, this method can effectively reduce the complexity of the data, and has a wider range of superiority and universality [Donoho (2006)]. The whole signal processing is shown in Fig. 1.

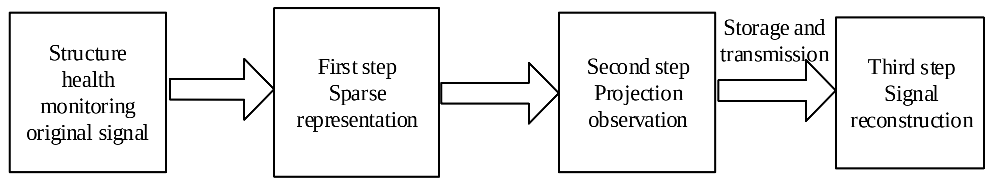


Figure 1: Signal processing of compressive sensing

2.1 Sparse representation

In the structural health monitoring signal acquisition process, the signal obtained through the piezoelectric array has continuous amplitude. The number of non-zero values in the sparse coefficient vector can be reduced by selecting the appropriate sparse transform base, and a sparse representation of the phased array signal can be obtained. In the process of sparse representation, the signal is projected onto an orthogonal base. If the obtained transformation vector satisfies the characteristics of sparsity, it can be used as a sparse representation of the original signal, which is the prior condition of the compressive sensing. That is to say if a signal satisfies certain conditions, the sparse representation of the signal can be obtained by selecting one transformation base. In general, the commonly used transform bases are: Fourier Transform, Discrete Cosine Transform, Wavelet Basis [Song, Guo and Zhu (2014)], Curvelet Basis [Shahidi, Tang and Ma (2013)], and so on. At present, in addition to the traditional Fourier Analysis, the advanced signal processing methods are widely used in the study, such as Wavelet Analysis [Soni, Jain and Roshan (2013)], HHT analysis method [Petrov (2016)], and so on.

Discrete Cosine Transform (DCT) is an orthogonal transformation method, which has the best correlation, and is often considered as a quasi-optimal transformation, moreover, it is also often used to lossy data compression of signals or image [Bayer and Cintra (2017)]. As the widely use of the special integrated circuit, the discrete cosine transform is constantly strengthened.

Discrete Fourier Transform (DFT) is a discrete form of Fourier transform in time domain and frequency domain. The time domain sampling of signal is transformed into its DFT frequency domain sampling [West, Harwell and McCall (2017)].

Principal Component Analysis (PCA) can be used to deal with the data of high dimension, noise and high correlation by projecting the data into the low dimensional space and the most possible features of the original data [Singh, Sharma and Dandapat (2016)].

In compressive sensing, many complex signals in real life can expressed in a more concise way, or can be more succinctly expressed under some orthogonal basis transforms. This indicates that these complex signals are sparse under some orthogonal bases. In many cases, there is only a small amount of useful information in the high-dimensional signal, and therefore, the key to obtain a sparse representation is to remove the redundant degree of signal [Zhu (2014); Gleichman and Eldar (2011)]. That is, by compressing the signal, the sparse representation of the signal can be obtained.

Suppose R^N is a set of real numbers space, any signal in this space can be expressed by a linear combination of a plurality of $N \times 1$ dimensional matrix vector $\{\varphi_i\}_{i=1}^{N}$. Assume that these basis vectors are orthogonal. $N \times N$ -dimensional sparse matrix transform base can be seen as based on ψ_i column, that is $\Psi = [\psi_1, \psi_2, \ldots, \psi_N]$. Therefore any one of a limited length of one-dimensional discrete signal $x \in R^N$ can be expressed as:

$$x = \Psi\Theta = \sum_{i=1}^{N} \theta_i \psi_i \tag{1}$$

where Θ is the sparse coefficient vector. If most of the elements of Θ are zero, the one dimensional discrete time signal $x \in R^N$ is sparse. At this time, the sparse coefficients are satisfied:

$$\Theta = \Psi^T x \text{ or } \theta_i = \langle x, \psi_i \rangle = \psi_i^T x \tag{2}$$

where the signal x is the expression in time domain, $<\cdot>$ is the inner product, T is transpose of a vector, the sparse coefficient Θ represents the form of x by the sparse transform Ψ .

Set the signal $x = \{x_1, x_2, \ldots x_N\}$

$$\|x\|_p = \left(\sum_{i=1}^{N} |x_i|^p \right)^{1/p} \tag{3}$$

where $\|x\|_p$ is the l_p - norm of x.

For any real number, when $\|\Theta\|_p \le k$, if $0 < p < 2$, $0 < k << N$, the vector Θ is considered to be k sparse in sparse transform base Ψ [Jiang (2015)].

2.2 Projection observation

The sparse coefficients can be obtained after sparse decomposition of the original signal, according to the sparse coefficients [Feng, Zhang and Liu (2016)]. Thus we can construct the observation matrix which is not related to the transform of the signal. A projection observation vector y_i is obtained by the inner product of the projection matrix Φ which contains a M row vectors $\{\phi\}_{i=1}^N$ and the spares coefficient vector Θ , where the dimensions of the projection vectors is M .

$$y_i = \langle \Theta, \phi_i \rangle \tag{4}$$

Set the projection observation vector $y = (y_1, y_2, \cdots, y_M)$,

$$Y = \Phi\Theta = \Phi\Psi^T x = \Xi x \tag{5}$$

In Eq. (5), the solution of Θ is an undetermined problem. However, because of the sparsity $k << M$, the problem can be solved, and the original signal can be restored with high probability.

The matrix Φ for projection transformation needs to be irrelevant to the sparse transformations matrix Ψ and to satisfy the finite equidistant principle. It is found that the Gauss random observation matrix has the characteristics that it is not related to the majority of the fixed orthogonal bases, and meet the compressive sensing requirement. Therefore, the Gauss matrix can be worked as the projection observation matrix.

2.3 Signal reconstruction

Signal reconstruction process uses the known M -dimensional projection observation vector to accurately reconstruct the N -dimensional original signal x , where the $N >> M$. Candes has proved that the signal reconstruction can be realized by solving the minimum l_0 -norm [Zhu (2014)] problem. It means the solution of the undetermined equation $Y = \Phi\Theta = \Phi\Psi^T x = \Xi x$ can be replaced by solving the minimum l_0 -norm.

$$\min\|\Theta\|_0 \ s.t. \ Y = \Phi\Theta = \Phi\Psi^T x \tag{6}$$

where *s.t.* means the constraint. Eq. (6) is the solution of a Linear Programming (LP) problem, which is a convex optimization problem [Li (2015)]. If the reconstruction error is taken into account, the Eq. (7) can be converted to the minimum l_1 -norm problem:

$$\min\|\Theta\|_1 \ s.t. \ \|\Phi\Theta - Y\|_2 \le \varepsilon \tag{7}$$

In the process of signal reconstruction, the most important task is to find the sparse solution to meet Eq. (7). In order to solve this problem, there are two approaches can be considered, which are convex optimization and greedy algorithm [Xia, Wang, Sun et al. (2015)]. The convex optimization algorithm obtains the sparsest mainly by increasing the constraint. Its norm bound form is expressed as:

$$\min\|\alpha\|_p, \quad s.t. \quad y = \Phi\Psi\alpha \tag{8}$$

Where $p \in R$, $\left(\|\alpha\|_p\right)^p = \sum_{i=1}^{N} |\alpha_i|^p$.

Literature shows that the $l_p (0 < p \le 1)$ has a good application in the sparse representation [Zhang, Xu, Li et al. (2016)]. Commonly used algorithms are basis pursuit algorithm based on the linear programming and the gradient projection sparse reconstruction algorithm. The greedy algorithm is to use matching pursuit algorithm, combined with the local optimization approach to find the non-zero coefficient. On this basis, the orthogonal matching pursuit method is developed [Needell and Vershynin (2010)].

Basis Pursuit (BP) algorithm [Xiao, Zhao and Li (2013)], which is represented in the form of norm sparsity signal, namely by means of minimizing the number of signal norm sparse representation problem into a constrained extremum problems, thereby the problem is transformed into a linear programming problem.

In the reconstruction algorithm, the minimum l_1 norm and minimum l_0 norm have equivalence, and can be interchangeable under certain conditions. Then the Eq. (8) is converted to an optimization problem under the minimum l_1 -norm:

$$\min_{\alpha}\|\alpha\|_{l_1} \quad s.t. \quad y = \Phi\Psi\alpha \tag{9}$$

Because the number of observations is far less than the length of the signal, it is difficult to solve a set of undetermined equations in the signal reconstruction process. The undetermined equation has infinitely many solutions, the solution process is difficult to achieve. However, since the signal passes through the sparse transformation, the issue can be resolved. Restricted Isometry Property (RIP) is also a theoretical guarantee for the solution of the observation matrix. The basis pursuit algorithm is based on the above principle to reconstruct the original signal.

3 Experimental verification

In this study, a sensor linear array which consists of 8 piezoelectric sensors is arranged in a 800 mm×800 mm×3 mm plate. The diameter and the thickness of the piezoelectric sensors are 8 mm and 0.48 mm, respectively. The distance between two adjacent piezoelectric sensors is 12 mm.

We use the phased array to scan the aluminum plate structure and obtain 8×7×181 groups data, here we select one group to process the data, which is emitted by the No. 0 array element and received by the No. 1 array element in the 90 degree direction; other angles take the same approach. The time domain waveform of the data obtained from the group is shown in Fig. 2.

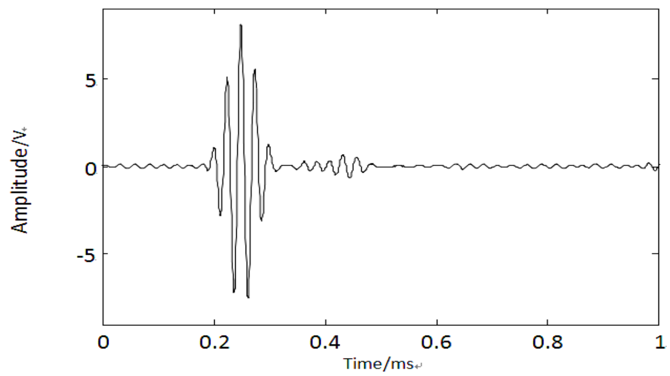


Figure 2: Original signals in the time domain waveform

3.1 Sparse representation based on discrete cosine transforms

Let X be a time series of length N, $X=\{X(0),X(1),\cdots,X(N-1)\}$, $N\in Z$. The one-dimensional DCT transform $Y(k)$ and its inverse transform $X(\mathrm{n})$ are presented in Eq. (10) and Eq. (11):

$$Y(k)=a_k\sum_{n=0}^{N-1}X(n)\cos\frac{\pi(2n+1)k}{2N} \tag{10}$$

$$X(n)=\sum_{k=0}^{N-1}a_kY(k)\cos\frac{\pi(2n+1)k}{2N} \tag{11}$$

where $n=0,1,\cdots,N-1$; $Y(k)$ is one-dimensional DCT transformed component, a_k is the one-dimensional DCT transform coefficient, the expression of a_k is shown in the Eq. (12):

$$\mathrm{a}_k=\begin{cases}1/\sqrt{N} & (k=0)\\ 2/\sqrt{N} & (k>0)\end{cases} \tag{12}$$

According to the Eq. (10), $Y(0)$ is the maximum constant after one-dimensional DCT transform. According to the Eq. (11), after removing the constant, the shape of the signal does not change, only a certain phase shift occurs, which can prove that the DC component $Y(0)$ is not suitable for a signal characteristics. Then, in the process of selecting the AC component, the AC component with larger amplitude is chosen as the characteristic of the signal.

DCT is used to process the waveforms received by the 0-array-element to receive No. 1 array element in 90 degree direction. Fig. 3 shown in the sparse representation of the original signal is obtained, In Fig. 3, the value of most sparse coefficients of the signal after DCT is equal to zero or close to zero, which is consistent with the nature of sparse signal.

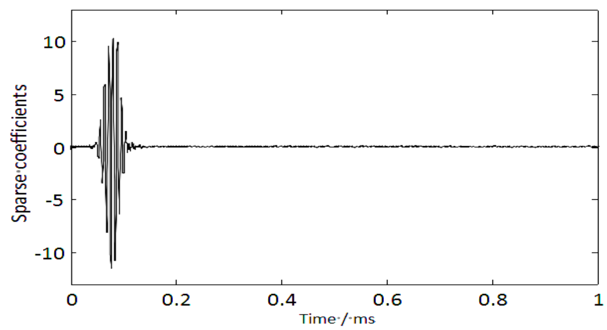


Figure 3: DCT sparse coefficients of the original signal

3.2 Design the observation matrix for the phased array signals

For the processing of ultrasonic signals, the Gauss random matrix is multiplied with the sparse coefficient of the signal, and the observation vector of the signal can be obtained.

Let Φ is $M \times N$ matrix and its general term is:

$$\Phi(i, j) = \frac{1}{\sqrt{M}} h_{ij} \tag{13}$$

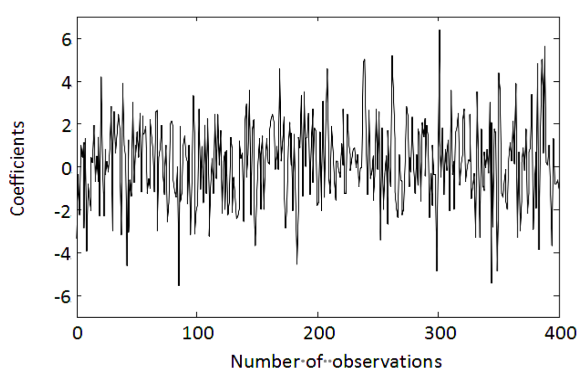
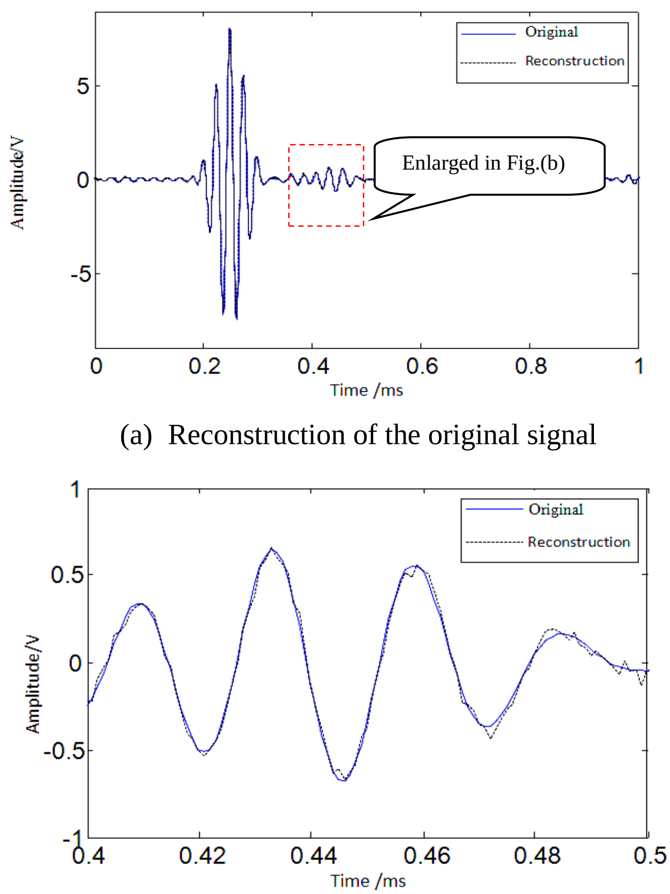


Figure 4: Signal obtained by Gaussian measurement

Each element in the matrix h_{ij} is independent of each other, and is subject to a mean of 0 and variance $1/\sqrt{M}$ Gaussian distribution. Studies show that the matrix cannot associated with most sparse orthogonal matrix [Wen, Zhang, Wong et al. (2014)], and the number of measurements value is relatively small. For the original data with length N and sparse degree K, only the $M \geq cK\log(N/K)$ measurements have a high possibility to restore the original signal. Where c is a very small constant value. In our example, the ultrasonic data length N is 1024, the projection observation of the signal uses the number of observed values 400. The resulting pattern is shown in Fig. 4.

3.3 Signal reconstruction based on the basis pursuit

We use BP algorithm to minimize the number of signal norm sparse representation problem into a constrained extremum problem, thereby the problem is transformed into a linear programming problem.



(b) The partial enlargement of the reconstructed signal

Figure 5: The reconstruction of basis pursuit

According to the basis pursuit to deal with the phased array signal, you can get the results as shown in Fig. 5. Fig. 5(a) shows the reconstruction of the original signal and Fig. 5(b) shows the reconstruction of BP algorithm.

3.4 Signal reconstruction error analysis

Fig. 5 shows the corresponding reconstructed signal obtained by using the basis pursuit algorithm. Tab. 1 shows the effectiveness of the reconstruction algorithm. In Tab. 1, the absolute error of ΔV is expressed as:

$$\Delta V = |V_0 - V_1| \tag{14}$$

Where, V_0 is the amplitude of the maximum point of the reconstructed signal; V_1 is the amplitude of the original phased array signal. The relative error of δ is present in Eq. (15).

$$\delta = \Delta V / V_1 * 100\% \tag{15}$$

Table 1: Reconstruction error

Algorithm name	BP algorithm
$\Delta V / V$	0.0985
$\delta / \%$	0.284

Table 2: The error comparison

Orthogonal Transformation	PCA	DCT	DFT
Absolute error/V	0.1871	0.8943	21.0229
Relative error/%	0.39	1.88	1.27

Tab. 2 shows the error comparison of some common transform base. It can be seen from the experimental error analysis that the BP algorithm used in this paper has low error.

4 Conclusion

This paper studies phased array signal reconstruction for structural health monitoring using the basis pursuit algorithm. The proposed method consists of three steps: Signal sparse representation, observation matrix design, signal reconstruction. Finally making error analysis of reconstructed signal. The signal from a sensor liner array on an aluminum plat is used as a study case to demonstrate how this approach effectively saves storage space and reduce the data complexity. The experimental also process its superiority in signal processing method by comparison to other commonly used algorithm such as PCA, DCT and DFT.

Acknowledgement: This project is supported by the National Natural Science Foundation of China (Grant No. 51305211), Natural Science Foundation of Jiangsu (Grant No. BK20160955), Jiangsu Government Scholarship for Overseas Studies, College students practice and innovation training project of Jiangsu province (Grant No. 201710300218), and the PAPD.

References

Abbate, A.; Frenkel, J.; Das, P. (1995): Wavelet transform signal processing for dispersion analysis of ultrasonic signals. *Ultrasonics Symposium*, vol. 1, no. 1, pp. 751-755.

Andrea, B.; Pietro, G.; Marco, C.; Capineri, L. (2016): An integrated acousto/ultrasonic structural health monitoring system for composite pressure vessels. *IEEE Transactions on Ultrasonics, Ferrorlectrics, and Frequency Control*, vol. 63, no. 6, pp. 864-873.

Bayer, F. M.; Cintra, R. J. (2017): DCT-like transform for image compression requires 14 additions only. *Electronics Letters*, vol. 48, no. 15, pp. 919-921.

Bassoli, E.; Forghieri, M.; Vincenzi, L.; Bovo, M.; Mazzotti, C. (2017): Structural health monitoring of a historical masonry bell tower using operational modal analysis. *Key Engineering Materials*, vol. 747, pp. 440-447.

Cao, H.; Thaker, S. K.; Oseng, M. L.; Nguyen, C. M.; Jebali, C. et al. (2015): Development and characterization of a novel interdigitated capacitive strain sensor for dtructure health monitoring. *IEEE Sensor Journal*, vol. 15, no. 11, pp. 6542-6548.

Chen, Y. H. (2011): Research of image compression based on discrete cosine transforms. *Modern Electronic Technology*, vol. 21, pp. 86-88.

Donoho, D. L. (2006): Compressed sensing. *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289-1306.

Duarte, M. F.; Davenport, M. A.; Takhar, D.; Laska, J. N.; Sun, T. et al. (2008): Single-pixel imaging via compressive sampling. *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 83-91.

Feng, J.; Zhang, G.; Liu, Y. (2016): Improved sparse signal reconstruction algorithm based on SL0 norm. *Journal of Data Acquisition and Processing*, vol. 31, no. 1, pp. 178-183.

Friedland, S.; Li, Q.; Schonfeld, D. (2014): Compressive sensing of sparse tensors. *IEEE Transactions on Image Processing*, vol. 23, no. 10, pp. 4438-4447.

Gleichman, S.; Eldar, Y. C. (2011): Blind compressive sensing. *IEEE Transactions on Information Theory*, vol. 57, no. 10, pp. 6958-6975.

Haupt, J.; Bajwa, W. U.; Raz, G.; Nowaak, R. (2010): Toeplitz compressed sensing matrices with applications to sparse channel estimation. *IEEE Transactions on Information Theory*, vol. 56, no. 11, pp. 5862-5875

Jiang, J. (2015): *Data Compression Sampling and Damage Identification of Phased Array Structural Health Monitoring (Ph.D. Thesis)*. Nanjing University of Information Science and Technology.

Jin, S. J.; Yang, X. X.; Chen, S. L.; Huang, Y. Q.; Wei, G. (2014): Development and application of ultrasonic phased array inspection technology. *Journal of Electronic Measurement and Instruments*, vol. 9, pp. 925-934.

Li, J.; Li, X. L.; Yang, B.; Sun, X. M. (2015): Segmentation-based image copy-move forgery detection scheme. *IEEE Transactions on Information Forensics and Security*, vol. 10, no. 3, pp. 507-518.

Li, J. X. (2015): *Study on Algorithm of Signal Reconstruction Based on Compressive Sensing (Ph.D. Thesis)*. Beijing Institute.

Liu, X. R. (2012): *Research on Ultrasonic Phased Array Technology Testing and Evaluation Methods (Ph.D. Thesis)*. Nanchang Aeronautical University.

Nagarajaiah, S. (2107): Sparse and low-rank methods in structural system identification and monitoring. *Procedia Engineering*, vol. 199, pp. 62-69.

Needell, D.; Vershynin, R. (2010): Signal recovery from incomplete and inaccurate measurements via regularized orthogonal matching pursuit. *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 2, pp. 310-316.

Petrov, A. (2016): *Detection of Climatic Fluctuations by Hilbert-Huang Method in the Data of Zagreb-Grič Observatory (Ph.D. Thesis)*. Sveučilište u Zagrebu, Zagreb.

Shahidi, R.; Tang, G.; Ma, J.; Herrmann, F. J. (2013): Application of randomized sampling schemes to curvelet-based sparsity-promoting seismic data recovery. *Geophysical Prospecting*, vol. 61, no. 5, pp. 975-997.

Singh, A.; Sharma, L. N.; Dandapat, S. (2016): Multi-channel ECG data compression using compressed sensing in eigenspace. *Computers in Biology & Medicine*, vol. 73, pp. 24-37.

Soni, A.; Jain, J.; Roshan, R. (2013): Image steganography using discrete fractional fourier transform. *International Conference on Intelligent Systems & Signal Processing*, vol. 978, no. 81, pp. 99-103.

Song, Y. J.; Guo, R.; Zhu, L. J. (2014): Selection and threshold method of speech based on wavelet denoising algorithm. *Prospect of Science and Technolog*y, vol. 11, pp. 133-138.

Sun, Y. J.; Ji, S. (2015): Analysis, realization and experiment of Lamb wave phased arrays for damage detection and imaging in carbon composite structures. *Journal of Vibroengineering*, vol. 17, no. 1, pp. 188-202.

Sun, Y. J.; Zhang, Y. H.; Qian, C. S.; Zhang, Z. J. (2013): Near-field ultrasonic phased array deflection focusing based CFRP wing box structural health monitoring. *International Journal of Distributed Sensor Networks*, vol. 2014, no. 4, pp. 1-6.

Wen, C. K.; Zhang, J.; Wong, K. K.; Chens, J. C.; Yuen, C. (2014): On sparse vector recovery performance in structurally orthogonal matrices via LASSO. *IEEE Transactions on Signal Processing*, vol. 64, no. 17, pp. 4519-4533.

West, N. E.; Harwell, K.; McCall, B. (2017): DFT signal detection and channelization with a deep neural network modulation classifier. *IEEE International Symposium on Dynamic Spectrum Access Networks*, pp. 1-3.

Xiao, H. G.; Zhao, Y. B.; Li, J. C. (2013): A compressed sensing ultra wideband channel estimation method based on greedy tracking algorithm. *Mobile Communication*, vol. 2, pp. 70-76.

Xia, Z. H.; Wang, X. H.; Sun, X. M.; Wang, Q. (2015): A secure and dynamic multi-keyword ranked search scheme over encrypted cloud data. *IEEE Transactions on Parallel and Distributed Systems*, vol. 27, no. 2, pp. 340-352.

Yu, K.; Li, Y.; Wang, Z.; Bao, M.; Cai, C. C. (2012): New acoustic signal acquisition method based on compressive sensing. *Chinese Journal of Scientific Instrument*, vol. 1, pp. 105-112.

Zhang, Z.; Xu, Y.; Li, X.; Zhang, D. (2016): A survey of sparse representation: algorithms and applications. *IEEE Access*, vol. 3, pp. 490-530.

Zhu, X. (2014): *Research on Signal Processing Method Based on Compressive Sensing for Wireless Communications (Ph.D. Thesis)*. Ningbo University.