# Designing and Optimization of Fuzzy Sliding Mode Controller for Nonlinear Systems

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**Abstract:** For enhancing the control effectiveness, we firstly design a fuzzy logic based sliding mode controller (FSMC) for nonlinear crane systems. On basis of overhead crane dynamic characteristic, the sliding mode function with regard to trolley position and payload angle. Additionally, in order to eliminate the chattering problem of sliding mode control, the fuzzy logic theory is adopted to soften the control performance. Moreover, aiming at the FSMC parameter setting problem, a DE algorithm based optimization scheme is proposed for enhancing the control performance. Finally, by implementing the computer simulation, the DE based FSMC can effectively tackle the overhead crane sway problem and avoid unexpected accident greatly.

Keywords: Sliding mode control, fuzzy logic theory, systems optimization.

# **1** Introduction

Nonlinear crane systems is a class commonly used lifting appliance for the heavy cargoes transportation in harbours, construction site and industrial factories. According to the control requirement, the payload should rapidly and accurately arrive at the given site, the residual oscillation time and amplitude require shorter and smaller as far as possible to against the unexpected impact. In addition, the control difficulty will greatly increase due to the its underactuation trait. Hence, it is significant to develop the effective control method for solving the overhead crane systems payload swing problem.

Recently, a series of studies are made for overhead crane systems to damp the payload oscillations. In Sun et al. [Sun, Wang, Bi et al. (2015b); Yu, Li and Panuncio (2014)], the optimized PID controller are designed for damping the load vibration. The PID controllers performance are promoted by heuristic algorithm and neural compensation. To compare with nonlinear control method, the linear control method can not effectively tackle the nonlinear feature and easily gives rise to the unsatisfied anti-swing control performance

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for complex control environment. Aiming at this problem, the nonlinear control methods [Sun, Fang and Chen (2017); Sun and Fang (2014); Sun, Fang, Chen et al. (2014, 2016); Sun, Wu, Fang et al. (2017)] are successfully applied in overhead crane systems and get satisfied control performance.

Among aforementioned nonlinear control methods, the sliding mode control method [Cheng (2016); Chwa (2017); Du, Yang, Li et al. (2018); Li, Shi, Yao et al. (2016)] is regarded as a kind of the effective control approach because of the advantages of the easy designing and high robustness, and widely utilized various engineering fields. But it also suffers the chattering problem coming from the discontinuous switching characteristic around the predefined manifold. Hence, in order to figure out this problem, researchers incorporate the sliding-mode with fuzzy logic uncertainty observer to realize the overhead crane systems efficient control [Park, Chwa and Eom (2014)]. In Pezeshki et al. [Pezeshki, Badamchizadeh, Ghiasi et al. (2015)], a T-S fuzzy logic based sliding mode controller is developed to tackle the dynamic performance of overhead crane. But the parameter configuration will play an important role for control performance. Hence, to design an efficient parameter learning scheme is necessary to enhance the control performance. The optimization approaches are widely utilized in various domain for improving the system performance [Takahashi, Shibata, Motoyama et al. (2017); Li, Niu, Liu et al. (2018); Efe (2018)]. To compare with traditional optimization approaches, evolutionary computation begin to get a lot of attentions for system parameter identification and optimization because of the outstanding optimization capability [Santucci, Baioletti and Milani (2016); Fan and Yan (2016); Sun, Wang, Srinivasan et al. (2014); Sun, Wang, Bi et al. (2015a); Sabar, Abawajy and Yearwood (2017); Wang and Tang (2016); Suganthi, Devaraj, Ramar et al. (2018)]. On basis of the powerful optimization capability, we propose a optimization scheme by incorporating DE algorithm to set the FSMC parameter.

# 2 Fuzzy sliding mode controller designing

Sliding mode control (SMC) is a kind of commonly used lonlinear control method and has been successfully utilized in complex nonlinear systems. It also is a special kind of nonlinear control which the nonlinearity is expressed as the discontinuity of the control. The control strategy of SMC is different to compare with other control method, in which the system structure is purposefully changed in the dynamic process according to the current state of the system.

Given the typical nonlinear control systems as follows.

$$\dot{X} = F(\mathbf{x}, t) + G(\mathbf{x}, t)u + d(t) \tag{1}$$

The nonlinear functions (F and G), control signal (u) and external disturbance (d(t)) construct the nonlinear system. And  $\mathbf{x} = [x, \dot{x}, \dots, x^{(n)}]^T$  are denoted as the system state variables. Here, we denoted  $x_d$  as reference state track, and the error of system is obtained as follows.

$$\mathbf{E}(t) = \mathbf{x}_d(t) - \mathbf{x}(t) \tag{2}$$

$$s(\mathbf{E}) = \mathbf{C} * \mathbf{E} \tag{3}$$

In order to let the  $\mathbf{E}(t)$  arrive at the sliding mode surface and move to the origin, the control process is separate two stages which are the approaching phase( $s(\mathbf{E}) \neq 0$ ) and the sliding phase( $s(\mathbf{E}) = 0$ ). For  $s(\mathbf{E}) \neq 0$ , the control law should satisfy the condition of  $s(\mathbf{E})\dot{s}(\mathbf{E}) < 0$  so that control law can drive the system error  $\mathbf{E}$  to the sliding mode surface. Then, the corresponding switching control law  $u_{sw}$  can be depicted as follows.

$$u_{sw} = u_0 sgn\bigg(s(\mathbf{E})\bigg) \tag{4}$$

Notes that, sgn() and  $u_0$  represent the sign function and control signal initial value.

For  $s(\mathbf{E}) = 0$ , the equivalent control law  $u_{eq}$  is usually adopted to let the dynamic characteristics of the system remain on the sliding surface. The corresponding control force  $u_{eq}$  is described to let  $\dot{s}(\mathbf{E}) = 0$ .

$$\dot{s} = \mathbf{C}\dot{\mathbf{E}}$$

$$= \mathbf{C}\frac{\partial s}{\partial \mathbf{x}}(\dot{\mathbf{x}}_d - \dot{\mathbf{x}})$$

$$= \mathbf{C}\left(\frac{\partial s}{\partial \mathbf{x}}(\dot{\mathbf{x}}_d) - \frac{\partial s}{\partial \mathbf{x}}\left(F(\mathbf{x}, t) + G(\mathbf{x}, t)u + d(t)\right)\right)$$

$$= 0$$
(5)

Here, we assume that  $\frac{\partial s}{\partial \mathbf{x}}G(\mathbf{x},t)$  is non-singular.

$$u_{eq} = \left(\frac{\partial s}{\partial \mathbf{x}}(G(\mathbf{x},t))\right)^{-1} \left(\frac{\partial s}{\partial \mathbf{x}}(\dot{\mathbf{x}}_d) - \frac{\partial s}{\partial \mathbf{x}}\left(F(\mathbf{x},t) + d(t)\right)\right)$$
(6)

Then, the expression can be acquired as:

,

$$u = u_{sw} + u_{eq}$$

$$= u_0 sgn\left(s(\mathbf{E})\right) + \left(\frac{\partial s}{\partial \mathbf{x}}(G(\mathbf{x},t))\right)^{-1}$$

$$* \left(\frac{\partial s}{\partial \mathbf{x}}(\dot{\mathbf{x}}_d) - \frac{\partial s}{\partial \mathbf{x}}\left(F(\mathbf{x},t) + d(t)\right)\right)$$
(7)

In order to figure out the chattering problem, the fuzzy theory is utilized to soft the discontinuous switching around the predefined manifold. To consider a second-order nonlinear systems:

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = F(x_1, x_2) + G(x_1, x_2)u \end{cases}$$
(8)

where,  $F(x_1, x_2)$  and  $G(x_1, x_2)$  are linear and nonlinear functions, u is the control force. Based on the aforementioned description,  $s(x_1, x_2)$  is depicted as follows.

$$s(x_1, x_2) = C_1 x_1 + x_2 \tag{9}$$



Figure 1: Fuzzification of sliding mode function

In order to soft the control output, we adopt five fuzzy language partitions with the form of  $NB, NS, \dots, PB$  for fuzzification, and the corresponding fuzzification of sliding mode function is shown in Fig. 2.

#### **3 Fuzzy sliding mode controller optimization**

# 3.1 Differential evolution algorithm

Differential evolution algorithm is a commonly used global real value optimization approach for parameter identification and system optimization. It includes three basic operations which are mutation, crossover and selection. At the beginning evolution stage, NP individuals  $\mathbf{x} = {\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{NP}}$  are randomly produce. And the mutation, crossover and selection operations are implemented orderly based on the following scheme.

## 3.1.1 Mutation

During the mutation process,  $\mathbf{x}_{r1}, \mathbf{x}_{r2}, \mathbf{x}_{r3}(\mathbf{x}_{r1} \neq \mathbf{x}_{r2} \neq \mathbf{x}_{r3})$  are randomly choice. And these picked out individuals do not conformity. The produced new individual can be obtained according to the following formula.

$$\mathbf{v}_i = \mathbf{x}_1 + F_m \cdot (\mathbf{x}_2 - \mathbf{x}_3) \tag{10}$$

Notes that, the scaling parameter  $(F_m)$  is used for controlling the amplification of  $\mathbf{v}_i$ .

#### 3.1.2 Crossover

In order to incorporate the population information, the new mutant individuals need to perform the crossover operation with origin individuals. The new individual  $\mathbf{u}_i = [u_{i,1}, u_{i,2}, \ldots, u_{i,D}]$  can be produced crossing operation the between mutant  $\mathbf{v}_i$  and  $\mathbf{x}_i$ .

$$u_{i,j} = \begin{cases} v_{i,j}; if(r_j \le p_c) orj = rand\\ x_{i,j}; if(r_j > p_c) orj \neq rand \end{cases}$$
(11)

The *j*th random number  $(r_j)$  belongs to 0 and 1, and the crossover probability  $(p_c)$  is a constant among the range of [0, 1]. The rand number (rand) belongs to [1, D] to ensure the  $v_{i,j}$  element be obtained.

#### 3.1.3 Selection

In order to select the high quality individual from population, the selection operation is employed to choose the excellent individual from  $\mathbf{u}_i$  and  $\mathbf{x}_i$ . That is the individual  $\mathbf{u}_i$  should picked out if the evaluation value of  $\mathbf{u}_i$  is better than the  $\mathbf{x}_i$ , if not, the individual  $\mathbf{x}_i$  is remained.

$$\mathbf{x}_{i}^{t+1} = \begin{cases} \mathbf{u}_{i}; f(\mathbf{u}_{i}) < f(\mathbf{x}_{i}^{t}) \\ \mathbf{x}_{i}^{t}; f(\mathbf{u}_{i}) \ge f(\mathbf{x}_{i}^{t}) \end{cases}$$
(12)

# 3.2 Parameter learning scheme

According to DE algorithm optimization process, the parameters of fuzzy sliding mode controller firstly are encoded. These parameter can be divided into two classes which are unknown sliding mode function and fuzzfication parameters. For evaluating the control performance of FSMC, the evaluation function (Fun) is given on basis of control requirement.

$$Fun = \int_0^t |x| dt + 2 * \int_0^t |\theta| dt$$
 (13)

Here, the trolley displacement (x) and payload swing angle  $(\theta)$  are used for evaluating control performance.

The corresponding parameter learning process is described in below.

Step 1: Define the leaning parameters range and initialize DE algorithm parameters ( $G = 500, NP = 30, p_c = 0.5, F_m = 0.7$ );

Step 2: Produce *NP* groups FSMC parameters and perform control process for each group of parameters;

Step 3: Evaluate the control performance value of each group of parameters;

Step 4: Let iter = iter + 1 and i = 1;

Step 5: Based on (eq.(10)), do the mutation and acquire new mutant individual  $\mathbf{v}_i$ ;

Step 6: Perform the crossover between  $\mathbf{x}_i$  and  $\mathbf{v}_i$  to get new crossover individual  $\mathbf{u}_i$ ;

Step 7: Let i = i + 1 and return to the 5th step until i = NP;

Step 8: Evaluate the new NP groups parameters by implementing the control process;

Step 9: Choose the best parameter to remain next generation and let iter = iter + 1;

Step 10: Go back to the 4th step when the end condition is satisfied, otherwise end the optimization process.



Figure 2: Nonlinear overhead crane systems

# 4 Description of overhead crane systems and simulation experiment

#### 4.1 Nonlinear overhead crane systems

This section gives the brief description of nonlinear overhead crane, in which the under-actuated feature bring the difficulties for controlling payload oscillation. From the Fig. 2, we can see that it consists on trolley and payload basic parts. According to the overhead crane dynamic feature, the mechanism model is described as follows.

$$(M+m)\ddot{x} + ml(\dot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) = F \tag{14}$$

$$\cos\theta + l\ddot{\theta} + g\sin\theta = 0 \tag{15}$$

Here, M and m are the trolley and load weight,  $\theta$  is used for representing the swing angle, x is the trolley displacement, l is the rope length, F represents the driving force from control system.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(\mathbf{x}) + g_1(\mathbf{x})u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(\mathbf{x}) + g_2(\mathbf{x})u \end{cases}$$
(16)

where,  $\mathbf{x} = [x_1, x_2, x_3, x_4]^T$ ,  $x_1 = x$ ,  $x_2 = \dot{x}$ ,  $x_3 = \theta$  and  $x_4 = \dot{\theta}$  are trolley displacement, trolley velocity, payload swing angle and payload angular velocity; u represents the control force.  $f_1, f_2, g_1$  and  $g_2$  are described as follows.

$$f_1(\mathbf{x}) = \frac{mlx_4^2 sinx_3 + mgsinx_3 cosx_3}{M + msin^2 x_3}$$
(17)

$$g_1(\mathbf{x}) = \frac{1}{M + msin^2 x_3} \tag{18}$$

$$f_2(\mathbf{x}) = \frac{(M+m)gsinx_3 + mlx_4^2sinx_3cosx_3}{(M+msin^2x_3)l}$$
(19)

$$g_1(\mathbf{x}) = \frac{\cos x_3}{(M + m \sin^2 x_3)l} \tag{20}$$

#### 4.2 Simulation result discussions

For evaluating the optimal tunning FSMC effectiveness, this section gives the simulation experiment under different operation conditions. According to the description of overhead crane, we construct trolley position and payload angle sliding functions. By incorporating this two sliding functions, the overall sliding function is derived.

$$s_1(x_1, x_2) = C_1 * x_1 + x_2 \tag{21}$$

$$s_2(x_3, x_4) = C_2 * x_3 + x_4 \tag{22}$$

$$s(x_1, x_2, x_3, x_4) = \lambda_1 * s_1 + \lambda_2 * s_2 = \lambda_1 * C_1 * x_1 + \lambda_1 * x_2 + \lambda_2 * C_2 * x_3 + \lambda_2 * x_4$$
(23)

where  $\lambda_1, \lambda_2, C_1, C_2$  are overall sliding mode function coefficient which are adjustable.

To confirm the proposed method effectiveness, two different operation conditions  $(\text{Con1}:m = 3kg, x_d = 3, x_d = 9m, \text{Con2}:m = 9kg, x_d = 3, x_d = 9m)$  simulation are performed and do the comparisons with the optimal PID controllerSun, Wang, Bi et al. (2015b) and sliding mode controller.



Figure 3: The simulation results under the first condition

From Fig. 3 and Fig. 4, we can infer that the three control methods can rapidly and accurately drive the trolley to the given point. Comparing with the optimal PID and SMC, the FSMC doesn't take much adjustment. For damping the payload oscillation, the



Figure 4: The simulation results under the second condition

performance of the optimal PID controller is unsatisfactory, and the payload vibration time will last for a long time. On the contrary, the SMC and FSMC can effectively eliminate the payload residual vibration. Note that, the FSMC is superior no matter in oscillation amplitude control or eliminate residual vibration.

## **5** Conclusions

In this paper, we presented a fuzzy logic based sliding mode controller for nonlinear overhead crane systems to solve the payload oscillation problem. In order to reasonably configure the FSMC parameter, DE algorithm is incorporated for tunning the corresponding FSMC parameters so as to improve the control performance. Finally, for demonstrating the proposed method effectiveness, the corresponding simulation experiment is done at two different operation conditions. By comparing with commonly used methods, the DE based FSMC method shows the excellent anti-swing control performance. This proposed method also can be used in other engineering fields such as robot control, power system control and chemical control.

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