Joint Spectrum Partition and Performance Analysis of Full-Duplex D2D Communications in Multi-Tier Wireless Networks

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Abstract: Full-duplex (FD) has been recognized as a promising technology for future 5G networks to improve the spectrum efficiency. However, the biggest practical impediments of realizing full-duplex communications are the presence of self-interference, especially in complex cellular networks. With the current development of self-interference cancellation techniques, full-duplex has been considered to be more suitable for device-to-device (D2D) and small cell communications which have small transmission range and low transmit power. In this paper, we consider the full-duplex D2D communications in multi-tier wireless networks and present an analytical model which jointly considers mode selection, resource allocation, and power control. Specifically, we consider a distance based mode selection scheme. The performance analysis of different D2D communications modes are performed based on stochastic geometry, and tractable analytical solutions are obtained. Then we investigate the optimal resource partitions between dedicated D2D mode and cellular mode. Numerical results validate the theoretical anlaysis and indicate that with appropriate proportions of users operated in different transmission modes and optimal partitioning of spectrum, the performance gain of FD-D2D communication can be achieved.

Keywords: Full-duplex, device-to-device communications, HetNets, power control, spectrum partition.

1 Introduction

The next generation cellular network (i.e., 5G) aims to provide significant improvement on system capacity, data rate, and spectrum/energy efficiency. A critical solution for satisfying the challenging requirements in 5G is to bring the transmitter and receiver closer by dense deployment of small cells [Lee and Quek (2015)] and enabling device-to-device (D2D) communications [Lin, Andrews, Ghosh et al. (2014); Andrews, Buzzi, Choi et al. (2014)]. Specifically, small cell provides high data rate in a smaller coverage with reduced transmit power [Wang, Ju, Gao et al. (2018)]. Small cell and traditional macro cell constitute the multi-tier heterogeneous wireless networks (HetNets). Also, D2D communication as an underlay coexistence with cellular networks add another tier to the HetNets. Both small

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cell and D2D communications can help improving the spectrum utilization efficiency, energy efficiency, and offloading traffic from base station.

Furthermore, full-duplex (FD) has been introduced to perform bidirectional communication on the same temporal and spectral resources for enhancing the spectrum efficiency [Sabharwal, Schniter, Guo et al. (2014); Choi, Jain, Srinivasan et al. (2010)]. Theoretically, enabling wireless devices transmit and receive simultaneously can double the spectral efficiency, which raised great interest for next-generation wireless networks. However, the presence of self-interference is still the key challenge in implementing FD communication, and becomes the main restriction for wide application [Sultan, Song and Han (2014)]. Considering the present status of self-interference cancellation (SIC) techniques, FD is potentially suitable in heterogeneous networks due to the applications of small cell as well as the low transmit power [Lee and Quek (2015)].

Note that D2D communications can be operated in different modes, i.e., cellular mode, reuse mode, and dedicated mode. In cellular mode, the device performs exactly the same as a cellular device which still requires relaying via the base station. In reuse mode, D2D communications in performed in an underlay manner which reuse the spectrum allocated for cellular communications. In dedicated mode, dedicated spectrums are allocated for D2D communications to avoid interference. Naturally, when devices are operated in D2D mode, the question of spectrum resource sharing arises. The spectrum sharing is similar to cognitive radio networks. In general, the D2D spectrum sharing can be clarified into two types: spectrum overlay and spectrum underlay [Lin, Andrews and Ghosh (2014); Zhu and Hossain (2015)]. With the overlay spectrum sharing, cellular and D2D transmitters use orthogonal spectrum, the underlay spectrum sharing refers to the scenario that D2D devices use the cellular spectrum occupied by cellular users in a reuse mode. Although the underlay spectrum sharing improve the spectral efficiency, it also complicates the network interference management. Comparing with the underlay approach, dedicated D2D mode has better SINRs since resources allocated to D2D and cellular links are orthogonal. Most existing research on D2D mainly focus half-duplex communication [Lin, Andrews and Ghosh (2014); Zhu and Hossain (2015); ElSawy, Hossain and Alouini (2014); Lin and Andrews (2013)], and only few work consider the full-duplex D2D communications [Ali, ElSawy and Alouini (2016); Mach, Becvar and Vanek (2015); Ali, Rajatheva and Latva-aho (2014)]. However, only single-tier cellular network and one D2D mode are considered. More recently, small cell use cases have been identified in Wang et al. [Wang, Tian, Svensson et al. (2015)], and Ye et al. [Ye, Al-Shalash, Caramanis et al. (2014); Wu, Cai, Hu et al. (2015); Cho, Koufos, Jäntti et al. (2015)] designs the spectrum sharing with cooperative communication. In our previous work [Xia, Zhu, Chen et al. (2017)], we provide performance analysis on the D2D communications in multi-tier networks. However, the different modes and spectrum sharing are not considered.

In this work, we consider a system of full-duplex D2D communications in multi-tier wireless networks and present an analytical model which jointly considers mode selection, power control, and spectrum sharing. Specifically, we propose a distance based mode selection scheme. The performance analysis of different D2D communication modes are performed based on stochastic geometry which has been widely used to analyze the performance of heterogeneous cellular network [François and Bartłomiej (2009)]. The transmit power, coverage probability, and rate are analyzed and tractable analytical solutions are obtained. Then, we investigate the optimal spectrum partitions between dedicated D2D mode and cellular mode. Numerical results show the validity of the obtained analytical solutions. Also, the results show that with appropriate proportions of users operated in different transmission modes and optimal partitioning of spectrum, the performance gain of FD-D2D communication can be achieved.

The rest of this paper is organized as follows: In Section 2, we present the system model and the channel model. In Section 3, the transmit power is analyzed. In Section 4, the detailed coverage and rate analysis are provided. The simulation results and analysis are presented in Section 5. Section 6 concludes the paper. A list of major symbols and notations used in this paper is provided in Tab. 1.

Table 1: List of symbols and notations

Symbol	Definition
Φ_B	Describing the locations of MBS
Φ_s	PPP describing the locations of SBS
Φ_c	PPP describing the locations of MBS cellular UEs
Φ	Unmarked PPP describing the locations of cellular UEs
$\widetilde{\Phi}$	Marked PPP describing the locations of cellular UEs
λ	Intensity of UEs
λ_b	Intensity of MBS
λ_s	Intensity of SBS
λ_d	Intensity of D2D UEs
λ_c	Intensity of MBS cellular UEs
p	The probability of a cellular UE select the SBSs
$\widetilde{D}$	D2D transmit threshold
D_i	Distance between a UE and its receiver
ρ_c	Power control cutoff threshold at MBS cellular receiver
ρ_f	Power control cutoff threshold at f-D2D
ρ_r	Power control cutoff threshold at r-D2D
ρ_s	Power control cutoff threshold at SBS cellular receiver
η	Path-loss exponent
h_0	Small-scale fading channel gain
g_0	Fading channel power gain
δ^2	Noise power
ξ	Self-interference factor
P_c	Transmit power of MBS cellular UE
P_f	Transmit power of f-D2D UE
P_r	Transmit power of r-D2D UE
P_s	Transmit power of SBS cellular UE

2 System model

2.1 Network model

We consider a large D2D-enabled two-tier uplink cellular network, and the MBSs are regularly placed according to a hexagonal grid with density λ_b. The SBSs are randomly distributed in the network which is modeled by an independently marked Poisson point process (mPPP) with intensity λ_s, and the transmitting UEs also distributed in the given geographical area randomly which is modeled as an mPPP. To simplify analysis, we focus on a single cell A and approximated as a circle with radius $R = \sqrt{\frac{1}{\pi\lambda_b}}$. As depicted in Fig. 1, the cellular users connect to the base station (i.e., MBS, SBS), the D2D pairs communicate with each other directly. If the D2D users transmit on the reuse mode, it will reuse the channel with existing cellular users, and causing co-channel interference between D2D and cellular users. If the D2D users transmit on the dedicated mode, the co-channel interference would not occur. Similar to the definition in Zhu et al. [Zhu and Hossain (2015)] the transmitting UEs can be represented as $\widetilde{\Phi} = \{(X_i, y_i, D_i)\}$, where

- $\widetilde{\Phi} = \{X_i\}$ denotes the spatial locations of the UEs which is an unmarked homogeneous PPP in $\mathbb{R}^2$ with density λ.
- $\{y_i\}$ represents the location of the receiver of transmitting UE X_i .
- $\{D_i\}$ denotes the length of radio links where $D_i = \|X_i - y_i\|$, for notational simplicity D_i is an independently and identically distributed (i.i.d.) random variable.

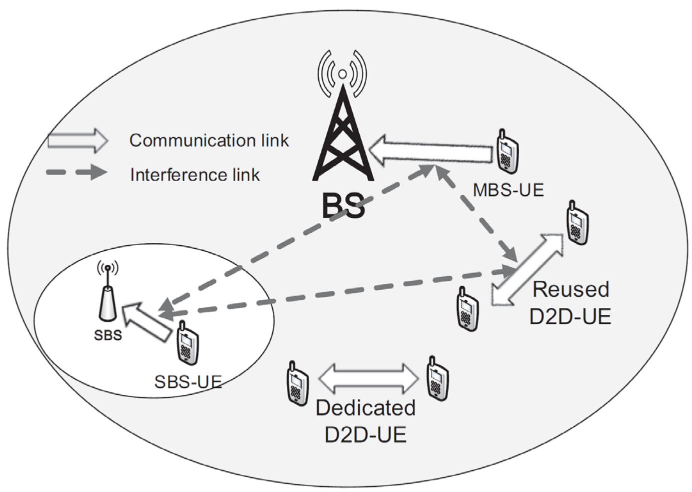

Figure 1: An illustrative example of a typical MBS cell in HetNet

From the PPP assumption, we denote the cellular link distance distribution by

$$f_{r_c}(x) = 2\,\pi\,\lambda_b x \cdot \mathbb{I}_{x\in[0,\sqrt{\frac{1}{\pi\lambda_b}}]}\,, \tag{1}$$

where r_c is a random variable which denotes the distance between a cellular UE and it transmitting base station. Moreover, the D2D pair transmit distance is Rayleigh distributed with pdf $f_{r_d} = 2\pi\lambda r_d e^{-\pi\lambda r_d{}^2}$, and the SBS-cellular link length r_s is given by $f_{r_s} = 2\pi\lambda r_s e^{-\pi\lambda r_s{}^2}$. Also, we consider a general power-law path-loss model for both cellular and D2D link, which the signal power decays at the rate $r^{-\eta}$ with the distance r, η is the path-loss exponent where $\eta \geq 2$. In this paper, we assume that the cellular and D2D links experience same propagation conditions. Accordingly, a target receiver can receive power $P \cdot h \cdot r^{-\eta}$ at a typical position with a distance r from its transmitter, P and h are the transmit power and power gain, respectively. In a Rayleigh fading network, we denote

$h \sim \exp(\mu)$ is an exponentially distributed random variable, and all channel gains are assumed to be i.i.d.

2.2 Mode selection

Here we consider a flexible distance based mode selection scheme. The cellular mode is used if $D \leq \widetilde{D}$ where a UE is a D2D UE. If the distance to its receiver is less than threshold $\widetilde{D}$, otherwise, D2D mode is selected. Note that, $\mathbb{P}[\, D_i \leq \widetilde{D}]$ denote the probability of a UE to be a D2D UE, and $1 - \mathbb{P}[\, D_i \leq \widetilde{D}$ to be cellular UE. When it comes to the selections of cellular users, the proportions denoted by a flexible selection criterion which based on the bias factor β to impose a tunable selection for the SBSs. The probability of a cellular UE select the SBSs is:

$$
\begin{aligned}
p &= \mathbb{P}[\, UE\ select\ SBS\ from\ MBS\ and\ SBS] \\
&= \mathbb{P}[r_s < \beta r_c] \\
&= \boldsymbol{E}\big[\mathbb{P}[r_s < \beta r_c | r_c]\big] \\
&= \int_0^R \mathbb{P}[r_s < \beta r_c]\, f_{r_c}(r_c) dr_c \qquad (2) \\
&= \int_0^R \int_0^{\beta r_c} f_{r_s}(r_s) dr_s \cdot f_{r_c}(r_c) dr_c
\end{aligned}
$$

For the sake of simple presentation, we define $= \left(\frac{\rho_s}{\rho_c}\right)^{\frac{1}{\eta}}$. In this case, the intensities of cellular UEs in SBS mode can be denoted by $\lambda_{sc} = p\, \lambda \mathbb{P}[\, D > \widetilde{D}]$ \$, and the MBS cellular intensities is $\lambda_c = (1-p)\, \lambda \mathbb{P}[\, D > \widetilde{D}]$. Moreover, we denote by x_r, x_d the proportions of D2D users transmit in reuse and dedicated mode, so $\lambda_r = x_r \cdot \lambda \mathbb{P}[\, D \leq \widetilde{D}]$ and $\lambda_d = x_d \cdot \lambda \mathbb{P}[\, D \leq \widetilde{D}]$ represent the intensities of reuse D2D mode and dedicated D2D mode.

2.3 Spectrum partitioning

Here, we consider the spectrum partitioning which specifically refers to the cellular link resource. In this paper, we divide the channel into F frequency subchannels, and the spectrum is divided into two orthogonal portions. We aim to determine the optimal partitioning (θ, 1-θ), where a fraction θ are allocate to dedicated D2D communication while the rest are assigned for cellular and reuse D2D users. Moreover, we defined the θ as the D2D spectrum partition factor.

3 Transmit power analysis

Due to PPP assumptions we modeled, the transmit power of the MBS-cellular, SBS-cellular, reuse D2D, and dedicated D2D modes are all random variables. Moreover, we employ channel inversion based power control scheme, and the purpose is that the transmitting devices can compensate for the large scale path-loss. Note that for ease of analysis, we employ the minimum power of ρ received signal at the terminals. In this section we analyze the transmit power distribution for ease of the follow references.

Proposition 1. The average transmit power of a typical MBS-UE, a SBS-UE and a D2D-mode UE are respectively given by:

$$\boldsymbol{E}[P_c] = \frac{\rho_c}{\left(1+\frac{\eta}{2}\right)\pi^{\frac{\eta}{2}}\lambda_b^{\frac{\eta}{2}}}, \tag{3}$$

$$\boldsymbol{E}[P_s] = \rho_s(\lambda\pi)^{-\frac{\eta}{2}}\gamma(1+\frac{\eta}{2}, \lambda\pi R_s^2), \tag{4}$$

$$\boldsymbol{E}[P_\kappa] = \frac{\rho_\chi(\lambda\pi)^{-\frac{\eta}{2}}}{1-e^{-\lambda\pi\widetilde{D}^2}}\gamma(1+\frac{\eta}{2}, \lambda\pi\widetilde{D}^2), \tag{5}$$

where, $\kappa \in \{f, r\}$ represent the different D2D link.

Proof. According to the approximate approach in the above section, we obtain the typical distribution of the active cellular network and we obtain the PDF of different transmit link. And the average MBS-cellular transmit power is:

$$\boldsymbol{E}[P_c] = \boldsymbol{E}[\rho_c r_c^\eta] = \int_0^{\frac{1}{\sqrt{\pi\lambda_b}}} 2\pi\lambda_b\rho_c \cdot x^{\eta+1}dx = \frac{\rho_c}{\left(1+\frac{\eta}{2}\right)\pi^{\eta/2}\lambda_b^{\eta/2}}.$$

Accordingly, the average SBS-cellular transmit power is:

$$\boldsymbol{E}[P_s] = \boldsymbol{E}\left[\rho_s r_{f_b}^\eta\right] = \int_0^{R_f} \rho_s 2\pi\lambda x^{\eta+1}e^{-\pi\lambda x^2}dx = \rho_s(\lambda\pi)^{-\frac{\eta}{2}}\gamma\left(\frac{\eta}{2}+1, \lambda\pi R_f^2\right).$$

And the average D2D transmit power is:

$$\boldsymbol{E}[P_\kappa] = \boldsymbol{E}\left[\rho_\kappa r_\kappa^\eta | D < \widetilde{D}\right]$$

$$= \frac{1}{\mathbb{P}(D < \widetilde{D})}\int_0^{\widetilde{D}} 2\pi\lambda x^{\eta+1}\rho_\kappa e^{-\lambda\pi x^2}dx$$

$$= \frac{\rho_\kappa(\lambda\pi)^{-\frac{\eta}{2}}}{1-e^{-\lambda\pi\widetilde{D}^2}}\gamma\left(\frac{\eta}{2}+1, \lambda\pi\widetilde{D}^2\right)$$

4 Coverage probability and rate analysis

In this section, we analyze the average rate of transmit users in different modes. Accordingly, we denote the point process $\widetilde{\Phi_c^l} \subset \Phi_c$ and $\widetilde{\Phi_s^l} \subset \Phi_s$ the set of interfering MBS-cellular UEs and the set of interfering SBS-cellular UEs in the same subchannel, and $\widetilde{\Phi_c}$ and $\widetilde{\Phi_{f_c}}$ are both PPP.

4.1 Average rate for MBS-cellular mode

We assume an orthogonal resource access scheme and only one active uplink transmitter exist. For a MBS-cellular UE, the interference comes from the SBS-cellular UEs transmit in the same subchannel and D2D UEs in reuse mode accessing the same channel. In a uplink MBS cellular mode transmission, the base station is the receiver and the aggregate interference experienced by the MBS in the MBS-cellular mode on channel α is: $\mathcal{I}_c^\alpha = \mathcal{I}_{cc}^\alpha + \mathcal{I}_{cf}^\alpha + \mathcal{I}_{cr}^\alpha + \mathcal{I}_{cs}^\alpha$

$$= \sum_{X_i \in \widetilde{\Phi_c^\alpha}} P_c g_0 \|X_i\|^{-\eta} + \sum_{X_i \in \Phi_f^\alpha} P_c g_0 \|X_i\|^{-\eta} + \sum_{X_i \in \Phi_r^\alpha} P_c g_0 \|X_i\|^{-\eta} + \sum_{X_i \in \widetilde{\Phi_s^\alpha}} P_c g_0 \|X_i\|^{-\eta}, \tag{6}$$

where g_0 represents the fading gain from interferes to the tagged receiver, $\mathcal{I}_{c\chi}$ is the interference of UEs operated in mode $\chi \in \{c, f, r, s\}$, and similarly, P_χ is the interference power. The SINR of the receiver is denoted as:

$$SINR_c^\alpha = \frac{\rho_c h_0}{\delta^2 + \mathcal{I}_c^\alpha}, \tag{7}$$

where h_0 denotes the fading gain to the tagged receiver from its transmitter, δ^2 denotes the noise power, ρ_c denotes the received threshold of the tagged receiver. Then, the average spectrum efficiency of a UE in the MBS-cellular mode is:

$$\overline{\tau_c} = \boldsymbol{E}_{h_0, g_0, \Phi_\chi^\alpha}[\log(1 + SINR_c^\alpha]. \tag{8}$$

Then, we can express the following proposition.

Proposition 2. The average spectrum efficiency of a MBS-cellular UE is given by:

$$\overline{\tau_c} = \int_0^\infty \frac{p_c(\lambda_c^\alpha, v)}{v+1} dv \tag{9}$$

where,

$$p_c(\lambda_c^\alpha, v) = \int_0^\infty \exp(-sW)\, \mathcal{L}_{I_{cc}}(s_c)\mathcal{L}_{I_{cf}}(s_c)\mathcal{L}_{I_{cr}}(s_c)\mathcal{L}_{I_{cs}}(s_c)\, f_{r_c}(r_c) dr_c, \tag{10}$$

where $s_c = \mu \frac{v}{\rho_c}$, and the Laplace Transform in (10) can be expressed as follows:

$$\mathcal{L}_{I_{cc}}(s_c) = \exp\left(-2\pi\lambda_b s_c^{\frac{2}{\eta}} \boldsymbol{E}\left[P_c^{\frac{2}{\eta}}\right] \int_{\frac{R}{(sP_c)^{\frac{1}{\eta}}}}^{\infty} \frac{y}{1 + y^\eta} dy\right),$$

$$\mathcal{L}_{I_{c\chi\setminus\{c\}}}(s_c) = \exp\left(-\lambda_{\chi\setminus\{c\}} s_c^{\frac{2}{\eta}} \boldsymbol{E}\left[P_{\chi\setminus\{c\}}^{\frac{2}{\eta}}\right] \cdot K(\eta)\right),$$

and $K(\eta) = \frac{2\pi\Gamma\left(\frac{2}{\eta}\right)\Gamma\left(1 - \frac{2}{\eta}\right)}{\eta}$.

In the MBS-cellular mode, the transmit resources are allocated by the base station in a round-robin manner, and the probability of a MBS-cellular UE transmit on channel α is $1/N_c$, and $N_c = \frac{\lambda_c}{\lambda_b}\left(1 - \exp^{-\frac{\lambda_c}{\lambda_b}}\right)^{-1}$ epresent the expect MBS-cellular UEs numbers [Zhu and Hossain (2015)]. Then, the expected channels are:

$$\mathrm{F_c} = \sum_{\alpha=1}^{(1-\theta)F} \frac{1}{N_c} \tag{11}$$

Then, the average spectrum efficiency of a MBS-cellular UE can be denoted as $\mathrm{R_c} = \mathrm{F_c}\overline{\tau_c}$.

4.2 Analysis of SBS-cellular mode

For an SBS-cellular UE accessing channel α, the interference comes from the MBS-cellular mode and reused D2D mode users. In this case, the aggregate interference in a typical SBS UEs receiver, specifically, the SBS experienced interference in channel α is given by:

$$\mathcal{I}_s^\alpha = \sum_{X_i \in \widetilde{\Phi_c^\alpha}} P_c g_0 \|X_i\|^{-\eta} + \sum_{X_i \in \Phi_f^\alpha} P_c g_0 \|X_i\|^{-\eta} + \tag{12}$$

$$\sum_{X_i\in \Phi_r^\alpha} P_c g_0 \|X_i\|^{-\eta} + \sum_{X_i\in \widetilde{\Phi_s^\alpha}} P_c g_0 \|X_i\|^{-\eta}.$$

Accordingly, the SINR at the tagged SBS receiver is:

$$SINR_s^\alpha = \frac{\rho_s h_0}{\delta^2 + \mathcal{I}_s^\alpha}. \tag{13}$$

Then, we derive the average spectrum efficiency of a SBS-cellular UE as

$$\overline{\tau_s} = \boldsymbol{E}_{h_0, g_0, \Phi_\chi^\alpha}[\log(1 + SINR_s^\alpha], \tag{14}$$

and we can obtain the following proposition.

Proposition 3. The average spectrum efficiency of a SBS-cellular UE is given by

$$\overline{\tau_s} = \int_0^\infty \frac{p_s(\lambda_s, v)}{v+1} dv \tag{15}$$

where

$$p_s(\lambda_s, v) = \int_0^\infty \exp(-s_s W)\, \mathcal{L}_{I_{sc}}(s_s)\mathcal{L}_{I_{sf}}(s_s)\mathcal{L}_{I_{sr}}(s_s)\mathcal{L}_{I_{ss}}(s_s)\, f_{r_s}(r_s) dr_s, \tag{16}$$

where $s_s = \mu \frac{v}{\rho_s}$, and the Laplace Transform in (16) can be expressed as follows:

$$\mathcal{L}_{I_{sc}}(s_s) = \exp\left(-2\pi\lambda_b s_s^{\frac{2}{\eta}} \boldsymbol{E}\left[P_c^{\frac{2}{\eta}}\right] \int_{\frac{R}{(sP_c)^{\frac{1}{\eta}}}}^{\infty} \frac{y}{1+y^\eta} dy\right),$$

$$\mathcal{L}_{I_{s\chi\backslash\{c\}}}(s_s) = \exp\left(-\lambda_{\chi\backslash\{c\}} s_s^{\frac{2}{\eta}} \boldsymbol{E}\left[P_{\chi\backslash\{c\}}^{\frac{2}{\eta}}\right] \cdot K(\eta)\right),$$

and $K(\eta) = \frac{2\pi\Gamma\left(\frac{2}{\eta}\right)\Gamma\left(1-\frac{2}{\eta}\right)}{\eta}$.

The SBS-cellular user is modeled to reuse the spectrum resource and the expect spectrum can be denoted by kp, where k denotes the amount of subchannels of a SBS-UE chosen among the $(1-\eta)F$ subchannels, and for each subchannel the SBS users decided to transmit in this channel with probability p. Then, we can obtain the average rate is $R_s = kp\overline{\tau_s}$.

4.3 Analysis of reused full-duplex D2D mode

When analyze the FD communication, the self-interference need to be raised. And in this paper we only consider the residual self-interference which the imperfect interference cancellation is modeled, hence we approximate ξ denote the SI factor. We characterize the reused FD-D2D in both f-D2D links and r-D2D links, for notational convenience we set $\kappa \in (f, r)$ where f, r denote the f-D2D and the r-D2D links. Moreover, the FD-D2D pairs reuse the spectrum resource allocated to the MBS UEs, and the aggregate interference is

$$\mathcal{I}_\kappa^\alpha = \sum_{X_i\in \widetilde{\Phi_c^\alpha}} P_c g_0 \|X_i\|^{-\eta} + \sum_{X_i\in \Phi_f^\alpha} P_c g_0 \|X_i\|^{-\eta} + \tag{17}$$

$$\sum_{X_i\in \Phi_r^\alpha} P_c g_0 \|X_i\|^{-\eta} + \sum_{X_i\in \widetilde{\Phi_s^\alpha}} P_c g_0 \|X_i\|^{-\eta} + \xi\rho_\kappa r_\kappa^\eta.$$

The SINR at the tagged D2D receiver is given by

$$SINR_{\kappa}^{\alpha} = \frac{\rho_{\kappa} h_0}{\delta^2 + \mathcal{I}_{\kappa}^{\alpha}}. \tag{18}$$

Accordingly, the average spectrum efficiency of a FD-D2D pair is

$$\overline{\tau_d} = \sum_{\kappa \in (f,r)} \boldsymbol{E}_{h_0, g_0, \Phi_{\chi}^{\alpha}} [\log{(1 + SINR_{\kappa}^{\alpha}}], \tag{19}$$

Proposition 4. The average spectrum efficiency of a FD-D2D pair is given by

$$\overline{\tau_d} = \sum_{\kappa \in (f,r)} \int_0^{\infty} \frac{p_{\kappa}(\lambda_{\kappa}, v)}{v+1} dv \tag{20}$$

and

$$p_{\kappa}(\lambda_{\kappa}, v) = \int_0^{\infty} \exp(-s_{\kappa} W)\, \mathcal{L}_{I_{\kappa c}}(s_{\kappa}) \mathcal{L}_{I_{\kappa f}}(s_{\kappa}) \mathcal{L}_{I_{\kappa r}}(s_{\kappa}) \mathcal{L}_{I_{\kappa s}}(s_{\kappa}) \exp{(-s_{\kappa} \xi \rho_{\kappa} r_{\kappa}^{\eta})} dr_{\kappa},$$

(21)

where $s_{\kappa} = \mu \frac{v}{\rho_{\kappa}}$, and the Laplace Transform in (16) can be expressed as follows:

$$\mathcal{L}_{I_{kc}}(s_{\kappa}) = \exp\left(-2\pi \lambda_b s_{\kappa}^{\frac{2}{\eta}} \boldsymbol{E}\left[P_c^{\frac{2}{\eta}}\right] \int_0^{\infty} \frac{y}{1 + y^{\eta}} dy\right),$$

$$\mathcal{L}_{I_{\kappa\chi\backslash\{c\}}}(s_{\kappa}) = \exp\left(-\lambda_{\chi\backslash\{c\}} s_{\kappa}^{\frac{2}{\eta}} \boldsymbol{E}\left[P_{\chi\backslash\{c\}}^{\frac{2}{\eta}}\right] \cdot K(\eta)\right),$$

and $K(\eta) = \frac{2\pi\Gamma\left(\frac{2}{\eta}\right)\Gamma\left(1-\frac{2}{\eta}\right)}{\eta}$.

The FD-D2D users in reuse mode reuse spectrum resources with kp similar to SBS users, and we can obtain the average rate is $\mathrm{R_d} = kp\overline{\tau_d}$

4.4 Analysis of dedicated mode

In this section, we analyze the average rate of FD-D2D links in the dedicated network. We treat the existing interference as noise, and we use Shannon's capacity formula to approximate the rate, i.e., $R_{dd} = F_d \bar{\tau} = F_d \boldsymbol{E}[\log(1 + SINR)]$, where $\mathrm{F_d}$ is available allocated bandwidth, $\bar{\tau}$ can simply denote the average spectrum efficiency. In this case, the spectrum allocation is implemented by the MBS in a round-robin manner, and $\mathrm{F_d}$ can be denoted as:

$$\mathrm{F_d} = \sum_{\alpha=1}^{\theta F} \frac{1}{N_d} \tag{22}$$

which $N_d = \frac{\lambda_d}{\lambda_b}\left(1 - \exp^{-\frac{\lambda_d}{\lambda_b}}\right)^{-1}$ is the expect number of dedicated users.

5 Numerical results and spectrum partition

In this section, we firstly validate out model by simulations and present some numerical results. Then, we analyze the optimizing spectrum partition and obtain the optimal spectrum partition factor $\theta^{\star}$. Note that, $\theta^{\star}\mathrm{F}$ subschannels are allocated for dedicated D2D user while the remains are for reused UEs.

5.1 Analytical results validation

In this part, we validate the analytical results based on the parameters we set, and all the results are functions of SINR which we derived in Section IV. Moreover, we compare the derived distribution to its corresponding Monte Carlo simulations of 2000 runs under the same parameters. Fig. 2 and Fig. 3 show the SINR coverage, the former is conducted under perfect SIC while the latter with residual SIC factor ξ. From Fig. 2, we can see that the simulation results match the analytical results well, which validated that the proposed framework capture the FD-D2D based HetNet coverage features well. Besides, the Fig. 3 shows the proportions of dedicated D2D users influence the network performance and with the increases of dedicated D2D users the coverage probability increased.

Table 2: Parameter values

Parameter	Value	Parameter	Value
λ_b	5 MBS/km^2	ρ_F, ρ_r	-90 dBm
λ_s	20 SBS/km^2	ρ_s	-100 dBm
λ	50 UE/km^2	η	4
ρ_c	-80 dBm	δ^2	-90 dBm

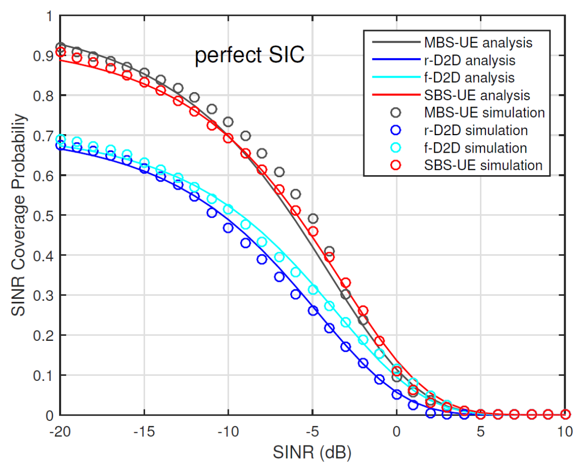

Figure 2: Coverage probability without SI in a two-tier HetNets

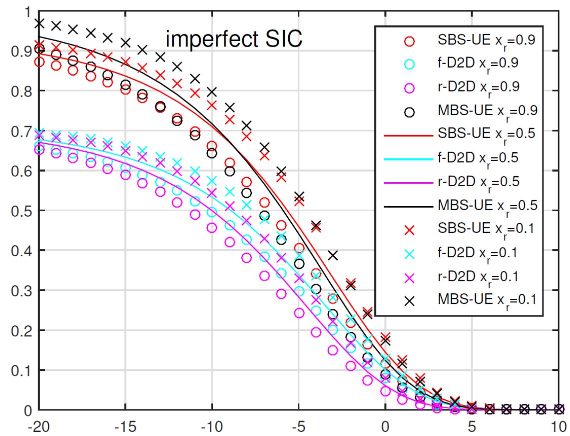

Figure 3: Coverage probability with imperfect SIC, SIC factor $\boldsymbol{\xi = 10^{-8}}$

Moreover, we investigate the D2D mode selection threshold $\widetilde{D}$ which impacts the average rate of the network. As shown in Fig. 4, the average network throughput firstly increases with $\widetilde{D}$ due to the D2D offloading gain, because with increasing $\widetilde{D}$ more D2D UEs can be scheduled. But then, increased $\widetilde{D}$ motivates more cellular UEs to transmit in D2D mode, which decrease the transmit performance due to the increased D2D co-channel interference, so Fig. 4 demonstrated that with appropriate choice of $\widetilde{D}$, D2D-based cellular network can achieve much higher rate than traditional cellular network.

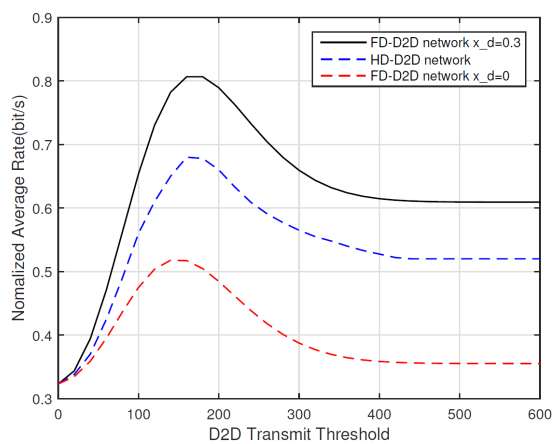

Figure 4: Network throughput of FD-D2D, HD-D2D, and Network without D2D

5.2 Optimizing spectrum partition

In this part we analyze the optimal spectrum partition factor $\theta^{\star}$

$$\theta^{\star} = \arg\ max_{\theta\in(0,1)} u(T_r, T_d), \tag{23}$$

where T_r and T_d represent the reused and the dedicated rate expression, and from the above section we denote the rate expression as following:

$T_d = R_{dd};$

$T_r = R_c + R_s + R_d$.

And in this paper, we use the weighted proportional fair function:

$u(T_r, T_d) = \omega_r T_r + \omega_d T_d$,

where weight factors $\omega_r, \omega_d > 0$ and $\omega_r + \omega_d = 1$. From Lin et al. [Lin, Andrews and Ghosh (2014)], we obtain the following optimal weighted proportional spectrum partition factor. **Lemma 1.** The optimal weighted proportional spectrum partition $\theta^\star$ is given by

$$\theta^\star = 1 - \frac{\omega_c}{\omega_c+\omega_d} \cdot \frac{1}{1-\left(e^{\lambda\pi\tilde{D}^2}-1\right)^{-1} T_r/T_d} \tag{24}$$

From Fig. 5, we plot the utility value versus θ under different values of x_r, which is the proportion of reuse D2D UEs, and we obtained that the optimal spectrum partition $\theta^\star \approx 0.4$ and it can be seen that the optimal $\theta^\star$ are equal under different x_r. However, in this paper we only consider a fixed mode selection threshold $\widetilde{D}$, and $\theta^\star(\widetilde{D})$ we will analyze it future.

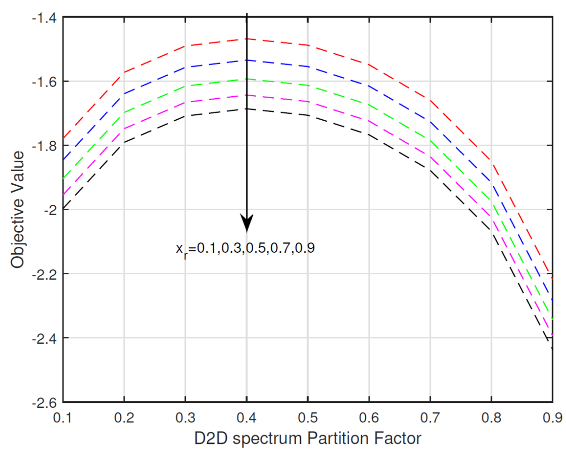

Figure 5: The utility value versus D2D spectrum partition factor θ under different values of $\boldsymbol{x_r}$

6 Conclusion

In this work, we have considered the full-duplex D2D communications in multi-tier cellular networks and have presented a tractable framework for analyzing the performance different communication modes. We have obtained tractable expressions for important performance metrics such as coverage probability and average rate. Based on the theoretical analysis, we derive the optimal spectrum partitioning for dedicated communication mode. Then numerical simulations have been performed which demonstrate the effectiveness of the proposed analytical framework. Also, the results show that with appropriate partition of the spectrum the network provide better performance. For the future work, the game theory based spectrum partition can be studied.

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