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Article

Small-World Networks with Unitary Cayley Graphs for Various Energy Generation

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Abstract: Complex networks have been a prominent topic of research for several years, spanning a wide range of fields from mathematics to computer science and also to social and biological sciences. The eigenvalues of the Seidel matrix, Seidel Signless Laplacian matrix, Seidel energy, Seidel Signless Laplacian energy, Maximum and Minimum energy, Degree Sum energy and Distance Degree energy of the Unitary Cayley graphs [UCG] have been calculated. Low-power devices must be able to transfer data across long distances with low delay and reliability. To overcome this drawback a small-world network depending on the unitary Cayley graph is proposed to decrease the delay and increase the reliability and is also used to create and analyze network communication. Small-world networks based on the Cayley graph have a basic construction and are highly adaptable. The simulation result shows that the small-world network based on unitary Cayley graphs has a shorter delay and is more reliable. Furthermore, the maximum delay is lowered by 40%.

Keywords: Seidel energy; Seidel Signless Laplacian eigenvalues; Distance degree energy; Unitary Cayley graphs

1 Introduction

Complex networks have recently gained popularity in a variety of disciplines and research areas. The Internet has altered the way we deal with everything in our daily lives. Computer experts were fascinated by the idea of mastering the wheel of controlling the Internet's complexity and massive expansion. The data magnitude of social networks is unpredictable and unmanageable by social scientists. The biological interactions that characterize a cell metabolism are believed to establish its pathways and supply biologists with information [1]. To be able to manage networks before networks manipulate our needs, new science is required [2].

The small-world phenomenon has recently become a hot area of theoretical and practical research, attracting the attention of multidisciplinary academics. "Short chains of acquaintances" or "six degrees of separation" are synonymous with the term "small world." [3,4] refers to the graph of a human social network, in which nodes replace people and edges between nodes simulate if the two matching people know each other by first name [5]. Because any two random pairs of nodes are separated by a small number of nodes, usually less than 6, the graph is known as a "small world." Although the first-name basis criterion for edge definition is just a little naive, the resulting graph acts as a real-world network.

Because of the adoption of the constraints of either of the end extreme network types, random networks and regular lattices, small-world networks are extremely important. Small-world networks have shown that they can be utilized as frameworks for studying complex system interaction networks [6]. The most significant goal of the small-world study is to confirm the notion that all networks have a qualitatively similar structure across different areas. Although nodes in the network exhibit a high degree of clustering, a common aspect of vast networks is the existence of short pathways between most node pairs. Nodes can be reached and navigated without requiring a comprehensive of the entire network. Such qualities aided in the description of large-scale social network behaviour, as well as providing vital insights into the internal architecture of decentralized peer-to-peer systems.

The main contribution of this research, works a new small world network-based unitary Cayley graphs is proposed to design and analyse the communication of the network and also it reduces the delay and gives better reliability. The remainder of the research is organized as follows: Section 2 shows the connected research efforts as well as the research work basis. The explained various energy generation theorems in Section 3. Section 4 clearly illustrates the proposed small world network-based unitary Cayley graph. Section 5 shows the result and discussion of the small-world network. Section 6 depicts the conclusion.

2 Literature Survey

This section explains various surveys related to small-world networks studied throughout the years. The present state of the small-world network is discussed and an overview is provided in this study.

Baysal et al. [7] presented the autapse effects on the chaotic resonance (CR) phenomena in single neurons and small world neural networks. When the autaptic delay time matches the half period signal, the multiple CR develops when the autaptic time delay for ideal chaotic current. The autapse considerably improves the chaotic resonance of the suitable autaptic values, according to the results. Dhaya et al. [8] presented the topology of the deep learning (DL) design is changed in such a way that optimal cross-layer connection is achieved. This modification takes advantage of our crucial insight that a Small-World Network's topology crosses the border convergence is fastest for a given level of accuracy. The results show that the proposed strategy is both accurate and quick to respond to for training.

Qiu et al. [9] presented a data propagation strategy with the small world characteristics used for data transfer in IoV. As the hop distance decreases, the time required to disseminate a message decrease. The suggested methodology has a low average data packet delay, according to simulation results. Furthermore, because of the increased robustness, the packet delivery ratio is higher. Pandey et al. [10] presented the small-world network for data transfer with minimal latency and energy balance based on a wireless sensor network. The average path length of a small world wireless sensor network (SW-WSN) is short, and the average clustering coefficient is large. The suggested small world network accomplishes power exchanging, boosts the network, increases power economy as well as minimises data, according to experimental outcomes.

Qi et al. [11] presented the small-world network is linked to decreased alert attention after total sleep deprivation. The findings show that topological features of networks are disturbed, and that abnormal temporal and salience network topology may operate as neural markers underpinning vigilant attention problems following total sleep deprivation (TSD). Gu et al. [12] presented the small-world network model to explain how a small-world network with a select group of people. The addition of theories boosted the network model prediction capacity greatly, it helped to reveal factors which influence an establishment of a small-world industrial network focused on exclusive groups.

3 Various Energy Generation

The various energies generations are Seidel matrix, Seidel Signless Laplacian matrix, Seidel energy, Seidel Signless Laplacian energy, Maximum and Minimum energy, Degree Sum energy and Distance Degree energy of the Unitary Cayley graph have been calculated.

3.1 Seidel Energy of Unitary Cayley Graph X_n

Here, [13] we obtain Seidel eigenvalues and Seidel energy of X_n.

The Seidel matrix $SM(X_n) = (s_{ij})$ of a *Unitary Cayley graph* X_n on n vertices and $\frac{n\phi(n)}{2}$ edges is a real symmetric matrix as

$$s_{ij} = \begin{cases} -1 & \textit{if } v_i \textit{ and } v_j \textit{ are adjacent vertices} \\ 1 & \textit{if } v_i \textit{ and } v_j \textit{ are not adjacent vertices} \\ 0 & \textit{elsewhere} \end{cases} \tag{1}$$

Let $s_1, s_2, \ldots, s_n$ be the eigenvalues of $SM(X_n)$.The Seidel energy of X_n is defined as $SE(X_n) = \sum_{i=1}^{n} |s_i|$.

Theorem 1: The Seidel eigenvalues of the *Unitary Cayley graph* X_n are $n - 1 - 2\phi(n)$ and

$-1 - 2\phi(n)\frac{\mu(t_i)}{\phi(t_i)},\ 1 \le i \le n-1$ where $t_i = \frac{n}{\gcd(i,n)}$.

Also, the Seidel energy $SE(X_n)$ is $|\ n - 1 - 2\phi(n)| + \sum_{i=1}^{n-1}\left|1 + 2\phi(n)\frac{\mu(t_i)}{\phi(t_i)}\right|$.

Proof: The Seidel matrix $SM(X_n) = A(\overline{X_n}) - A(X_n)$, where $A(\overline{X_n})$ is the adjacency matrix of the complement of UCG X_n.

The eigenvalues of X_n are $\phi(n)\frac{\mu(t_i)}{\phi(t_i)},\ 0 \le i \le n-1$, where $t_i = \frac{n}{\gcd(i,n)}$.

Let $\eta_0 \ge\ \eta_1 \ge, \ldots, \ge \eta_{n-1}$ be the eigenvalues of X_n.

Let $\eta_0 = \phi(n)$.

The eigenvalues of the complement of the UCG $\overline{X_n}$ are $n - 1 - \eta_0,\ -1 - \eta_1,\ -1 - \eta_2 \ldots,\ -1 - \eta_{n-1}$.

Hence the eigenvalues of $SM(X_n)$ are $n - 1 - 2\phi(n)$ and $-1 - 2\phi(n)\frac{\mu(t_i)}{\phi(t_i)},\ 1 \le i \le n-1$

$$\text{The Seidel energy of } X_n \text{ is } SE(X_n) = \sum\nolimits_{i=0}^{n-1} |\eta_i| \tag{2}$$

$$= |\eta_0| + \sum\nolimits_{i=1}^{n-1} |\eta_i| \tag{3}$$

$$= |(n-1) - 2\phi(n)| + \sum_{i=1}^{n-1}\left|-1 - 2\phi(n)\frac{\mu(t_i)}{\phi(t_i)}\right| \tag{4}$$

$$= |n - 1 - 2\phi(n)| + \sum_{i=1}^{n-1}\left|1 + 2\phi(n)\frac{\mu(t_i)}{\phi(t_i)}\right| \tag{5}$$

3.2 Seidel Signless Laplacian Energy of UCG X_n

The Seidel matrix of *Unitary Cayley graph* X_n on n vertices and $\frac{n\phi(n)}{2}$ edges are $S(X_n)$.

Let $D_S(X_n) = diag(n-1-2\phi(n),\ n-1-2\phi(n),\ \ldots,\ n-1-2\phi(n))$ be the diagonal matrix.

The Seidel Signless Laplacian matrix $S_{SL}M(X_n) = D_S(X_n) + S(X_n)$ of a *Unitary Cayley Graph* X_n on n vertices and $\frac{n\phi(n)}{2}$ edge is a real symmetric matrix defined as [14]:

$$a_{ij} = \begin{cases} -1 & \text{if } i \neq j \text{ and } v_i,\ v_j \text{ are adjacent} \\ 1 & \text{if } i \neq j \text{ and } v_i,\ v_j \text{ are not adjacent} \\ n-1-2\phi(n) & \text{if } i = j \end{cases} \tag{6}$$

Let $\sigma_0, \sigma_1, \ldots, \sigma_{n-1}$ be the eigenvalues of $S_{SL}M(X_n)$. The spectrum of the $S_{SL}M(X_n)$ is the set of its eigenvalues together with their multiplicities.

The Seidel Signless Laplacian energy of X_n is defined as $S_{SL}E(X_n) = \sum_{i=0}^{n-1} |\sigma_i + 2\phi(n) - n + 1|$.

Theorem 2: Seidel Signless Laplacian eigenvalues of the *Unitary Cayley Graph* X_n are $2(-1-2\phi(n))$ and $n-2-2\phi(n)\left(1+\frac{\mu(t_i)}{\phi(t_i)}\right)$, $1 \leq i \leq n-1$ where $t_i = \frac{n}{\gcd(i,n)}$. Also, the Seidel Signless Laplacian energy $S_{SL}E(X_n)$ is $|\ n-1-2\phi(n)| + \sum_{i=1}^{n-1}\left|1+2\phi(n)\frac{\mu(t_i)}{\phi(t_i)}\right|$.

Proof: The Seidel Signless Laplacian matrix $S_{SL}M(X_n) = D_S(X_n) + S(X_n)$ where $S(X_n)$ is the Seidel matrix of UCG X_n. $D_S(X_n) = diag(n-1-2\phi(n),\ n-1-2\phi(n),\ \ldots,\ n-1-2\phi(n))$ is a diagonal matrix.

The eigenvalues of X_n are $\phi(n)\frac{\mu(t_i)}{\phi(t_i)}$, $0 \leq i \leq n-1$, where $t_i = \frac{n}{\gcd(i,n)}$.

The eigenvalues of $D_S(X_n)$ are $n-1-2\phi(n)$ (n times).

The Seidel Signless Laplacian energy of X_n is

$$S_{SL}E(X_n) = \sum_{i=0}^{n-1} |\sigma_i + 2\phi(n) - (n-1)| \tag{7}$$

$$= |\ n-1-2\phi(n)| + \sum_{i=1}^{n-1}\left|1+2\phi(n)\frac{\mu(t_i)}{\phi(t_i)}\right|. \tag{8}$$

3.3 Maximum Degree Energy of UCG X_n

The Maximum Degree of energy of UCG X_n is the sum of the absolute eigenvalues of the Maximum Degree matrix, [15] where Maximum Degree matrix $MDM(X_n) = (m_{ij})$ of a *Unitary Cayley Graph* X_n on n vertices and $\frac{n\phi(n)}{2}$ edge is a real symmetric matrix as

$$m_{ij} = \begin{cases} \max(d_i, d_j) & \text{if } v_i, v_j \text{ are adjacent} \\ 0 & \text{elsewhere.} \end{cases}$$

where d_i, d_j are the degree of vertices v_i and v_j.

Theorem 3: The Maximum degree energy of *Unitary Cayley Graph* X_n is $2\phi(n)(n-1)$.

Proof: The Maximum degree matrix of X_n is

$$MDM(X_n) = \begin{pmatrix} 0 & \phi(n) & \phi(n) & \ldots & \phi(n) \\ \phi(n) & 0 & \phi(n) & \ldots & \phi(n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi(n) & \phi(n) & \phi(n) & \ldots & 0 \end{pmatrix}_{n\times n} \tag{9}$$

$$= \phi(n)\begin{pmatrix} 0 & 1 & 1 & \ldots & 1 \\ 1 & 0 & 1 & \ldots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \ldots & 0 \end{pmatrix} \tag{10}$$

$$= \phi(n)\left[\begin{pmatrix} 1 & 1 & 1 & \ldots & 1 \\ 1 & 1 & 1 & \ldots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \ldots & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \end{pmatrix}\right] \tag{11}$$

$$= \phi(n)(J_n - I_n) \tag{12}$$

The spectrum of J_n is $\begin{pmatrix} 0 & n \\ n-1 & 1 \end{pmatrix}$. The spectrum of I_n is $\begin{pmatrix} 1 \\ n \end{pmatrix}$.

Hence the spectrum of (X_n) is $\phi(n)\begin{pmatrix} -1 & n-1 \\ n-1 & 1 \end{pmatrix}$.

The Maximum Degree of energy of UCG X_n is $2\phi(n)(n-1)$.

3.4 Minimum Degree Energy of UCG

The Minimum Degree of energy of UCG X_n is the sum of the absolute eigenvalues of the Minimum Degree matrix [16], where the Minimum Degree matrix $Min(X_n) = (m_{ij})$ of a *Unitary Cayley Graph* X_n on n vertices and $\frac{n\phi(n)}{2}$ edges is a real symmetric matrix as

$m_{ij} = \begin{cases} \min(d_i, d_j) & \text{if } v_i, v_j \text{ are adjacent} \\ 0 & \text{elsewhere.} \end{cases}$ where d_i, d_j are the degree of vertices v_i and v_j.

Theorem 4: Minimum Degree energy of the *Unitary Cayley Graph* X_n is $2\phi(n)(n-1)$.

Proof: The *Unitary Cayley graph* X_n is a $\phi(n)$ regular graph.

Therefore, the maximum degree of X_n is equal to the minimum degree. Hence the Minimum Degree of energy of X_n is $2\phi(n)(n-1)$.

3.5 Degree Sum Energy of UCG X_n

The sum of the absolute eigenvalues of the [17] Degree Sum matrix is the Degree Sum energy of UCG X_n, where the Degree Sum matrix $DSM(X_n) = (m_{ij})$ of a *Unitary Cayley Graph* X_n on n vertices and $\frac{n\phi(n)}{2}$ edge is a real symmetric matrix as $\begin{cases} 2\phi(n) & \text{if } i \neq j \\ 0 & \text{elsewhere} \end{cases}$.

Theorem 5: The Degree Sum energy of *Unitary Cayley Graph* X_n is $4\phi(n)(n-1)$.

Proof: The Degree sum matrix of X_n is

$$DSM(X_n) = \begin{pmatrix} 0 & 2\phi(n) & 2\phi(n) & \dots & 2\phi(n) \\ 2\phi(n) & 0 & 2\phi(n) & \dots & 2\phi(n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 2\phi(n) & 2\phi(n) & 2\phi(n) & \dots & 0 \end{pmatrix}_{n\times n} \tag{13}$$

$$= 2\phi(n)\begin{pmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & 0 & 1 & \dots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \dots & 0 \end{pmatrix} \tag{14}$$

$$= 2\ \phi(n)\left[\begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 1 & 1 & \dots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \dots & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}\right] \tag{15}$$

$$= 2\phi(n)(J_n - I_n) \tag{16}$$

The spectrum of J_n is $\begin{pmatrix} 0 & n \\ n-1 & 1 \end{pmatrix}$. The spectrum of I_n is $\begin{pmatrix} 1 \\ n \end{pmatrix}$.

Hence the spectrum of (X_n) is $2\phi(n)\begin{pmatrix} -1 & n-1 \\ n-1 & 1 \end{pmatrix}$.

The Degree Sum energy of UCG X_n is $4\phi(n)(n-1)$.

3.6 Degree Distance Energy of Unitary Cayley Graphs X_n

Computed distance energy of *Unitary Cayley graphs*. Motivated by these papers we determine the degree of distance energy of Unitary Cayley Graphs (UCG) [18].

The degree distance matrix of X_n, denoted by $DDM(X_n)$ can be defined as

$DDM(X_n) = \begin{cases} (d_i + d_j)\ d(v_i, v_j), & if\ v_i \neq v_j \\ 0 & if\ v_i = v_j \end{cases}$ where d_i = degree of v_i and $d(v_i, v_j)$ is the shortest distance between the vertices v_i and v_j. Let $\lambda_1, \lambda_2, \dots, \lambda_n$ be the degree distance eigenvalues of DDM (X_n). The degree of distance energy of X_n, $DDE(X_n) = \sum_{i=1}^{n}|\lambda_i|$.

Theorem 6: Let X_n be the UCG, where n is a prime number. The spectrum of a degree distance matrix X_n is $\begin{pmatrix} -2(n-1) & 2(n-1)^2 \\ n-1 & 1 \end{pmatrix}$. The degree of distance energy is $4(n-1)^2$.

Proof: For UCG X_n, n being prime, X_n is a complete graph.

Therefore, $d_i = degree(v_i) = n-1, \forall i.$

$d(v_i, v_j) = 1,\ \forall\ v_i, v_j,\ d_i + d_j =\ degree(v_i) + degree(v_j)$ = 2 (n−1)

The degree distance matrix

$$DDM(X_n) = \begin{pmatrix} 0 & 2(n-1) & 2(n-1) & \dots & 2(n-1) \\ 2(n-1) & 0 & 2(n-1) & \dots & 2(n-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 2(n-1) & 2(n-1) & 2(n-1) & \dots & 0 \end{pmatrix}_{n \times n} \tag{17}$$

$$= 2(n-1)(J_n - I_n) \tag{18}$$

The spectrum of J_n is $\begin{pmatrix} 0 & n \\ n-1 & 1 \end{pmatrix}$. The spectrum of I_n is $\begin{pmatrix} 1 \\ n \end{pmatrix}$.

Hence the Spectrum of $(J_n - I_n)$ is $\begin{pmatrix} -1 & n-1 \\ n-1 & 1 \end{pmatrix}$

The degree of distance energy $DDE(X_n) = \sum_{i=1}^{n} |\lambda_i|$ (19)

$$= |-2(n-1)|(n-1) + \left|2(n-1)^2\right| \times 1 \tag{20}$$

$$= 2(n-1)^2 + 2(n-1)^2 = 4(n-1)^2. \tag{21}$$

4 Small-World Network Based on the Unitary Cayley Graph

Cayley graphs are well-known for being good models for computer networks with multiple connections. Cayley graphs are used to create a variety of well-known and practical connectivity networks. To begin, define a few terms and symbols. V vertices and E arcs or directed edges make up a $digraph = (V, E)$. $V \times V's$ elements are a subset of $(u,\ v)E$. If the subset E is symmetric, $(u, v)\ E$ implies (v,u) E, the undirected edge can be used to identify two opposing arcs (u, v) *and* (v, u). Assume G is a finite group with S as a subset. (This study does not address infinite groups.)

For $g \in G, s \in S,\ Cay(G, S)$ is a Cayley graph of a group G and its subset S with G elements at the vertices and ordered pairs (g, gs) at the arcs. Cay (G, S) is a simple (undirected) graph if $1 \notin S$ represents the identity member of $S = S^{-1},\ Cay(G, S)$. It is recommended that additional concepts and fundamental outcomes on graphs and groups, as well as connectivity networks. G is a finite group; hence the clustering coefficient is 3. Consider the situation when $1 \notin S, S = S^{-1}, = Cay$ for some G generating set S.

Then there is a Cayley graph with constant degree $d = |S|$, in which each node v has precisely d neighbours, and these d neighbours have at most $\frac{d(d-1)}{2}$ edges between them. As a result, the only thing we need to think about is node 1's clustering coefficient, which is G's identity element.

Node 1 has S neighbours. If $s_1,\ s_2 \in S,$ then s_1 *and* s_2 are adjacent if and only if $s_2 = s_1 s$. Assume H is a subset of S and $H \cup 1$ is a subgroup of G. Then $s_1\ s_2 \in H$ when $s_1,\ s_2 \in H$. As a result, the set S of neighbours of node 1 has at least $|H|(|H| - 1)/2$ edges. As a result, if H can be set to a big value, the clustering coefficient will be high.

5 Experimental Results

Numerical simulations are used to examine the performance of small world network-based Unitary Cayley graphs.

Fig. 1 depicts the delay of regular node density at the various sectors. When delay increases as the number of regular nodes increases. Furthermore, the small world network-based unitary Cayley latency is reduced. The latency grows as the value increases when the regular node is fixed. The maximum delay of small world network-based Unitary Cayley graphs has been reduced by 40%, according to simulation data.

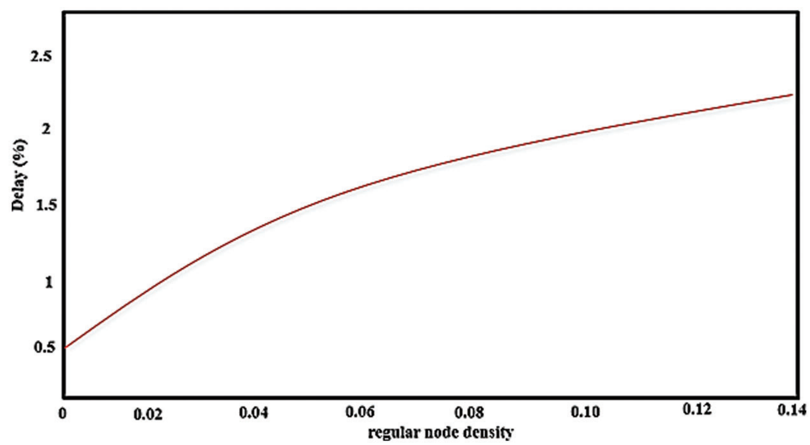

Figure 1: Delay function of regular node density in proposed small world network-based Unitary Cayley graphs

The reliability factor of a regular node with varying sectors is shown in Fig. 2. The reliability factor drops as regular node density improves. Furthermore, the small world network-based unitary Cayley has a greater reliability factor. When the regular node density is kept constant, the reliability factor drops.

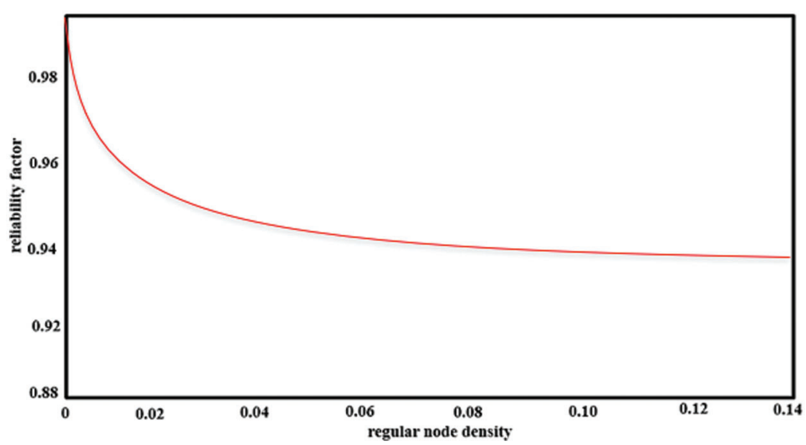

Figure 2: Reliability factor as a function of a regular node with proposed small world network-based Unitary Cayley graphs

The consensus speed and power consumption of our proposed topology control solutions were assessed. The simulations were run on 100 host graphs, each with sensors evenly and randomly spread throughout a 100 m 100 m region.

Fig. 3 depicts the consensus protocol's energy consumption. With power model 0 and power model 1, BCG-1 used 2% less energy than BCG-0. With power model 1, BCG-1 used 2% less energy than BCG-0.

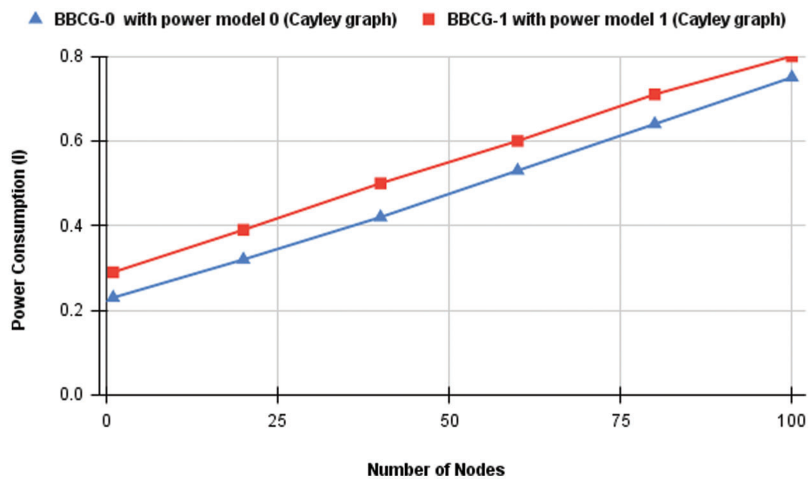

Figure 3: Power consumption

For N = 1081, the distance created by the suggested node ID assignment approaches is shown in Fig. 4. The right-skewed distribution of the CR assignment (BCG-1) histogram shows a significant frequency of short edges. All connection distances are under 80 meters, according to the Dist-swap assignment (BCG-2).

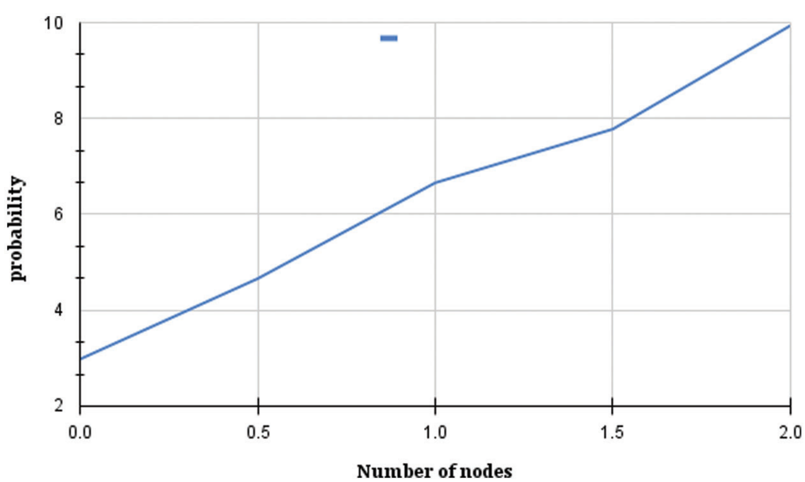

Figure 4: Distance of probability

6 Conclusion

One of the key concepts of spectral graph theory, which integrates organic chemistry with mathematics, is graph energy. Graph energies and limits have been determined using eigenvalues of graph matrices. In this paper, small-world computer networks, for which Cayley graphs have been demonstrated to be good models. Small-world networks based on the Cayley graphs have a straightforward topology and are very adaptable. Furthermore, small-world computer networks based unitary Cayley graph have a 40% lower delay and higher reliability. The Cayley-graph model can be used to design, analyse, and train communication and other real-world networks. Cayley graphs are therefore useful method for small-world computer networks. This graph method is easy to conceptualize, construct and compare since they have a simpler structure. Cayley-graph employed to recreate a broad variety of real-life networks in the biological, social, and technological domains by carefully defining the parameters.

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