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Designing Adaptive Multiple Dependent State Sampling Plan for Accelerated Life Tests

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Abstract: A novel adaptive multiple dependent state sampling plan (AMDSSP) was designed to inspect products from a continuous manufacturing process under the accelerated life test (ALT) using both double sampling plan (DSP) and multiple dependent state sampling plan (MDSSP) concepts. Under accelerated conditions, the lifetime of a product follows the Weibull distribution with a known shape parameter, while the scale parameter can be determined using the acceleration factor (AF). The Arrhenius model is used to estimate AF when the damaging process is temperature-sensitive. An economic design of the proposed sampling plan was also considered for the ALT. A genetic algorithm with nonlinear optimization was used to estimate optimal plan parameters to minimize the average sample number (ASN) and total cost of inspection (TC) under both producer's and consumer's risks. Numerical results are presented to support the AMDSSP for the ALT, while performance comparisons between the AMDSSP, the MDSSP and a single sampling plan (SSP) for the ALT are discussed. Results indicated that the AMDSSP was more flexible and efficient for ASN and TC than the MDSSP and SSP plans under accelerated conditions. The AMDSSP also had a higher operating characteristic (OC) curve than both the existing sampling plans. Two real datasets of electronic devices for the ALT at high temperatures demonstrated the practicality and usefulness of the proposed sampling plan.

Keywords: Accelerated life test; acceleration factor; adaptive of multiple dependent state sampling plan; average sample number; total cost of inspection; weibull distribution

1 Introduction

An acceptance sampling plan (ASP) is essential to ensure that product quality conforms to standards. Generally, producers decide to accept or reject lot sentencing which depends on a single sampling plan

(SSP) that is often used in various industries. An SSP is easy to use but usually results in a larger sample size than other sampling plans [1]. Sometimes, an SSP cannot decide whether a lot will be accepted or rejected. Moreover, when using an SSP, the producer is often at a psychological disadvantage because the rejected lots are not given a second chance [2]. In these cases, the producer should apply a double sampling plan (DSP) for the inspection process. If the results of the first sample are not definitive for acceptance or rejection, a second sample is taken, which then leads to a decision on the disposition of the lot. For this reason, a DSP is more economical and reliable than an SSP for lots with very low or very high proportions of defects because a decision can be made after taking the first sample [3]. Producers can also apply more effective sampling plans, such as the multiple dependent state sampling plan (MDSSP), to assist in lot acceptance decisions. Wortham et al. [4] introduced the MDSSP to deliver lots for serial inspection in a continuous production process. Sample size can be reduced by implementing the MDSSP since the decision regarding the disposition of the current lot is made using the results of samples drawn from both current and previous lots. The MDSSP has been used by numerous researchers in a variety of situations. Govindaraju et al. [5] proposed an MDSSP design to minimize the sum of producer and consumer risks within the acceptable quality level limits, while Balamurali et al. [6] investigated the MDSSP under normal distribution using a variable sampling plan. The Bayesian approach was employed by Balamurali et al. [7] to analyze the MDSSP. Some studies [8–12] applied MDSSP concepts to design control charts. Rao et al. [13] presented a generalization of the MDSSP called the GMDSSP and discovered that this was more efficient in lowering sample size. The mean lifetime of products using a GMDSSP was investigated by Aslam et al. [14] for the gamma, Burr type XII and Birnbaum-Saunders distributions, while Aslam et al. [15] developed the MMDSSP, a modified version of the MDSSP that they claimed was more adaptable and effective than the existing MDSSP in terms of sample size and inspection cost over time truncated life. Charongrattanasakul et al. [16] proposed a novel adaptive version of the MDSSP that accepted the current lot if the quality of the product was excellent, good or moderate, while existing sampling plans only operated at two levels. They claimed that their proposed sampling plan was more flexible and efficient in terms of average sample number than the MDSSP and MMDSP. Other studies [17–19] applied the Bayesian approach to design a group chain sampling plan that considered inspection based on preceding and succeeding lots.

Nowadays, many products, including electronic devices and electrical appliances, are highly reliable and testing each item does not ensure the mean lifetime of the product. Therefore, life testing is used to determine product lifetime under specified conditions. Several studies presented an ASP for the truncated life test under various lifetime distributions. Tripathi et al. [20] presented an SSP for generalized half-normal distribution, where the lifetime experiment was truncated at a specified time, while Rao et al. [21] presented an MDSSP that decreased the life of the product under exponentiated half-logistic distribution. A novel ASP for length-biased weighted Lomax distribution was created by Al-Omari et al. [22] based on a truncated life test, while an attribute-modified chain sample inspection plan was created by Tripathi et al. [23] based on a time-truncated life test under the Darna distribution. Abushal et al. [24] developed an ASP for the power-inverted Topp-Leone distribution, which is a truncated life test that takes advantage of the median life of products. Life testing analyzes failure times of test units under normal operating conditions.

Generally, products are manufactured using high-quality processes to ensure long lifetimes. Therefore, collecting failure statistics for these products under use conditions is very difficult. As a result, the accelerated life test (ALT) is becoming more popular as this gives information on a highly reliable product lifetime in a short time. The ALT analyzes a product under conditions that are more extreme than general use (temperature, strain, stress, etc.), thus forcing the product to fail faster. These tests speed up the detection of various product defects and failure types. Producers now realize that the ALT plays an essential role in rapidly inspecting finished products. Several authors have proposed sampling plans under the ALT. Kim et al. [25] suggested an SSP for ALT under the Weibull distribution. They considered the case where the life test was hybrid censored, while Gao et al. [26] discussed the design of an ALT sampling plan for an exponential distribution using time-censoring. Some studies suggested other sampling plans. Aslam et al. [27] developed an SkSP-V sampling plan for ALT when product lifetime followed the Weibull distribution, while Aslam et al. [28] proposed a group skip-lot sampling plan using ALT resampling when the product lifetime followed the Weibull distribution. Statistical inference for ALTs under various types of stress (constant-stress, step-stress and progressive-stress) has also been proposed for censoring data under various lifetime distributions. For more details, see [29–33].

The economic design of the inspection process should consider all costs associated with implementing the plan such as inspection costs, internal failure costs and outgoing failure costs. Changes in these cost parameters will affect the total cost of the inspection. Several authors studied the economic designs of various ASPs using different approaches. Hsu et al. [2] proposed an economic model for an SSP to determine the minimum appropriated cost for both producer and consumer, while Aslam et al. [34] proposed an economic design of a group ASP to ensure the lifetime of products following the Weibull distribution using a Bayesian approach. Fallahnezhad et al. [35] proposed repeating group ASPs that included give-away cost per unit of extra sold material and inspection error, while Balamurali et al. [36] presented an economic design of a quick switching sampling system to minimize total cost while meeting the risk requirements of both producers and consumers. Finally, Hakamipour [37] compared constant-stress and step-stress predictors under a cost constraint for progressive Type I censoring.

This study focused on designing sampling plans for ALT based on continuous production processes and delivering lots for serial inspection. To the best of our knowledge, an ASP design for the ALT using the MDSSP has not been previously presented. Many ALT studies used only a single sample to decide whether to accept or reject a lot. In some situations, producers cannot decide to accept or reject a lot based on a single sampling plan because the quality level of the first sample can be both good and bad. Therefore, here, a novel adaptive version of the MDSSP (AMDSSP) for the ALT was used to decrease sample size when the mean lifetime followed the Weibull distribution. This novel sampling plan was designed based on the concept of the DSP together with the existing MDSSP. An economic model of the AMDSSP for ALT was also developed. The proposed sampling plan was compared with existing sampling plans in terms of average sample number, probability of current lot acceptance and total cost of inspection.

The remainder of this paper is arranged as follows. A brief explanation of the ALT and the Weibull distribution is provided in Section 2, with the proposed operating method and design of the AMDSSP for the ALT given in Section 3. An economic design of the AMDSSP for the ALT is presented in Section 4, with a numerical illustration and application of two real datasets in Section 5. A discussion and conclusions drawn are provided in Section 6.

2 Accelerated Life Test and Weibull Distribution

2.1 Accelerated Life Test

The accelerated life test (ALT) is a technique for testing and analyzing systems and components that are predominantly electrical, electromechanical and mechanical to determine or improve their quality. A manufacturer can collect failure data by increasing the stress levels on a component to induce failure more rapidly. In practice, the ALT often mimics the real-world environments a product is likely to experience such as thermal changes, humidity and power cycling.

This study considered the Arrhenius model for temperature stress of an electronic device. This model is used when the damaging mechanism is temperature-sensitive (especially for integrating circuits, LEDs, dielectrics, semiconductors, battery cells and insulating tapes) [38]. The Arrhenius reaction rate equation is typically used to describe whether temperature affects the device as follows:

$$r = Ae^{-\frac{E_a}{kT}} \tag{1}$$

where E_a is the activation energy and a low value denotes a small temperature dependency, k is Boltmann's constant 8.6171×10^{-5} eV/K, A is a nonthermal constant and T is temperature in degrees Kelvin. Assuming that device life (L) is proportional to the inverse reaction rate of the process, then Eq. (1) can be rewritten as $L = Ae^{E_a/kT}$. The acceleration factor (AF) is the ratio between product life under use conditions (L_U) and life under accelerated conditions (L_A). The thermal acceleration factor can be expressed as [38]:

$$AF = \frac{L_U}{L_A} = e^{\frac{E_a}{k}\left(\frac{1}{T_U} - \frac{1}{T_A}\right)} \tag{2}$$

where T_U is the use condition temperature ($°C + 273$) and T_A is the accelerated temperature ($°C + 273$). Thus, AF is greater than 1 in the case where T_A is greater than T_U.

2.2 Weibull Distribution

The ALT of products has been the subject of several engineering studies where the lifetime is based on the Weibull distribution. This study presented a novel adaptive sampling plan for the ALT in the case of a Weibull distributed product lifetime. Let t_U be the lifetime of a product under use condition following the Weibull distribution with shape parameter δ and scale parameter λ_U. The cumulative distribution function under the use condition can be defined by:

$$F_U(t_U) = 1 - e^{-\left(\frac{t_U}{\lambda_U}\right)^{\delta}}, \quad t_U \geq 0,\ \lambda_U > 0,\ \delta > 0, \tag{3}$$

Kim et al. [25] suggested t_A as the lifetime of a product under accelerated conditions following the Weibull distribution, with shape parameter δ and scale parameter λ_A. Suppose $\lambda_A = \lambda_U/AF$, where AF is the acceleration factor. Then, the cumulative distribution function and mean lifetime under the accelerated condition can be defined by:

$$F_A(t_A) = 1 - e^{-\left(\frac{t_A}{\lambda_A}\right)^{\delta}}, \quad t_A \geq 0,\ \lambda_A > 0,\ \delta > 0, \tag{4}$$

and

$$\mu = \left(\frac{\lambda_A}{\delta}\right)\Gamma\left(\frac{1}{\delta}\right). \tag{5}$$

From Eq. (4), the failure probability of a product at censoring time τ_A when $\lambda_A = \lambda_U/AF$ under the accelerated condition is shown by Eq. (6):

$$p = 1 - e^{-\left(\frac{\tau_A \times AF}{\lambda_U}\right)^{\delta}} \tag{6}$$

Suppose $\tau_A \times AF$ is equivalent to the censoring time at the use condition [25]. The value of censoring time under the accelerated condition τ_A can be written in terms of the specified mean lifetime μ_0, e.g., $\tau_A = a\mu_0$ for an experiment termination ratio (a). From Eq. (6), the failure probability of the product at the censoring time τ_A can be rewritten as:

$$p = 1 - e^{-a^{\delta} AF^{\delta}\left(\frac{\mu_0}{\mu}\right)^{\delta}\left(\frac{1}{\delta}\Gamma\left(\frac{1}{\delta}\right)\right)^{\delta}}. \tag{7}$$

Eq. (7) shows the failure probability in terms of the experiment termination ratio, shape parameter, acceleration factor and mean lifetime based on the Weibull distribution.

3 Design of an Adaptive Multiple Dependent State Sampling Plan for Accelerated Life Test

Wortham et al. [4] presented the multiple dependent state sampling plan (MDSSP) as an attribute of the inspection process. This sampling plan requires continuous sampling from both the current and previous lots and decides whether to accept or reject the current lot. This approach results in reduced sample size and is often used when manufacturing is continuous with multiple lots, and each lot is submitted sequentially for inspection. However, the MDSSP considers data from previous lots using a minimum sample size to eliminate a current lot of moderate quality. The existing MDSSP will accept or reject the current lot under a single sampling but in some cases, a single sampling may not be sufficient to accept or reject the current lot. Therefore, this presents an opportunity to increase the producer's risk and reduce the consumer's risk.

This study put the MDSSP and the DSP concepts into practice using a novel adaptation of the MDSSP called the AMDSSP. If the quality of the first sample is undecided, the second sample must be inspected before deciding whether to accept the model. This proposed sampling plan can reduce the sample size of the MDSSP by recognizing that it will accept the current lot if it is of good or moderate quality. Therefore, the proposed sampling plan is more flexible than the existing MDSSP, with reduced sample size. Suppose the AMDSSP operates under the same conditions and procedures as the existing MDSSP by considering the following conditions. The product inspected consists of serial lots produced by continuous processes. Every lot inspected should be of the same quality. A certain number of samples are taken from each lot. The current lot will be of the same quality as the previous lot, and customers trust that the manufacturer is being honest.

This research used the AMDSSP to design an optimal sampling plan for the ALT when the lifetime of the product followed the Weibull distribution. This proposed plan consisted of five parameters including n_1, n_2, c_1, c_2 and m. The AMDSSP for ALT has the following operational steps:

Step 1. Choose the first random sample size n_1 for the known AF under accelerated conditions at time 0 from the current lot. Sample items should be put through the life test under accelerated conditions. Count the nonconforming items before the censoring time τ_A, which is denoted by d_1.

Step 2. The current lot is accepted as being of good quality if $d_1 \le c_1$, and it is rejected if $d_1 > c_2$ or when the censoring time τ_A is reached, whichever comes first. Otherwise, go to Step 3.

Step 3. Choose the second sample size n_2 if $c_1 < d_1 \le c_2$. Sample items should be put through the life test under accelerated conditions. Count the nonconforming items which expire before the censoring time τ_A and denote as d_2. If $d_1 + d_2 \le c_2$ and the remaining m previous lots are of good quality, consider the current lot to be of moderate quality. Otherwise, reject the current lot.

Let c_1 be the maximum acceptable number of nonconforming items for unconditional acceptance $c_1 \ge 0$ and c_2 be the maximum acceptable number of additional nonconforming items for conditional acceptance $c_2 > c_1$. We can summarize the above steps in a flow chart, as presented in Fig. 1.

The probability of current lot acceptance when it is of good quality without considering the quality of m previous lots is denoted by P_I (p) and given as follows:

$$P_I(p) = P(d_1 \le c_1). \tag{8}$$

The probability of current lot acceptance when it is of moderate quality provided previous lots are of good quality ($d_1 \le c_1$) is denoted by P_{II} (p) and obtained as follows:

$$P_{II}(p) = P(c_1 < d_1 \le c_2)P(d_1 + d_2 \le c_2)[P(d_1 \le c_1)]^m. \tag{9}$$

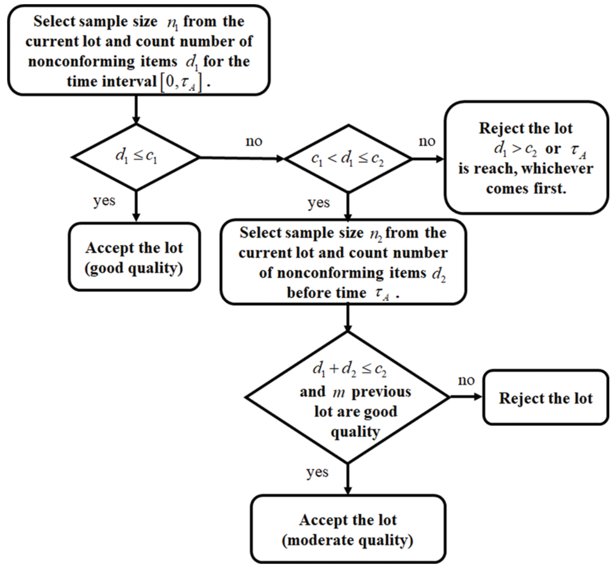

Figure 1: Operating procedure of the AMDSSP for ALT

As a result, the operating characteristic (OC) function is described by

$$\begin{aligned} P_a(p) &= P_I(p) + P_{II}(p) \\ &= P(d_1 \le c_1)\left[1 + P(c_1 < d_1 \le c_2)P(d_1 + d_2 \le c_2)[P(d_1 \le c_1)]^{m-1}\right]. \end{aligned} \tag{10}$$

The binomial distribution can be used to derive the OC function from Eq. (11) as follows:

$$P_a(p) = \sum_{d_1=0}^{c_1} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} \times$$
$$\left(1 + \left(\sum_{d_1=c_1+1}^{c_2} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1}\right) \cdot \left(\sum_{d_2=0}^{c_2-d_1} \binom{n_2}{d_2} p^{d_2}(1-p)^{n_2-d_2}\right) \cdot \left(\sum_{d_1=0}^{c_1} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1}\right)^{m-1}\right). \tag{11}$$

Another important point to note is that we can switch AMDSSPs to SSPs and DSPs by considering $m \to \infty$, the AMDSSP is reduced to SSP with an acceptance number c_1, while at $m \to 0$, the AMDSSP is reduced to DSP with acceptance numbers c_1 and c_2. The average sample number (ASN) of the AMDSSP is derived by:

$$\begin{aligned} ASN &= n_1 + n_2(1 - P_I);\ P_I = P(d_1 \le c_1) + P(d_1 > c_2) \\ &= n_1 + n_2 \sum_{d_1=c_1+1}^{c_2} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} \end{aligned} \tag{12}$$

Sample size and costs are necessary for the inspection process to provide good quality electronic products. Effective economic sampling plans can reduce the sample size and cost of the inspection process. The AMDSSP for the ALT is designed to obtain a lower ASN and total cost of the inspection process than existing sampling plans. This proposed sampling plan ensures that the mean lifetime (μ) and the ratio between true mean lifetime and specified mean lifetime (μ_0) or mean ratio are essential. It was observed that $\mu/\mu_0 \ge 1$. If μ/μ_0 is increased, then the true mean lifetime is longer than the specified mean lifetime.

In quality control studies, there is no explicit guarantee when using a sampling method that all products will be of good quality. Two types of risks can occur. The producer's risk (α) denotes the probability that a good lot will be rejected as performing unsatisfactorily, while the consumer's risk (β) denotes the probability of accepting a lot of poor quality. The producer's risk can be controlled directly, while the consumer's risk depends on the sample size used in the test. A larger sample size gives a more negligible consumer's risk. The consumer's risk is often difficult to control because of the lack of flexibility in choosing the sample size.

The mean ratio, which affects the quality level of the product, is related to failure probability. The acceptable quality level (AQL or p_1) and the limiting quality level (LQL or p_2) are taken into consideration when determining the requirements for producer's risks (alpha) and consumer's risks (beta). The AMDSSP for the ALT is practical for two points (AQL, 1−α) and (LQL, β) and is considered for changes in the OC curve. A producer expects that the probability of current lot acceptance should be greater than 1−α at p_1. On the other hand, a customer expects that the probability of current lot acceptance should be less than β at p_2. The nonlinear optimization technique is used to determine the optimal parameters resulting in reduced size of the *ASN* and total cost at p_1 under the ALT.

4 Economic Design of an AMDSSP for the Accelerated Life Test

This section presents an economic design of an AMDSSP for the ALT following the concepts of Hsu et al. [2] and Hakamipour [37]. Performance indicators of the proposed sampling plan, such as $P_I(p)$, $P_{II}(p)$ and $P_a(p)$ are given in Eqs. (8)–(10) and average total inspection (*ATI*) values are used, where *ATI* is defined as [39]:

$$ATI = n_1 P_I(p) + (n_1 + n_2) P_{II}(p) + N(1 - P_a(p)) \tag{13}$$

Three cost components are considered in the AMDSSP for ALT under the current lot inspection as the cost of the accelerated life test, the expected cost of internal failure per lot and the expected cost of external failure per lot.

First component: Let C_A be the cost of the accelerated life test for each lot of products, as shown in Eq. (14).

$$C_A = C_s + C_u ATI + C_o \tau_A \tag{14}$$

where C_s represents the fixed cost for setting up an accelerated life test experiment for the time interval [0, τ_A], C_u represents the sampling cost per unit and C_o represents the operating cost of conducting an accelerated life test per unit time.

Second component: Let C_I be the cost of internal failure for each lot of products, as shown in Eq. (15).

$$C_I = C_d(ATI \cdot p + (1 - P_a(p))(N - ATI)p) \tag{15}$$

where C_d represents the cost of replacement per unit, N represents the lot size and $ATI \cdot p + (1-P_A(p))(N - ATI)$ p represents the expected number of nonconforming items detected per lot.

Third component: Let C_E be the cost of external failure for each lot of products, as shown in Eq. (16).

$$C_E = C_{nd} \cdot (N - ATI)p \cdot P_a(p) \tag{16}$$

where C_{nd} represents the cost of an outgoing nonconforming per unit and $(N - ATI)\, p \cdot P_a(p)$ represents the expected number of nonconforming items not detected per lot. Therefore, the total cost for inspection of the products per lot under the AMDSS plan for ALT is given by:

$$\begin{aligned} TC &= C_A + C_I + C_E \\ &= C_s + C_u ATI + C_o \tau_A + C_d(ATI \cdot p + (1 - P_a(p))(N - ATI)p) + C_{nd} \cdot (N - ATI)p \cdot P_a(p) \end{aligned} \tag{17}$$

In this research, the nonlinear optimization technique was used to determine the optimal parameters for reduced size of the *ASN* and total cost at p_1 under the ALT. The genetic algorithm (GA) method with nonlinear optimization was applied using the MATLAB program. The procedure for applying the GA method to determine the optimal parameters is shown as a flow chart in Fig. 2.

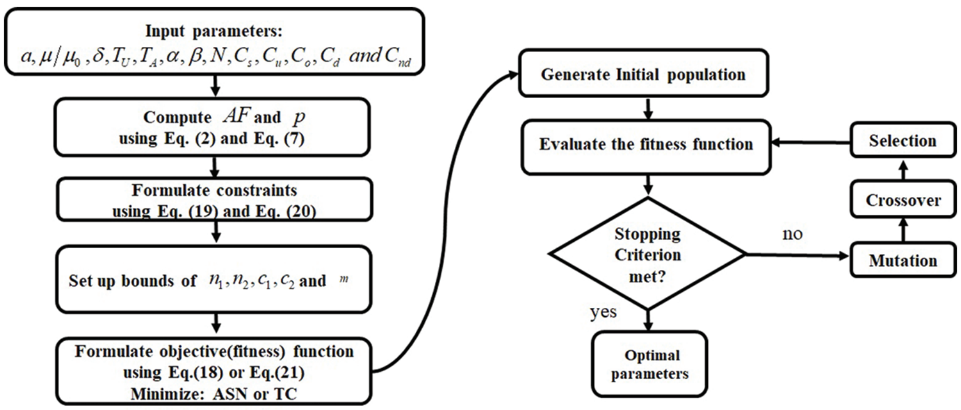

Figure 2: Flow chart of the GA method to determine the optimal parameters based on the AMDSS plan for the ALT

5 Numerical Illustration

5.1 Numerical Illustration of the AMDSS Plan for the ALT

In this section, plan parameters of the AMDSSP for ALT were determined by supposing that the mean ratio of the producer's risk is $\mu/\mu_0 = 2, 4, 6, 8$. By contrast, consumers expect to receive good products. The mean ratio $\mu/\mu_0 = 1$ was assumed as the consumer's risk. Under the Weibull distribution, values of p_1 and p_2 were calculated using Eq. (7) for different values of μ/μ_0. The optimal parameters (n_1, n_2, c_1, c_2, m) of the AMDSSP for ALT under the Weibull distribution were determined and selected to simultaneously satisfy both the producer's and consumer's risks with the minimum *ASN*. The producer's risk was set at $\alpha = 0.05$ with different consumer's risk values at $\beta = 0.10$ and 0.05. Two values of shape parameters under the Weibull distribution were considered as $\delta = 2.5$ and 3. In the ALT process, spending minimum censoring time under accelerated conditions is necessary. Thus, for $\tau_A = a\mu_0$, a should be closer to 0, resulting in a lower τ_A. Then, a was defined as 0.1, 0.2 and 0.5. The Arrhenius model was used for temperature stress to compute the thermal *AF*. In the used condition, temperature T_U was 50°C with different values for the accelerated temperature T_A = (120, 125, 130, 135) °C. From Eq. (2), the *AF* values are 6.80, 7.60, 8.47 and 9.41. The optimal plan parameter of the AMDSSP for ALT to minimize *ASN* can be determined using the nonlinear optimization problem as follows:

Objective function: Minimize $ASN(p) = n_1 + n_2 \sum_{d_1=c_1+1}^{c_2} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1}$ (18)

Subject to:

$$P_a(p_1) = \sum_{d_1=0}^{c_1} \binom{n_1}{d_1} p_1^{d_1}(1-p_1)^{n_1-d_1} + \left(\sum_{d_1=c_1+1}^{c_2} \binom{n_1}{d_1} p_1^{d_1}(1-p_1)^{n_1-d_1}\right) \cdot \left(\sum_{d_2=0}^{c_2-d_1} \binom{n_2}{d_2} p_1^{d_2}(1-p_1)^{n_2-d_2}\right) \cdot \left(\sum_{d_1=0}^{c_1} \binom{n_1}{d_1} p_1^{d_1}(1-p_1)^{n_1-d_1}\right)^m \geq 1-\alpha, \tag{19}$$

$$P_a(p_2)=\sum_{d_1=0}^{c_1}\binom{n_1}{d_1}p_2^{d_1}(1-p_2)^{n_1-d_1}$$
$$+\left(\sum_{d_1=c_1+1}^{c_2}\binom{n_1}{d_1}p_2^{d_1}(1-p_2)^{n_1-d_1}\right)\cdot\left(\sum_{d_2=0}^{c_2-d_1}\binom{n_2}{d_2}p_2^{d_2}(1-p_2)^{n_2-d_2}\right)\cdot\left(\sum_{d_1=0}^{c_1}\binom{n_1}{d_1}p_2^{d_1}(1-p_2)^{n_1-d_1}\right)^m \tag{20}$$
$$\leq\beta$$

$n_1,\ n_2>1,\quad m\geq 1,\quad c_1>c_2\geq 0.$

Results in Tables 1–3 show that for fixed values of a, AF and μ/μ_0 the ASN decreased with either an increment in β or a decrement in δ. For fixed values of δ, β, a and μ/μ_0 the ASN increased with an increment in AF. For instance, $\delta=3$, $\beta=0.05$, $a=0.1$ and $\mu/\mu_0=2$, as shown in Table 1. The ASN increased from 17.6808 to 26.0128 when AF changed from 6.80 to 9.41, while ASN increased when δ increased for fixed values of β, a, μ/μ_0 and AF. Also, ASN decreased if either the value of a or μ/μ_0 increased.

Table 1: Optimal plan parameters of the AMDSSP for ALT with a = 0.1

δ	AF	μ/μ_0	β = 0.05							β = 0.10						
			n_1	n_2	c_1	c_2	m	ASN	$P_a(p_1)$	n_1	n_2	c_1	c_2	m	ASN	$P_a(p_1)$
2.5	6.80	2	15	9	1	3	1	16.4334	0.9527	12	3	1	2	1	12.2855	0.9592
		4	13	5	0	1	1	13.5143	0.9792	11	10	0	2	1	11.9252	0.9908
		6	12	7	0	1	3	12.2596	0.9946	9	9	0	1	2	9.2527	0.9973
		8	12	8	0	1	2	12.1473	0.9989	9	9	0	1	1	9.1249	0.9995
	7.60	2	**18**	**4**	**2**	**3**	**1**	**18.3161**	**0.9506**	17	3	2	3	1	17.2110	0.9619
		4	17	6	1	3	3	17.0976	0.9991	16	7	2	3	2	16.0053	0.9999
		6	15	3	0	2	1	15.1845	0.9962	14	6	1	3	2	14.0094	1.0000
		8	15	4	1	2	1	15.0017	1.0000	12	4	0	2	1	12.0978	0.9994
	8.47	2	20	13	3	5	1	20.9607	0.9638	19	12	3	5	1	19.7628	0.9719
		4	19	5	2	3	1	19.0133	0.9996	17	5	2	3	1	17.0096	0.9997
		6	18	9	1	3	1	18.0398	1.0000	15	10	0	2	1	15.7984	0.9933
		8	16	5	0	2	2	16.2117	0.9965	14	6	0	2	4	14.2229	0.9948
	9.41	2	22	5	4	5	1	22.2659	0.9506	20	6	4	5	1	20.2353	0.9645
		4	20	7	3	4	1	20.0037	0.9999	19	9	4	5	1	19.0002	1.0000
		6	19	4	1	4	3	19.0327	0.9998	17	6	1	2	4	17.0379	0.9993
		8	18	6	1	2	1	18.0107	0.9999	16	4	1	2	1	16.0056	1.0000

(Continued)

Table 1 (continued)

δ	AF	μ/μ_0	$\beta = 0.05$							$\beta = 0.10$						
			n_1	n_2	c_1	c_2	m	ASN	$P_a(p_1)$	n_1	n_2	c_1	c_2	m	ASN	$P_a(p_1)$
3	6.80	2	**17**	**10**	**1**	**2**	**1**	**17.6808**	0.9686	16	15	1	2	3	16.9266	0.9618
		4	16	7	0	1	1	16.3711	0.9945	15	2	0	1	3	15.0998	0.9912
		6	14	14	0	1	2	14.2003	0.9993	13	13	0	2	1	13.1740	0.9998
		8	13	11	0	1	2	13.0622	0.9999	10	3	0	1	2	10.0131	0.9999
	7.60	2	22	13	2	4	2	22.6377	0.9879	19	8	2	3	2	19.2334	0.9855
		4	19	8	1	3	1	19.0307	1.0000	18	18	2	3	1	18.0016	1.0000
		6	18	10	1	3	2	18.0032	1.0000	15	4	1	2	2	15.0009	1.0000
		8	17	6	1	2	2	17.0003	1.0000	14	5	0	2	1	14.0426	0.9999
	8.47	2	23	10	2	4	2	24.1197	0.9547	20	8	2	3	1	20.5307	0.9541
		4	21	8	0	3	3	22.0588	0.9541	18	6	2	3	1	18.0014	1.0000
		6	19	4	0	2	3	19.1494	0.9960	17	5	0	1	4	17.1647	0.9950
		8	18	9	0	2	2	18.1359	0.9995	15	8	0	2	2	15.1008	0.9997
	9.41	2	25	11	3	5	1	**26.0128**	0.9609	22	12	3	4	1	22.6031	0.9514
		4	23	7	1	2	3	23.1241	0.9967	21	7	2	3	1	21.0062	0.9999
		6	22	8	2	3	3	22.0002	1.0000	19	5	2	3	3	19.0001	1.0000
		8	20	4	1	2	3	20.0010	1.0000	17	8	2	3	1	17.0000	1.0000

The OC curves present the effect of various m values based on the probability of current lot acceptance with the same values n_1, n_2, c_1 and c_2. The optimal plan parameters were considered for fixed $a = 0.1$, $\delta = 3$, $\mu/\mu_0 = 2$, $AF = 6.8$, $\alpha = 0.05$ and $\beta = 0.05$. Results in Table 1 show that the optimal plan parameters were (n_1, n_2, c_1, c_2, m) = (17, 10, 1, 2, 1) and the OC function for the AMDSSP with $m = 1$, 2, 3 and 4 is shown in Fig. 3. For $m = 2$, 3 and 4, the probability of current lot acceptance was lower than $m = 1$. As a result, high probability of accepting the current lot depended only on accepting the previous lot. In addition, if the proportion of nonconformity increased, the value of m did not significantly affect the probability of current lot acceptance.

Example: To apply the AMDSSP for the ALT, we used results in Tables 1–3. Suppose that the producer intends to apply the AMDSSP in the inspection process where the lifetime is based on the Weibull distribution with $\delta = 2.5$. Let $\mu_0 = 1{,}000$ and $\tau_A = 100$; then, $a = 0.1$. Also, we assumed that $\alpha = 0.05$, $\beta = 0.05$, $AF = 7.60$ and $\mu/\mu_0 = 2$. Table 1 gives the optimal plan parameters for the AMDSSP as $n_1 = 18$, $n_2 = 4$, $c_1 = 2$, $c_2 = 3$ and $m = 1$, with the probability of current lot acceptance 0.9565 and ASN 18.3161. The inspection procedure is as follows:

Step 1: Choose an initial random sample of 18 items at time 0 under accelerated conditions. Conduct the ALT on each sampled item and count the number of nonconforming items (d_1) before $\tau_A = 100$ h.

Step 2: If $d_1 \leq 2$, the current lot will be accepted regardless of the quality of the previous lot, and called **good quality**. If $d_1 > 3$, the current lot will be rejected. Otherwise, go to step 3.

Table 2: Optimal plan parameters of the AMDSSP for ALT with $a = 0.2$

δ	AF	μ/μ_0	$\beta = 0.05$							$\beta = 0.10$						
			n_1	n_2	c_1	c_2	m	ASN	$P_a(p_1)$	n_1	n_2	c_1	c_2	m	ASN	$P_a(p_1)$
2.5	6.80	2	12	6	5	6	1	12.2269	0.9557	11	5	5	6	1	11.1254	0.9739
		4	9	6	2	3	1	9.0433	0.9975	9	7	3	5	1	9.0041	1.0000
		6	8	7	2	3	1	8.0021	1.0000	8	8	3	5	1	8.0001	1.0000
		8	8	8	2	3	1	8.0003	1.0000	7	5	3	4	1	7.0000	1.0000
	7.60	2	18	11	9	19	2	18.2095	0.9729	15	13	8	9	1	15.1918	0.9804
		4	16	9	2	4	3	16.6804	0.9710	13	13	2	4	1	13.5862	0.9861
		6	15	7	1	2	2	15.3016	0.9851	12	7	2	3	1	12.0165	0.9995
		8	13	5	0	3	1	13.7037	0.9801	11	8	2	3	2	11.0019	1.0000
	8.47	2	18	10	10	11	2	18.3025	0.9547	17	12	10	11	1	17.2303	0.9730
		4	17	8	3	3	1	17.0000	0.9531	15	8	3	4	3	15.1992	0.9807
		6	16	7	2	3	1	16.0770	0.9965	14	4	2	3	1	14.0305	0.9983
		8	14	4	2	3	1	14.0043	0.9999	12	6	2	3	1	12.0040	0.9999
	9.41	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	20	11	4	5	1	20.4314	0.9564	17	4	3	5	2	17.3710	0.9663
		6	19	6	3	4	1	19.0324	0.9981	15	3	2	3	1	15.0536	0.9952
		8	17	3	2	3	1	17.0118	0.9995	14	5	3	4	1	14.0006	1.0000
3	6.80	2	15	8	6	7	1	15.1122	0.9839	14	6	5	6	1	14.1958	0.9637
		4	10	5	2	3	1	10.0104	0.9996	10	7	3	4	1	10.0007	1.0000
		6	10	7	2	3	1	10.0004	1.0000	9	5	3	4	1	9.0000	1.0000
		8	9	6	2	4	1	9.0003	1.0000	8	8	3	4	1	8.0000	1.0000
	7.60	2	19	10	8	9	1	19.2936	0.9569	16	12	7	8	1	16.3416	0.9599
		4	18	6	2	3	1	18.1533	0.9898	15	3	2	3	2	15.0481	0.9956
		6	17	9	2	3	1	17.0079	0.9999	14	8	2	3	2	14.0039	0.9999
		8	16	7	1	2	1	16.0186	0.9998	12	7	2	3	2	12.0002	1.0000
	8.47	2	21	12	11	12	1	21.2532	0.9679	18	11	10	11	1	18.1700	0.9783
		4	20	9	2	3	1	20.5970	0.9521	17	8	2	3	1	17.3723	0.9714
		6	18	6	2	2	1	18.0000	0.9973	15	8	2	3	1	15.0121	0.9997
		8	17	8	0	2	2	17.8670	0.9772	14	7	0	2	1	14.6314	0.9915
	9.41	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	21	5	3	4	1	21.2214	0.9701	18	8	3	4	2	18.2263	0.9790
		6	20	6	3	4	1	20.0046	0.9998	16	4	2	2	1	16.0000	0.9953
		8	18	3	1	2	4	18.0337	0.9986	15	8	1	2	3	15.0634	0.9989

Table 3: Optimal plan parameters of the AMDSSP for ALT with $a = 0.5$

δ	AF	μ/μ_0	$\beta = 0.05$							$\beta = 0.10$						
			n_1	n_2	c_1	c_2	m	ASN	$P_a(p_1)$	n_1	n_2	c_1	c_2	m	ASN	$P_a(p_1)$
2.5	6.80	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	8	6	5	6	1	8.2194	0.9580	7	5	5	6	2	7.0749	0.9849
		6	7	5	3	4	1	7.0741	0.9893	6	5	2	3	1	6.2581	0.9600
		8	7	5	3	4	1	7.0066	0.9995	6	6	2	3	1	6.0540	0.9956
	7.60	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	14	12	10	11	1	14.1886	0.9801	12	10	9	10	1	12.1142	0.9865
		6	13	9	5	6	1	13.2583	0.9651	11	7	5	6	1	11.0868	0.9872
		8	10	8	3	4	2	10.1183	0.9885	8	5	2	3	1	8.2032	0.9743
	8.47	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	15	15	12	13	1	15.2131	0.9828	13	9	10	11	1	13.2874	0.9601
		6	14	7	6	7	1	14.2635	0.9509	12	4	5	7	1	12.2718	0.9661
		8	12	4	3	4	1	12.2290	0.9534	10	4	3	4	1	10.1315	0.9770
	9.41	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		6	17	13	9	10	1	17.2452	0.9735	14	4	7	8	1	14.1580	0.9509
		8	16	4	6	7	1	16.0451	0.9904	12	6	4	5	1	12.2177	0.9632
3	6.80	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	9	8	5	7	1	9.4415	0.9504	9	8	5	7	1	9.4415	0.9504
		6	9	6	3	4	2	9.0863	0.9898	8	6	2	3	1	8.3155	0.9599
		8	8	8	2	4	2	8.0549	0.9993	7	5	2	3	1	7.0212	0.9987
	7.60	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	15	13	10	11	1	15.2798	0.9711	12	10	8	9	2	12.3059	0.9605
		6	14	5	4	5	1	14.2438	0.9512	11	5	4	5	1	11.0968	0.9838
		8	11	2	2	3	1	11.0711	0.9874	9	4	2	3	1	9.0843	0.9914
	8.47	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	16	14	12	13	2	16.4211	0.9597	14	14	11	12	2	14.2793	0.9756
		6	15	6	0	1	1	15.0000	0.9691	13	7	5	5	1	13.0000	0.9524
		8	12	9	3	4	1	12.1933	0.9822	11	6	2	4	1	11.5239	0.9751
	9.41	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		4	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		6	18	5	8	9	1	18.1635	0.9562	15	4	7	8	1	15.1184	0.9653
		8	17	7	5	5	3	17.0000	0.9803	13	6	4	5	1	13.1081	0.9850

Notes: -There is no optimal plan.

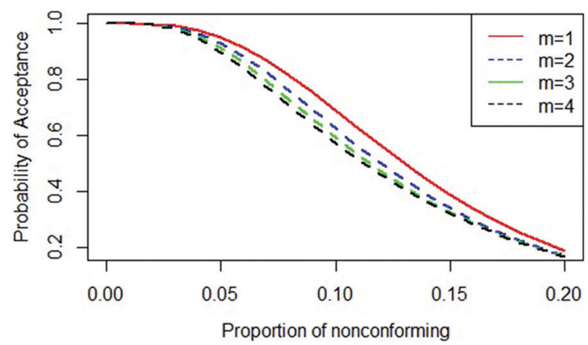

Figure 3: OC curves of the AMDSSP for ALT with different m values

Step 3: If $2 < d_1 \leq 3$, choose a second sample size of 4 items. Conduct the ALT on each of the 4 items and count the number of nonconforming items (d_2) before $\tau_A = 100$ h. Accept the current lot if $d_1 + d_2 \leq 3$ and the previous lot is of good quality, which is called **moderate quality**. Otherwise, reject the current lot.

5.2 Numerical Illustration of the Economic Design of an AMDSSP for the ALT

Optimal plan parameters of the AMDSSP for ALT to minimize the total cost of inspection are determined using the nonlinear optimization problem as follows:

Objective function: Minimize

$$TC = C_s + C_u ATI + C_o \tau_A + C_d(ATI \cdot p + (1 - P_a(p))(N - ATI)p) + C_{nd} \cdot (N - ATI)p \cdot P_a(p) \quad (21)$$

Subject to: $P_a(p_1) \geq 1 - \alpha, \quad P_a(p_2) \leq \beta,$

$$n_1,\ n_2 > 1, \quad m \geq 1, \quad c_1 > c_2 \geq 0.$$

Let $P_a(p_1)$ and $P_a(p_2)$ be the probabilities for lot acceptance at p_1 and p_2 obtained using Eqs. (19) and (20).

The optimal parameters of an economic AMDSSP for the ALT along with corresponding $P_a(p)$, *ATI* and *TC* are reported in Tables 4 and 5. Suppose the input parameters are $\delta = 3$, $N = 1{,}000$, $C_s = 10$, $C_u = 0.1$, $C_o = 1$, $C_d = 2$ and $C_{nd} = 10$. The fixed values of producer's risk and consumer's risks are assumed to be $(\alpha, \beta) = (0.05, 0.05)$. The failure probability of product (p) corresponding to the mean ratios $\mu/\mu_0 = 2$, 4, 6, 8 are considered as p_1 and the probability of failure at the ratio $\mu/\mu_0 = 1$ is taken as p_2. Considering different values of $a = 0.1$ and 0.2, $AF = 6.80$, 7.6, 8.47 and 9.41, while $\tau_A = 250$ and 500 h, respectively.

ATI and *TC* increased if *AF* increased and τ_A decreased for fixed value of a and μ/μ_0, as shown in Tables 4 and 5. For fixed values of a, *AF* and τ_A, *ATI* and *TC* decreased when μ/μ_0 increased. *ATI* and *TC* increased when a increased for fixed values of μ/μ_0, *AF* and τ_A. From Eq. (17), *ATI* was correlated in the same direction as *TC*; if *ATI* decreased, *TC* also decreased. From results in Table 4, $\tau_A = 250$, $a = 0.1$ and $\mu/\mu_0 = 4$, *TC* increased from 296.8704 to 353.1590 when *AF* changed from 6.80 to 9.41. This means that *TC* also increased if the manufacturer increased the *AF* or τ_A in the ALT.

Table 4: Optimal plan parameters of economic AMDSSP for the ALT with $\tau_A = 250$

a	AF	μ/μ_0	n_1	n_2	c_1	c_2	m	ATI	TC	$P_a(p)$
0.1	6.80	2	30	20	2	4	1	39.2613	528.9737	0.9893
		4	27	6	1	2	1	27.1049	**296.8704**	0.9998
		6	25	7	0	2	1	25.0201	272.6496	0.9993
		8	21	5	0	1	2	21.0331	266.4010	0.9998
	7.60	2	32	12	2	4	2	74.4586	614.6518	0.9536
		4	29	11	1	2	1	29.6408	310.4968	0.9991
		6	28	8	0	1	1	29.6357	277.0508	0.9973
		8	26	8	0	1	1	26.0315	268.5772	0.9996
	8.47	2	35	15	3	5	1	76.7705	744.0898	0.9544
		4	33	10	1	2	1	35.3959	328.8507	0.9969
		6	30	8	0	2	1	31.7121	282.6216	0.9965
		8	28	7	0	1	1	28.2848	271.0785	0.9991
	9.41	2	38	15	5	6	4	78.0871	915.8169	0.9580
		4	35	11	2	3	3	35.5709	**353.1590**	0.9993
		6	32	9	2	3	1	32.0011	289.9289	1.0000
		8	29	10	0	2	1	29.1469	274.2164	0.9989
0.2	6.80	2	16	10	5	8	1	55.4907	2120.7000	0.9592
		4	15	2	1	3	2	22.6613	531.3983	0.9914
		6	14	11	0	1	1	14.8127	343.0270	0.9991
		8	**12**	**6**	**0**	**2**	**1**	13.2281	295.8288	0.9983
	7.60	2	-	-	-	-	-	-	-	-
		4	25	4	3	4	2	28.9791	635.9169	0.9957
		6	23	9	2	3	3	23.3033	375.2564	0.9996
		8	21	7	2	3	1	21.0049	310.0035	1.0000
	8.47	2	-	-	-	-	-	-	-	-
		4	29	11	3	5	1	29.0886	776.1599	0.9976
		6	26	15	2	3	2	28.1491	417.9212	0.9976
		8	24	7	1	2	1	24.9004	328.4656	0.9988
	9.41	2	-	-	-	-	-	-	-	-
		4	30	5	4	7	1	34.3628	955.2131	0.9939
		6	28	8	2	4	2	29.3074	474.8614	0.9981
		8	25	9	4	5	1	25.0003	352.9332	1.0000

Table 5: Optimal plan parameters of economic AMDSSP for the ALT with $\tau_A = 500$

a	AF	μ/μ_0	n_1	n_2	c_1	c_2	m	ATI	TC	$P_a(p)$
0.1	6.80	2	29	25	3	4	2	33.3615	780.9831	0.9954
		4	26	24	2	3	4	26.0076	546.7969	1.0000
		6	23	6	0	2	1	23.0170	522.4667	0.9994
		8	18	12	0	2	4	18.1022	516.1181	0.9998
	7.60	2	30	8	2	4	2	59.2285	871.6112	0.9676
		4	29	20	1	3	4	29.1316	560.4850	0.9996
		6	27	23	2	3	1	27.0001	526.8486	1.0000
		8	25	10	0	2	4	25.5198	518.5264	0.9991
	8.47	2	32	11	3	5	2	60.9639	1004.3730	0.9683
		4	30	25	1	3	4	31.0435	578.7286	0.9984
		6	29	29	2	3	4	29.0011	532.4476	1.0000
		8	26	10	0	2	4	27.2673	520.9778	0.9982
	9.41	2	34	10	4	6	1	67.0336	1173.895	0.9638
		4	31	11	1	3	2	32.6831	602.9407	0.9973
		6	30	13	2	3	1	30.0018	539.7728	1.0000
		8	27	10	0	2	2	28.0477	524.1097	0.9981
0.2	6.80	2	-	-	-	-	-	-	-	-
		4	14	10	1	3	1	19.4360	782.2997	0.9938
		6	13	10	2	3	3	13.0130	593.0231	1.0000
		8	11	11	2	3	3	11.0002	545.7157	1.0000
	7.60	2	-	-	-	-	-	-	-	-
		4	24	9	3	4	1	28.8092	885.7226	0.9949
		6	22	9	2	3	1	22.2548	625.2532	0.9997
		8	20	15	1	3	4	20.0075	559.9394	0.9999
	8.47	2	-	-	-	-	-	-	-	-
		4	27	27	5	6	1	28.8813	1026.4200	0.9981
		6	25	17	2	3	3	27.0945	667.9599	0.9977
		8	23	11	1	3	4	23.2612	578.4253	0.9995
	9.41	2	-	-	-	-	-	-	-	-
		4	28	10	5	7	1	31.1281	1208.2020	0.9965
		6	26	16	2	4	1	27.4049	725.0103	0.9981
		8	24	10	3	4	1	24.0065	602.9066	1.0000

Notes: -There is no optimal plan.

5.3 Comparative Study

A comparative study was undertaken between the proposed AMDSSP, MDSSP and an SSP for the ALT when lifetime followed the Weibull distribution. A literature review determined no previous research regarding an MDSSP for the ALT. An SSP for the ALT was proposed by Kim et al. [25]. The performances of AMDSSP, MDSSP and SSP were compared in terms of OC curve, *ASN* and *TC* for the same values of specified parameters. All three plans were considered based on the ALT for the Weibull distribution, where $\delta = 2.5$, $a = 0.1$, $AF = 7.60$, $\mu/\mu_0 = 6$, $\alpha = 0.05$ and $\beta = 0.05$. The parameters considered were $n_1 = 15$, $n_2 = 3$, $c_1 = 0$, $c_2 = 2$ and $m = 1$ for AMDSSP; $n = 18$, $c_1 = 0$, $c_2 = 2$ and $m = 1$ for MDSSP and $n = 18$ and $c = 0$ for SSP. The OC curve displayed the performance of the three plans as the difference in probabilities of accepting the lot under the same parameters. Fig. 4 shows that the AMDSSP had a higher OC curve compared to MDSSP and SSP for ALT. Moreover, the OC curve of the AMDSSP was consistent with the OC curve of the other two sampling plans when product failure probability increased.

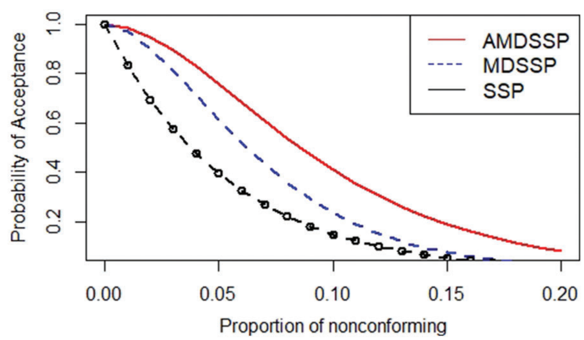

Figure 4: OC curves for AMDSSP, MDSSP and SSP

Results in Table 6 show that performances of AMDSSP, MDSSP and SSP for the ALT followed the Weibull distribution considered under the mean ratio $\mu/\mu_0 = 2, 4, 6, 8$, $\alpha = 0.05$ and $\beta = 0.05$. To illustrate the effectiveness of the AMDSSP for the ALT, we compared the values of *ASN* and *TC* of AMDSSP with MDSSP and SSP. The existing MDSSP and SSP used a single sample to inspect a current lot. Therefore, the value of *ATI* used in Eq. (17) differed from the proposed plan, that is, $ATI = n + (1 - P_a(p))(N - n)$ [39]. Results showed that both *ASN* and *TC* of the AMDSSP were smaller than the MDSSP and SSP for several set parameters. For instance, when $AF = 9.41$ and $\mu/\mu_0 = 2$, the *ASN* of the AMDSSP was 23.0565, while *ASN* values of MDSSP and SSP were 47 and 62, respectively. In the same way, the *TC* of the AMDSSP was 933.0939, while the *TC* of MDSSP and SSP were 935.0012 and 940.0031, respectively. Therefore, the *ASN* and the total cost for inspection under the accelerated condition decreased when implementing AMDSSP rather than MDSSP and SSP.

5.4 Application of Real Data (Electronic Device)

Two real datasets were used to investigate the performance of the AMDSSP for the ALT under the Weibull distribution. First, the Weibull distribution fit was checked for both datasets. Unknown parameters were estimated using the maximum likelihood method, while the goodness of fit test value was judged using the Kolmogorov-Smirnov (K-S) test. Model-fitting results for two real datasets are shown in Table 7.

Table 6: *ASN* and *TC* of the AMDSSP, MDSSP and SSP for ALT under the Weibull distribution with $\delta = 3$, and $a = 0.1$

AF	μ/μ_0	AMDSSP			MDSSP			SSP		
		ASN	TC	$P_a(p)$	ASN	TC	$P_a(p)$	ASN	TC	$P_a(p)$
6.80	2	16.1235	525.6044	0.9761	25	528.0172	0.9579	37	529.3647	0.9814
	4	14.6541	296.1078	0.9946	22	296.5094	0.9999	30	297.0961	0.9998
	6	12.0737	271.4841	0.9996	16	271.8521	0.9997	22	272.3999	0.9998
	8	9.0353	265.2421	0.9999	14	265.7302	0.9999	18	266.8521	0.9922
9.41	**2**	**23.0565**	**933.0939**	**0.9707**	**47**	**935.0012**	**0.9608**	**62**	**940.2031**	**0.9879**
	4	18.3074	351.4103	0.9764	24	352.9070	0.9999	39	353.2997	0.9999
	6	16.0009	288.6789	0.9999	18	288.8361	0.9999	33	290.0069	0.9999
	8	11.0003	272.5797	0.9999	15	272.9427	0.9999	24	273.7593	0.9999

Table 7: Model fitting results for two real datasets

Dataset	Parameter estimate	L-L	AIC	BIC	K-S	p-value
1	$\hat{\delta} = 0.9188$, $\hat{\lambda} = 9{,}290.9320$	−303.8968	611.7510	614.5534	0.0946	0.9279
2	$\hat{\delta} = 0.9297$, $\hat{\lambda} = 4{,}586.7004$	−235.9190	475.8381	478.2758	0.1147	0.8603

Dataset 1. As demonstrated by Pham [38], silicon carbide (SiC) can be used in place of silicon for semiconductor devices, particularly those that operate at high temperatures and electric fields. Temperatures as high as 145°C are used to conduct thorough accelerated life experiments on 6H-SiC metal-oxide-silicon (MOS) capacitors. The following data were recorded for 30 failure time (hours) observations:

75	359	701	722	738	1,015	1,388	2,285	3,157	3,547
3,986	4,077	5,447	5,735	5,869	6,242	7,804	8,031	8,292	8,506
8,584	11,512	12,370	16,062	17,790	19,767	20,145	21,971	30,438	42,004

From Eq. (2), for use condition temperature (T_U) 50°C and accelerated temperature (T_A) 145°C, $AF =$ 11.54. Results in Table 7 show that the K-S test result was 0.0946 with a p-value of 0.9279. Therefore, this dataset fitted to the Weibull distribution. The maximum likelihood estimate of shape parameter $\hat{\delta} = 0.92$ gave $\hat{\mu} = 9{,}287.30$. The cumulative distribution function of this data was $F(t) = 1 - e^{-\left(\frac{t}{9290.93}\right)^{0.92}}$ where t is failure time. Given that μ_0 is 9,287.30 h and τ_A is 928.73 h, then a is 0.1. A nonlinear optimization technique was used to solve the optimization problem in Eqs. (18)–(20) by supposing that $\alpha = 0.05$, $\beta = 0.05$ and $\mu/\mu_0 = 6$. The result gave optimal parameters for the AMDSS as (n_1, n_2, c_1, c_2, m) = (11, 6, 4, 5, 2), probability of current lot acceptance 0.9556 and ASN 11.2469. For illustration purposes, the first random sample selected 11 items from the dataset and sample items were placed on the ALT as follows:

1,015 3,986 4,077 **738** 5,735 **701** 4,200 48,506 11,512 2,285 30,438

From the 11 items, two failures ($d_1 = 2$) were recorded before the censoring time under accelerated condition $\tau_A = 928.73$ h. Results showed $d_1 < 4$ ($d_1 < c_1$). The current lot was accepted as good quality.

Dataset 2. In MOS devices, gate oxide is frequently the cause of device failure, particularly in high-density arrays that require thin gate oxides. For more details, see Pham [38]. A manufacturer ran a 150°C stress test on 25 devices, and the following failure time observations were made:

162	188	288	350	392	681	969	1,303	1,527	2,526	3,074	3,652	3,723
3,781	4,182	4,450	4,831	4,907	6,321	6,368	7,489	8,312	13,778	14,020	18,640	

Suppose $T_U = 50$°C and $T_A = 145$°C. By substituting in Eq. (2), the thermal acceleration factor $AF = 12.73$. Results in Table 7 show that the K-S test result was 0.1147 with a p-value of 0.8603. Therefore, this dataset fitted the Weibull distribution. The maximum likelihood estimate of shape parameter $\hat{\delta} = 0.93$ gave $\hat{\lambda} = 4{,}636.56$. The cumulative distribution function of this data was $F(t) = 1 - e^{-\left(\frac{t}{4586.70}\right)^{0.93}}$ where t is failure time. Given that μ_0 is 4,636.56 h and τ_A is 463.66 h, then a is 0.1. A nonlinear optimization technique in Eqs. (18) and (21) was used to solve this problem subject to Eqs. (19), (20) by supposing that $\alpha = 0.05$, $\beta = 0.05$, $\mu/\mu_0 = 8$ and $N = 25$. The result gave optimal parameters for the AMDSS as (n_1, n_2, c_1, c_2, m) = (8, 3, 2, 4, 1), $P_a = 0.9556$, $ASN = 8.4101$ and $TC =$ 505.1462. For illustration purposes, the inspection procedure was as follows:

Step 1: Select 8 items from the dataset for the first random sample, then put each sample item on the ALT and count the number of nonconforming items (d_1) before $\tau_A = 463.66$ h.

Step 2: If $d_1 < 2$, the current lot will be accepted as good quality. If $d_1 > 4$, the current lot will be rejected. Otherwise, go to step 3.

Step 3: If $2 < d_1 < 4$, select 4 items from the dataset for the second random sample. Put each sample item on the ALT and count the number of nonconforming items (d_2) before $\tau_A = 463.66$ h. Accept the current lot as moderate quality if $d_1 + d_2 < 4$ and the previous lot is good quality. Otherwise, reject the current lot.

6 Discussion and Conclusion

This paper proposed an adaptive multiple dependent state sampling plan (AMDSSP) for ALT when the lifetime of the product followed the Weibull distribution. We developed the proposed sample plan using the DSP concept together with the existing MDSSP. Under the accelerated condition, the effect of temperature on the electronic device was considered using the acceleration factor (AF) of the Arrhenius model. The nonlinear optimization technique determined the optimal plan parameters to satisfy consumer's risk and producer's risk simultaneously. Tables for optimal plan parameters are presented for values of δ, β, a, AF, μ/μ_0 and τ_A. The ASN value was used to judge the performance of the AMDSSP for ALT. Studies showed that the scale parameters were associated with the AF under accelerated conditions. Higher AF as higher temperature tests, increased the ASN. The illustrative example showed operational process for the AMDSSP for ALTs. An economic design of AMDSSP for ALT was also proposed under three costs for the current lot inspection as cost of the accelerated life test, the expected cost of internal failure per lot and the expected cost of external failure per lot. Optimal proposed plan parameters to achieve the lowest total cost of inspection gave values of a, AF, μ/μ_0 and τ_A. The ATI and TC for inspection determined the effectiveness of the proposed economic model. As AF increased and τ_A decreased, the ATI and the TC for inspection also increased. Efficacy studies of AMDSSP, MDSSP and SSP for ALT were compared when the lifetime of the product followed the Weibull distribution. Results showed that ASN and TC reduced under accelerated conditions using the AMDSSP rather than MDSSP and SSP. The application considered two real datasets on failure time of electronic devices under temperature stress. These datasets were applied to demonstrate the usability and utility of the proposed sampling plan for ALT. We concluded that the AMDSSP was more flexible, efficient and economical than MDSSP and SSP for ALT following the Weibull distribution. Future studies will consider the AMDSSP under various types of stress for accelerated testing techniques.

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