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Heat and Mass Transfer Along of a Vertical Wall by Natural convection in Porous Media

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Abstract: This work treats heat and mass transfer by natural convection along a vertical wall in porous media imbibed by fluid, using an integral method. The problem governing parameters are the buoyancy ratio, N, and the Lewis number, Le. The results for the local Nusselt and Sherwood numbers are presented for a large range of these parameters. The concentration and thermal boundary layer thickness are also determined. We observe that our results are in good agreement with those obtained by Bejan and Khair (1985).

Keywords: Integral method, Transfer, natural convection, thermal boundary layer thickness, porous media.

1 Introduction

In several fields such as drying processes, agriculture and building energy analysis, it is important to know the dynamics of temperature and moisture content distributions and how they relate to each other to evaluate heat flux and mass transfer through porous media depending on applications on a variety of geophysical and technological problems.

Free convection flows arising as a consequence of combined thermal and solutal buoyancy effects in porous media are of importance because of the fundamental nature of the problem and broad range of applications (relating to the manufacture and industrial process such as geothermal systems, fibro, stock age of the nuclear products, the dispersion of chemical contaminate ...). In particular, free convection about a vertical impermeable surface embedded in a porous medium, belongs to a family of heat transfer phenomena which have a wide range of applications in many geophysical and industrial fields. These problems have been treated by Ene and Poliševski,(1987), Kays and Crawford (1993), and a great deal of effort has

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been devoted during the past decades to the understanding of the convective heat transfer in a fluid-saturated porous medium subject to various additional effects.

An exhaustive review of the topic of convective flow in porous media can be found in the books of Nield and Bejan (1999), Ingham and Pop (1998, 2002), Vafai (2000), Pop and Ingham (2001).

Hydrodynamic instability due to density differences was analyzed in the works of Wooding (1962), Bachmat et al. (1970), Bejan (1980), Bues et al. (1991).

The dependence of dispersion on density and viscosity contrast of the miscible fluids was discussed by Bouhroum (1985) and Moser (1995).

Bejan and Khair (1985) used the Darcy low to study the flow characteristics in the boundary layer, caused by thermal and concentration gradients. Lai and Pulaski (1991) reexamined this type of convection (along a vertical wall) with a wall injection effect.

The heat and mass transfer caused by natural convection near vertical wall in porous media was investigated by Nakajima and Hessian (1995), by Singh and Queenly (1997) using the boundary layer approximation. It is also worth mentioning Nield and Bejan (1999) and Singh (2006).

Other investigations developed models for moisture transport in porous materials. Cunningham (1988) developed a mathematical model for hygroscopic materials in flat structures that uses an electrical analogy with resistance for the vapor flow and an exponential approximation function with constant mass transport coefficients. Keresteciogl o and Gu (1989) investigated the phenomenon using evaporation condensation theory in the pendulum state. The application of this theory is limited to low moisture content. The Darcy model, with the Boussinesq approximation, was used by Mamou et al. (1995) to study double-diffusive natural convection in an inclined porous layer subject to transverse gradients of heat and solute. A wide range of controlling parameters was investigated in this study. A good agreement was observed between the analytical predictions and the numerical simulations. Khanafer and Vafai (2000) are focused their study on the analysis of heat and mass transfer in a square enclosure using the generalized model of the momentum equation. Khanafer and Chamkha (1978) investigated laminar, mixed-convection flow in an enclosure filled with a Darcian fluid-saturated uniform porous medium in the presence of internal heat generation. All these authors are neglected the Dufour and Soret effects on the basis that they are of a smaller order than the effects described by Fourier's and Fick's laws. Ranganathan and Viskanta (1988) investigated both analytically and numerically natural convection in a two-dimensional square cavity filled with a binary gas due to combined temperature and concentration gradients. The analysis indicated that the velocities at the vertical walls were inversely proportional to the concentration parameter. Goyeau, Songbe, and Gobin, (1996) performed a numerical study on double-diffusive natural convection in a porous cavity using the Darcy-Brinkman formulation. Their numerical results for mass transfer were in excellent agreement with the scaling analysis over a wide range of the controlling parameters, while their heat transfer results showed that the boundary-layer analysis was not a suitable method to predict the correct scales for heat transfer in the same domain. Chen and Chen (1996) considered double-diffusive fingering convection in a porous medium. The Darcy equation including Brinkman and Forchheimer terms to account for viscous and inertia effects was used for the momentum equation. Only recently flow control problems have been addressed in systematic, rigorous manner by scientists and engineers as Hou and Svobodny (1991), Abergel and Ternam (1990) and Fattorini and Sritharan (1992). But these recent analyses, which combine modern computational fluid dynamics and rigorous optimization methods, are usually very complicated mathematically and there are still many technical difficulties to be overcome before they become practical design tools. An intention of this work is to analyse this problem by an integral method. The results are compared with those obtained by Bejan and Khair (1985), for several buoyancy ratio values and give good agreements.

2 Mathematical formulation

We consider a slow two-dimensional laminar flow on a vertical wall in a porous environment imbibed by a Darcy fluid. The physical model of this problem is shown in fig. 1.

For modeling this problem we assume the following assumptions:

- The physical properties are considered constant, except for the density term that is associated with the body force.
- The flow is sufficiently slow so that the convecting fluid and the porous media matrix are in local equilibrium
- Darcy's low the Boussinesq and boundary layer approximations hold.

With these assumptions, the governing equations of this problem are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u=\frac{gK}{\nu}(\beta_T(T-T_\infty)+\beta_C(C-C_\infty)) \tag{2}$$

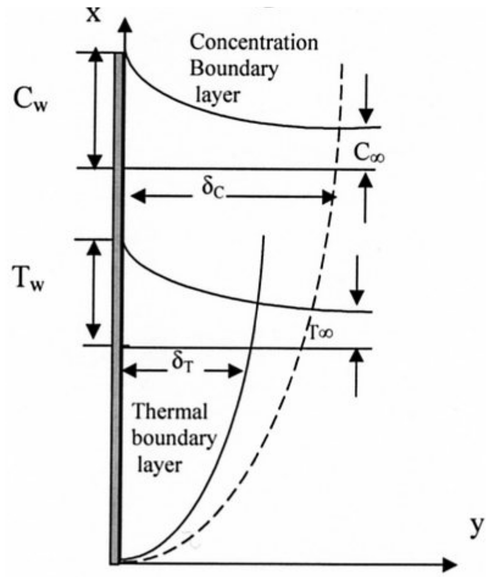

Figure 1: Physical model

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\alpha\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The boundary conditions at the wall are

$$y=0, \quad v=0, \quad T=T_w, \quad C=C_w, \tag{5}$$

and at infinity are:

$$y\rightarrow\infty, \quad u=0, \quad T\rightarrow T\infty, \quad C\rightarrow C\infty. \tag{6}$$

3 Integral method

The equations (2), (3), (4) with the boundary conditions have been solved by an integral method. The partial derivative equations have been converted into ordinary ones by making use the following transformations:

$$\eta = \frac{y}{x}(Ra_x)^{\frac{1}{2}},$$
$$\psi = \alpha(Ra_x)^{\frac{1}{2}} f(\eta)$$
$$\theta = \frac{(T - T_w}{(T_w - T_\infty)}, \quad (7)$$
$$\phi = \frac{(C - C_w}{(C_w - C_\infty)}$$

Where $Ra_x = \frac{g\beta_T K x(T_w - T_\infty)}{\alpha\nu}$ is the modified local Raleigh number, Ψ is the stream function. These new variables transform the above equations in the form:

$$f'(\eta) - \theta'(\eta) - N\phi'(\eta) = 0$$
$$\theta''(\eta) + \frac{1}{2}f(\eta)\phi'(\eta) = 0 \quad (8)$$
$$\phi''(\eta) + \frac{1}{2}Lef(\eta)\cdot\phi'(\eta) = 0$$

With boundary conditions:

$$f(0) = 0, \theta(0) = \phi(0) = 1$$
$$f'(\infty) = \theta(\infty) = \phi(\infty) = 0 \quad (9)$$

Here, f' represent the non-dimensional velocity related to the stream function $\Psi(x,y)$.

In the above equations (8), N and Le are the buoyancy ratio and Lewis number, respectively, they read:

$$N = \frac{\beta_C(C_w - C_\infty}{\beta_T(T_w - T_\infty)} \quad ; \quad Le = \frac{\alpha}{D} \quad (10)$$

From equations (8) we obtain the following relations

$$-\theta'(0) = \frac{1}{2}\int_0^\infty f'\phi d\eta$$

$$-\phi'(0) = \frac{Le}{2}\int_0^\infty f'\phi d\eta \quad (11)$$

Now, we introduce exponential temperature and concentration profiles as follows:

$$\theta(\eta) = \exp(-\eta/\delta_T)$$

$$\phi(\eta) = \exp(-\xi\eta/\delta_T) \tag{12}$$

Where δ_T is an arbitrary scale of the thermal boundary layer thickness whereas, ξ is its ratio to the concentration boundary layer thickness δ_C. By using the first equation of the system (8) and first equation of the system (10), we obtain two distinct expressions

$$\begin{aligned} \delta_T^2 &= \frac{4(\xi+1)}{1+2N+\xi} \\ \delta_T^2 &= \frac{4\xi^2(\xi+1)}{(N(\xi+1)+2\xi)Le} \end{aligned} \tag{13}$$

From the two last equations of system (8) we also obtain the following cubic equation for determining the boundary layer thickness ratio ξ as:

$$\xi^3 + (1+2N)\xi^2 - [(2+N)Le]\xi - NLe = 0 \tag{14}$$

Equation (14) permits to compute the local Nusselt and Sherwood numbers,

$$\begin{aligned} \frac{Nu}{(Ra_x)^{1/2}} &= 0.5\left[\frac{\xi+1+2N}{1+\xi}\right]^{1/2} \\ \frac{Sh}{(Ra_x)^{1/2}} &= 0.5\xi\left[\frac{\xi+1+2N}{1+\xi}\right]^{1/2} \end{aligned}$$

The results acquired in the framework of the above approach may be critically examined by comparing them against those obtained by Bejan and Khair (1985). We show a small error of 5%, which depends on the assumed profile. This situation can be corrected by an adjustment of the multiplicative constant (replacing 1/2 by 0.444). Thus we propose the following approximate formula as:

$$\begin{aligned} \frac{Nu}{(Ra_x)^{1/2}} &= 0.444\left[\frac{\xi+1+2N}{1+\xi}\right]^{1/2} \\ \frac{Sh}{(Ra_x)^{1/2}} &= 0.444\xi\left[\frac{\xi+1+2N}{1+\xi}\right]^{1/2} \end{aligned}$$

4 Results and discussions

Equations (15) give the local Nusselt and Sherwood number values as 0.444 for $N = 0$ and $Le = 1$. In order to show clearly the influence of the governing problem parameters on the combined heat and mass transfer along vertical wall due to natural convection, we have done calculations for a large range of these parameters.

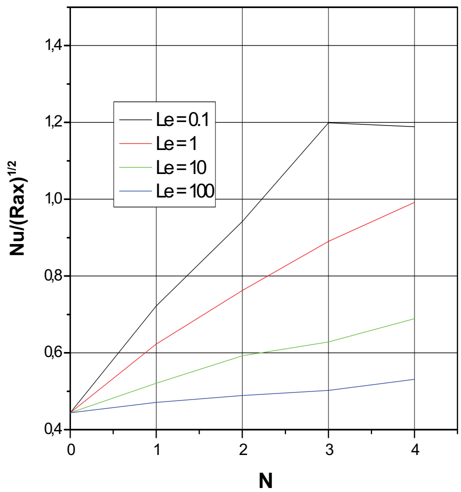

Figure 2: Heat transfer coefficient as a function of buoyancy

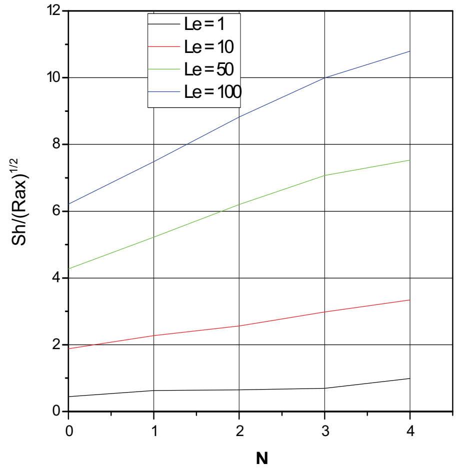

Figure 3: Heat transfer coefficient as a function of buoyancy

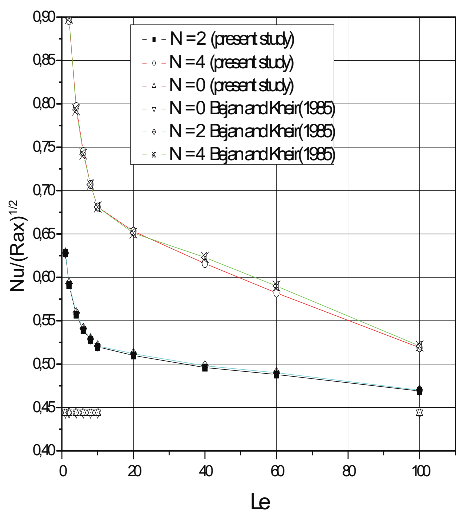

Figure 4: Thermal transfer results

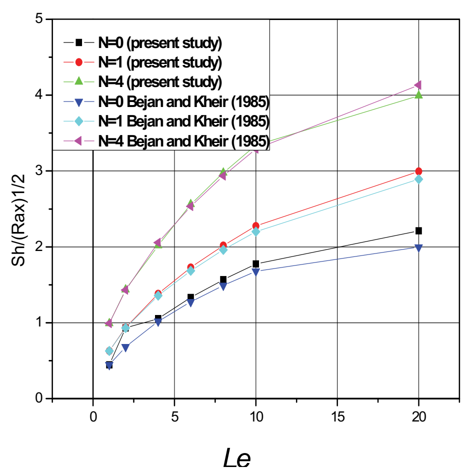

Figure 5: Mass transfer results

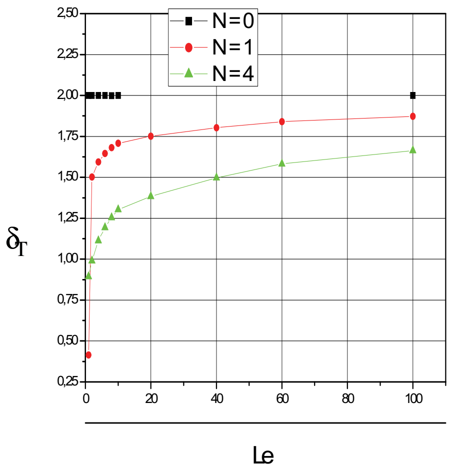

Figure 6: Thermal boundary layer thickness

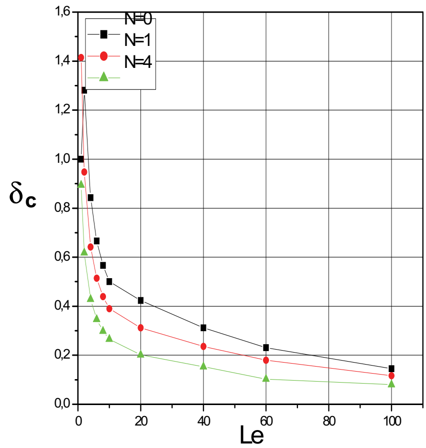

Figure 7: Concentration boundary layer thickness

Fig.2 presents the local Nusselt number as a function of the buoyancy ratio, N, for various values of the Lewis number, Le. We observe that the rate of heat transfer increases with increasing Lewis number.

In fig.3, the same influence on the local Sherwood number is noticed for all $N > 0$. The local Nusselt number is also plotted in fig.4 as a function of Lewis number, Le, for various values of the buoyancy ratio, N. This graph shows that local Nusselt number decreases with increasing Lewis number for all the considered values of N.

In fig, 5 we remark the same result for the local Sherwood number.

Fig.4 and fig.5 show that the local Nusselt and Sherwood numbers are in good agreement with those obtained by Bejan and Kheir (1985).

We can observe, in fig. 6, that thermal boundary layer thickness δ_T presents an increasing trend for N = 1 and 4 for increasing values of Lewis number. Whereas, fig.7, shows that concentration boundary layer thickness, decreases for N= 0, 2, 4, with increasing Lewis number values. From fig. 6 and 7, it is evident that the Lewis number has a more pronounced effect on the concentration than on the thermal field.

Nomenclature

u Darcy velocity in the direction x, [m/s].
v Darcy velocity in the direction y, [m/s].

D Massy diffusivity of porous media, [m^2/s]
f Non dimensional stream function
h Local heat transfer coefficient [w/m^2 K]
N Buoyancy of porous media
k Thermal conductibility [w/m^2°K]
T Temperature [°K]
C Concentration
g Gravity acceleration [m/s^2]
K Permeability

Greek symbols

β_C Concentration expansion coefficient
β_T Thermal diffusivity of porous media, m^2/s]
ψ Stream function
ξ Ratio between δ_T and δ_C
ν Viscosite cinematic, [m^2/s]
δ_T Thermal boundary layer thickness [m]
δ_C Concentration boundary layer thickness [m]
Φ Non dimensional concentration
α Thermal diffusivity m^2/s

Indexes

∞ Condition at infinity
w Condition at wall

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