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Stability of Marangoni Convection in a Composite Porous-Fluid with a Boundary Slab of Finite Conductivity

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Abstract: A linear stability analysis is used to investigate the onset of Marangoni convection in a three-layer system comprising an incompressible fluid saturated porous layer over which lies a layer of the same fluid and below which lies a solid layer. The lower boundary is subjected to a fixed heat flux, while the upper free surface of the fluid is non-deformable. At the interface between the fluid and the porous layer, the Beavers-Joseph slip condition is used and the Darcy law is employed to describe the flow in the porous medium. The asymptotic analysis of the long-wavelength is performed and the results are compared with those for the case of porous-fluid layer system. The effects of the thermal conductivity and the thickness of the solid plate on the onset of convective instability are studied. It is found that the solid plate with a higher relative thermal conductivity or higher thickness ratio tends to stabilize the system.

Keywords: Linear stability, Marangoni convection, Porous medium, Regular Perturbation Method.

1 Introduction

The problem of thermoconvective instability in a horizontal fluid layer driven by the surface tension effects (Marangoni convection) has been studied extensively by many researchers. The first theoretical study on the steady Marangoni convection in a horizontal fluid layer was made by Pearson (1958). The convective instability of a fluid overlying a porous region saturated with the fluid subjected to uniform temperature gradient has been investigated extensively by several authors [Nield(1977,1998), Nield and Bejan(2006), Straughan (2001), Taslim and Narusawa (1989)]. Nield (1977) considered a layered model and employed an empirical interfacial condition at the fluid-porous interface suggested by Beavers and Joseph

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(1967). Chen and Chen (1988) produced a classical paper in which they have studied the thermal convection in two-layer system composed of a porous layer saturated with fluid over which lay the same fluid. The work of Chen and Chen (1988) employed the fundamental model for convection in a porous-fluid-layer system developed originally by Nield (1977). McKay (1998) examined the onset of Bénard convection in a layer of fluid on top of a saturated porous layer. He reported that the relative thickness of the two layers determined whether this convection is concentrated in the fluid layer or in the porous layer. The presence of thin porous layers was found to have a small destabilizing influence on the system. Shivakumara, Suma and Chavaraddi (2006) studied the onset of Marangoni convection in a composite porous-layer system and the Beavers-Joseph slip condition is used at the interface while the Darcy law is employed to describe the flow in the porous medium. They showed that the linear stability curves for the onset of Marangoni convection depend on the parameter in their analysis, which is the number

$$\zeta = \frac{\text{depth of fluid layer}}{\text{depth of porous layer}}.$$

They interpreted their findings by showing that for ζ small, the instability was initiated in the porous medium, whereas for larger ζ, the instability was controlled by the fluid layer. They also suggested that the regular perturbation technique with small wave number a as a perturbation parameter can conveniently be used in solving convective instability problems for the case of insulating boundaries.

In this paper, we extend Shivakumara, Suma and Chavaraddi (2006) work to the problem of the Marangoni convection in a composite porous-fluid layer with a boundary slab of finite conductivity. We use the regular perturbation technique to obtain the asymptotic solutions of the long-wavelength.

2 Physical Formulation

Consider an infinite horizontal porous layer of thickness d_p underlying a liquid layer of thickness d_f and overlying a solid layer of thickness d_s. The physical configuration is shown in Fig. 1. The lower boundary is subjected to a fixed heat flux, while the upper surface of the fluid is free and is assumed to be non-deformable.

Based on the above assumptions together with the Boussinesq approximation, the governing equations for the continuity, momentum and energy in the fluid layer are respectively

$$\nabla \cdot u_f = 0 \tag{1}$$

$$\rho_0 \left(\frac{\partial}{\partial t} + u_f \cdot \nabla \right) u_f = -\nabla p_f + \nu \nabla^2 u_f \tag{2}$$

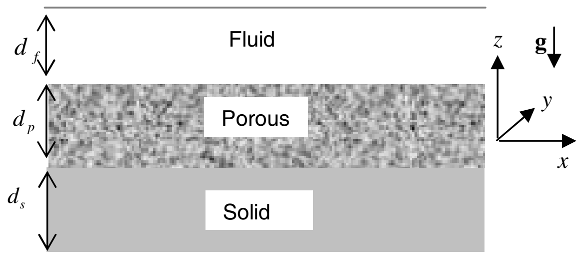

Figure 1: Physical Model

$$\left(\frac{\partial}{\partial t}+u_f\cdot\nabla\right)T_f=\kappa_f\nabla^2T_f, \tag{3}$$

and for the porous layer, the equations are

$$\nabla\cdot u_p=0 \tag{4}$$

$$\frac{\rho_o}{\Phi}\frac{\partial u_p}{\partial t}=-\nabla_p p_p-\frac{\mu}{K}u_p \tag{5}$$

$$H\frac{\partial T_p}{\partial t}+(u_p\cdot\nabla_p)T_p=\kappa_p\nabla_p^2T_p \tag{6}$$

while for the solid layer, the energy equation takes the form

$$\frac{\partial T_s}{\partial t}=\kappa_s\nabla^2T_s \tag{7}$$

where u is the velocity vector, T is the temperature, p is for pressure, K is the permeability of the porous medium, H is the ratio of heat capacity, μ is the fluid viscosity, κ is the thermal diffusivity and ρ_o is the reference fluid density. The subscripts f, p and s refer to the quantities in the fluid, porous and solid layers, respectively.

We may introduce the infinitesimal disturbances to the governing equations by setting

$$(u_f,u_p,u_s,\rho,p,\mu,T_f,T_p,T_s)=$$
$$(0,0,0,\rho,\bar{p},\bar{\mu},\bar{T}_f,\bar{T}_p,\bar{T}_s)+\left(u'_f,u'_p,u'_s,\rho',p',\mu',T'_f,T'_p,T'_s\right)$$

where the primed quantities are the perturbed ones over their equilibrium counterparts. The variables are then nondimensionalized using d_f, d_f^2/κ_f, κ_f/d_f, ΔT_f as the units of length, time, velocity and temperature, respectively in the fluid layer and d_p, d_p^2/κ_p, κ_p/d_p, ΔT_p as the corresponding characteristic quantities in the porous layer. The detailed flow fields in both the fluid and porous layers can be clearly discerned for all depth ratios, $\zeta = d_f/d_p$. Then the linearized perturbation equations in dimensionless forms for the fluid layer are

$$\left[\frac{1}{P_r}\frac{\partial}{\partial t} - \nabla^2\right]\nabla^2 w_f = 0, \tag{8}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)\theta_f = w_f, \tag{9}$$

and for the porous layer are

$$\left(\frac{Da}{P_{r_p}}\frac{\partial}{\partial t} + 1\right)\nabla_p^2 w_p = 0, \tag{10}$$

$$\left(H\frac{\partial}{\partial t} - \nabla_p^2\right)\theta_p = w_p. \tag{11}$$

For the fluid layer, $P_r = \upsilon/\kappa$ is the Prandtl number, $\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}$ is the Laplacian operator and for the porous layer, $P_{r_p} = \upsilon/\kappa_p\phi$ is the Prandtl number, $Da = K/d_p^2$ is the Darcy number and $\nabla_p^2 = \frac{\partial}{\partial x_p^2} + \frac{\partial}{\partial y_p^2} + \frac{\partial}{\partial z_p^2}$.
The linearized pertubation equation in dimensionless forms for the solid layer is

$$\frac{\partial \theta_s}{\partial t} = \frac{\kappa_s}{\kappa_p}\nabla^2\theta_s. \tag{12}$$

Also, the linearized perturbed boundary conditions in dimensionless forms at the non-deformable and insulating upper surface (at $z = 1$), are

$$w_f = 0, \tag{13}$$

$$\frac{\partial \theta_f}{\partial z} = 0, \tag{14}$$

$$\frac{\partial^2 w_f}{\partial z^2} = \mathrm{M}\nabla_h^2\theta_f \tag{15}$$

where $\mathrm{M} = \sigma\Delta T_f d_f/\mu\kappa_f$ is the Marangoni number and $\nabla_h^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$ is the horizontal Laplacian operator. At the porous-fluid interface ($z = 0$) the boundary conditions are

$$w_f - \frac{\varsigma}{\varepsilon_T}w_p = 0, \tag{16}$$

$$\theta_f - \frac{\varepsilon_T}{\varsigma}\theta_p = 0, \tag{17}$$

$$\frac{\partial \theta_f}{\partial z}-\frac{\partial \theta_p}{\partial z_p}=0, \tag{18}$$

$$\frac{\partial^2 w_f}{\partial z^2}-\chi\nabla_h^2 w_f-\frac{\alpha\varsigma}{\sqrt{Da}}\frac{\partial w_f}{\partial z}+\frac{\alpha\varsigma^3}{\varepsilon_T}\left[\frac{1}{\sqrt{Da}}\right]\frac{\partial w_p}{\partial z_p}=0, \tag{19}$$

$$\left(3\nabla_h^2+\frac{\partial^2}{\partial z^2}\right)\frac{\partial w_f}{\partial z}-\frac{1}{P_r}\frac{\partial}{\partial t}\left(\frac{\partial w_f}{\partial z}\right)+\frac{\varsigma^4}{\varepsilon_T}\left[\frac{1}{Da}\right]\frac{\partial w_p}{\partial z_p}+\frac{1}{P_{r_p}}\frac{\partial}{\partial t}\left(\frac{\partial w_p}{\partial z_p}\right)=0, \tag{20}$$

where χ is a constant taking the value 0 for the Beavers-Joseph condition and 1 for the Jones condition (see Shivakumara, Suma and Chavaraddi (2006)) and $\varepsilon_T=\kappa_f/\kappa_p$ is the ratio of the thermal diffusivities. At the solid-porous interface ($z_p=-1$), the boundary conditions are

$$w_p=0, \tag{21}$$

$$\frac{\partial \theta_p}{\partial z_p}=0, \tag{22}$$

$$\theta_p=\theta_s, \tag{23}$$

$$\frac{\partial \theta_p}{\partial z_p}=\left(\frac{k_s}{k_p}\right)\frac{\partial \theta_s}{\partial z_p}. \tag{24}$$

At the bottom surface, at $z_p=-d_s/d_p$, a uniform heat flux is imposed and

$$\frac{\partial \theta_s}{\partial z_p}=0 \tag{25}$$

The perturbation quantities in terms of normal modes are expressed as

$$(w_f,\theta_f)=[\mathrm{W}_f(z),\Theta_f(z)]\exp\left[i\left(a_x x+a_y y\right)+\omega t\right] \tag{26}$$

$$(w_p,\theta_p,\theta_s)=\left[\mathrm{W}_p\left(z_p\right),\Theta_p\left(z_p\right),\Theta_s\left(z_p\right)\right]\exp\left[i\left(\tilde{a}_x x+\tilde{a}_y y\right)+\omega_p t\right] \tag{27}$$

where $a=\sqrt{(a_x^2+a_y^2)}$ is the dimensionless wave number in the fluid layer, while $a_p=\sqrt{(\tilde{a}_x^2+\tilde{a}_y^2)}$ is the dimensionless wave number in the porous layer. By substituting Eqs. (26) and (27) into Eqs. (8)-(12), and setting $\omega=0$, the governing equations of the perturbed state become

$$\left(D^2-a^2\right)^2\mathrm{W}_f=0, \tag{28}$$

$$\left(D^2-a^2\right)\Theta_f=-\mathrm{W}_f, \tag{29}$$

$$\left(D_p^2-a_p^2\right)\mathrm{W}_p=0, \tag{30}$$

$$\left(D_p^2 - a_p^2\right)\Theta_p = -\mathrm{W}_p, \tag{31}$$

$$\left(D_p^2 - a_p^2\right)\Theta_s = 0, \tag{32}$$

where D and D_p denote the differentiation with respect to z and z_p, respectively. The wave number in the fluid and porous layer must be the same, so that we have $a/d_f = a_p/d_p$ and hence $\zeta = a/a_p$. Using Eqs. (26) and (27), the boundary conditions given by Eqs. (13)-(25) now take the form at $z = 1$:

$$\mathrm{W}_f = 0, \tag{33}$$

$$D\Theta_f = 0, \tag{34}$$

$$D^2\mathrm{W}_f + Ma^2\Theta_f = 0, \tag{35}$$

at $z = 0$:

$$\mathrm{W}_f - \frac{\zeta}{\varepsilon_T}\mathrm{W}_p = 0, \tag{36}$$

$$D\Theta_f - D_p\Theta_p = 0, \tag{37}$$

$$\Theta_f - \frac{\varepsilon_T}{\zeta}\Theta_p = 0, \tag{38}$$

$$\left[D^2 + \chi a^2 - \frac{\alpha\zeta D}{\sqrt{Da}}\right]\mathrm{W}_f + \frac{\alpha\zeta^3}{\varepsilon_T\sqrt{Da}}D_p\mathrm{W}_p = 0, \tag{39}$$

$$\left[D^2 - 3a^2\right]D\mathrm{W}_f + \frac{\zeta^4}{\varepsilon_T Da}D_p\mathrm{W}_p = 0, \tag{40}$$

at $z_p = -1$:

$$W_p = 0, \tag{41}$$

$$D_p\mathrm{W}_p = 0, \tag{42}$$

$$\Theta_p = \Theta_s, \tag{43}$$

$$D\Theta_p = k_r D\Theta_s, \tag{44}$$

and at $z_p = -d_r$:

$$D\Theta_s = 0, \tag{45}$$

where $k_r = k_s/k_p$ is the ratio of the thermal conductivity of the solid plate to that of the porous layer, and $d_r = d_s/d_p$ is the ratio of the solid plate thickness to the porous layer thickness. Solving the perturbation equation (32) for the solid layer, together with the boundary conditions (43)-(45), the thermal boundary condition at the solid-porous interface, at $z_p = -1$ becomes

$$D\Theta_p = k_r a_p \tanh(a_p d_r)\Theta_p \tag{46}$$

3 Long Wavelength Asymptotic Analysis

As the fluid is subjected to a uniform heat flux below ($k_r = 0$ or $d_r = 0$) and above ($B_i = 0$), the critical wave number vanishes, $a_c \to 0$. For studying the validity of the small wave number analysis, the dependent variables in both the fluid and porous layer are now expanded in powers of a^2 in the form

$$(\mathrm{W}, \Theta) = \sum_{i=0}^{N} \left(a^2\right)^i (\mathrm{W}_i, \Theta_i), \tag{47}$$

$$(\mathrm{W}_p, \Theta_p) = \sum_{i=0}^{N} \left(\frac{a_p^2}{\zeta^2}\right)^i (\mathrm{W}_{pi}, \Theta_{pi}). \tag{48}$$

Substitution of Eqs. (47) and (48) in Eqs (28)-(31) and collecting the terms of zeroth order, we obtain

$$D^4 \mathrm{W}_{f0} = 0, \tag{49}$$

$$D^2 \Theta_{f0} = -\mathrm{W}_{f0}, \tag{50}$$

$$D_p^2 \mathrm{W}_{p0} = 0, \tag{51}$$

$$D_p^2 \Theta_{p0} = -\mathrm{W}_{p0}, \tag{52}$$

and the boundary conditions (33)-(42) and (46) become at $z = 1$:

$$\mathrm{W}_{f0} = D\Theta_{f0} = D^2 \mathrm{W}_{f0} = 0, \tag{53}$$

at $z = 0$:

$$\mathrm{W}_{f0} = \frac{\zeta}{\varepsilon_T} \mathrm{W}_{p0}, \tag{54}$$

$$\Theta_{f0} = \frac{\varepsilon_T}{\zeta} \Theta_{p0}, \tag{55}$$

$$D\Theta_{f0} = D_p \Theta_{p0}, \tag{56}$$

$$D^2 \mathrm{W}_{f0} - \frac{\alpha\zeta}{\sqrt{Da}} D\mathrm{W}_{f0} + \frac{\alpha\zeta^3}{\varepsilon_T \sqrt{Da}} D_p \mathrm{W}_{p0} = 0, \tag{57}$$

$$D^3 \mathrm{W}_{f0} = -\frac{\zeta^4}{\varepsilon_T Da} D_p \mathrm{W}_{p0}, \tag{58}$$

and at $z_p = -1$:

$$\mathrm{W}_{p0} = D_p \Theta_{p0} = 0. \tag{59}$$

The terms of order a^2 are

$$D^4 \mathrm{W}_{f1} = 0, \tag{60}$$

$$D^2 \Theta_{f1} - \frac{\varepsilon_T}{\zeta} = -\mathrm{W}_{f1}, \tag{61}$$

$$D_p^2 \mathrm{W}_{p1} = 0. \tag{62}$$

$$D_p^2 \Theta_{p1} - 1 = -\mathrm{W}_{p1}, \tag{63}$$

and the corresponding boundary conditions at $z = 1$:

$$\mathrm{W}_{f1} = D\Theta_{f1} = 0, \tag{64}$$

$$D^2 \mathrm{W}_{f1} + M\Theta_{f0} = 0, \tag{65}$$

at $z = 0$:

$$\mathrm{W}_{f1} = \frac{1}{\zeta \varepsilon_T} \mathrm{W}_{p1}, \tag{66}$$

$$\Theta_{f1} = \frac{\varepsilon_T}{\zeta^3} \Theta_{p1}, \tag{67}$$

$$D\Theta_{f1} = \frac{1}{\zeta^2} D_p \Theta_{p1}, \tag{68}$$

$$D^2 \mathrm{W}_{f1} - \frac{\alpha \zeta}{\sqrt{Da}} D\mathrm{W}_{f1} + \frac{\alpha \zeta}{\varepsilon_T \sqrt{Da}} D_p \mathrm{W}_{p1} = 0, \tag{69}$$

$$D^3 \mathrm{W}_{f1} = -\frac{\zeta^2}{\varepsilon_T Da} D_p \mathrm{W}_{p1}, \tag{70}$$

and at $z_p = -1$,

$$\mathrm{W}_{p1} = 0, \tag{71}$$

$$D_p \Theta_{p1} = \frac{k_r d_r}{\zeta^2} \Theta_{p0}. \tag{72}$$

We use the symbolic algebra package MAPLE 10 running on a Pentium PC to carry out much of the tedious algebraic manipulations to obtain the critical Marangoni number M_c as

$$\begin{aligned} M_c = \Big[& 864 \alpha k_r d_r Da^{3/2} + A_1 \zeta + A_2 \zeta^2 + A_3 \zeta^3 + \\ & A_4 \zeta^4 + A_5 \zeta^5 + A_6 \zeta^6 \Big] \Big/ \left[\varepsilon_T A_7 \left(\alpha \zeta + 6\sqrt{Da} \right) \right] + \frac{48 \alpha^2 \zeta^7}{A_7 \left(\alpha \zeta + 6\sqrt{Da} \right)} \end{aligned} \tag{73}$$

where

$$
\begin{aligned}
A_1 =& 864\alpha Da^{3/2}(1+k_r d_r)+144\alpha^2 k_r d_r Da \\
A_2 =& Da\left(864\alpha\sqrt{Da}\,(1+\varepsilon_T)+144\alpha^2\,(1+k_r d_r)\right) \\
A_3 =& Da\Big(864\varepsilon_T\alpha\sqrt{Da}+144\alpha^2\,(1+\varepsilon_T)+864k_r d_r\Big) \\
A_4 =& 144\varepsilon_T\alpha^2 Da+864Da+432\alpha k_r d_r\sqrt{Da} \\
A_5 =& 864\varepsilon_T Da+432\alpha\sqrt{Da}+48\alpha^2 k_r d_r \\
A_6 =& 48\alpha^2+432\varepsilon_T\alpha\sqrt{Da} \\
A_7 =& \alpha\zeta^6+6\zeta^5\sqrt{Da}+12\alpha\zeta^3 Da+48\alpha\zeta^2 Da+(36\varepsilon_T\alpha Da+72Da^{3/2})\zeta \\
& +72\varepsilon_T Da^{3/2}
\end{aligned}
$$

As $k_r = 0$ or $d_r = 0$, Eq. (73) can be reduced to the result of Shivakumara, Suma and Chavaraddi (2006).

Table 1: The critical Marangoni numbers for different values of thickness d_r and ζ in the case of $\varepsilon_T = 0.725$ and $Da = 0.003$

ζ	Shivakumara et. al. [10]	Present Study ($k_r = 1$)			
		$d_r = 0.0$	$d_r = 1.0$	$d_r = 10.0$	$d_r = 100.0$
0.001	0.33229	0.33229	332.384	3320.854	33205.559
0.01	2.39090	2.38979	239.649	2374.980	23728.319
1.0	90.3660	90.3660	142.752	614.227	5328.975
2.0	77.6648	77.7225	94.788	239.381	1685.306
3.0	68.7142	68.7153	76.371	141.677	794.735
5.0	60.7071	60.7166	63.447	87.116	323.807
10	54.4200	54.4401	55.100	61.040	120.428
100	48.6459	48.6460	48.653	48.712	49.308
1000	48.0646	48.0646	48.0647	48.065	48.071

4 Results and discussion

The onset of Marangoni convection corresponds to a vanishing small wave number in a three-layer system, comprising of an incompressible fluid saturated porous layer over which lies a layer of the same fluid and below which lies a solid layer, is investigated theoretically. As the fluid is subjected to a uniform heat flux below and above ($B_i = 0$), the critical wave number (a_c) is vanishing. The regular perturbation technique is used to obtain the analytical expression for the critical Marangoni

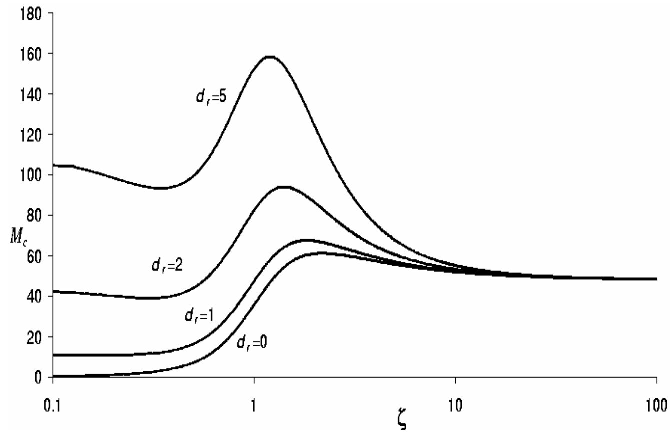

Figure 2: Variation of M_c with ζ for different values of $d_r$in the case $\varepsilon_T = 0.725$, $k_r = 1$, $\alpha = 1$ and $Da = 0.003$

numbers, M_c for long-wavelength ($a \to 0$). The critical Marangoni number M_c depends on the depth ratio $\zeta = d_f/d_p$ and $d_r = d_s/d_p$, the thermal conductivity ratio k_r, the Darcy number Da and α.

The critical Marangoni numbers obtained for different values of ζ and d_r when $\alpha = 1.0$, $\varepsilon_T = 0.725$ and $Da = 0.003$ are presented in Table 1. From the table, we note that the asymptotic solutions are in good agreement with Shivakumara, Suma and Chavaraddi (2006) for $d_r = 0$. As $\zeta \to \infty$, the critical Marangoni number M_c attains a constant value 48, which is the exact value known for the case of single fluid layer (Pearson (1958)). Also, we note that M_c increases for all values of d_r with ζ and for higher value of ζ, requiring higher value of d_r, for the system to become more stable.

Figure 2 shows the variation of the critical Marangoni number M_c with the depth ratio ζ for various values of d_r in the case of $\varepsilon_T = 0.725$ and $k_r = 1$. All curves have the regular trends with ζ, in which they increase, then they reach the peak and finally they decrease. Further, increasing the value of d_r leads to a higher curve due to the increase of the critical Marangoni number M_c and it is found to have a stabilizing influence on the system. In addition, we have also used different values of Da in order to see their effects on the critical Marangoni number as shown in Fig. 3 for various values of d_r. For all values of Da considered, increasing the value of d_r gives a stabilizing system.

Fig. 4 shows the variation of the critical Marangoni number M_c with the thickness ratio ζ for various values of k_r and $d_r = 1$. From the figure, it is obvious that for an

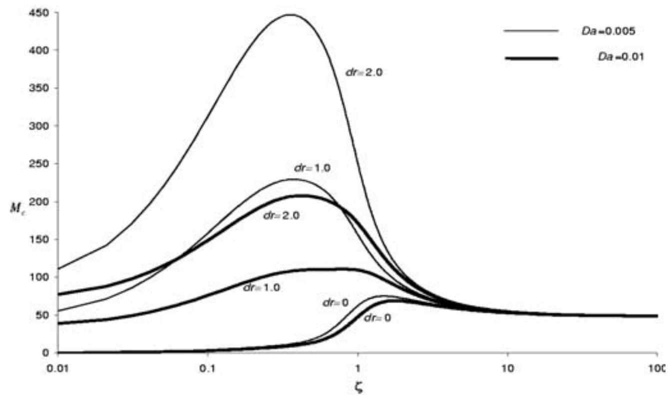

Figure 3: Variation of M_c with $\zeta$for different values of $Da$and $d_r$in the case of $\varepsilon_T = 0.725$, $k_r = 1$, $\alpha = 1$ and $Da = 0.003$

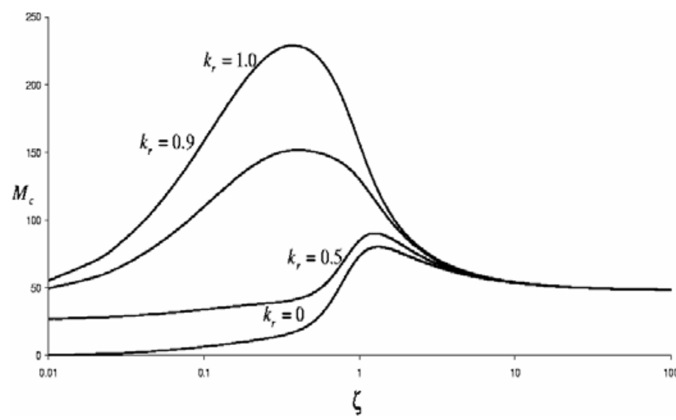

Figure 4: Variation of m_c with ζ with for different values of k_r in the case $\varepsilon_t = 0.725$, $\alpha = 1$,$d_r = 1$and $da = 0.003$

increasing value of k_r, the curve is higher due to the increasing critical Marangoni number M_c. In all of the cases above, we use the values of the parameter similar to Shivakumara, Suma and Chavaraddi (2006) and also, we recover their results for the case of $d_r = 0$or $k_r = 0$.

5 Conclusion

The onset of Marangoni convection corresponds to a vanishingly small wave number in a fluid-porous-solid layer system is studied. We found that it is possible to control the onset of Marangoni convection effectively by appropriately choosing the values of ζ, d_r, k_r and Da. A larger depth ratio d_r is stabilizing and the critical Marangoni number M_c increases with d_r. Also, an increase in the thermal conductivity ratio k_r results in a stabilizing state, since thermal disturbances are easily dissipated deep into the solid layer and the critical Marangoni number M_c increases.

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