

On the Behavior of an Interface under Molecular Diffusion: A Theoretical Prediction and Experimental Study

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Abstract: A theoretical model has been developed to predict the expansion of a salty gradient (i.e. the interface) layer under natural diffusion. The salty gradient layer is initially sandwiched between two homogeneous miscible layers of varying salinity, which may or may not have the same thickness. The model describes the concentration profile of the salty gradient layer (expressed by analytical solutions of the diffusion equation) as the boundaries of this interfacial layer move into the adjacent (hitherto homogeneous) regions. The lifetime of the adjacent layers is also predicted. An experimental study for a configuration with salty water below and distilled water above (of the same thicknesses) is carried out to verify the theoretical predictions of the model.

Keywords: Stratified miscible layers; interface; moving boundaries; lifetime of the stratification.

1 Introduction

Recently, there has been growing interest in the development of renewable energy in general and solar energy in particular. The solar pond is an engineering process for the storage of solar energy [Tabor (1981)]. It consists of three layers: (1) a lower homogeneous layer which has high concentration of salt, (2) an upper homogeneous layer that is composed of distilled water and (3) an intermediate layer which is called the salty gradient layer (or interface). The efficiency of the solar pond depends on the effect of double-diffusive convection on the salty gradient layer; consequently, double-diffusion in the gradient layer has been extensively studied in the laboratory in order to understand the mechanisms of the growth of disruptive instabilities [Bergman et al. (1985); Bergman et al. (1987); Linden and Shirtcliffe (1978); Gau et al. (1992)]. Also, some experimental studies have shown

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that the formation of mixed zones separated by thin interfaces [Turner (1968)] enhance the growth of this instability. However, other noteworthy observations in solar ponds have shown that if the salt gradient layer of a solar pond is locally disturbed, the gradient layer reconstructs under salt diffusion. From these observations arises a question: How is the salty gradient reconstructed? Indeed, it is even worthwhile to identify the initial condition of a salty gradient layer if the results of experimental studies and solar-pond observations are to be understood. Such questions/discrepancies lead us to think that more investigations need to be done in this field. Therefore, this paper is aimed at giving some answers to the above issues.

Construction of a salty gradient layer. A salty gradient layer in laboratories or in solar ponds is built using various techniques. The Oster method [Oster (1965)] is one of these techniques. It consists of two communicating vessels containing the same depths of distilled water and salty water. The solution in the vessel initially containing distilled water is vigorously stirred while it is gradually drained into the bottom of an experimental tank. Another method similar to Oster's technique was used by Menzkirch [Menzkirch and Peters (1992)], however these techniques are not practical if the required volume is immense, for example in solar ponds. Thus, another more practical and effective technique is used. It consists of creating a series of homogeneous layers [Poplowsky et al. (1981)] of salty solutions of varying concentrations. This technique of stratification produces thin gradient layers (i.e. interfaces) between each of the homogeneous layers. The interfaces are the main elements that control the evolution of the overall stratification. While the salt profile of the stratification is non-linear, the thicknesses of the salty gradient layers expand under salt diffusion phenomenon and eventually merge to form a single salty gradient layer throughout the whole stratification, which subsequently evolves in time and space until it reaches a homogeneous state. This implies that the evolution of the overall salt profile has two typical steps: the first one is achieved when a linear salty profile takes place in the whole stratification and the second one is achieved when the salt gradient layer reaches a homogeneous state.

Although, there are many mathematical books dealing with the problem of isothermal diffusion [Crank (1975); Bejan (1987); Carslaw and Jaeger (1960); Fourier (1978)], there is no mathematical model describing the evolution of the salty stratified layers; i.e. the moving boundaries of a linear salty gradient layer expanding into isothermal stratified layers have not been clearly modeled. Therefore, in view of the importance of the phenomenon, we felt that a closer look is necessary in order to predict mathematically this evolution and the lifetime of the stratified layers. Moreover, an experimental study has been performed to verify the predictions of the model.

2 Analytical investigation

In the following, we present some classical solutions of the diffusion equation that will be used in the theoretical predictions. The system under study is a set of horizontal homogeneous, vertically stratified liquid layers of varying salinity. Between each layer is a salty gradient layer (i.e. the interface), as sketched in Fig. 1. The initial conditions are: h_o is the thickness of the interface and has a salt concentration profile indicated by the diagonal lines. z is the vertical coordinate varying from $z = \frac{h_o}{2}$ to $z = -\frac{h_o}{2}$.

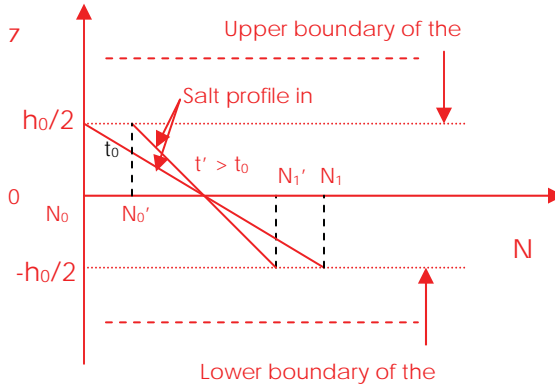


Figure 1: schematic of layer considered

Owing to the system studied, it has been assumed that the liquid layers are isothermal, the diffusion coefficient D_m is constant and the salt diffusion has one dimension (z). With these assumptions, the dimensionless salt diffusion equation into the layer is defined by:

$$\frac{\partial N^*(z^*, t^*)}{\partial t^*} = \frac{\partial^2 N^*(z^*, t^*)}{\partial z^{*2}} \quad (1)$$

$$-\frac{1}{2} < z^* < \frac{1}{2}, \quad t^* > 0 \quad (2)$$

Where $t^* = \frac{D_m}{h_o^2} t$, $N^* = \frac{N}{N_1}$, $z^* = \frac{z}{h_o}$.

The technique of separation of variables has been used to solve Eqs.1 and 2, and we now consider four particular solutions that will be utilized later in the theoretical predictions.

2.1 Both boundaries impermeable

The initial condition:

$$N^*(z^*, t = 0) = \frac{1}{2} - z^* \quad -\frac{1}{2} \leq z^* \leq \frac{1}{2} \quad (3)$$

is associated to the following boundary conditions:

$$\lim_{z^* \rightarrow \pm \frac{1}{2}} \frac{\partial N^*(z^*, t)}{\partial z^*} = 0 \quad (t > 0) \quad (4)$$

The solution, satisfying the differential Eq.1 and the initial value-boundary value problem Eqs.3 and 4, has the following form:

$$N^*(z^*, t) = \frac{1}{2} + \sum_{k=0}^{+\infty} \frac{4(-1)^{k+1}}{(2k+1)^2 \pi^2} \sin((2k+1)\pi z^*) \exp\left(-\frac{t}{\theta_k}\right) \quad (5)$$

Where

$$\Theta = \frac{h_0^2}{(2k+1)^2 \pi^2 Dm} \quad (6)$$

is the relaxation time.

2.2 Both boundaries permeable

The initial condition is still the same:

$$N^*(z^*, t = 0) = \frac{1}{2} - z^* \quad -\frac{1}{2} \leq z^* \leq \frac{1}{2} \quad (7)$$

The boundary conditions are now:

$$\left(\frac{\partial N^*(z^*, t)}{\partial z^*}\right)_{z^* = -\frac{1}{2}} = \left(\frac{\partial N^*(z^*, t)}{\partial z^*}\right)_{z^* = \frac{1}{2}} \quad (t > 0) \quad (8)$$

Therefore, the solution that satisfies the differential Eq.1 and the initial value-boundary value problem Eqs.7 and 8, has the following form:

$$N^*(z^*, t) = \frac{1}{2} - \sum_{k=0}^{+\infty} \frac{2\sqrt{2}}{(2k+1)\pi} \left\{ \frac{4}{(2k+1)\pi} \left(\cos\left(k\frac{\pi}{2}\right) + \sin\left(k\frac{\pi}{2}\right) \right)^2 - \left(\cos\left(k\frac{\pi}{2}\right)^2 - \sin\left(k\frac{\pi}{2}\right)^2 \right) \right\} \times \sin\left((2k+1)\frac{\pi}{2} z^*\right) \exp\left(-\frac{t}{\theta'_k}\right) \quad (9)$$

Where

$$\Theta'_k = 4\Theta_k = \frac{4h_0^2}{(2k+1)^2\pi^2 Dm} \quad (10)$$

is the relaxation.

2.3 Constant concentration at the upper boundary and lower boundary impermeable

Once again, the initial condition remains the same, but the boundary conditions are:

$$\lim_{z^* \rightarrow -\frac{1}{2}} \frac{\partial N^*(z^*, t)}{\partial z^*} = 0 \quad (t > 0) \quad (11)$$

$$N^*\left(z^* = \frac{1}{2}, t\right) = 0 \quad (t > 0) \quad (12)$$

and the solution of the differential Eq.1 that satisfied the Eqs. 11 and 12 is:

$$N^*(z^*, t) = \sum_{k=0}^{k=+\infty} \frac{2(-1)^k \sqrt{2}}{(1+4k)\pi} \left(1 + \frac{4}{(1+4k)^2\pi^2}\right) \times \left(\cos\left((1+2k)\frac{\pi}{2}z^*\right) - \sin\left((1+2k)\frac{\pi}{2}z^*\right)\right) \exp\left(-\frac{t}{\theta'_k}\right) \quad (13)$$

Where

$$\Theta'_k = 4\Theta_k = \frac{4h_0^2}{(2k+1)^2\pi^2 Dm} \quad (14)$$

2.4 Impermeable upper boundary and constant concentration at the lower boundary

We now take for the boundary conditions:

$$N^*(z^* = -\frac{1}{2}, t) = 1 \quad (t > 0) \quad (15)$$

$$\lim_{z^* \rightarrow \frac{1}{2}} \frac{\partial N^*(z^*, t)}{\partial z^*} = 0 \quad (t > 0) \quad (16)$$

And find:

$$N^*(z^*, t) = 1 - \sum_{k=0}^{k=+\infty} \frac{2(-1)^k \sqrt{2}}{(1+4k)\pi} \left(1 - \frac{2}{(1+4k)\pi}\right) \times \left(\cos\left((1+2k)\frac{\pi}{2}z^*\right) + \sin\left((1+2k)\frac{\pi}{2}z^*\right)\right) \exp\left(-\frac{t}{\theta'_k}\right) \quad (17)$$

For which

$$\Theta'_k = 4\Theta_k = \frac{4h_0^2}{(2k+1)\pi^2 Dm} \quad (18)$$

is the relaxation time.

3 Theoretical prediction of the expansion rate of an interface's thickness

Experimental observations reveal that three kinds of stratification can occur, and all others are a combination thereof. The three basic forms are: (1) A salty gradient layer in which the profile of salt concentration is linear in z ; (2) Two homogeneous miscible layers (i.e. salty water below and distilled water above) of the same thicknesses and between them a salty gradient layer (i.e. an interface), and (3) two homogeneous miscible layers (i.e. salty water below and distilled water above), which have different thicknesses, and between them a salty gradient layer (i.e. an interface). We now consider each form in turn.

Case 1. For the first stratification, the salt concentration profile evolves until a homogeneous state is achieved, and is described by the classical problem of diffusion. Equation 5 describes the evolution of concentration profile and Eq.6 gives the theoretical lifetime of such stratification.

Case 2. For the second stratification, the thickness of the interface expands in a symmetric way until the salt concentration profile is linear within the stratification. Then, it evolves just as in the first case.

Case 3. For the third stratification, the thickness of the interface expands in a symmetry way until one of two new forms of stratification appears: a gradient layer either beneath or above a homogeneous layer. In both cases, the gradient layer subsequently expands from the side adjacent to the homogeneous layer until a linear salt concentration profile appears within the stratification. Finally, it evolves as in the first case.

Following these basic ideas we now deduce, from a more geometric point of view, a mathematical model, which predicts the rate of expansion of the interface's thickness and the lifetime of the overall stratification.

3.1 *The interface sandwiched between two homogeneous layers with the same thicknesses*

3.1.1 *Time-dependent interface thickness*

Obviously, the thickness of the salty gradient layer increases as the interface expands under the diffusion phenomenon in a symmetric manner and replaces the

adjacent homogeneous layers with a gradient layer. Subsequently the linear salt profile of the gradient layer continues evolving under the diffusion phenomenon until the salt concentration profile becomes homogeneous.

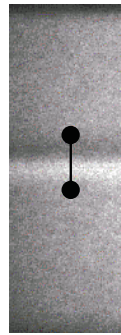
The physics of the mechanisms of diffusion across the interface is well known. At the upper edge of the interface ($z = +h/2$), a thin boundary layers appears due to the salt diffusion into the overlaying fresh water layer. Also, at the lower edge of the interface ($z = - h/2$), a thin boundary layer appears due to the diffusion of salt into the gradient layer from the underlying high salinity layer. As these two thin layers appear, they increase the thickness of the interface.

The salt concentration profile of the expanding interface layer can be reconstruct from the evolution of the salt concentration profile of a gradient layer bounded by rigid and impermeable layers (see section 2-4). This assumption is reasonable because the salt fluxes at the edges of the interface fulfill the conservation law. Thus, the expansion of the interface is symmetric.

Thus, it appears that the boundaries of the interface are moving and the thickness of interface is expanding. This phenomenon continues while it is sandwiched between two homogeneous layers. After the two homogeneous layers are eroded away, the profile is a linear throughout the stratification, and it subsequently evolves under the diffusion phenomenon until the salt concentration profile becomes homogeneous.

We use the salt profile given in section 2-4, to deduce the profile of an expanding interface. When we considered a fixed thickness of an interface then the salt flux diffused from the edges is the same quantity of salt required to create the rate of the expanding interface. Therefore, when we superimposed the two cases, it seems that profile of the interface of moving boundaries is nothing else than the extrapolation of the profile given in section 2-4. Visualization of this process by a shadowgraph system (see section 4) has been performed to prove the manner of the expansion of the gradient layer. Additionally, the thickness of the gradient layer has been measured by this same system. The gray levels of the stratification given by shadowgraph system (see Fig.2-a) were analyzed. It reveals that the interface expands in front of its two edges in a symmetry way (see Fig.2-b).

Figure 3 shows a sketch of the evolution of the thickness of the interface. It evolves in a symmetric way due to the diffusion phenomenon in the two sides. It is worthwhile to note that the concentrations at the moving boundaries of the gradient layer remain unchanged while the homogeneous layers exist. If we consider a fixed zone from the expanding salty gradient layer as for example the initial thickness of the gradient layer, then the evolution of the salt concentration profile through the fixed zone of interface is also described by Eq.9. It is a worthwhile to note that this observation allows us to deduce the new concentration profile from a geometrical point

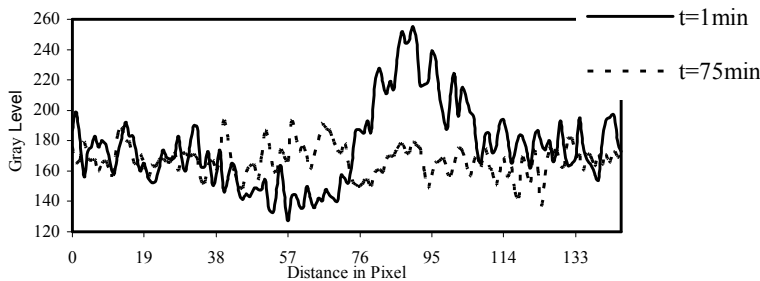


Upper layer: distillate water

Interface thickness: gradient layer

Lower layer: salty water

a) Shadowgraph visualization



b) Evolution of the gray level across the interface thickness

Figure 2: Shadowgraph visualisation and evolution of grey level across the interface

of view. If we extrapolate linearly at any time the salt profile of the fixed zone (i.e. the initial thickness of gradient layer) up to the concentrations of the homogeneous layers, then we obtain the salt profile of the expanding gradient at the same time. Hence, we can conclude that from a geometric point of view and Eq.9, the position of moving boundaries at any time is as follows. The concentration at point A_1 (see Fig.3) remains unchanged at any time. At time t , the concentration on point A at the edge of fixed zone is given by Eq.9. The salt concentration across the interface is assumed linear. Then, at a time t , the salt concentration profile of the expanding interface (see Fig. 3) is described by the segment AA_1 , where: $A(N^*(z^* = \frac{1}{2}, t), z^* = \frac{1}{2})$ and $A_1(N^*(z^* = 0, t), z^* = 0)$ from which $N^*(z^* = \frac{1}{2}, t)$ and $N^*(z^* = 0, t)$ are given by Eq.9. Then the slope of the segment is given as follows:

$$tg(\alpha) = \frac{1}{1 - 2N^*(z^* = \frac{1}{2}, t)} \quad (19)$$

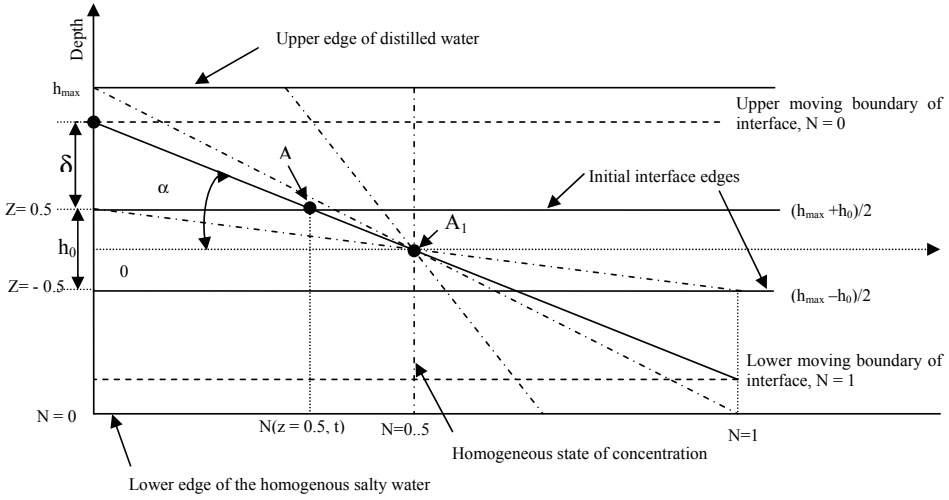


Figure 3: Sketch of expansion of the interface case, it is sandwiched between two homogeneous layers of the same thicknesses

If we extend the segment (AA_1) , it intersects with the straight line $N = 0$; and defines the salt concentration profile of the upper layer. Also, $tg(\alpha)$ can also be expressed (see Fig. 3) by:

$$tg(\alpha) = 1 + 2\delta^*(t) \tag{20}$$

Where $2\delta^*(t)$ represents the dimensionless expansion rate of the interface thickness ($\delta^*(t) = \frac{\delta(t)}{h_0}$).

Combining the Eqs.19 and 20, we deduce that:

$$1 + 2\delta^*(t) = \frac{1}{1 - 2N^*(z^* = \frac{1}{2}, t)} \tag{21}$$

Equation 21 is reduced in dimensional form as:

$$h(t) = h_0 - 2\delta(t) = \frac{h_0}{1 - 2N^*(z^* = \frac{1}{2}, t)} \tag{22}$$

Where $h(t) = h_0 + 2\delta(t)$ is the expanding interface thickness. When we substitute

the value of $N^*(z = \frac{1}{2}, t)$ given by Eq.9 into Eq.22, we get:

$$h(t) = h_0 \div \left[2 \sum_{k=0}^{+\infty} \frac{2}{(2k+1)\pi} \left\{ \frac{4}{(2k+1)\pi} \left(\cos\left(k\frac{\pi}{2}\right) + \sin\left(k\frac{\pi}{2}\right) \right)^2 - \left(\cos\left(k\frac{\pi}{2}\right)^2 - \sin\left(k\frac{\pi}{2}\right)^2 \right) \right\} \exp\left(-\frac{t}{\theta'_k}\right) \right] \quad (23)$$

As it can be seen from Fig.3, the expanding interface thickness phenomenon is modeled by a mathematical formulation using geometrical concepts. It can be noticed that essentially the same physical laws of the diffusion equation govern all models and cases listed above.

3.1.2 The lifetime of the stratification

The lifetime of the stratified layers is the time necessary for the stratification to vanish. The evolution of the stratified layers reveals that this time is the sum of two times. The first one is the time required for the concentration profile of the initial stratification layer to evolve until it reaches a linear concentration profile throughout the whole stratification. The second one is the time required for the linear profile to completely vanish. It is worthwhile to note that the relaxation time given by Eq.6 has no physical meaning in general, but does represent one when $k=0$. This occurs when Eq.5 is estimated by the first term alone, an approximation valid when the time is large enough that $\exp(-\frac{t}{\theta_k})$ is small and the other exponential functions are even smaller. Thus the mass fraction $N^*(z^*, t)$ may be well approximated by the first order (or at most by the first few terms of the series). Finally, as $t \rightarrow \infty$, the terms associated to exponential disappear completely. Therefore, for the following, we assume that $k=0$. The use of only the first term is appropriate because this term dominates the overall evolution of the system.

Eventually, the homogeneous upper and lower layers erode and a single gradient layer is established (see Fig. 3). If we considered the thickness of the stratified system (h_{max}), then from Eq.23 we have,

$$\frac{h_{max}}{h_0} = \frac{\pi^2}{4(4-\pi)} \exp\left(\frac{t_{in}}{\Theta_0}\right) \quad (24)$$

Therefore, the required time for the linear salt profile to be established within the stratified layers is deduced from Eq.24, and we obtain:

$$t_{in} = \frac{h_0^2}{\pi^2 D_m} \log \left[\frac{4}{\pi} \left(\frac{4}{\pi} - 1 \right) \frac{h_{max}}{h_0} \right] \quad (25)$$

Thereafter, it requires a time (t_{real}) for the gradient layer formed throughout the whole stratification to become homogeneous. As t_{theo} is the theoretical relaxation time, the t_{real} can be taken five times the relaxation time ($t_{real}=5.t_{theo}$); this is the time required for the exponential function to reach a value of 0.01. Then, from the Eq.6 we have Eq 26:

$$t_{real} = 5t_{theo} = \frac{5h_{max}^2}{\pi^2 D_m} \quad (26)$$

Finally, from Eqs.25 and 26, the lifetime of the whole process is:

$$t = t_{in} + t_{real} = \frac{h_{max}^2}{\pi^2 D_m} \left[5 + \frac{h_0^2}{h_{max}^2} \log \left(\frac{4}{\pi} \left(\frac{4}{\pi} - 1 \right) \frac{h_{max}}{h_0} \right) \right] \quad (27)$$

3.2 The interface sandwiched between two homogeneous layers with different thickness

In such state, the interface thickness expands under the diffusion phenomenon until the thinner homogeneous layer is completely eroded away by the expanding interface (i.e. by the gradient layer).

Based on Eq.25 above, we can deduce the time required for this thinner layer to be eroded. At that time, a new kind of stratification appears. It consists of a linear salt gradient layer beneath or above a homogeneous layer, depending on the initial relative thicknesses of the two homogeneous layers. In such stratification, the gradient layer continues to expand into the remaining homogeneous layer. The diffusion of salt carries on until a gradient layer is established within the remaining layer. Ultimately, under the diffusion phenomenon the salt concentration distribution evolves until it reaches a homogeneous state.

In such a system, the only issue is to predict the lifetime of a homogeneous layer, which overlies a gradient layer, as the other kind of stratification is presented above.

3.2.1 Time-dependent interface thickness

Figure 4 sketches how to deduce from the theoretical and geometrical point of view (see section 2-3) the expanding interface thickness after the less thickness homogeneous layer has disappeared. The basic idea used to solve this problem is to consider that the salt flux transfer between the homogeneous layer and the gradient layer supports the increasing gradient layer thickness as the moving boundary expands. According to section 2.3, the concentration has the same value for either the moving or fixed boundary case (see point B in Figure 4). However, in case of the moving boundary, the concentration at $z^* = \frac{1}{2}$ does not have the same value as

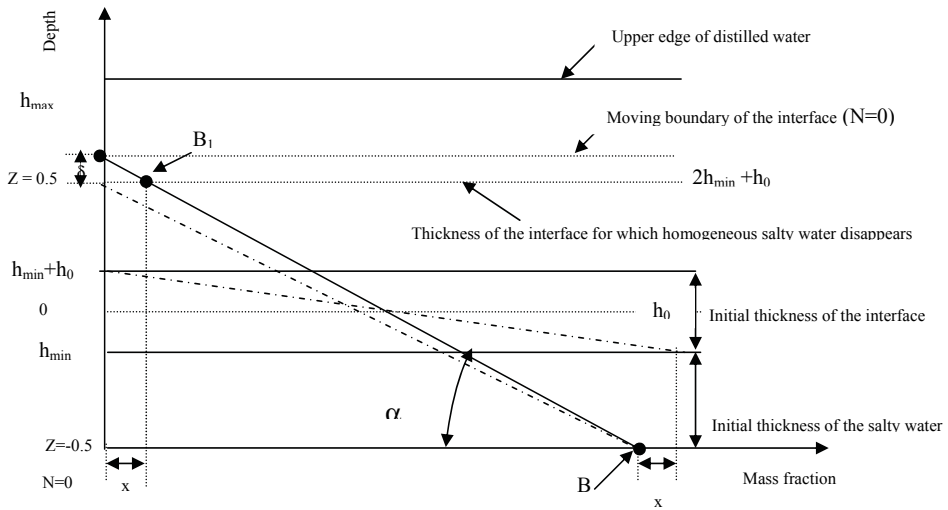


Figure 4: Sketch of expansion of the interface case, it is sandwiched between two homogeneous layers of different thicknesses

for a fixed boundary where it is zero at the point B_1 (see Fig 4). From this figure we have: $x = 1 - N^*(z^* = -\frac{1}{2}, t)$.

Then the salt concentration at point BB_1 is: $x = 1 - N^*(z^* = -\frac{1}{2}, t)$ (see Fig.4).

If we assume that the salt concentration profile is linear, then the concentration profile of the moving boundary is the segment B_1B where B ($N^*(z^* = -\frac{1}{2}, t), -\frac{1}{2}$) and $B_1(1 - N^*(z^* = -\frac{1}{2}, t), \frac{1}{2})$ (see Fig. 4).

The segment (BB_1) is extended from B_1 until it intersects the horizontal line for which $N^*(z^*, t) = 0$. The point of intersection represents the new edge of the gradient layer. The extended segment is a salt concentration profile of the expanding gradient layer. Then, the slope of the salt concentration profile is given by:

$$tg(\alpha) = \frac{1}{2N^*(z^* = -\frac{1}{2}, t) - 1} \tag{28}$$

When we consider the expanding thickness of the interface (see Fig 4), the slope is also given by:

$$tg(\alpha) = \frac{1 + \delta^*}{N^*(z^* = -\frac{1}{2}, t)} \tag{29}$$

Combining Eqs.28 and 29, the rate of increase of the interface $(1 + \delta^*)$ becomes:

$$\frac{1 + \delta^*}{N^* \left(z^* = -\frac{1}{2}, t \right)} = \frac{1}{2N^* \left(z^* = -\frac{1}{2}, t \right) - 1} \quad (30)$$

When N^* from Eq.1 is substituted into Eq.30, and reduced to dimensional form it results in the following:

$$\begin{aligned} h(t) = h_0 & \left[\sum_{k=0}^{k=+\infty} \frac{2(-1)^k \sqrt{2}}{(4k-1)\pi} \left(1 + \frac{4}{(1+4k)^2 \pi^2} \right) \right. \\ & \left. \left(\cos \left((1+2k) \frac{\pi}{4} \right) - \sin \left((1+2k) \frac{\pi}{4} \right) \right) \exp \left(-\frac{t}{\theta'_k} \right) \right] \\ & \div \left[2 \sum_{k=0}^{k=+\infty} \frac{2(-1)^k \sqrt{2}}{(1+4k)\pi} \left(1 + \frac{4}{(1+4k)\pi} \right) \right. \\ & \left. \left(\cos \left((1+2k) \frac{\pi}{4} \right) - \sin \left((1+2k) \frac{\pi}{4} \right) \right) \exp \left(-\frac{t}{\theta'_k} \right) - 1 \right] \quad (31) \end{aligned}$$

3.2.2 The lifetime of the stratification

The time required for the thinner layer (h_{\min}) to disappear is given by Eq.31 with $k=0$, as follows:

$$t_g = \frac{h_0^2}{\pi^2 D_m} \log \left[\frac{4}{\pi} \left(\frac{4}{\pi} - 1 \right) \frac{2h_{\min} + h_0}{h_0} \right] \quad (32)$$

The time required for the remaining homogeneous layer to disappear is given by Eq.3 with $k = 0$ as follows:

$$t_{in} = \frac{4(2h_{\min} + h_0)^2}{\pi^2 D_m} \log \left(\frac{4}{\pi} \left(1 + \frac{4}{\pi^2} \right) \frac{2h_{\max} - 1}{h_{\max}} \right) \quad (33)$$

The relaxation time of a gradient layer is given by Eq.26. Therefore, the lifetime for the whole stratification is given by

$$\begin{aligned} t = t_g + t_{in} + t_{real} \\ = \frac{h_0^2}{\pi^2 D_m} \log \left[\frac{4}{\pi} \left(\frac{4}{\pi} - 1 \right) \frac{2h_{\min} + h_0}{h_0} \right] \\ + \frac{4(2h_{\min} + h_0)^2}{\pi^2 D_m} \log \left[\frac{4}{\pi} \left(1 + \frac{4}{\pi^2} \right) \frac{2h_{\max} - 1}{h_{\max}} \right] + \frac{5h_{\max}^2}{\pi^2 D_m} \quad (34) \end{aligned}$$

4 Experimental methods

An experimental study was performed in a vessel of a dimension (135mm×40mm×50mm). The stratification consisted of two homogeneous layers separated by a gradient layer. The lower layer was a solution of sodium chloride of 5%wt mass fraction. The upper one was distilled water. The gradient layer had a linear concentration profile decreasing upwards from 5%wt to 0%wt. More details of the experimental setup are cited in [Abdeljabar and Safi (2001)].

The measurements of concentration were done by a conductivity probe that traveled the depth of the vessel at a sufficiently low speed that the stratification was not disturbed.

The initial thicknesses of layers of the stratification were measured by a shadowgraph system [Goldstein (1974); Merzkirch (1987)]. The shadowgraph visualization is sensitive to the second derivative of the vertical density structure ($\frac{\partial^2 \rho}{\partial z^2} \neq 0$); therefore light rays passing along the two homogeneous layers are not deflected, however. Light passes along the interface (i.e. the salty gradient layer) are deflected, so the interface thickness's appears dark.

The visualization by the shadowgraph system showed that each homogeneous layer of the stratification has a thickness of 15mm±1mm and the gradient layer (interface layer) has a thickness of 4mm±1mm.

5 Experimental results

The shadowgraph system was used to measure the thickness of the layers. As the visualization shows a shadow, it implies that the concentration profile is not linear within the stratification. However, the time for the shadow to disappear is the time required for a linear profile to become established within the stratification. The visualization by shadowgraph system reveals a time t of 2h 46 min, in good agreement with the real theoretical time predicted by Eq.25 ($t=2h46min$). In the next phase of the experiment, the salt concentration was measured with a probe, which traveled slowly through the depth of the vessel. The measurements were begun at a time $t=2h46min$, the time at which the gradient layer was established throughout the whole stratification. This time is considered as start time in the next set of measurements. The other measurements of concentration are given in the following table.1:

Table 1: Times of salt profile measurements

Measurement	1	2	3
Time (min)	2657	5710	6990

The experimental measurements were reduced to dimensionless form to compare with the theoretical results. The evolution of the experimental and the theoretical salty profiles were plotted and shown in Figures 5 and 6. Figures 5 and 6 reveal that the same plots in the two figures have some discrepancies; initially, the theoretical profile evolves more quickly than experimental profile. This discrepancy can be explained by the first order approximation in the theoretical model.

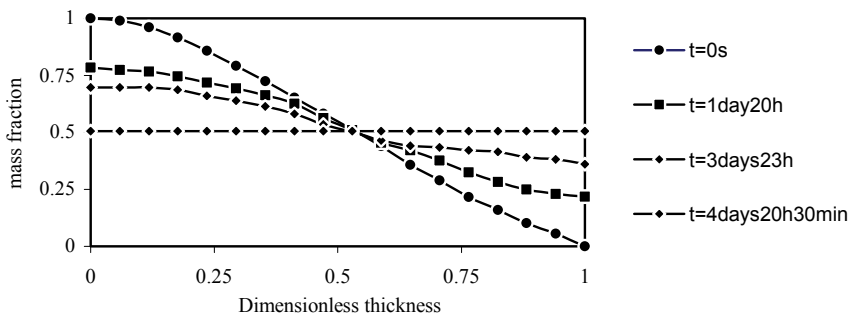


Figure 5: Experimental profile of salt concentration

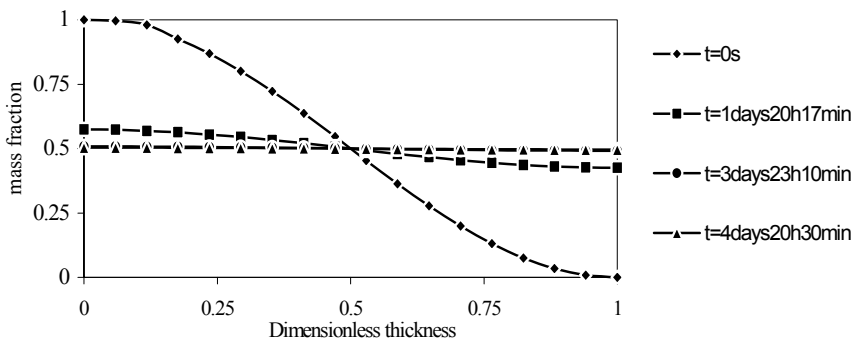


Figure 6: Theoretical Profile of salt Concentration

To agree with each other, the time should be greater (i.e. time t is greater than θ , or we consider more than one term).

For a giving thickness of $h=34\text{mm}$ and diffusion coefficient $D_m = 1.4 \cdot 10^{-5} \text{cm}^2/\text{s}$, the theoretical lifetime deduced from Eq.26 is $t_{theo} = \Theta = 23.24$ hours and the real lifetime calculated from Eq.26 is $t_{real} = 5\Theta = 116.20$ hours. Though, the experimental measurement showed that the lifetime of the stratification is $t = 116.30$

hours. A comparison between the experimental lifetime and that predicted by the model reveals that both results agree within 0.9 %. Thus, the two results are in a good agreement.

Although it seems to be a tendency for a theoretical lifetime to be a bit lower than experimental results, theoretical prediction describes well the experimental results.

6 Conclusion

We present mathematical models to describe the evolution of isothermal, stratified miscible layers under diffusion phenomena. The theoretical predictions of this model were confirmed by experimental studies. These mathematical models allow defining with accuracy the initial conditions and evolution of stratifications. These results are useful for experimental investigations in laboratories. Moreover the model can predict the lifetime of some natural reserves of freshwater that lie on seawater.

Nomenclature

D_m	salt diffusivity
h	the interface thickness in time
h_{\max}	the overall depth of the stratification
h_{\min}	the depth of the less thickened homogeneous layer
h_0	initial interface thickness
N	salt mass fraction
N_1	initial maximum salt mass fraction
t	lifetime of the stratification
t_g	time required for which a less thickness homogeneous layer disappears
t_{in}	time required for which the overall thickness of the stratification converted into a gradient layer
t_{theo}	relaxation time of a gradient layer
t_{real}	life time of a gradient layer ($5 * t_{theo}$)
%wt	percent weight

Greek Symbols

Θ	relaxation time
$\Theta' = 4\Theta$	relaxation time

$2.\delta = h - h_0$ the rate growth of the interface
depth

Superscript

* dimensionless

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